The origins of quantum drama and the critical view of Paul Ehrenfest

R. Guzmán
Tecnológico de Monterrey,
Av. Eugenio Garza Sada Sur 2501, Mty, N.L., México,
email: rguzman@itesm.mx

J.A. Cervera
El Colegio de México, Camino al Ajusco,
20. Pedregal de Santa Teresa. 10740 Ciudad de México
email: jacervera@colmex.mx

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In this paper we address the quantum drama that emerged from tensions around the different paradigms of physical explanation that had been developed throughout the nineteenth century. We will draw special attention to the relevance that, in order to understand this revolution in physics, have the scientific dimensions that go beyond its experimental and logical aspects, namely the important role played by the imagination, the use of mental experiments, the use of simple models, analogies, metaphors, etc. The criticism and contribution of Paul Ehrenfest on the quantum hypothesis, which he was interested in, unlike other physicists, from the very beginning, is a very valuable resource for studying the origins and these facets of quantum theory. In his writings on the subject, Ehrenfest strives to clarify the validity of the analogies and methodologies used and cares about the nature of the hypothesis, assumptions and conditions used, giving his thoughts a character of epistemological interest. We will show in what way the success attained by physics in these times of conceptual struggle, probably depended on the existence of a balance between the conservative attitude of some scientists and the more open nature of others, and between imagination and creativity of certain physicists and the skeptical and critical character of others.

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1. Introduction

By the last quarter of the nineteenth century, the mechanical view of the physical world, which had worked largely as means of intelligibility of phenomena, allowing a material and spatial-temporal image of them, became no longer unquestionable. The electromagnetic theory that had been constructed by several physicists, including James Clerk Maxwell (1831-1879), was emerging as an alternative to mechanics. On the other hand, there was also a very successful line of research that sought a mechanical explanation of heat, *i.e.*, thermodynamics phenomena, along with its fundamental principles of energy and entropy, where the work of scientists such as Ludwig Boltzmann (1844-1906) could be found. From these studies, statistical mechanics emerged. The quantum drama we refer to in the title of this article, stems from the convergence of different ideas around these three mainstays of nineteenth-century physics, particularly from Max Planck's search for an explanation of entropy based on Maxwell's electrodynamics, and from the dissatisfaction that Albert Einstein (1879-1955) experienced for being unable to achieve his ideal unified view of physics, having to consider the existence of discrete material particles on one side, and a continuous field on the other. At the end, the introduction of the energy quantum allowed physics to overcome various apparent dead ends, due to theoretical and experimental works but also encouraged by both the imagination and creativity of a group of scientists.

In his professional duties and decision makings, the scientist cannot isolate himself from psychological and social influences in order to guide his work only by empirical data and logical machinery [1]. The use of metaphors, analogies, and other imaginative resources subject to a variety of interpretations by their own nature, remains necessary for the advancement of science, even if it implies a certain risk for its objectivity. Our body of metaphors and other products of the imagination, ultimately determine to a large extent what can be thought in any field. In this article we highlight the fact that the study of black body and the eventual introduction of the quantum were guided by a series of analogies, idealizations, mental experiments and analysis methods adopted from the study of different phenomena rather than radiation, and how this constituted the origin of a radical transformation in physics.

Paul Ehrenfest (1880-1933) is probably the best personification of the drama that took place in physics in the early twentieth century. We find in him a singular character, a great teacher, a critic of his discipline and especially a critic of himself, whose life was entangled with the crucial issues of physics of the early twentieth century. In the conclusions of an article he wrote with his wife Tatiana in 1911 for the Encyklopädie, Ehrenfest claimed that the application of Boltzmann's ideas on the phenomena of radiation posed serious unsolved difficulties [2]. It will be discussed how Ehrenfest had dealt with these problems since 1905 [3] and how his criticism and analysis of the topic is concerned with the validity of the evoked analogies, the adopted methodologies, and with the nature of the hypotheses, assumptions and conditions used, providing his reflections with an epistemic dimension. Hence the contribution and criticism of Paul Ehrenfest to the quantum drama have been chosen as our way to delve in this story and to determine the continuities and fractures regarding classical physics in the studies of black body radiation and in the introduction of quantum discontinuity that was carried out in the late years of the nineteenth century and the beginning of the twentieth. In opposition to Thomas Kuhn's non-accumulative characterization of science during scientific revolutions [4], we will argue that even in those cases in which profound conceptual changes occur, such as the quantum revolution, continuity of ideas can still be traced.

