Temperature distribution in a *p-n* thermoelectric module: Quadratic approximation with respect to an electrical current

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The temperature distribution in a *p-n* thermoelectric module in quadratic approximation with respect to an electrical current, when the Joule and Thomson surface and bulk effects are presented, is studied. Here were taken into account the temperature dependences of the thermal conductivity and the Seebeck coefficient, the presence of the Peltier and Seebeck surface coefficients. The partial cases are considered and the applicability criteria of the quadratic approximation in these cases was established.

Keywords: Thermoelectric cooling; Minimal temperature; p-n junction

1. Introduction

Nowadays thermoelectric cooling is associated exclusively with the Peltier effect, discovered in 1834 by Peltier [1]. The effect is usually defined as the absorption or evolution of heat (in addition to the Joule heat) at the junction between two conductors through which a dc electrical current passes (see, for example [2–6]). The absorption or evolution of heat depends on the direction of the electrical current, and it is equal per unit time to

$$Q_\Pi = (\Pi_1 - \Pi_2)J \qquad (1)$$

where $\Pi_{1,2}$ are the Peltier coefficients of the conducting materials and J is the electrical current.

However in [7–9] it has been shown that the thermodynamic process of cooling (heating) can be explained by considering the Le Chatelier–Braun principle [10]. To summarize the content of [7] and [9] the change in the drift heat flux

$$\boldsymbol{q}_{dr} = \Pi \boldsymbol{j} \qquad (2)$$

(*j* the electrical current density) in a heterogeneous system causes a thermodiffusion heat flux

$$\boldsymbol{q}_{diff} = -\kappa \nabla T \qquad (3)$$

(where κ is the thermal conductivity and T the temperature) compensating this change. Due to this thermodiffusion heat flux, a temperature heterogeneity arises that means the

cooling (heating) of the system depending on the electrical current direction and material properties. When the temperature in the system is below the equilibrium temperature, we have the effect of thermoelectric cooling, and when the temperature is above the equilibrium temperature, we have the thermoelectric heating effect. The full heat flux is

$$\boldsymbol{q} = \boldsymbol{q}_{dr} + \boldsymbol{q}_{diff} . \qquad (4)$$

In the linear approximation with respect to an electrical current the stationary energy balance equation is as follows [7]:

$$\operatorname{div} \boldsymbol{q} = 0 . \qquad (5)$$

In [7,11] on the basis of the energy balance equation (5) the investigation of thermoelectric cooling in the semiconductor structure in the linear approximation with respect to an electrical current is realized.

In the quadratic approximation with respect to an electrical current the Joule heat must be taken into consideration:

$$Q_J = \rho j^2 \qquad (6)$$

(*ρ* is an electrical resistivity) and the Thomson heat too:

$$Q_{Th} = \alpha \boldsymbol{j} \nabla T \qquad (7)$$

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(α is a Seebeck coefficient) [8]. It is important to remember that the Thomson heat arises not due to the temperature gradient externally caused but due to the temperature gradient induced by the Peltier effect that acts in the sample. In this case the stationary energy balance equation is as follows [8]:

$$\text{div}\, \boldsymbol{q} = Q_{\text{J}} + Q_{\text{Th}} . \tag{8}$$

In [8] on the basis of the energy balance equation (8) the thermoelectric cooling in the quadratic approximation with respect to an electrical current in a homogeneous sample is studied. In this work the temperature dependences of heat conductivity κ and the Seebeck coefficient α are taken into account. Also in [8] the surface Peltier effect is taken into consideration.

In [12] the precise temperature distribution in the single-stage *p-n* thermoelectric module is established in the case when the kinetic coefficients are constants and the value of the temperature minimum at a *p-n* junction is found. However, this result is just erroneous. As will be shown below, it is impossible to find the temperature minimum at the interface in the quadratic approximation with respect to an electrical current.

In general case of the thermoelectric cooling in a *p-n* structure the electrons (holes) and phonons can have different temperatures [13]. In order to be able to use the one-temperature approximation the certain conditions must be carried out. One of these conditions is when the sample thickness of a *p-n* structure is much larger than the cooling length of electrons (holes) [13, 14].

A one-temperature approximation under certain conditions can be utilized for the thin samples when its thicknesses are of the cooling length order or much thinner than the cooling length [15]. In this paper we shall investigate the case when the conditions of the one-temperature approximation are carried out.

