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Notes

Contrast of the Generalized M5 Method to estimate Extreme Predictions and *PMP* in 24 hours, in the state of Zacatecas, Mexico

Contraste del Método M5 Generalizado para estimar Predicciones Extremas y *PMP* en 24 horas, en el estado de Zacatecas, México

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Abstract

Probable maximum precipitation (PMP) is the basis for the estimation of the probable maximum flood, with which large hydraulic works are dimensioned and hydrologically revised. There are two groups of methods to estimate *PMP*: meteorological and statistical. Meteorological methods are the most reliable, but require a lot of data that is usually not available. Statistical methods are much simpler and only use annual *maximum daily precipitation (PMD)* values. The classic method of this group is the David M. Hershfield method, published in 1961. Subsequently, in England (NERC, 1975) another statistical method was developed based on the prediction of duration 24 hours and return period (*Tr*) 5 years, designated M5; this approach allows predictions with various *Tr*. Jónas Elíasson (Elíasson, 1997; Elíasson, 2000) generalized the M5 method, in a *regional technique* that only requires two statistical parameters: the M5 and the coefficient of variation (*Cv*). In this study, the results of the generalized M5 method (MM5G) are compared with those of the Hershfield method, previously calculated based on the *PMD* data for 81 localities in the state of Zacatecas, Mexico. The MM5G was applied using the available values of M5 and *Cv*.

Results allow the recommendation of the use of MM5G to estimate predictions of *PMD* with *Tr* less than 100 years, in no-data sites in the state of Zacatecas, Mexico. It is also recommended for estimations of *PMP* of 24 hours duration, remembering that such method underestimates less than 16.4%, with respect to the result of the Hershfield method, when the *Cv* is less than 0.251 and overestimates, of the order of 38.0% when the *Cv* exceeds 0.386.

Keywords: Probable maximum precipitation, probable maximum flood, extreme predictions, relative error, coefficient of variation.

Resumen

La *precipitación máxima probable (PMP)* es la base para la estimación de la creciente máxima probable, con la cual se dimensionan y revisan hidrológicamente las grandes obras hidráulicas. Existen dos grupos de métodos para estimar la *PMP*: meteorológicos y estadísticos. Los primeros son más confiables, pero requieren muchos datos que, por lo general, no están disponibles. Los métodos estadísticos son mucho más simples y sólo utilizan valores de la *precipitación máxima diaria (PMD)* anual. El método clásico de este grupo es el de David M. Hershfield, expuesto en 1961. Posteriormente, en Inglaterra (NERC, 1975), se desarrolló otro método estadístico basado en la predicción de duración 24 horas y periodo de retorno (*Tr*) de cinco años, designado M5; este enfoque permite realizar predicciones con diversos *Tr*. Jónas Elíasson (Elíasson, 1997; Elíasson, 2000) generalizó el método M5 en una *técnica regional* que sólo requiere dos parámetros estadísticos: el M5 y el coeficiente de variación (*Cv*). En este estudio, se contrastan en 81 localidades del estado de Zacatecas, México, los resultados del método M5 generalizado (MM5G) contra los del método de Hershfield, previamente calculados con base en los datos de *PMD*. El MM5G se aplicó utilizando los valores puntuales de M5 y *Cv* disponibles. Los resultados orientan a recomendar el MM5G para estimar predicciones de *PMD* con *Tr* menores de 100 años en sitios sin datos dentro del estado de Zacatecas, México. También se recomienda para estimaciones de la *PMP* puntual de duración 24 horas, recordando que tal método subestima menos de 16.4% con respecto al resultado del método de Hershfield, cuando *Cv* es menor de 0.251 y sobreestima, del orden de un 38.0%, cuando *Cv* excede a 0.386.

Palabras clave: precipitación máxima probable, creciente máxima probable, predicciones extremas, error relativo, coeficiente de variación.

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Introduction

Estimation of the *Probable Maximum Flood* (*CMP*, for its Spanish initials) is used in the hydrological dimension of large reservoirs, or of those that, due to their location above population centers, are highly dangerous. The *CMP* also allows the evaluation of flood risks in nuclear power plants and in important hydroelectric power plants (Jakob, 2013; Salas, Gavilán, Salas, Julien, & Abdullah, 2014). Linsley, Kohler and Paulhus (1988) consider it appropriate to locate above the flood level defined by the *CMP*, potable water supply plants, wastewater treatment plants and other essential public facilities, such as hospitals, airports and access highways. Thus, when the *CMP* occurs, the damages will be substantial and extensive, but will not be increased due to failures in the vital systems of the city.

The *CMP* is not estimated based on the *hydrological analysis of frequencies* (AHF), because being the maximum extreme event; it implies an extrapolation far beyond the reliable limit achievable with the records of annual floods available. In addition, the probabilistic models currently used in AHFs do not have an upper limit, case of the Log-Normal and Wakeby distributions; as well as the most common cases of General of Extreme Values, Log-Pearson type III and Generalized Logistics and Pareto models (Jakob, 2013; Salas *et al.*, 2014).

