

# Improvement of the vertical “scatter degree” method and its application in evaluating water environmental carrying capacity

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## Abstract

Zheng, Z. H., & Yu, Y. X. (March-April, 2017). Improvement of the vertical “scatter degree” method and its application in evaluating water environmental carrying capacity. *Water Technology and Sciences* (in Spanish), 8(2), 71-76.

The “scatter degree” method is an objective evaluation method based on a “difference drive”. However, it is currently perceived as incapable of properly reflecting the subjective information of the evaluator. In order to reflect the objective law of raw data and assist decision-makers to control major factors of the water environmental carrying capacity (WECC), this paper proposes a method to process the weight of the indicator by combining the analytic hierarchy process (AHP) and the vertical “scatter degree” method. The computing results of this improved method are virtually identical to those of the old method, but the former is more scientific and reliable.

**Keywords:** Water environmental carrying capacity; analytic hierarchy process; processing weight; improved vertical “scatter degree” method.

## Resumen

Zheng, Z. H., & Yu, Y. X. (marzo-abril, 2017). Mejoramiento del método de “grado de dispersión” vertical y su aplicación en la evaluación de la capacidad de carga ecológica del agua. *Tecnología y Ciencias del Agua*, 8(2), 71-76.

El método de “grado de dispersión” vertical es un método de evaluación objetiva; sin embargo, actualmente se percibe como incapaz de reflejar de forma adecuada la información subjetiva del evaluador. Con el fin de reflejar la ley objetiva de datos brutos y ayudar a los tomadores de decisiones a controlar los principales factores de la capacidad de carga ecológica del agua (WECC), este trabajo propone un método para procesar la ponderación del indicador al combinar el proceso de jerarquía analítica (AHP) y el método de “grado de dispersión” vertical. Los resultados de este método mejorado son prácticamente idénticos a los obtenidos con el antiguo método, pero el primero es más científico y confiable.

**Palabras clave:** capacidad de carga ecológica del agua, proceso de jerarquía analítica, ponderación, método de “grado de dispersión” vertical mejorado.

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## Introduction

Water is the primary nature resource for human survival and social development. The water environment is the synthesis of water bodies, and it is closely related to various kinds of natural factors and social factors. WECC (Water Environmental Carrying Capacity) is based on the relationship between sustainable development and water environment, and its size determines the scale and speed of development of human society. Many scholars, both domestic and overseas, have researched WECC and its significance

to humans (Long & Jiang, 2003). Various methods have been used for its evaluation, such as the system dynamics model (Han, 2005), the vector norm method (Huang & Song, 2013), the fuzzy comprehensive evaluation method (Fang, 2014), the multi-objective model (Wang, Zhou, & Yu, 2015), the artificial neural network model (Wang, Sun, Li, & Hou, 2007), the radial basis function (RBF) artificial neural network model (Hu, Xu, Xu, & Fu, 2015), and the catastrophe progression method (Wu, Wang, Cao, Wang, & Zhao, 2015), and so on. The evaluation of the water environmental carrying capacity involves hydrology in

addition to analysis of social, environmental, and ecological factors. These factors can be multi-layered, dubious and complex. Because of this complexity, this paper combines the analytic hierarchy process (AHP) and the vertical "scatter degree" method, which considers objective and subjective information fully in its evaluation. Through the comparison with the vertical "scatter degree" method, this method is determined to be more reasonable. In addition, it can provide technical support for improving regional WECC.

## Improvement of "scatter degree" method

### *The theory of the "scatter degree" method (Guo, 2012)*

The synthetic function is the linear function of greatest-type indicators ( $x_1, x_2, \dots, x_m$ ):

$$y = \omega_1 x_1 + \omega_2 x_2 + \dots + \omega_m x_m = \omega^T x \quad (1)$$

where  $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$  is an  $m$ -dimensional pending positive vector.