2. The emergence of quantum

Although it has been traditionally accepted that the quantum of energy concept was introduced in a work by Planck from late 1900 and published in early 1901, we will assume the position sustained by Kuhn [5] regarding the fact that Planck only conceived a real physical energy discontinuity until many years later, and that Albert Einstein and Paul Ehrenfest were the ones who in fact introduced an important conceptual change during the period 1905-1906. Planck was a physicist working fully within traditional paradigms during that period of time.

This story, however, starts long before, especially if we uphold the opinion that there is still a continuity of ideas in spite of the revolutionary conceptual changes. It starts with spectroscopy and the introduction of the black body concept, and continues in the year 1895 when a closer approximation to the new concepts is achieved and three significant facts concur: Planck began his works on black body radiation that same year, the new areas of physics that will undermine its own foundations (radioactivity, X rays, relativity, the quantum, etc.) could not be anticipated yet, and finally, the rapid development of technology made it possible to perform novel experiments that would test those new fields of physics, which were so far accessible only through mere speculation [6].

2.1. Spectroscopy and the concept of black body radiation

Paul Ehrenfest takes "Gustav Kirchhoff's (1824-1887) assertion on the universality of black body radiation" [7] as a starting point in his analysis of Planck's works. In this section, we will briefly recount some background in order to better contextualize Plancks'work on black body radiation.

Electromagnetic theory, developed in the nineteenth century, proved spectacularly successful in describing the propagation of light and other forms of radiation. Nevertheless, this theory was not able to explain the ways in which matter emitted or absorbed radiation, as shown in different experiments. This situation constituted a real dilemma for physical sciences. The study of light (and later of other forms of radiation) in terms of its spectral content (colors) has its own history that dates back to Newton and his prism experiments. However, we will place ourselves in the early nineteenth century, when light began to be described in terms of waves, that is, as vibration propagated through ether [8].

Spectroscopy was developed in the nineteenth century as well, and it revealed extremely odd information about the characteristics of radiation emitted or absorbed by matter. It also showed the inability of already known theoretical tools to explain mechanisms of interaction between the two physical entities that the paradigms of science were studying in that century: matter and radiation. In 1814, Joseph Fraunhofer (1787- 1826) had studied a peculiarity in the continuous spectrum of sunlight, which consisted of presenting a series of dark stripes. Subsequently, some ways of artificially producing dark lines in the spectrum of light were found by making light pass through various substances (the resulting spectra were called 'absorption spectra'). The several efforts to explain this kind of phenomena started to pay off in the second half of the century, thanks to the work of scientists such as Gustav Kirchhoff and Robert Wilhelm Bunsen (1811-1899). Just as absorption spectra were identified, emission spectra detected in flames that contained various substances (sodium, for example, gave rise to bright lines that coincided in position with some of the dark Fraunhofer lines) were also identified. The collaborative work between these two scientists, a physicist and a chemist, yielded important conclusions: 1) the spectrum could be used as a way to identify chemical elements, 2) a substance capable of emitting a certain spectral line, has a great capability to absorb it, 3) this opened a whole new research field: astrophysics. This type of scientific results on the emission and absorption of radiation by matter was very successful, although it obviously did not explain the mechanisms by which these phenomena occurred. According to existing theories, the light emission of a given frequency would require the existence of an oscillating electric charge (electrical oscillators), but there was no model to explain their existence and way of operating.

On the other hand, the black body concept arose from a number of theoretical considerations that were developed by Kirchhoff himself, which will only be outlined here in order to specify some terminology [9]. Kirchhoff claimed that the absorption (A) and emission (E) capabilities may be different for each body that can emit and absorb energy, but the ratio between emission and absorption is the same for all bodies ($E/A = K$). Hence this is a universal function that depends only on temperature T and frequency ν, that is, $K = K(T, \nu)$ [10]. Kirchhoff imagined the existence of a body whose absorption capability A equals 1, which he named 'black body' (the one that absorbs all radiation incident upon it). This body would have an emission capability equal to the universal function $K(T, \nu)$ [11].