The estimation of the *CMP* is made based on a design storm, of critical duration and of magnitude equal to the *PMP*, feasible to occur in the basin under study, according to the meteorological knowledge and the hydrological processes that occur under extreme conditions. The *CMP* has the following three basic characteristics generated by the *PMP*: (1) it is the maximum flood theoretically possible to occur in the basin under study; (2) it generates extremely high risks for any hydraulic work and (3) it is feasible to occur in such locality, at a specific time of the year

and under modern meteorological conditions (WMO, 2009; Jakob, 2013).

PMP is defined as (WMO, 2009): theoretically, it is the highest precipitation for a given duration, which is physically possible to occur over a basin area or in a storm area, in a specific geographic location, in a certain time of the year and under modern weather conditions.

The *PMP* estimation methods are divided into two groups, meteorological and statistical. The first ones are the most reliable; they are based mainly on the maximization of humidity and in the transposition of the observed storms and their combination. Their accuracy depends on the quantity and quality of the available data (WMO, 2009). Due to the absence of meteorological data in many sites and watersheds, statistical methods have reached universality. The first of them dates from the beginning of the 1960s (Hershfield, 1961; Hershfield, 1965), there are other more recent approaches, such as Koutsoyiannis (1999), Elíasson (2000) and Koutsoyiannis and Papalexiou (2017). Unfortunately, the statistical methods of estimating the *PMP* are only recommended for preliminary evaluations and the development of large vision projects or pre-feasibility studies.

The *objective* of this study consisted of carrying out a comparison between the predictions and the punctual *PMP* of 24-hour duration, obtained with statistical method of Jónas Elíasson and the same estimations calculated previously in 81 localities of the state of Zacatecas, Mexico, based on an AHF and the Hershfield method, processing the available series of annual maximum daily precipitation (*PMD*), with more than 25 data until year 2012. The procedure developed by Elíasson (1997) must be highlighted; it is a regional method that allows estimations in sites without data, obtaining only of the nearby rain-gauge stations, two statistical values: the *PMD* of the 5 year-return period and the coefficient of variation (quotient between the standard deviation and the mean).

Generalization of the M5 method

Overview

The original method M5 to estimate extreme precipitations and *PMP* values was proposed by the NERC (1975) of England, it uses: (1) the precipitation of 24-hour duration and 5 year-return period, designated M5 as *index variable*; (2) a regional estimate of the coefficient of variation (C_v) and (3) the value of the reduced variable (y) of the Gumbel or General Extreme Value Distribution (GVE) type I, whose shape parameter k is equal to zero. It is a *regional method* that allows estimations in sites without data, based on the map of isovalue curves of the M5 in the British Isles.

Eliasson (1994) indicates that both the method of David M. Hershfield (Hershfield, 1961; Hershfield, 1965) and M5, which statistically estimate the *PMP*, are incorrect when using a probability distribution not bounded at its right end, since by definition the *PMP* has a physical upper limit. Eliasson (1994) also points out that the use of regional envelope curves of extreme precipitation values, to which the Gumbel distribution is fitted, tends in the high return periods, to a limit value of the reduced variable, which can be used to estimate the *PMP*.

Elíasson (1997) points out that when the Gumbel distribution, which is a straight line in the paper of extreme probability, is fitted to series of annual maximum daily precipitation (*PMD*), usually the data are close in the middle part of such model, but they show deviations in the low and large values. These deviations and the lack of an upper limit in the Gumbel model, detract from the reliability of the original M5 method to estimate the *PMP*.

Estimation of Extreme Predictions

In the generalization of the M5 method, due to Elíasson (1997), its anomalies cited, are eliminated them by transforming the random variable of the Gumbel distribution and delimiting the reduced variable (y_{lim}), to obtain a truncated Gumbel model. Now the M5 method depends on another local parameter called *slope factor* (C_i) that is a function of the C_v ; its equations are (Elíasson, 2000):

$$X_{Tr} = M5 \cdot [1 + C_i \cdot (y - 1.50)] \quad \text{with } y \leq y_{lim} \quad (1)$$

being:

$$C_i = \frac{0.78}{\frac{1}{C_v} + 0.72} \quad (2)$$

$$y = -\ln [-\ln (1 - 1/Tr)] \quad (3)$$

when $25 < M5_d < 200$ mm/day, then:

$$y_{lim} = 10.70 - 0.0071 \cdot M5_d \quad (4)$$

In the above expressions, the undefined variables are: X_{Tr} is an extreme precipitation in 24 hours or random variable with return period Tr , $M5$ is the maximum precipitation of 24-hour duration and $Tr = 5$ years, that is with $y \cong 1.50$ and a probability of 0.80 of non-exceedance; on the other hand, $M5_d$ is a daily value. The C_i tends to a value of 0.19 with a standard deviation of 0.035, according to the curve of C_i against $M5$ for the processed values (Elíasson, 1997).