Traditionally studies of the Peltier effect do not consider the nonequilibrium charge carriers [3, 4, 6–8, 16, 17] so that only majority charge carriers and their electrical current are taken into account in the expressions for thermal fluxes in *n*- and *p*-regions, in spite of the fact that the current of minority charge carriers near the *p-n* junction has the same order of magnitude as the current of majority charge carriers [18]. In this way, the thermal generation and extraction of minority charge carriers must take place near the interface to allow the flow of electrical current [18]. As a consequence, nonequilibrium charge carriers will arise. In [19] the full system of equations, which describes the thermoelectric cooling in a *p-n* semiconductor structure and takes into account the presence of nonequilibrium charge carriers, is established. The influence of the nonequilibrium charge carriers can be ignored if the rate of electron (hole) recombination [20] is sufficiently large. In this paper we assume that the electron (hole) recombination rate is sufficient to neglect by the influence of the nonequilibrium charge carriers upon the thermoelectric cooling phenomenon.

At the present time the general structure of the boundary conditions in an electrical current contact has been already established [21, 22]. This structure of boundary conditions takes into account the presence of the nonequilibrium charge carriers and the opportunity that the electrons and phonons can have different temperatures. Since in this paper we consider a one-temperature approximation and neglect the presence of nonequilibrium charge carriers, the structure of boundary conditions in an electrical current contact simplifies to the equality of the electrical currents from two sides of the interface [9].

The purpose of this paper is to obtain the temperature distribution in the single-stage *p-n* thermoelectric module in the quadratic approximation with respect to the electrical current taking into account the temperature dependence of the kinetic coefficients, the surface Peltier and Thomson effects, and to obtain the criteria of utilizing this quadratic approximation.

2. Heat balance equation and boundary conditions

Let us represent a thermoelectric module (TEM) by the structure composed of two uniform different semiconductors of *n*- and *p*-types (see figure 1). The metal plate between two branches of TEM can be ignored for the thermoelectric processes, since the Seebeck coefficient for metals is negligibly small in comparison with that of semiconductors [17]. For this reason, we do not take the metal plate at $x = 0$ into consideration.

We suppose that the electrical contacts $x = \pm d$ are kept at the equilibrium temperature T_0:

$$T^{\text{n}}\left(-d\right) = T_0 \tag{9a}$$

$$T^{\text{p}}\left(d\right) = T_0 \tag{9a}$$

(here and below in the text the overhead characters "*n*" and "*p*" mean *n*- and *p*- semiconductors accordingly), the lateral surfaces are adiabatically insulated, and the structure's cross-sectional area is equal to the unit measurement everywhere. The current flows along the normal to the interface ($x = 0$) between the layers (direction x-axis). In this case, the problem becomes one-dimensional.

The temperature distribution in the one-dimensional case can be obtained from the heat balance equation (8):

$$\frac{\mathrm{d}q_x^{\text{n,p}}}{\mathrm{d}x} = \rho^{\text{n,p}} j_x^2 + \alpha^{\text{n,p}} j_x \frac{\mathrm{d}T^{\text{n,p}}}{\mathrm{d}x} . \tag{10}$$

Taking into account (2), (3), and the equation $\Pi^{\text{n,p}} = \alpha^{\text{n,p}} T^{\text{n,p}}$, (10) transforms to the following equation:

$$\kappa^{\mathrm{n,p}}\frac{\mathrm{d}^2 T^{\mathrm{n,p}}}{\mathrm{d}x^2} - T^{\mathrm{n,p}} j_x \frac{\mathrm{d}\alpha^{\mathrm{n,p}}}{\mathrm{d}T^{\mathrm{n,p}}}\frac{\mathrm{d}T^{\mathrm{n,p}}}{\mathrm{d}x} + \frac{\mathrm{d}\kappa^{\mathrm{n,p}}}{\mathrm{d}T^{\mathrm{n,p}}}\left(\frac{\mathrm{d}T^{\mathrm{n,p}}}{\mathrm{d}x}\right)^2 = -\rho^{\mathrm{n,p}} j_x^2$$

. $\qquad\qquad\qquad\qquad\qquad$ (11)