Inserting the standard observed values ( $x_{i1}, x_{i2}, \dots, x_{im}$ ) of evaluation object  $S_i$  into Eq. (1), Eq. (2) is acquired:

$$y = \omega_1 x_{i1} + \omega_2 x_{i2} + \dots + \omega_m x_{im} = \sum_{j=1}^m \omega_j x_{ij} \quad (2)$$

If and, Eq. (2) is converted to Eq. (3):

$$y = A\omega \quad (3)$$

The variance of evaluation objects  $y = \omega^T x$ :

$$s_2 = \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2 = \frac{y^T y}{n} - \bar{y}^2 \quad (4)$$

The principle of determining the weight coefficient of the "scatter degree" method is getting the linear function  $\omega^T x$  of index vector  $x$  and making the variance of the evaluated object's values as high as possible;  $y = A\omega$  is

substituted into Eq. (4). When  $\bar{y} = 0$ , the new formula is as follows:

$$ns^2 = \omega^T A^T A \omega = \omega^T H \omega \quad (5)$$

where  $H = A^T A$  is a real symmetric matrix.

When  $\omega$  is unlimited, formula (5) can be an arbitrarily large value. Here,  $\omega^T \omega = 1$ , and the maximum value of formula (5) is acquired. Namely, the value  $\omega$  is acquired to meet the constraints:

$$\begin{aligned} &\max \omega^T H \omega \\ &s.t. \begin{cases} \omega^T \omega = 1 \\ \omega > 0 \end{cases} \end{aligned} \quad (6)$$

For formula (5), when  $\omega$  is standard feature vector of the largest eigenvalues of  $H$ ,  $\omega^T H \omega$  achieves its maximum value. The weight coefficient vector  $(\omega_1, \omega_2, \dots, \omega_m)^T$  is obtained by normalizing  $\omega$ , and . The comprehensive values  $y_i$  of the evaluation object are obtained by using Eq. (2).

### *The improvement of the "scatter degree" method*

The "scatter degree" method emphasizes that the method should highlight the differences among the evaluation objects as a whole, and it runs under the premise of the same importance of the evaluation target to every evaluation indicator. In fact, the degree of the importance is not same. The "function drive" theory provides the weighting coefficient  $r_j (j = 1, 2, \dots, m)$  of each indicator  $x_j$ . On this basis, each indicator is weighted by the following formula:

$$x_{ij}^* = r_j x_{ij}, j = 1, 2, \dots, m; i = 1, 2, \dots, n \quad (7)$$

where  $x_{ij}$  are the standard observed values. The mean value and mean square deviation of  $x_{ij}^*$  are 0 and  $r_j^2$ , respectively. The weighting coefficient of weighted data  $\{x_{ij}^*\}$  is acquired by using the "scatter degree" method. Then, the comprehensive evaluation values can be obtained.

### Analytic Hierarchy Process (Deng, Li, Zeng, Chen, & Zhao, 2012)

In this article, the Analytic Hierarchy Process (AHP) is used to confirm the weighting coefficients  $r_j$  of each indicator.

AHP was discovered by Satty while performing operational research in the 1970's. The indicators are classified into several levels of objective, criterion, and index. This method is based on both quantitative and qualitative approaches.

The steps of AHP include building a hierarchical model, building a matrix of each level, performing a consistent check, and verifying the total arrangement of each hierarchy with a consistent check.

### The process and results of the evaluation

#### The evaluating data and its standards

Reference data was adopted from (Geng, 2012). All the nine indicators are classified into positive and negative classes, and need to be standardized to remove the influence of inverse indices and different dimensions.

For the positive index:

$$x_{ij}^* = \frac{x_{ij} - m_j}{M_j - m_j} \quad (i = 1, 2, \dots, n; j = 1, 2, \dots, m) \quad (8)$$

For the negative index:

$$x_{ij}^* = \frac{M_j - x_{ij}}{M_j - m_j} \quad (i = 1, 2, \dots, n; j = 1, 2, \dots, m) \quad (9)$$

where  $M_j = \max_i \{x_{ij}\}$  and  $m_j = \min_i \{x_{ij}\}$

Matrix A is calculated based on the raw data as well as Eqs. (8) and (9). The results shown as follows:

$$A = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.785 & 0.000 & 0.832 & 0.436 & 0.604 \\ 0.000 & 0.137 & 0.256 & 0.293 & 1.000 & 0.247 & 1.000 & 0.000 & 1.000 \\ 0.448 & 0.360 & 0.587 & 0.698 & 0.981 & 0.258 & 0.000 & 0.876 & 0.328 \\ 0.069 & 0.801 & 0.877 & 0.788 & 0.238 & 0.852 & 0.857 & 1.000 & 0.000 \\ 0.552 & 1.000 & 1.000 & 1.000 & 0.000 & 1.000 & 0.814 & 0.907 & 0.029 \end{bmatrix}$$