Estimation of the *PMP*

Elíasson (1997) found that applying equation 4 extreme predictions values of the NERC (1975) and of the *PMP* of the US National Weather Service for the state of Washington in USA, in a 25.9 km² area are reproduced. This is the other generalization of the $M5$ method. y_{lim} is a regional parameter and $M5$ and C_i are local parameters that have the same probabilistic performance as the mean and C_v in the frequency analysis. Equation 4 defines extreme values of y_{lim} of 10.5225 with a Tr of 37142.5 years and of 9.280 with a Tr of 10721.9 years. To estimate the punctual *PMP* of 24-hour duration, y_{lim} is evaluated with expression 4, then such value is substituted in equation 3 and the respective Tr is

cleared, which is finally replaced in the following expression by Alfnes and Förland (2006):

$$PMP = M5 \cdot \exp\{C_i \cdot [\ln(Tr) - 1.50]\} \quad (5)$$

Topics related to the contrast

Available Extreme Predictions

During a study conducted in 2013, Campos-Aranda (2014) processed 98 records of annual maximum daily precipitation (*PMD*) of the state of Zacatecas, Mexico, whose minimum amplitude was 25 years and the maximum of 68 data. The lapses of such records varied from 1943 to 2012 and the statistical quality tests detected that 17 series showed deterministic components, reason why they were eliminated, leaving 81 records to be processed. In columns 2 and 3 of Table 1, the names of the rain-gauge stations and the amplitudes of each series of annual *PMD* are indicated. In columns 5 and 6 of Table 1, their respective values of the $M5_d$ and Cv are shown.

Table 1 (first part). Contrast of daily predictions (X_{Tr}) and of the *PMP* in 24 hours, both in millimeters, of the generalized $M5$ method in 81 rain-gauge stations in the state of Zacatecas, México.

1	2	3	4	5	6	7	8	9
No.	Name of the station:	n^1	FDP ²	$M5_d$ ₃	Cv^4	$Tr = 100 \text{ years}$		
						PMD_{Tr}^5	X_{Tr}^6	ER^7
1	Achimec	50	GVE	55.5	0.324	90.3	102.6	0.5
2	Agua Nueva	43	GVE	45.5	0.335	71.9	85.0	4.6
3	Ameca La Vieja	30	GVE	54.6	0.290	83.2	97.5	3.7
4	Boca del Tesorero	44	GVE	53.6	0.302	87.0	96.9	-1.4

5	Calera de V. Rosales	51	GVE	50.2	0.295	77.8	90.1	2.5
6	Camacho	31	LP3	34.6	0.301	59.7	62.5	-7.4
7	Cañitas de Felipe P.	37	GVE	48.3	0.410	92.3	96.4	-7.6
8	Coapas	41	GVE	50.8	0.280	76.6	89.7	3.7
9	Chalchihuites	47	LP3	55.8	0.386	98.9	109.1	-2.4
10	Cedros	38	GVE	39.5	0.299	68.6	71.2	-8.2
11	Col. Glz. Ortega	37	LP3	58.8	0.417	131.4	118.0	-20.5
12	Concep. del Oro	47	LP3	49.0	0.419	87.8	98.5	-0.8
13	Corrales	33	GVE	54.9	0.387	108.9	107.4	-12.7
14	El Arenal	39	LP3	63.4	0.396	112.1	125.0	-1.3
15	El Cazadero	55	GVE	53.8	0.398	106.3	106.3	-11.5
16	El Nigromante	29	GVE	54.8	0.365	92.0	105.2	1.2
17	El Romerillo	30	GVE	57.2	0.296	89.0	102.8	2.2
18	El Salvador	25	GVE	58.6	0.361	107.0	112.1	-7.3
19	El Sauz	66	GVE	46.7	0.287	73.6	83.1	-0.1
20	Espíritu Santo	28	GVE	59.1	0.365	108.7	113.5	-7.6
21	Excamé	66	GVE	64.0	0.267	94.6	111.5	4.3
22	Felipe Ángeles (S)	26	GVE	57.1	0.345	81.5	107.6	16.8
23	Fresnillo	54	LP3	55.6	0.354	90.2	105.7	3.7
24	García de la Cadena	27	GVE	69.6	0.237	88.8	117.2	16.8
25	Genaro Codina	28	GVE	52.5	0.251	75.2	89.8	5.7
26	Gral. Gpe. Victoria	45	GVE	54.4	0.364	96.9	104.3	-4.7
27	Guadalupe	31	LP3	55.3	0.367	89.1	106.4	5.6
28	Huanusco	36	GVE	58.7	0.257	80.4	101.1	11.3
29	Huitzila	25	GVE	75.0	0.271	112.2	131.2	3.5
30	Jalpa	34	LP3	58.8	0.210	79.2	95.8	7.0
31	Jerez	42	GVE	49.5	0.286	80.3	88.0	-3.0
32	Jiménez del Teúl	40	GVE	46.6	0.361	81.5	89.1	-3.2
33	Juan Aldama	34	GVE	59.4	0.345	94.1	112.0	5.3
34	Juchipila	59	GVE	55.8	0.284	92.0	99.0	-4.8
35	La Florida	53	GVE	53.3	0.270	81.7	93.2	0.9
36	La Villita	53	LP3	62.9	0.206	85.8	101.9	5.1