Here we need only to pay attention to the fact that there is not the Thomson heat in the right side. This term was reduced with the term in the left side of the equation that we obtain from

$$\frac{\mathrm{d}q_{\mathrm{d}rx}^{\mathrm{n,p}}}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\alpha^{\mathrm{n,p}} T^{\mathrm{n,p}} j_x\right) = T^{\mathrm{n,p}} j_x \frac{\mathrm{d}\alpha^{\mathrm{n,p}}}{\mathrm{d}T^{\mathrm{n,p}}}\frac{\mathrm{d}T^{\mathrm{n,p}}}{\mathrm{d}x} + \alpha^{\mathrm{n,p}} j_x \frac{\mathrm{d}T^{\mathrm{n,p}}}{\mathrm{d}x}.$$

The boundary conditions at the interface ($x = 0$) are as follows (see [9]):

$$q_{sx} - q_x^{\mathrm{n}}(0) = Q_s^{\mathrm{n}} \qquad\qquad\qquad (12a)$$

$$q_x^{\mathrm{p}} - q_{sx} = Q_s^{\mathrm{p}}. \qquad\qquad\qquad (12b)$$

Here

$$q_{sx} = j_x \Pi_s + \eta\left(T^{\mathrm{n}}(0) - T^{\mathrm{p}}(0)\right) \qquad\qquad (13)$$

is the thermal flux at the surface $x = 0$ (Π_s is the surface Peltier coefficient [9], η is the surface thermal conductivity [23]);

$$Q_s^{\mathrm{n,p}} = j_x^2 \rho_s^{\mathrm{n,p}} + j_x \alpha_s^{\mathrm{n,p}}\left(T^{\mathrm{p}}(0) - T^{\mathrm{n}}(0)\right) \qquad (14)$$

is the heat dissipation power at the interface. In this equation ρ_s^{n} (ρ_s^{p}) and α_s^{n} (α_s^{p}) are the surface electrical resistance and the surface Seebeck coefficient from the side of n- (p-) material respectively. $\rho_s = \rho_s^{\mathrm{n}} + \rho_s^{\mathrm{n}}$ is an electrical surface resistance and $\alpha_s = \alpha_s^{\mathrm{n}} + \alpha_s^{\mathrm{p}}$ is the surface Seebeck coefficient [9,20]. It is worth to notice that $\Pi_s \neq \alpha_s T_0$ in a general case [8]. The first term ($Q_{sJ}^{\mathrm{n,p}} = j_x^2 \rho_s^{\mathrm{n,p}}$) in the right side of (14) is the Joule heat that is being dissipated at the interface from the side of n- (p-) material, and the second term ($Q_{sTh}^{\mathrm{n,p}} = j_x \alpha_s^{\mathrm{n,p}}\left(T^{\mathrm{p}}(0) - T^{\mathrm{n}}(0)\right)$) is the Thomson heat which is being dissipated from the side of the n- (p-) material [9].

Hence, the boundary condition for (11) are (9a), (9b), (12a), and (12b).

3. Temperature distribution in the thermoelectric module

The solution of (11) in the quadratic approximation with respect to an electrical current is as follows:

$$T^{\mathrm{n,p}}(x) = T_0 + \delta T_1^{\mathrm{n,p}} + \delta T_2^{\mathrm{n,p}}. \qquad\qquad (15)$$

Here $\delta T_1 \sim j_x$ and $\delta T_2 \sim j_x^2$.

3.1. Temperature distribution in the lineal approximation

In the lineal approximation with respect to the electrical current equation (11) is as follows:

$$\kappa_0^{\mathrm{n,p}}\frac{\mathrm{d}^2 \delta T_1^{\mathrm{n,p}}}{\mathrm{d}x^2} = 0. \qquad\qquad\qquad (16)$$

Here $\kappa_0^{\mathrm{n,p}} = \kappa^{\mathrm{n,p}}(T_0)$.

In this approximation the boundary conditions (9a), (9b), (12a), and (12b) simplify to

$$\delta T_1^{\mathrm{n}}(-d) = 0 \qquad\qquad\qquad (17a)$$

$$\delta T_1^{\mathrm{p}}(d) = 0 \qquad\qquad\qquad (17b)$$

$$j_x \Pi_0^{\mathrm{n}} - \kappa_0^{\mathrm{n}}\frac{\mathrm{d}\delta T_1^{\mathrm{n}}}{\mathrm{d}x} = j_x \Pi_s + \eta\left(\delta T_1^{\mathrm{n}}(0) - \delta T_1^{\mathrm{p}}\right) \quad (17c)$$

$$j_x \Pi_0^{\mathrm{p}} - \kappa_0^{\mathrm{p}}\frac{\mathrm{d}\delta T_1^{\mathrm{p}}}{\mathrm{d}x} = j_x \Pi_s + \eta\left(\delta T_1^{\mathrm{n}}(0) - \delta T_1^{\mathrm{p}}\right). \quad (17d)$$