Efforts to achieve the fit between theory and experiment, feeding each other, are the engine for conceptual changes and technological progress. By 1895, experimental results regarding black body radiation (the universal function K) were not up to expectations, but it had, in fact, been determined that this function had a maximum radiation at a frequency ν_m or wavelength λ_m for each temperature. However, near Berlin, the Physikalische-Technische Reichsanstalt (Imperial Physical-Technical Institute), which was specifically created to address the applied aspects of physics, undertook the study of issues related to black body radiation, offering very reliable experimental results to theorists [12]. By 1896, there were a couple of theoretical results, with classical foundations indeed, corroborated with experiments that restricted, but had not yet determined the real distribution of black body radiation precisely. The first of these results is the Stefan-Boltzmann law of radiation, which states that the total energy radiated, integrated over all frequencies, is directly proportional to T^4. This relationship was found empirically by Stefan and proven theoretically by Boltzmann [13]. The second result is the Wien's displacement law, which states that the spectral distribution function must have the form $K(T,\nu) = \nu^3 f(\nu/T)$, from which can be deduced, regardless of the temperature value T, that the product $\lambda_m T$ is always the same, that is, if the temperature is doubled, then the wavelength for which the maximum in the radiation spectrum occurs, is reduced by half. In addition to these results, we can mention other two that offer an explicit form for the spectral distribution, as a function of frequency: one, deduced by Wien himself, although lacking sufficiently rigorous reasoning, which fits well with experimental results for high frequencies but fails at low ones, and the other one known as the Rayleigh-Jeans distribution, which is deduced directly from classical principles and foundations, providing good results for low frequencies but not for high ones, where it grows indefinitely and it would imply the presence of infinite energy.

Several examples in the history of science have shown how a researcher might cling to certain beliefs in order to support his world vision, which sooner or later have to be abandoned for the sake of empirical evidence. As it is well known, from the Greeks to Copernicus, the circle had been a guide to understanding the motion of celestial objects, until Kepler, who had at his disposal more precise observations of the motion of Mars, definitely abandoned this idea, not before persisting on it for a long time and trying to save it through explanations on how phenomena -what is presented to our senses- may differ from true reality. In a similar way, the classical schemes, especially the mechanical and continuous view of the world, were a test for evidence of intelligibility of the physical world in the nineteenth century. This belief, however, had to collapse when faced with the existence of more precise experimental data (at first, regarding the black body radiation, and then other phenomena) and the need to adjust the theoretical schemes in order to "save the appearances". And we say adjust, precisely because, unlike what a "naïve falsifiability" view would suggest, a theory does not fall down due to the existence of refuting experimental data, but first a series of auxiliary hypotheses emerge in search of these adjustments, and they can only in the end lead to a total paradigm shift.

2.2. Planck's quantum and Ehrenfest's critique

Philosophical convictions held by scientists generally play an important heuristic role and to some extent determine what each of them consider the goal of their discipline and its particular methods, which often represent a significant point of dispute between peers [14]. This philosophical dimension became especially important for physicists involved in the genesis of quantum physics, since fundamental questions around the traditional paradigms of continuity, causality, etc. arose in this process. The path followed by those scientists thus depended on how intensely they were committed to their discipline's traditional forms of work, on their conceptions of knowledge, what they believed a scientific theory should be and their particular notion of reality. In this section we will refer to Max Planck's contribution and Paul Ehrenfest's analysis of it. Ehrenfest's profound reflections on the directions physics was following during the first decade of the twentieth century enabled him to later contribute significantly to this transition process.

2.2.1. Planck's contribution

Letting aside the debate on whether or not to grant Planck with the introduction of the concept of energy quantum, it is undeniable that his work is fundamental in the history of the origins of quantum ideas, thus considering his philosophical beliefs is very revealing. Planck was above all a realist, as shown in the following extract from his scientific autobiography:

> My original decision to devote myself to science was a direct result of the discovery which has never ceased to fill me with enthusiasm since my early youth - the comprehension of the far from obvious fact that the laws of human reasoning coincide with the laws governing the sequences of the impressions we receive from the world about us; that, therefore, pure reasoning can enable man to gain an insight into the mechanism of the later. In this connection, it is of paramount importance that the outside world is something independent from man, something absolute, and the quest for the laws which apply to this absolute appeared to me as the most sublime scientific pursuit in life [15].