37	Las Ánimas	28	LP3	52.0	0.280	75.6	91.9	7.5
38	Loreto	46	GVE	64.0	0.349	110.3	121.1	-2.8
39	Los Campos	30	GVE	60.8	0.356	110.9	115.8	-7.6
40	Luis Moya	27	GVE	65.4	0.336	103.4	122.3	4.6
41	Mesillas	30	GVE	54.7	0.351	84.7	103.7	8.3
42	Mezquital del Oro	26	GVE	68.4	0.262	118.3	118.5	-11.4
43	Momax	25	LP3	55.8	0.320	103.6	102.7	-12.3
44	Monte Escobedo	44	GVE	58.5	0.228	82.2	97.4	4.9
45	Moyahua de Estrada	31	LP3	60.1	0.240	89.3	101.5	0.6
46	Nochistlán	58	GVE	61.3	0.359	100.0	117.1	3.6
47	Nuevo Mercurio	38	LP3	44.7	0.422	92.5	90.0	-13.9
48	Ojo Caliente	50	LP3	53.6	0.380	88.8	104.3	3.9
49	Palmillas	27	LP3	55.0	0.319	88.4	101.1	1.2
50	Pinos	55	GVE	60.7	0.326	98.9	112.4	0.6
51	Pino Suárez	28	GVE	60.5	0.374	113.1	117.1	-8.4
52	Presa El Chique	59	GVE	53.5	0.255	75.9	92.0	7.2
53	Presa Palomas	44	GVE	58.0	0.260	90.0	100.2	-1.4
54	Presa Santa Rosa	62	GVE	50.9	0.335	90.4	95.1	-6.9
55	Puerto de San Fco.	40	LP3	51.5	0.262	80.5	89.2	-1.9
56	Purísima de Sifuentes	28	LP3	52.0	0.426	99.5	105.1	-6.5
57	Río Grande	37	GVE	53.6	0.413	104.6	107.2	-9.3
58	Sain Alto	25	GVE	56.1	0.352	105.3	106.4	-10.6
59	San Andrés	36	LP3	61.9	0.410	110.3	123.5	-0.9
60	San A. del Ciprés	37	GVE	56.3	0.314	84.6	103.0	7.8
61	San Benito	28	GVE	61.3	0.434	115.4	124.7	-4.4
62	San Gil	36	GVE	46.7	0.417	81.2	93.7	2.1
63	San Isidro de los Glz.	33	GVE	48.2	0.297	78.7	86.7	-2.5
64	San Jerónimo	30	GVE	49.5	0.307	76.3	89.9	4.3
65	S. José de Llanetes	30	LP3	46.2	0.328	86.7	85.7	-12.5
66	S. Pedro de la Sierra	25	GVE	55.5	0.310	86.1	101.1	4.0
67	S. P. Piedra Gorda	68	GVE	50.9	0.266	67.6	88.6	15.9
68	San Tiburcio	37	GVE	51.7	0.384	92.2	100.9	-3.1

69	Sierra Hermosa	32	GVE	61.2	0.461	128.6	127.0	-12.6
70	Sombrerete	28	GVE	49.2	0.352	83.8	93.3	-1.4
71	Tayahua	44	GVE	53.9	0.280	77.7	95.2	8.5
72	Tecomate	50	GVE	55.1	0.247	78.0	93.8	6.5
73	Teúl de Glz. Ortega	44	GVE	63.1	0.302	100.0	114.1	1.0
74	Tlachichila	25	GVE	64.7	0.266	106.3	112.6	-6.3
75	Tlaltenango	56	GVE	63.7	0.256	94.6	109.6	2.5
76	Trancoso	54	GVE	55.0	0.277	69.3	96.9	23.7
77	Vicente Guerrero	25	GVE	58.1	0.266	80.0	101.1	11.8
78	Villa de Cos	45	GVE	62.5	0.359	104.7	119.3	0.9
79	Villa García	52	GVE	60.2	0.334	93.7	112.3	6.1
80	Villa Glz. Ortega	31	GVE	51.1	0.325	81.8	94.5	2.2
81	Villa Hidalgo	43	GVE	59.3	0.372	100.1	114.6	1.3
Minimum value:		25	19LP3	34.6	0.206	59.7	62.5	-20.5
Maximum value:		68	62GVE	75.0	0.461	131.4	127.0	23.7

Table 1 (second part). Contrast of daily predictions (X_{Tr}) and of the *PMP* in 24 hours, both in millimeters, of the generalized M5 method in 81 rain-gauge stations in the state of Zacatecas, México.

1	10	11	12	13	14	15	16	17	18
Sta.	$Tr = 1\ 000$ years			$Tr = 10\ 000$ years			<i>PMP</i>		
No.	PMD_{Tr}	X_{Tr}	ER	PMD_{Tr}	X_{Tr}	ER	MH ⁸	M5 ⁹	ER
1	114.7	132.2	2.0	137.7	161.8	4.0	344.7	381.0	10.5
2	88.8	109.9	9.6	103.7	134.9	15.1	278.9	333.1	19.4
3	101.1	124.1	8.7	116.5	150.7	14.5	330.0	320.9	-2.8
4	112.7	123.9	-2.7	139.1	150.9	-4.0	309.9	333.7	7.7
5	96.2	114.9	5.7	113.0	139.7	9.4	295.0	303.8	3.0
6	83.7	79.9	-15.5	114.1	97.3	-24.6	196.5	220.0	12.0
7	131.7	127.4	-14.4	177.8	158.5	-21.1	370.4	486.2	31.3
8	92.7	113.8	8.7	106.7	137.8	14.3	293.8	286.3	-2.6
9	133.3	143.4	-4.8	170.9	177.6	-8.0	360.6	501.8	39.2
10	96.1	91.0	-16.3	129.9	110.7	-24.6	241.7	247.3	2.3
11	224.9	156.3	-38.5	375.2	194.6	-54.1	449.1	597.9	33.1