Here $\Pi_0^{\mathrm{n,p}} = \Pi^{\mathrm{n,p}}(T_0)$. The surface coefficients $\alpha_s^{\mathrm{n,p}}$, $\rho_s^{\mathrm{n,p}}$, η, Π_s are considered to be constants. This is related to the situation that the microscopic theory, which would give the dependences of these coefficients upon the temperature (upon $T(0+)$ and upon $T(0-)$), is not now presented. Therefore, the dependences of these coefficients upon the temperature are not known at present.

The solution of (16) with boundary conditions (17a), (17b), (17c), and (17d) is as follows:

$$\delta T_1^{\mathrm{n,p}}(x) = j_x \frac{\kappa_0^{\mathrm{n,p}}\left(\Pi_0^{\mathrm{n,p}} - \Pi_s\right) + \eta d\left(\Pi_0^{\mathrm{n,p}} - \Pi_0^{\mathrm{p,n}}\right)}{\eta d\left(\kappa_0^{\mathrm{n}} + \kappa_0^{\mathrm{p}}\right) + \kappa_0^{\mathrm{n}}\kappa_0^{\mathrm{p}}}\left(x \pm d\right)$$

. $\qquad\qquad\qquad\qquad\qquad$ (18)

In (18) the upper sign (in this case the sign "+") corresponds to the first index (in this case the index "n") and the lower sign (in this case the sign "-") corresponds to the second index (in this case the index "p"). This distribution of the temperature coincides with the one obtained in the works [7] and [9].

3.2. Quadratic part δT2 of the temperature distribution

In the quadratic approximation with respect to the electrical current equation (11) is as follows:

$$\kappa_0^{n,p}\frac{d^2\delta T_2^{n,p}}{dx^2}-\alpha_{t0}^{n,p}T_0^{n,p}j_x\frac{d\delta T_1^{n,p}}{dx}+\kappa_{t0}^{n,p}\left(\frac{d\delta T_1^{n,p}}{dx}\right)^2=-\rho_0^{n,p}j_x^2.$$

$$(19)$$

Here $\alpha_{t0}^{n,p}=\left.\frac{d\alpha^{n,p}}{dT^{n,p}}\right|_{T=T_0}$, $\kappa_{t0}^{n,p}=\left.\frac{d\kappa^{n,p}}{dT^{n,p}}\right|_{T=T_0}$, $\rho_0^{n,p}=\rho^{n,p}(T_0)$.

We have not written the term $\kappa_{t0}^{n,p}\delta T_1\frac{d^2\delta T_1^{n,p}}{dx^2}$ in (19) because, as follows from (18), $\delta T_1^{n,p}$ is a linear function of x, and therefore, the second derivative of $\delta T_1^{n,p}$ with respect to x is equal to zero.

In this approximation the boundary conditions (9a), (9b), (12a), and (12b) are being simplified to:

$$\delta T_2^n(-d)=0 \qquad\qquad (20a)$$

$$\delta T_2^p(d)=0 \qquad\qquad (20b)$$

$$\eta\left(\delta T_2^n(0)-\delta T_2^p(0)\right)-j_x\delta T_1^n(0)\left(\alpha_0^n+T_0\alpha_{t0}^n\right)+$$

$$+\kappa_{t0}^n\delta T_1^n\left.\frac{d\delta T_1^n}{dx}\right|_{x=0}+\kappa_0^n\left.\frac{d\delta T_2^n}{dx}\right|_{x=0}=j_x\rho_s^n+j_x\alpha_s^n\left(\delta T_1^p(0)-\delta T_1^n(0)\right)$$

$$(20c)$$

$$-\eta\left(\delta T_2^n(0)-\delta T_2^p(0)\right)+j_x\delta T_1^p(0)\left(\alpha_0^p+T_0\alpha_{t0}^p\right)-$$

$$-\kappa_{t0}^p\delta T_1^p\left.\frac{d\delta T_1^p}{dx}\right|_{x=0}-\kappa_0^p\left.\frac{d\delta T_2^p}{dx}\right|_{x=0}=j_x\rho_s^p+j_x\alpha_s^p\left(\delta T_1^p(0)-\delta T_1^n(0)\right)$$