#### The application of vertical "scatter degree"

Symmetric matrix H is calculated by using the formula  $H = A^T A$ .

$$H = \begin{bmatrix} 1.5101 & 0.7685 & 0.8753 & 0.9191 & 1.2414 & 0.7261 & 1.3404 & 1.3987 & 0.7675 \\ 0.7685 & 1.7898 & 1.9485 & 1.9230 & 0.6820 & 1.8091 & 1.6378 & 2.0239 & 0.2852 \\ 0.8753 & 1.9485 & 2.1783 & 2.1757 & 1.0404 & 1.9615 & 1.8212 & 2.2984 & 0.4777 \\ 0.9191 & 1.9230 & 2.1757 & 2.1946 & 1.1658 & 1.9241 & 1.7826 & 2.3076 & 0.5514 \\ 1.2414 & 0.6820 & 1.0404 & 1.1658 & 2.6361 & 0.7035 & 1.8573 & 1.4407 & 1.7966 \\ 0.7261 & 1.8091 & 1.9615 & 1.9241 & 0.7035 & 1.8536 & 1.7918 & 1.9855 & 0.3614 \\ 1.3404 & 1.6378 & 1.8212 & 1.7826 & 1.8573 & 1.7918 & 3.0896 & 1.9588 & 1.5265 \\ 1.3987 & 2.0239 & 2.2984 & 2.3076 & 1.4407 & 1.9855 & 1.9588 & 2.7817 & 0.5780 \\ 0.7675 & 0.2852 & 0.4777 & 0.5514 & 1.7966 & 0.3614 & 1.5265 & 0.5780 & 1.4737 \end{bmatrix}$$

The maximal eigenvalue and eigenvector of matrix H' is calculated by using MATLAB:  $\lambda_{\max}(H) = 13.9332$ ,  $\omega(H) = (0.0768, 0.1124, 0.1280, 0.1287, 0.0980, 0.1144, 0.1391, 0.1436, 0.0591)^T$ .

The values of WECC in the years of 2005 and 2009 are calculated by plugging  $x_j(t_k)$  and  $w_j$  into Eq. (2). The results are shown in table 1.

#### The application of AHP-Vertical "scatter degree"

##### Calculation of the weight using AHP

1. A system of evaluation is established according to the reference, which is as follows: 1) Level of objective: WECC; 2) Level of indicators: ratio of water resource to utilization and exploitation  $X_1$ , gross regional output per capita  $X_2$ , water consumption of GDP  $X_3$ , water consumption of industrial output  $X_4$ , discharged volume of wastewater of industrial output  $X_5$ , discharged chemical oxygen demand of industrial output  $X_6$ , repeated utilization rate of industrial wastewater  $X_7$ , attainment rate of the industrial wastewater  $X_8$ , daily water consumption per capita  $X_9$ .
2. The weight of the index: The relative importance of all indicators was determined with

reference to the literature. Then, matrix B is acquired as follows:

$$B = \begin{bmatrix} & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 \\ X_1 & 1.00 & 1.00 & 0.33 & 0.25 & 0.33 & 0.33 & 0.50 & 0.50 & 0.25 \\ X_2 & 1.00 & 1.00 & 0.33 & 0.25 & 0.33 & 0.33 & 0.50 & 0.50 & 0.25 \\ X_3 & 3.00 & 3.00 & 1.00 & 0.50 & 1.00 & 1.00 & 2.00 & 2.00 & 0.50 \\ X_4 & 4.00 & 4.00 & 2.00 & 1.00 & 2.00 & 2.00 & 3.00 & 3.00 & 1.00 \\ X_5 & 3.00 & 3.00 & 1.00 & 2.00 & 1.00 & 1.00 & 2.00 & 2.00 & 0.50 \\ X_6 & 3.00 & 3.00 & 1.00 & 0.50 & 1.00 & 1.00 & 2.00 & 2.00 & 0.50 \\ X_7 & 2.00 & 2.00 & 0.50 & 0.33 & 0.50 & 0.50 & 1.00 & 1.00 & 3.00 \\ X_8 & 2.00 & 2.00 & 0.50 & 0.33 & 0.50 & 0.50 & 1.00 & 1.00 & 3.00 \\ X_9 & 4.00 & 4.00 & 2.00 & 1.00 & 2.00 & 2.00 & 0.33 & 0.33 & 1.00 \end{bmatrix}$$

Then, the weight of the indicator is obtained by using MATLAB:  $\lambda_{\max} = 9.886$ ,  $r = (0.043, 0.043, 0.127, 0.217, 0.127, 0.127, 0.092, 0.092, 0.133)^T$ .