12	116.5	130.5	-0.8	145.8	162.6	-1.3	356.2	511.0	43.5
13	166.4	141.2	-24.9	244.4	175.0	-36.7	395.7	496.4	25.5
14	149.3	164.8	-2.4	188.3	204.4	-3.9	448.3	586.9	30.9
15	160.4	140.1	-22.7	231.9	173.9	-33.6	386.1	510.2	32.2
16	117.3	137.4	3.7	140.9	169.6	6.5	384.6	451.2	17.3
17	110.0	131.2	5.5	128.8	159.5	9.6	345.4	344.6	-0.2
18	150.6	146.2	-14.1	201.8	180.3	-20.9	404.3	471.5	16.6
19	92.8	105.7	0.8	111.1	128.3	2.2	271.6	273.5	0.7
20	153.1	148.2	-14.3	205.1	182.9	-21.1	421.7	483.6	14.7
21	113.1	140.6	10.0	128.6	169.7	16.8	345.4	333.2	-3.5
22	91.5	139.7	35.1	97.7	171.8	56.2	381.3	429.6	12.7
23	114.0	137.6	6.8	137.4	169.4	9.1	363.2	436.0	20.0
24	95.7	145.8	34.8	99.4	174.4	55.3	363.2	311.0	-14.4
25	88.2	112.5	12.9	98.4	135.2	21.6	288.2	256.4	-11.0
26	133.5	136.2	-9.7	175.0	168.1	-15.0	351.3	446.6	27.1
27	111.3	139.0	10.5	132.0	171.6	15.0	374.9	459.1	22.5
28	90.5	127.0	24.2	97.1	152.8	39.3	323.2	293.1	-9.3
29	135.2	165.8	8.5	154.8	200.3	14.5	416.1	392.5	-5.7
30	91.9	117.6	13.2	103.6	139.3	19.0	277.2	231.8	-16.4
31	105.0	111.9	-5.7	131.2	135.7	-8.5	289.4	287.4	-0.7
32	109.7	116.3	-6.2	140.0	143.4	-9.4	309.9	382.1	23.3
33	115.4	145.4	11.5	133.4	178.7	18.5	403.4	445.2	10.4
34	123.5	125.8	-9.9	159.4	152.5	-15.4	308.1	318.4	3.3
35	102.6	117.7	1.5	122.9	142.1	2.3	291.7	285.3	-2.2
36	100.5	124.9	9.9	114.3	147.8	14.4	282.2	241.9	-14.3
37	90.3	116.5	14.2	103.7	141.1	20.4	286.2	292.4	2.2
38	147.6	157.4	-5.6	187.4	193.6	-8.6	398.8	484.8	21.6
39	157.3	150.8	-15.2	213.1	185.8	-22.8	400.4	477.2	19.2
40	126.7	158.2	10.5	146.5	194.1	17.3	429.0	466.9	8.8
41	101.3	134.9	17.8	114.2	166.0	28.6	351.5	424.0	20.6
42	174.0	149.1	-24.1	253.0	179.7	-37.1	357.4	345.9	-3.2
43	154.3	132.2	-24.2	224.2	161.7	-36.2	370.5	376.1	1.5

44	96.0	120.7	11.3	107.2	144.0	18.8	280.6	252.9	-9.9
45	111.6	126.5	0.3	135.4	151.5	-1.0	312.7	275.5	-11.9
46	125.0	152.6	8.0	147.2	188.1	13.1	391.4	486.9	24.4
47	142.7	119.5	-25.9	210.6	148.8	-37.5	349.9	475.5	35.9
48	113.0	136.8	7.1	136.3	169.2	9.9	365.1	471.8	29.2
49	113.4	130.2	1.6	139.4	159.1	1.0	341.5	369.5	8.2
50	125.7	145.0	2.1	151.1	177.5	4.0	370.9	417.4	12.5
51	161.9	153.3	-16.2	220.7	189.5	-24.0	418.6	513.2	22.6
52	88.2	115.4	15.8	97.8	138.8	25.6	278.7	266.2	-4.5
53	114.2	126.1	-2.3	138.9	151.9	-3.2	306.4	294.1	-4.0
54	125.9	123.0	-13.6	167.7	150.9	-20.4	327.8	369.6	12.8
55	104.7	112.3	-5.1	132.2	135.3	-9.4	256.5	265.7	3.6
56	141.6	139.6	-12.8	191.2	174.0	-19.5	365.6	554.8	51.8
57	152.9	141.9	-17.9	211.9	176.5	-26.3	400.2	541.2	35.2
58	161.7	138.5	-24.2	241.2	170.5	-37.5	354.3	435.7	23.0
59	146.3	163.3	-1.2	183.3	203.1	-1.9	461.4	608.1	31.8
60	100.2	132.3	16.9	112.3	161.6	27.3	341.0	369.0	8.2
61	158.9	165.9	-7.6	205.2	207.0	-10.7	493.3	664.1	34.6
62	104.4	124.2	5.2	125.6	154.5	8.9	350.0	485.1	38.6
63	102.6	110.7	-4.5	127.6	134.6	-6.6	285.4	295.4	3.5
64	92.0	115.2	10.9	105.1	140.5	18.3	310.3	317.1	2.2
65	130.6	110.6	-25.0	191.8	135.5	-37.5	308.5	327.5	6.2
66	105.0	129.8	9.4	121.1	158.3	15.7	356.6	357.4	0.2
67	74.3	111.7	33.0	78.3	134.7	52.3	262.7	267.9	2.0
68	124.4	132.5	-5.7	158.4	164.1	-8.3	368.3	464.2	26.0
69	196.2	170.1	-23.3	283.3	213.1	-33.4	507.0	737.2	45.4
70	110.3	121.4	-2.6	137.4	149.5	-3.7	320.9	386.4	20.4
71	90.4	120.8	18.2	100.1	146.3	29.3	290.9	302.4	4.0
72	90.8	117.3	14.4	100.8	140.8	23.6	280.0	263.0	-6.1
73	125.6	145.9	2.8	149.7	177.7	5.0	366.7	387.7	5.7
74	144.3	141.9	-13.0	189.4	171.3	-20.0	333.4	334.9	0.5
75	114.6	137.6	6.3	132.4	165.5	10.6	325.9	314.6	-3.5