$$(20d)$$

Taking into account (18), the solution for $\delta T_2^{n,p}(x)$ of (19) with boundary conditions (20a), (20b), (20c), and (20d) is as follows:

$$\delta T_2^{n,p}(x)=j_x^2(x\pm d)\times$$

$$\times\left[R^{n,p}+\frac{1}{\kappa_0^{n,p}}\left(\rho_0^{n,p}(d\mp x)+x\alpha_{t0}^{n,p}T_0\frac{K^{n,p}}{Z}-x\kappa_{t0}^{n,p}\frac{(K^{n,p})^2}{Z^2}\right)\right].$$

$$(21)$$

Here

$$R^{n,p}=\pm\frac{1}{Z}\left\{\eta d\left(\rho_s+\rho_0^{n,p}d\right)+\kappa_0^{n,p}\left(\rho_s^{n,p}-\rho_0^{n,p}d\frac{\eta d}{\kappa_0^{n,p}}\right)+\frac{d}{Z}\left[\eta d\left(K^{p,n}\alpha_0^{p,n}+K^{n,p}\alpha_0^{n,p}\right)-\right.\right.$$

$$\left.\left.-\left(\eta d\alpha_s+\kappa_0^{p,n}\alpha_s^{n,p}\right)\left(K^{p,n}+K^{n,p}\right)+K^{n,p}\kappa_0^{p,n}\right]\right\}.$$

$$(22)$$

In (21) and (22) $\alpha_s=\alpha_s^n+\alpha_s^p$, $\rho_s=\rho_s^n+\rho_s^p$,

$$Z=\eta d\left(\kappa_0^n+\kappa_0^p\right)+\kappa_0^n\kappa_0^p,$$

$$K^{n,p}=\kappa_0^{p,n}\left(\Pi_0^{n,p}-\Pi_s\right)+\eta d\left(\Pi_0^{n,p}-\Pi_0^{p,n}\right).$$

The interesting fact follows from (21): the temperature at the interface ($x=0$) does not depend upon the derivatives of both heat conductivity and Seebeck coefficient with respect to the temperature ($\kappa_{t0}^{n,p}$, $\alpha_{t0}^{n,p}$).

The criteria of the justice of the quadratic approximation use (see (15)) are following conditions:

$$\left|\delta T_1^{n,p}(x)\right|<<T_0 \qquad\qquad (23)$$

$$\left|\delta T_2^{n,p}(x)\right|<<\left|\delta T_1^{n,p}(x)\right|. \qquad\qquad (24)$$

It follows from criteria (23, 24) that the quadratic approximation can not be used for finding the extreme current (minimum in our case). Actually let us present $\delta T_1(0\pm)=a^\pm j_x$ and $\delta T_2(0\pm)=b^\pm j_x^2$. Then the temperature at the interface $T(0\pm)=T_0+a^\pm j_x+b^\pm j_x^2$ gets to its extremum at the current $j_x^m=-\frac{a^\pm}{2b^\pm}$. Under these circumstances the criteria (23, 24) reduces to the following criteria for the electrical current: $j<<j_1$ ($j_1=\frac{T_0}{|a^\pm|}$) and $j<<j_2$ ($j_2=\left|\frac{a^\pm}{b^\pm}\right|$) (here $j=|j_x|$). As we can see, the extreme electrical current j_x^m is of the same order as j_2. Hence, it is possible to state: 1) the necessary condition for the quadratic approximation is the requirement that the electrical current must be less than the extreme electrical current (the electrical current at which the temperature gets to its minimum value in the case of the thermal cooling); 2) the minimum temperature value and the electrical current at which the temperature gets its minimum can not be found in the limits of the quadratic approximation with respect to the electrical current.

4. Partial cases

To analyze the results obtained above let us consider estimating expressions, when $\kappa_0^{n,p}\sim\kappa_0$, $\alpha_0^p\sim-\alpha_0^n\sim\alpha_0$ ($\Pi_0^p\sim-\Pi_0^n\sim\Pi_0$), $\rho_0^{n,p}\sim\rho_0$, and $\rho_s^{n,p}\sim\tilde\rho_s$. In this case the temperature distributions (see (18) and (21)) are as follows:

$$\delta T_1^{n,p}(x)=j_x(x\pm d)\left(m\frac{\Pi_0}{\kappa_0}-\frac{\Pi_s}{\kappa_0+2\eta d}\right) \qquad (25a)$$