3. Consistency check: 1) consistency index:

$CI = \frac{\lambda_{\max} - n}{n - 1}$ , where  $n$  is the number of indicators, and  $CI = 0.11075$ ; 2) consistency ratio:  $CR = \frac{CI}{RI}$ , where  $RI$  is the mean random consistency index, and when  $n = 9$ ,  $RI = 1.46$ . So,  $CR = 0.07586 < 0.10$ , and the judgment matrix is consistent.

The WECC of AHP-Vertical "scatter degree". The matrix  $X'$  is acquired using Eq. (7).

$$X' = \begin{bmatrix} 0.430 & 0.000 & 0.000 & 0.000 & 0.997 & 0.000 & 0.765 & 0.401 & 0.803 \\ 0.000 & 0.059 & 0.325 & 0.635 & 1.270 & 0.314 & 0.920 & 0.000 & 1.330 \\ 0.193 & 0.155 & 0.745 & 1.515 & 1.246 & 0.328 & 0.000 & 0.806 & 0.437 \\ 0.030 & 0.344 & 1.113 & 1.711 & 0.303 & 1.082 & 0.788 & 0.920 & 0.000 \\ 0.237 & 0.430 & 1.270 & 2.170 & 0.000 & 1.270 & 0.749 & 0.835 & 0.039 \end{bmatrix}$$

A symmetric matrix  $H'$  is derived from the formula:  $H' = X'^T X'$

$$H' = \begin{bmatrix} 0.2792 & 0.1421 & 0.4780 & 0.8576 & 0.6779 & 0.3965 & 0.5303 & 0.5533 & 0.4389 \\ 0.1421 & 0.3309 & 1.0641 & 1.7944 & 0.3724 & 0.9879 & 0.6479 & 0.8007 & 0.1631 \\ 0.4780 & 1.0641 & 3.5134 & 5.9959 & 1.6781 & 3.1638 & 2.1279 & 2.6855 & 0.8069 \\ 0.8576 & 1.7944 & 5.9959 & 10.3342 & 3.2128 & 5.3027 & 3.5588 & 4.6069 & 1.5915 \\ 0.6779 & 0.3724 & 1.6781 & 3.2128 & 4.2517 & 1.1347 & 2.1701 & 1.6833 & 3.0346 \\ 0.3965 & 0.9879 & 3.1638 & 5.3027 & 1.1347 & 2.9897 & 2.0935 & 2.3199 & 0.6104 \\ 0.5303 & 0.6479 & 2.1279 & 3.5588 & 2.1701 & 2.0935 & 2.6150 & 1.6580 & 1.8678 \\ 0.5533 & 0.8007 & 2.6855 & 4.6069 & 1.6833 & 2.1399 & 1.6580 & 2.3544 & 0.7072 \\ 0.4389 & 0.1631 & 0.8069 & 1.5919 & 3.0346 & 0.6104 & 1.8678 & 0.7072 & 2.6068 \end{bmatrix}$$

The maximal eigenvalue and eigenvector of matrix  $H'$  is calculated using MATLAB:  $\lambda_{\max}'(H') = 22.22$ ,  $\omega'(H') = (0.0260, 0.0442, 0.1490, 0.2570, 0.1046, 0.1322, 0.1066, 0.1177, 0.0627)^T$ .

The values of WECC in the years of 2005 and 2009 were calculated by putting  $x_i(t_k)$  and  $w_j'$  into Eq. (2). The results are shown in table 1.

From the table and figures, it is clear that the results of the evaluation are basically consistent.

## Conclusions

By comparing the results of the two methods, the conclusions are as follows: (1) The trend of the two methods' calculation of the WECC is the same; namely, the WECC is enhanced. This enhancement in WECC is closely related to the strengthening of the environmental governance, upgrading of production equipment, and the improvement of water-saving and environmental awareness; (2) As the two groups of data were calculated and compared, the WECC of 2005 was found to be 0.3677 or 0.2947 with the two models, respectively. From this data, it shows that the WECC of 2006 increased 11.59% and 93.45% relative to the WECC of 2005 by vertical "scatter degree" method and AHP-vertical "scatter degree" method. Based on the original data of the reference, there were eight indicators' data of 2006 higher than them of 2005, and their increasing rates were mostly greater than

Table 1. The results of WECC by two different methods.