76	73.4	122.7	47.9	75.2	148.5	74.7	304.8	303.8	-0.3
77	90.1	127.5	25.2	96.7	153.8	40.7	334.4	303.3	-9.3
78	134.0	155.6	2.8	161.7	191.8	5.0	400.9	495.7	23.6
79	113.5	145.3	13.3	129.6	178.2	21.7	381.8	429.3	12.5
80	101.8	121.9	6.0	119.8	149.2	10.2	336.1	354.7	5.5
81	128.4	149.9	3.3	155.1	185.2	5.7	391.1	499.5	27.7
min	73.4	79.9	-38.5	75.2	97.3	-54.1	196.5	220.0	-16.4
Max	224.9	170.1	47.9	375.2	213.1	74.7	507.0	737.2	51.8

Symbols:

- ¹ number of years (data) processed.
- ² Probability distribution function adopted.
- ³ daily prediction of $Tr = 5$ years, in millimeters.
- ⁴ coefficient of variation, dimensionless.
- ⁵ maximum daily precipitation of the Tr indicated, in millimeters.
- ⁶ prediction in 24 hrs. of the method M5 of the Tr indicated, in mm.
- ⁷ relative error in percentage.
- ⁸ PMP estimated with the method of David M. Hershfield, in mm.
- ⁹ PMP estimated with the Jónas Elíasson M5 method, in mm.

The probabilistic processing of the 81 series of annual *PMD*, based on the General Extreme Value (GVE) and Log-Pearson type III (LP3) distributions, adopting (column 4) the one that reported a lower standard error of fitting (Kite, 1977), led to the extreme daily predictions of $Tr = 100$, 1000 and 10000 years, shown in columns 7, 10 and 13 of Table 1, from Campos-Aranda (2014). By multiplying the predictions of *PMD* by 1.13, those of 24-hour duration are obtained (Weiss, 1964; WMO, 2009).

Probable Maximum Precipitation (*PMP*) in 24 hours

The 24-hour punctual *PMP* of the state of Zacatecas, Mexico, was estimated based on the statistical method of David M. Hershfield

(Hershfield, 1961; Hershfield, 1965; Campos-Aranda, 1998a; Campos-Aranda, 1998b; WMO, 2009), whose results are in column 16 of Table 1, for the 81 series of annual *PMD* processed; such values come from Campos-Aranda (2014).

Quantitative measure of contrast

The *relative error* (ER) in percentage, calculated with following equation, was adopted as a basic indicator of the contrasts:

$$ER = \frac{X_{Tr} - 1.13 \cdot PMD_{Tr}}{1.13 \cdot PMD_{Tr}} 100 \quad (6)$$

in which, X_{Tr} is the prediction of 24-hour duration and return period (Tr) in years, estimated with equation 1 of the generalized M5 method by Jónas Elíasson and PMD_{Tr} is the daily extreme prediction of equal Tr . Both predictions are expressed in millimeters. When *PMP* is contrasted, the previous equation is transformed into the following:

$$ER = \frac{PMP_{M5} - PMP_{MH}}{PMP_{MH}} 100 \quad (7)$$

being, PMP_{M5} the punctual probable maximum precipitation in 24 hours estimated with equation 5 and PMP_{MH} that calculated with the Hershfield method, of 24-hour duration; both in millimeters. In expressions 6 and 7, when the estimates of the M5 method exceed the predictions previously calculated in the state of Zacatecas, Mexico (Campos-Aranda, 2014), positive relative errors are obtained. When predictions of M5 method are lower the ER are negative.

Discussion of results

Regarding predictions of $Tr = 100$ years

All the contrasts performed were analyzed based on the ER, considering that when such an indicator is less than 10.0%, a fairly approximate estimate was obtained. For the case of predictions of $Tr = 100$ years, as observed in column 9 of the Table 1, only in 15 rain-gauge stations or series of annual *PMD*, ERs greater than 10.0% were observed, whereby, in 81.5% of the processed records a fairly approximate prediction was achieved with the generalized M5 method. In the 15 records that have the highest dispersions with the predictions of the GVE or LP3 models, nine were by default and six by excess, with maximum values of -20.6% and 23.7%, shown in the last two lines of the last column of Table 1 (first part).