$$\delta T_2^{n,p}(x) = j_x^2 \frac{x \pm d}{\kappa_0}\left[\tilde{R}^{n,p} + \rho_0(d \mp x) + x\alpha_{t0}^{n,p}T_0 \frac{\tilde{K}^{n,p}}{\tilde{Z}} - x\kappa_{t0}^{n,p}\frac{\left(\tilde{K}^{n,p}\right)^2}{\tilde{Z}^2} \right]$$

$$(25b)$$

where

$$\tilde{R}^{n,p} = \pm \tilde{\rho}_s \pm d\left[\Pi_0 \frac{\alpha_0}{\kappa_0} + \Pi_s \frac{2\left(\eta d \alpha_s + \kappa_0 \alpha_s^{n,p}\right) \pm \kappa_0 \alpha_0}{\left(\kappa_0 + 2\eta d\right)^2} \right] \quad (26)$$

$$\tilde{K}^{n,p} = \mp\left(\kappa_0 + 2\eta d\right)\Pi_0 - \kappa_0 \Pi_s \quad (27)$$

$$\tilde{Z} = \kappa_0\left(\kappa_0 + 2\eta d\right). \quad (28)$$

$$1 >> \frac{\eta d}{\kappa_0}, \frac{|\Pi_s|}{\Pi_0}$$

In this partial case the linear part of the temperature distribution (25a) simplifies to the following equation:

$$\delta T_1^{n,p}(x) = \mp j_x \frac{\Pi_0}{\kappa_0}(x \pm d). \quad (29)$$

The result (29) can be explained easily. The reason of the coincidence of the temperatures on the contact in this case is the smallness of Π_s.

The quadratic part (see (25b)) of the temperature distribution simplifies to the following equation:

$$\delta T_2^{n,p}(x) = j_x^2 \frac{x \pm d}{\kappa_0} \times$$

$$\times\left[\pm \tilde{\rho}_s + \rho_0(d \mp x) \pm \frac{d}{\kappa_0}\left(\Pi_0\alpha_0 + 2\Pi_s\alpha_s^{n,p}\right) \mp x\alpha_{t0}^{n,p}T_0\frac{\Pi_0}{\kappa_0} - x\kappa_{t0}^{n,p}\frac{\Pi_0^2}{\kappa_0^2} \right]$$

$$(30)$$

As we can see from (29) and (30), the linear part of the temperature distribution is determined by only bulk Peltier coefficient, whereas the quadratic one is determined by both bulk and surface Peltier coefficients.

As it follows from the same equations the temperature at the interface ($x = 0$) in this partial case has the following form:

$$T^{n,p}(0) = T_0 - j_x \frac{\Pi_0 d}{\kappa_0} \pm j_x^2 \frac{d}{\kappa_0}\left[\pm \tilde{\rho}_s + \rho_0 d \pm \frac{d}{\kappa_0}\left(\Pi_0\alpha_0 + 2\Pi_s\alpha_s^{n,p}\right) \right]$$

$$. \quad (31)$$

As we can see from this formula the quadratic part of the temperature distribution can be discontinuous.

The criteria, (23) and (24), on the base of (29) and (30) induce the criteria for this partial case:

$$j << \frac{T_0\kappa_0}{\Pi_0 d}, \frac{\Pi_0}{\tilde{\rho}_s + \rho_0 d}, \frac{\kappa_0}{\alpha_0 d}, \frac{\Pi_0\kappa_0}{\left|\Pi_s\alpha_s^{n,p}\right|d} \ . \quad (32)$$

$$\frac{|\Pi_s|}{\Pi_0} >> 1 >> \frac{\eta d}{\kappa_0}$$

In this partial case the linear part of the temperature distribution (25a) simplifies to the following equation:

$$\delta T_1^{n,p}(x) = -j_x \frac{\Pi_s}{\kappa_0}(x \pm d) \quad (33)$$

The quadratic part (see (25b)) in this partial case simplifies to the following equation:

$$\delta T_2^{n,p}(x) = j_x^2 \frac{x \pm d}{\kappa_0} \times$$

$$\times\left[\pm \tilde{\rho}_s + \rho_0(d \mp x) + \left(d\left[\alpha_0 \pm 2\alpha_s^{n,p}\right] - x\alpha_{t0}^{n,p}T_0 \right)\frac{\Pi_s}{\kappa_0} - x\kappa_{t0}^{n,p}\frac{\Pi_s^2}{\kappa_0^2} \right].$$