For Planck, there is a real external world, but we can only get to know it indirectly in order to form an 'image' of it. The process is based on hypotheses, which as a constituent part of the physical image of the world are products of complete speculative freedom of human mind. That is precisely why the problem of black body radiation attracted Planck's attention, since Kirchhoff had made it clear that it was a universal function, an 'absolute' in the words of Planck. According to Kirchhoff, the state of heat radiation that can be reached in a cavity surrounded by emitting and absorbing substances of uniform temperature is completely independent of the nature and disposition of substances and a function solely of temperature and frequency, but by no means of the properties of substances. Let's consider in Planck's own words, taken from his Nobel lecture [16], the goal settled for himself and the conceptual means at his disposal for its achievement:

> (...) to find the solution to the problem of the distribution of energy in the normal spectrum of radiating heat (...) To attain this there was no other way but to seek out from all the different substances existing in Nature one of known emissive and absorptive power, and to calculate the properties of the heat radiation in stationary energy exchange with it.

According to Kirchhoff, these properties would be independent of the nature of the body (or substance). For this purpose, Planck chose Hertz's linear oscillators (which he called "resonators"). In the second of the first two articles in which Ehrenfest analyzes Planck's works [17], it is stated that one of his starting points was to extrapolate "Kirchhoff's law on the universality of black body radiation (...) to fictitious systems". Following Planck's idea that he recalled in his Nobel lecture, it could be said that:

> If a number of such Hertzian oscillators are set up within a cavity surrounded by a sphere of reflecting walls, then by analogy with audio oscillators and resonators, energy will be exchanged between them by the output and absorption of electromagnetic waves, and finally stationary radiation corresponding to Kirchhoff's Law, the so-called black-body radiation, should be set up within the cavity.

Rather than the analogy explicitly mentioned by Planck in this paragraph, we will see that it is the parallel established with the behavior of a gas in a chamber the basic reference for theoretical developments on the black body radiation, and how the critical analysis of the validity and limits of that parallelism and also on the nature of the hypotheses and assumptions in the explanation of these phenomena are the basis of the critical view posed by Ehrenfest. The main Planck's articles which Ehrenfest referred to in his 1905 text are: *Über irreversible Strahlungsvorgänge* (On irreversible radiation processes, which he identified as A), *Entropie und Temperatur Strahlende Wärme* (Entropy and temperature of radiant heat, B), *Über das Gesetz der im Energieverteilung Normalspektrum* (On the law of distribution of energy in the normal spectrum, C) and *Über irreversible Strahlungsvorgänge* (On irreversible radiation processes, D). In 1906, Ehrenfest writes an article reviewing a book entitled *Vorlesungen über die Theorie der Wärmestrahlung* (Lectures on the theory of heat radiation), published by Planck in that same year. Let us briefly identify the contributions made by Planck [18] in these works, so we can later present a description of the criticism of Ehrenfest.

The central theme in Planck's referred works is the black body radiation; however, we must frame them within his broader quest for an explanation of the concept of irreversibility and its relation to the increase of entropy. At first, the second law of thermodynamics had for Planck an absolute character, so he did not subscribe to Boltzmann's statistical methods, but was guided by what he considered a more inductive treatment of the subject. However, as it will be discussed below, he eventually had to adopt those statistical techniques overcoming intense personal resistance. Paradoxically, this decision, experienced as a failure in some sense, later became the element that assured Planck a place among the greatest physicists of all times [19].

The article referred to as (A) in Ehrenfest's nomenclature and sent for publication in November 1899 was actually a repetition for the Annalen der Physik of the last of a series of five articles Planck had been writing since February 1897. In the first of these articles he had "naively charming and agreeable expectations, that the laws of classical electrodynamics would, if approached in a sufficiently general manner with the avoidance of special hypotheses, be sufficient to enable us to grasp the most significant part of the process to be expected, and thus to achieve the desired aim" [20]. By the fourth delivery he already had to include a first statistical hypothesis that consisted in the introduction of the concept of "natural radiation" [21]. In the fifth delivery, and also in his article (A), following previous ideas, Planck defined an expression for entropy S (we call it Σ_1 to adjust to Ehrenfest's nomenclature) of an oscillator of frequency ν and energy E that was a function of E/ν (which was another way of expressing the law of displacement), and hence, recurring to electromagnetic and thermodynamic arguments, it shows that Σ_1 increases monotonically (demonstrating irreversibility) and that in a steady state (when Σ_1 remains constant), it reaches Wien's distribution of spectral radiation. It was still considered that this distribution was consistent with the experimental results. It could have lead Planck to promptly propose, convinced by his demonstration, that Wien's law had the same status and validity limits of the second law of thermodynamics. To his misfortune, shortly after his paper, new experimental results showed that Wien's law was not correct for the low frequency range. Therefore, in another article (which we call B) he reviewed some of his arguments, concluding that Wien's distribution law was not strictly necessary to satisfy the second law. But then, which was the correct formula for the spectral distribution?