Taking into account that the prediction of $Tr = 100$ years, extrapolates most of the processed records, initially it was sought to relate the 15 ER greater than 10.0% with the short records, but such dependence does not occur in a generalized manner. What can be observed is that positive ERs occur when the *Cv* of the annual *PMD* series is less or close to 0.260 and negative ERs are presented in *Cv* greater than 0.320; the anomalies of the previous one are in the Felipe Ángeles and Mezquital del Oro stations, both with short records of 26 years.

Regarding the predictions of $Tr < 100$ years

Based on the ER of the predictions of $Tr = 100$ years (column 9 of Table 1), the three stations with ERs negative maximums were selected, with ER minimums and ER positive maximums. In such stations a contrast was made of their predictions of Tr of 2, 5, 10, 25, 50 and 100 years, which is shown in Table 2. It is observed that all ERs are smaller in such predictions and close to zero, or less than 10%, in the Tr of 5, 10 and 25 years; except in Trancoso (Station No. 76). Therefore, it is concluded

that the generalized M5 method reproduces in a fairly approximate manner the predictions of $Tr < 100$ years.

Table 2. Contrast of predictions (X_{Tr}) in millimeters of the generalized M5 method in 9 rain-gauge stations of the state of Zacatecas, Mexico.

Sta No.	ER(%) M5 _d	FDP Cv	Prediction ER (%)	Return period (Tr) in years					
				2	5	10	25	50	100
11	-20.5	LP3	PMD_{Tr}	42.2	58.8	72.3	92.8	110.8	131.4
-	58.8	0.417	X_{Tr}	47.6	66.4	78.9	94.7	106.4	118.0
-	-	-	ER	-0.2	-0.1	-3.4	-9.7	-15.0	-20.5
47	-13.8	LP3	PMD_{Tr}	31.7	44.7	54.6	68.6	80.0	92.5
-	44.7	0.422	X_{Tr}	36.1	50.5	60.1	72.2	81.1	90.0
-	-	-	ER	0.8	-0.0	-2.6	-6.9	-10.3	-13.9
69	-12.6	GVE	PMD_{Tr}	41.8	61.2	75.4	95.2	111.3	128.6
-	61.2	0.461	X_{Tr}	48.0	69.2	83.2	100.9	114.0	127.0
-	-	-	ER	1.6	0.1	-2.3	-6.2	-9.4	-12.6
1	0.5	GVE	PMD_{Tr}	42.1	55.5	64.2	74.9	82.7	90.3
-	55.5	0.324	X_{Tr}	48.1	62.7	72.4	84.5	93.6	102.6
-	-	-	ER	1.1	0.0	-0.2	-0.2	0.2	0.5
19	-0.1	GVE	PMD_{Tr}	36.4	46.7	53.3	61.6	67.7	73.6
-	46.7	0.287	X_{Tr}	41.7	52.8	60.1	69.4	76.3	83.1
-	-	-	ER	1.4	0.1	-0.2	-0.3	-0.0	-0.1
59	-0.9	LP3	PMD_{Tr}	44.0	61.9	73.7	88.6	99.5	110.3
-	61.9	0.410	X_{Tr}	50.4	69.9	82.9	99.3	111.4	123.5
-	-	-	ER	1.4	-0.1	-0.5	-0.8	-0.9	-0.9
24	16.8	GVE	PMD_{Tr}	57.5	69.6	75.7	82.0	85.7	88.8
-	69.6	0.237	X_{Tr}	64.6	78.6	88.0	99.7	108.5	117.2
-	-	-	ER	-0.6	-0.1	2.9	7.6	12.0	16.8
22	16.8	GVE	PMD_{Tr}	43.3	57.1	64.6	72.4	77.3	81.5
-	57.1	0.345	X_{Tr}	48.8	64.5	75.0	88.1	97.9	107.6
-	-	-	ER	-0.3	-0.0	2.7	7.7	12.1	16.8
76	23.7	GVE	PMD_{Tr}	44.4	55.0	60.0	64.7	67.3	69.3
-	55.0	0.277	X_{Tr}	49.5	62.1	70.6	81.2	89.0	96.9
-	-	-	ER	-1.3	-0.1	4.1	11.1	17.0	23.7

Symbols: same as Table 1.

Regarding extreme predictions

In the Tr predictions of 1000 and 10000 years of the generalized M5 method, ERs lower than 10.0% are significantly reduced, with only 34.6% of the records in the greater Tr , according to column 15 of Table 1. In addition, the maximum values of such ERs increase considerably, being -54.1% and 74.7%, in the extreme predictions of $Tr = 10000$ years.

In a punctual study of each record with negative ER, it was detected that they occur in annual PMD series having maximum scattered values (*outliers*) and as a result, they are fitted to the GVE distribution with negative shape parameter (k) or Fréchet model. On the contrary, positive ERs occur when the records follow the Weibull model or GVE distribution with positive k , since there is now an upper limit. In summary, the generalized M5 method cannot reproduce the extreme elevated predictions of the distribution Fréchet, neither mark out or delimit those of the Weibull model.