$$(34)$$

As we can see from (33) and (34) the temperature distribution in this partial case does not depend upon the bulk Peltier coefficient. As it follows from (33) and (34) the temperature at the interface ($x = 0$) in this partial case is as follows:

$$T^{n,p}(0) = T_0 \mp j_x \frac{\Pi_s d}{\kappa_0} \pm j_x^2 \frac{d}{\kappa_0}\left[\pm \tilde{\rho}_s + \rho_0 d + \left(\alpha_0 \pm 2\alpha_s^{n,p}\right)\frac{\Pi_s d}{\kappa_0} \right].$$

$$(35)$$

As we can see from this formula the temperature distribution in the linear approximation undergoes the discontinuity at the interface ($x = 0$), whereas the quadratic part of the temperature distribution can be both discontinuous and continuous function at the interface.

The criteria (see (23) and (24)) on the base of equations (33) and (34) induce the criteria for this partial case:

$$j << \frac{T_0\kappa_0}{|\Pi_s|d}, \frac{|\Pi_s|}{\tilde{\rho}_s + \rho_0 d}, \frac{\kappa_0}{d\left(\alpha_0 + |\alpha_s^{n,p}|\right)} \ . \quad (36)$$

$$\frac{\eta d}{\kappa_0} >> 1, \frac{|\Pi_s|}{\Pi_0}$$

In this partial case the linear part of the temperature distribution (25a) simplifies to the following equation:

$$\delta T_1^{n,p}(x) = \mp j_x \frac{\Pi_0}{\kappa_0}(x \pm d). \quad (37)$$

The result (37) can be explained easily. The reason of the coincidence of the temperatures on the contact in this case is a large surface heat conductivity which is equalizing the temperatures at the contact.

It is interesting that the linear temperature distribution in this case is the same as in the first partial case. The reason of this is coincidence of the temperatures at the contact which takes place once because of small $|\Pi_s|$, another time because of large η.

The quadratic part (see (25b)) of the temperature distribution in this partial case simplifies to the following equation:

$$\delta T_2^{\mathrm{n,p}}(x) = j_x^2 \frac{x \pm d}{\kappa_0}\left[\pm \tilde{\rho}_s + \rho_0(d \mp x) \pm \right.$$
$$\pm d\left(\Pi_0 \frac{\alpha_0}{\kappa_0} + \Pi_s \frac{2\left(\eta d\alpha_s + \kappa_0\alpha_s^{\mathrm{n,p}}\right) \pm \kappa_0\alpha_0}{(2\eta d)^2}\right) \mp x\alpha_{\mathrm{t0}}^{\mathrm{n,p}}T_0 \frac{\Pi_0}{\kappa_0} - x\kappa_{\mathrm{t0}}^{\mathrm{n,p}}\frac{\Pi_0^2}{\kappa_0^2}\right]$$
$$. \tag{38}$$

As we can see from (37) and (38) the temperature distribution in the linear approximation is determined by the bulk Peltier coefficient whereas the quadratic part of the temperature distribution is determined by both surface and bulk Peltier coefficients.

As it follows from (37) and (38) the temperature at the interface ($x = 0$) in this partial case is as follows:

$$T^{\mathrm{n,p}}(0) = T_0 - j_x \frac{\Pi_0 d}{\kappa_0} + \frac{j_x^2 d}{\kappa_0}\left[\tilde{\rho}_s + \rho_0 d + d\left(\Pi_0 \frac{\alpha_0}{\kappa_0} + \Pi_s \frac{2\left(\eta d\alpha_s + \kappa_0\alpha_s^{\mathrm{n,p}}\right) \pm \kappa_0\alpha_0}{(2\eta d)^2}\right)\right]$$
$$. \tag{39}$$

As we can see from this formula the temperature distribution in the linear approximation is a continuous function at the interface ($x = 0$), whereas the quadratic part of the temperature distribution can be both continuous and discontinuous one at $x = 0$.