Planck focused on finding a formula consistent with the experimental results and he achieved it in two ways. The first of them, by an ad hoc heuristic reasoning in which a term of his theory (the inverse of the second derivative of entropy with respect to energy), that had a linear dependence on energy ($-aE$) and led to Wien's law, was changed to a quadratic dependence ($-aE - bE^2$), with no reference to any physical basis, which conducted to a new formula, now known as Planck's law [22]. Moved by the confirmation that his formula satisfied the experimental results, Planck spent subsequent days trying to elucidate its true physical nature.

This search led him to incorporate Boltzmann's ideas that relate entropy with probability and to later introduce them in the article that we have identified here as (C) [23].

In that article Planck tried to rectify his earlier theory, that had led him to the demonstration of Wien's law, and stated: "it will be necessary first to find in the set of conditions leading to Wien's energy distribution law that term which can be changed; thereafter it will be a matter of removing this term from the set and making an appropriate substitution for it" [24]. His strategy was then to find an expression for the entropy of the system of resonators replicating the statistical reasoning that Boltzmann used for the entropy of a gas of N molecules. Boltzmann associated entropy with the probability of the state of the gas, and the probability, in turn, depended on the number of configurations of the gas corresponding to that state. In order to 'count' the possible configurations, he had to introduce a discretization. Boltzman's molecules had only an energy value multiple of ε. Boltzmann counted the different forms or 'complexions' [25] in which a total energy E could be distributed in N molecules to give rise to a certain distribution [26]. Planck did something similar with resonators, considering that his system had N resonators and that among them a total energy $E = \varepsilon P$ was distributed (that is to say, if the total energy was E, it could be considered as distributed in a number of P packets with energy ε). Planck used a combinatorial way to calculate the number of 'complexions' R for a certain distribution as a function of N and P. Then, he applied the expression $S_{\mathrm{N}} = k \log R$ [27] as a formula for the entropy of the system of resonators, and after substituting and dividing by N to get the entropy of a resonator, he obtained an expression for S (which we shall call Σ_2 to use the nomenclature of Ehrenfest) that was different from the one he used before (Σ_1). This new expression was a function of E/ε and by comparison with the equivalent expression to the displacement law, which stated that entropy should be a function of E/ν, Planck realized that for them to be compatible it was required ε to be proportional to ν, that is to say, $\varepsilon = h\nu$, the now famous formula of quantization of energy, with the constant of proportionality h now called Planck's constant. With his new expression for entropy, was derived the (Planck's) distribution law, which he had earlier developed by different means.

Planck borrowed from Boltzmann the method he applied, but Boltzmann, at the end of the procedure in the analysis of a gas, applied the limit $\varepsilon \longrightarrow 0$, since the use of discretization was due to the need for counting possible configurations, but finally he had to restore the continuous nature of the system. Planck did not take the limit because, in that case, its distribution became Rayleigh's distribution (which made no sense or was not physically acceptable as it grew indefinitely at high frequencies). At what extent was Planck aware of being introducing a physical conception radically different to traditional paradigms? It seems that at that time quantization was a mere calculation artifice and had no real physical sense. Most contemporary physicists did not understand Planck's reasoning and what was really important for many of them was the fact that he had found an expression that matched perfectly with experimental results. But it was not so for some more critical physicists, including Einstein and Ehrenfest, who were certainly intrigued by the hidden meaning in the work of Planck [28].

After these events, in October 1901, Planck published one more article (D) aiming to supplement his researches on these topics. In this contribution he used the second expression (Σ_2) for the entropy once again, and even when admitting not being able to establish one single expression for entropy, he was confident that a more general treatment in the future might fix entropy in a single way.