Due to space limitations, rain-gauge stations with ERs negative maximums are not mentioned, nor are their respective k values cited, but they occur in the numbers: 11, 42, 47, 43, 65, 58 and 69, with k varying from -0.24 to -0.11. In contrast, the ERs positive maximums are in stations: 76, 22, 24, 67, 77 and 28, with k fluctuating from 0.35 to 0.17.

Regarding the PMP

According to the ERs shown in the final column of Table 1, 36 records have ERs lower than 10.0%, which corresponds to 44.4% of the 81 series of annual PMD , processed. In addition, in another 40 records the PMP estimated with the generalized M5 method, led to ER positive, that is, it overestimates the value of the PMP obtained with the Hershfield

method. Finally, negative ERs were obtained in the five missing records, with a maximum value of -16.4%. The above is generally considered an excellent approximation of a method that allows *PMP* estimates in 24 hours in sites without data, based only on two regional parameters, the $M5_d$ and the C_v .

With respect to the five underestimations of the *PMP*, they occur when the C_v of the annual *PMD* series is less than 0.251; On the other hand, significant overestimates, considered those of an ER greater than 30.0%, are generally associated with values greater than 0.386 of the C_v .

Regarding the regional analysis

In his study, Campos-Aranda (2014) carried out two regional analyses, one in the Hydrological Region No. 12 Partial (Rio Santiago) in the Juchipila River basin, which included 13 stations and another in Hydrological Region No. 37 (El Salado), which included 19 records. Selecting from Table 1 the lines of the stations belonging to each hydrological region, two tabulations were integrated. It was observed in both tabulations, that now their values tend to be similar; that is, they present less dispersion. The previous one was verified in its final lines of minimum and maximum values, which showed less amplitude or range. This generates confidence in the results of the application of the generalized M5 method, in areas or geographical zones of a hydrological region. Due to space limitations, just the final portion of the stations of the Juchipila river basin is shown in Table 3, which are: Excamé, Huanusco, Huitzila, Jalpa, Juchipila, La Villita, Los Campos, Mezquital del Oro, Moyahua de Estrada, Nochistlán, Teúl de González Ortega, Tlachichila and Tlaltenango.

Table 3. Values of Table 1 (second part) in the 13 rain-gauge stations of the Juchipila River basin of the state of Zacatecas, Mexico.

1	10	11	12	13	14	15	16	17	18
Sta.	$Tr = 1\ 000\ \text{years}$			$Tr = 10\ 000\ \text{years}$			<i>PMP</i>		
No.	PMD_{Tr}	X_{Tr}	ER	PMD_{Tr}	X_{Tr}	ER	MH ⁸	M5 ⁹	ER

21	113.1	140.6	10.0	128.6	169.7	16.8	345.4	333.2	-3.5
28	90.5	127.0	24.2	97.1	152.8	39.3	323.2	293.1	-9.3
29	135.2	165.8	8.5	154.8	200.3	14.5	416.1	392.5	-5.7
30	91.9	117.6	13.2	103.6	139.3	19.0	277.2	231.8	-16.4
34	123.5	125.8	-9.9	159.4	152.5	-15.4	308.1	318.4	3.3
36	100.5	124.9	9.9	114.3	147.8	14.4	282.2	241.9	-14.3
39	157.3	150.8	-15.2	213.1	185.8	-22.8	400.4	477.2	19.2
42	174.0	149.1	-24.1	253.0	179.7	-37.1	357.4	345.9	-3.2
45	111.6	126.5	0.3	135.4	151.5	-1.0	312.7	275.5	-11.9
46	125.0	152.6	8.0	147.2	188.1	13.1	391.4	486.9	24.4
73	125.6	145.9	2.8	149.7	177.7	5.0	366.7	387.7	5.7
74	144.3	141.9	-13.0	189.4	171.3	-20.0	333.4	334.9	0.5
75	114.6	137.6	6.3	132.4	165.5	10.6	325.9	314.6	-3.5
min	90.5	117.6	-24.1	97.1	139.3	-37.1	277.2	231.8	-16.4
Max	157.3	165.8	24.2	253.0	200.3	39.3	416.1	486.9	24.4

Conclusions

Based on the 81 punctual contrasts concentrated in Table 1, carried out in the state of Zacatecas, Mexico, the application of the M5 method generalized by Jónas Elíasson is recommended to estimate predictions of maximum daily precipitation (*PMD*) with return periods (*Tr*) less than 100 years, in localities without data within such state. The estimation of its two required statistical parameters, the daily prediction of $Tr = 5$ years ($M5_d$) and the coefficient of variation (*Cv*), will be made from the available values in the nearby rain-gauge stations, of its hydrological region or geographical area of the site under study.

The generalized M5 method is also recommended for making statistical estimates of the probable maximum precipitation (*PMP*) of 24-hour duration, in places without *PMD* data, within the state of Zacatecas, Mexico; taking into account that such a method underestimates values

when the C_v is less than 0.251 and that it overestimates magnitudes when the C_v is greater than 0.386, regardless of the value of the $M5_d$.

It is suggested to contrast the generalized M5 method, in other states or geographical regions of the country, to verify its universality and try to limit its *PMP* estimates in relation to the C_v , as was done in this work for the state of Zacatecas, Mexico.

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