The criteria (see (23) and (24)) on the base of (37) and (38) induce the criteria for this partial case:

$$j \ll \frac{T_0\kappa_0}{\Pi_0 d}, \frac{\Pi_0}{\tilde{\rho}_s + \rho_0 d}, \frac{\kappa_0}{\alpha_0 d}, \frac{\Pi_0\eta}{|\Pi_s\alpha_s|}, \frac{\Pi_0\eta^2 d}{\kappa_0|\Pi_s|\left(\alpha_0 + \alpha_s^{\mathrm{n,p}}\right)} . \tag{40}$$

$$\frac{|\Pi_s|}{\Pi_0} \gg \frac{\eta d}{\kappa_0} \gg 1$$

In this partial case the linear part of the temperature distribution (25a) simplifies to the following equation:

$$\delta T_1^{\mathrm{n,p}}(x) = -j_x \frac{\Pi_s}{2\eta d}(x \pm d). \tag{41}$$

The quadratic part (see (25b)) of the temperature distribution in this partial case simplifies to the following equation:

$$\delta T_2^{\mathrm{n,p}}(x) = j_x^2 \frac{x \pm d}{\kappa_0}\left[\pm \tilde{\rho}_s + \rho_0(d \mp x) \pm d\left(\Pi_0 \frac{\alpha_0}{\kappa_0} + \right.\right.$$
$$\left.+ \Pi_s \frac{2\left(\eta d\alpha_s + \kappa_0\alpha_s^{\mathrm{n,p}}\right) \pm \kappa_0\alpha_0}{(2\eta d)^2}\right) - x\alpha_{\mathrm{t0}}^{\mathrm{n,p}}T_0 \frac{\Pi_s}{2\eta d} - x\kappa_{\mathrm{t0}}^{\mathrm{n,p}}\frac{\Pi_s^2}{(2\eta d)^2}\right].$$
$$\tag{42}$$

As we can see from (41) and (42) the temperature distribution in the linear approximation is determined by the surface Peltier coefficient, whereas the quadratic part of the temperature distribution is determined by both the bulk Peltier coefficient and the surface one.

As it follows from (41) and (42) the temperature at the interface ($x = 0$) in this partial case is as follows:

$$T^{\mathrm{n,p}}(0) = T_0 \mp j_x \frac{\Pi_s}{2\eta} + j_x^2 \frac{d}{\kappa_0}\left[\tilde{\rho}_s + \rho_0 d + d\left(\Pi_0 \frac{\alpha_0}{\kappa_0} + \Pi_s \frac{2\left(\eta d\alpha_s + \kappa_0\alpha_s^{\mathrm{n,p}}\right) \pm \kappa_0\alpha_0}{(2\eta d)^2}\right)\right]$$
$$. \tag{43}$$

As we can see from this formula the temperature distribution in the linear approximation is the discontinuous function at the interface ($x = 0$), whereas the quadratic part of the temperature distribution can be both continuous and discontinuous function at $x = 0$.

The criteria (see (23), (24)) on the base of (41) and (42) induce the criteria for this partial condition:

$$j \ll \frac{T_0\eta}{|\Pi_s|}, \frac{\kappa_0|\Pi_s|}{\eta d(\tilde{\rho}_s + \rho_0 d)}, \frac{\kappa_0^2|\Pi_s|}{\eta d^2\Pi_0\alpha_0}, \frac{\kappa_0}{|\alpha_s|d}, \frac{\eta}{\alpha_0 + |\alpha_s^{\mathrm{n,p}}|} . \tag{44}$$

It follows from (39) and (43) that the quadratic parts of the temperature distribution in the third and fourth partial cases are identical at the interface ($x = 0$).

It follows an interesting result from all these partial cases: the temperature distribution in the second partial case depends upon only one Peltier coefficient, in this given case, upon the surface Peltier coefficient Π_s, whereas in all other partial cases the temperature distribution depends upon the both surface Π_s and bulk Π_0 Peltier coefficients.

5. Conclusions

The temperature distribution in the *p-n* thermoelectric module in the quadratic approximation with respect to the electrical current is obtained. The temperature dependences of the heat conductivity and the Seebeck coefficient, as well as the surface Seebeck coefficient, thermal conductivity and resistivity, have been taken into account. It is shown that the temperature dependences of the thermal conductivity and the Seebeck coefficient do not influence upon the value of the temperature at the *p-n* junction.

It is established that the quadratic approximation with respect to the electrical current is acceptable for electrical

current values, which are much less than the electrical current at which the temperature reaches its minimum value. Therefore, it is impossible to find both the extreme current and the value of the minimum temperature at the *p-n* junction using the quadratic approximation with respect to the electrical current.

The detailed temperature distributions in the quadratic approximation with respect to the electrical current in certain partial cases are obtained and the criteria, which determine the limits for the electrical current within which these distributions are acceptable, are established.

Acknowledgments

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