2.2.2. Ehrenfest's analysis

Paul Ehrenfest's first acquaintance with the black body radiation topic took place before his graduation, on a trip he made to Leiden with his friend Ritz in 1903 to attend some conferences delivered by Lorentz [29] The subject captivated him, but he had to wait until his graduation in 1904 to pay enough attention to it again. His efforts to elucidate Planck's work led to his 1905 and 1906 publications mentioned before.

The first of those works, where he quoted the articles we have mentioned before and that he identified as A, B, C and D, was devoted to the pursuit of physical assumptions underlying Planck's theory of black body radiation. The question that guides his analysis, posed by Ehrenfest at the very beginning, is: "What are the hypotheses - independent of each other - that allow this theory to generate, unambiguously, an energy distribution of black body radiation for each temperature?" [30]. In order to reach an answer, he first tries to demonstrate that based on Planck's theory it cannot be proven that "with a given total energy, Σ remains temporarily constant only when the stationary radiation state unambiguously holds a certain spectral distribution (corresponding to the total energy)" [31]. Unlike Boltzmann's theory of gases, which defined a unique function (-H) representing entropy and that could only grow to a stationary state corresponding to the Maxwell-Boltzmann velocity distribution, in Planck's theory two different functions had been used to represent entropy (Ehrenfest refers to papers A and D, where the functions Σ_1 and Σ_2 were both used) each one producing different stationary states. Thus, Ehrenfest affirms, there is "an essential discrepancy between Planck's function Σ in its behavior within the Planckian model and thermodynamic entropy's behavior in isolated thermal systems" [32] Through a dimensional analysis, based on the linearity of the electromagnetic equations and resonators, Ehrenfest proves that in Planck's theory, the stationary condition does not lead to a single spectral distribution without contradicting Wien's displacement law. Therefore Ehrenfest establishes that "for Planckian theory to provide a result without ambiguity, it will be necessary to incorporate an independent condition (...)" [33]-

Ehrenfest finds in the now famous paper C the additional hypotheses, which he had not considered in the first part of his analysis because the role they played in Planck's ideas seemed unclear. The new hypotheses were:

1) The hypothesis on probabilistic equality in the distribution of energy on the resonators.

2) The hypothesis that the radiation energy for different colors is composed by small particles of energy with value $E_\nu = \nu \times 6.55 \times 10^{-27}$ erg-sec [34].

Ehrenfest comments that the first hypothesis clearly has its analogue in Boltzmann's theory, but for the second hypothesis he states: "as far as I can see, there is no analogy for it in Boltzmann's theory" [35] And at this point he promises to address the issue in a further article.

In 1906, Ehrenfest received a copy of Planck's *Vorlesungen über die Theorie der Wärmestrahlung* (Lectures on the theory of heat radiation), which he comments on his second article on the subject, presented that same year. Ehrenfest opens his work by repeating the argument developed in his previous article, that is, that Planck's theory failed to offer a single definition for entropy in the cavity, so he had to introduce an additional hypothesis that deviates clearly from accepted principles in statistical mechanics. In order to explain Planck's theory limitations and staying faithful to his own style, he presents an "analogy with kinetic theory of gases" [36], to show and clarify the situation of a gas, which would be in fact equivalent to Planck's model [37]. The increase in entropy is caused by the collisions among molecules, and between them and the walls of the physical space enclosing the gas. Let us assume an idealized gas where the molecules do not collide with one another, but only with the walls. The speeds of the molecules will only change direction, preserving the energy distribution among them. Boltzmann's H function can grow in the rearrangement process of molecules' position and direction of speeds, but, since the speeds' magnitude remain the same (their initial distribution is kept), then a maximum depending on initial conditions is reached, but not the absolute maximum corresponding to Maxwell-Boltzmann distribution. The same happens with Planck's radiation model. Ehrenfest says:

> Therefore, radiation is not able, in general, in an isolated model, to achieve a state characterized by absolute stability. In the Planckian model there are infinite forms of perfectly stable non-black radiation. This can be proven in a simple way with the assistance of a dimensional method [38].