Year	2005	2006	2007	2008	2009
Vertical "scatter degree" method	0.3677	0.4103	0.5108	0.6925	0.7712
AHP-Vertical "scatter degree" method	0.2947	0.5701	0.8083	0.9886	1.1206

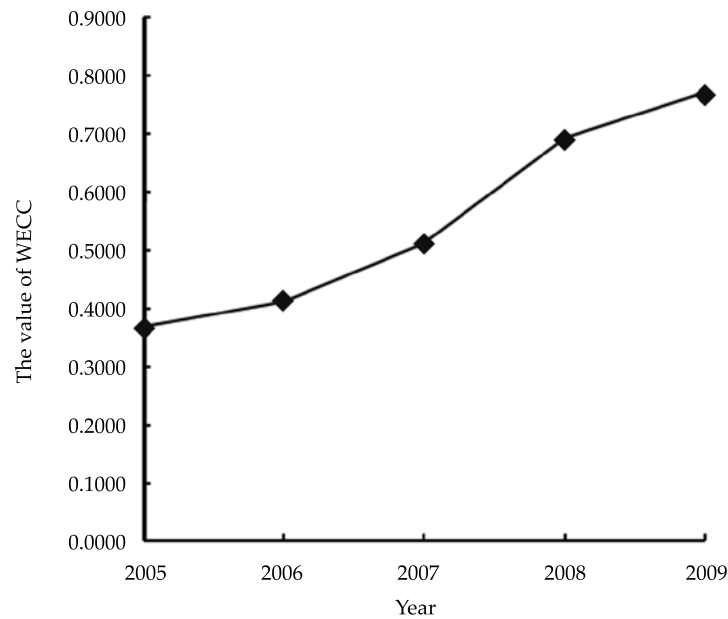


Figure 1. Water environmental carrying capacity of Shanxi Province spots (vertical "scatter degree" method).

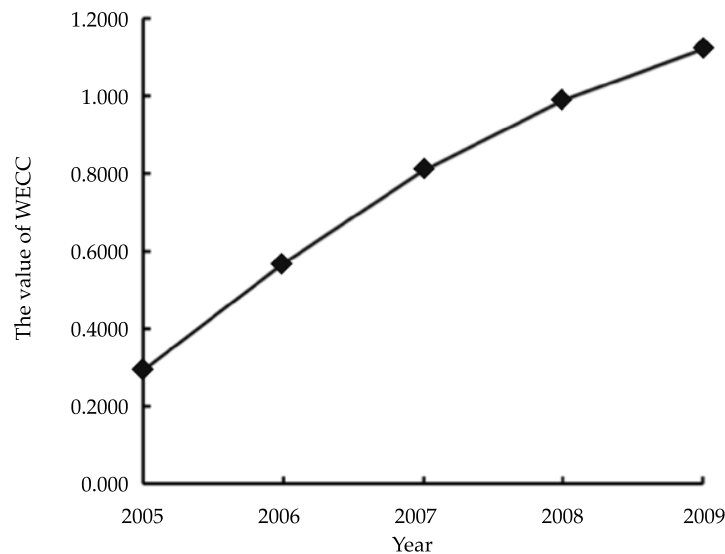


Figure 2. Water environmental carrying capacity of Shanxi Province spots (AHP-vertical "scatter degree" method).

10%. This indicated a clear increase in WECC from 2005 to 2006. Therefore, the result of the AHP-vertical "scatter degree" method is more reasonable. The other groups of the data in the table 1 had the same features.

Overall, the results show that the AHP-Ver-tical "scatter degree" method is more scientific and reasonable than the original method. It overcomes the weaknesses of the vertical scatter degree method in which the subjective

information from the evaluators cannot be reflected. By combining the subjective weighting and subjective weighting methods, the twice-weighted process not only expresses the degree of important for each of the indicators but also highlights the difference of every evaluating object. It has the advantage of remedying the single use of the subjective weighting method and the objective weighting method.

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