This dimensional method was the same he had used in his previous article. Now he could conclude with greater certainty that Planck's linear oscillators, interacting with the electromagnetic field, does not lead to a single spectral distribution. Just as in the idealized gas model the molecules do not interact to produce a redistribution of speeds, in Planck's model the oscillators with different frequencies do not exchange energy and therefore does not alter the frequency distribution in the radiation field. Then, having found this incompleteness in Planck's theory of resonators, Ehrenfest returns to what he calls the 'second' theory, which he considers 'independent' from the resonators' theory, and identifies it as the theory of 'complexions'. Ehrenfest argues on these ideas, but gets closer to the methods of Rayleigh and Jeans [39], that consisted on applying to radiation the same equipartition principle successfully used in the study of gases. Just as the total energy of the system would be divided equally (on average) among all the molecules of a gas, in the case of radiation, total energy would be divided equally among all possible vibration frequencies [40]. Applying the 'complexions' method, Ehrenfest reaches the same impossible results of the Rayleigh-Jeans law with an unbound spectral distribution function that grows indefinitely with the frequency, therefore impossible. Finally, Ehrenfest asks: Which are the means in Planck's theory to render the many ultraviolet superior tones of the cavity innocuous, so that they do not absorb all the energy as it happens with Rayleigh and Jeans, but rather make the spectral curve decrease beyond the ultraviolet? [41] Planck's way to solve the problem, but not the only one according to Ehrenfest, is to introduce an additional condition, considering the energy of the oscillators quantized and depending on the frequency. Thus the question could be solved formally, but leaving the physical nature of the hypothesis unsolved: being the discrete element dependent on frequency ($h\nu$), and given a finite total energy for the system, the capacity of high frequency vibrations to take additional energy, becomes automatically restricted.

Before closing this section it must be noted that even when Planck's function was widely accepted within the scientific community because of its agreement with the black body radiation particularities observed in laboratory research [42], the opposite happened to his justification methods. Although impeccable from a modern view, they remained mainly unintelligible at that time, and it was not until 1905, when other scientists, including Ehrenfest and Einstein, attempted to explore further on the physical implications of the new concept of the quantum.

2.3. Einstein's quantum: Ehrenfest pointing out a crucial difference

We will depart from a sequential presentation of the work of Ehrenfest regarding the emergence of the first quantum ideas, to comment an article he wrote later, in 1914, and which is another clear example of the sharp and precise style so typical of Ehrenfest to point out an idea in a simple way, stripping a concept of its complexities and making things crystal clear. But since the subject that Ehrenfest deals with in such paper is related to the difference between Planck's quantum and Einstein's quantum, we will refer briefly to the work of the latter in the field of quantum discontinuity and then return to Ehrenfest.

Let us look at Einstein's own words and review what his position was in relation to the fundamental work of Planck, in which he derived the distribution law and the energy element appears for the first time $\varepsilon = h\nu$ having a finite value because, as we have seen, Planck does not apply the limit $\varepsilon \to 0$:

> This form of reasoning does not make obvious the fact that it contradicts the mechanical and electrodynamics basis, upon which the derivation otherwise depends. Actually, however, the derivation presupposes implicitly that energy can be absorbed and emitted by the individual resonator only in "quanta" of magnitude $h\nu$, *i.e.*, that the energy of a mechanical structure capable of oscillations as well as the energy of radiation can be transferred only in such quanta, in contradiction to the laws of mechanics and electrodynamics [43].

The contradiction, according to Einstein, was fundamental to mechanics, but not so strong for electrodynamics, for he considered that the energy distribution expression derived by Planck was consistent with Maxwell's laws, although not a necessary consequence of them. However, Planck never considered to be contradicting the fundamental principles of his discipline, despite the 'acts of desperation' [44] in which he fell into in order to derive his distribution formula. But for Einstein, the electromagnetic theory was in a situation that demanded its reformulation. He wrote a paper in 1905 [45], later described as revolutionary by himself, in which he started emphasizing that a profound formal difference exists between the theoretical concepts that physicists have formed about gases and other ponderable bodies, and Maxwell's theory of electromagnetic processes in the so called empty space [46].

And he attempts to demonstrate that

> in the propagation of a light ray emitted from a point source, the energy is not distributed continuously over ever-increasing volumes of space, but consists of a finite number of energy quanta localized at points of space that move without dividing, and can be absorbed or generated only as complete units [47].

This means that Einstein's approach is indeed truly radical and revolutionary, in providing the quanta with an explicit physical character, unlike Planck, for whom the quanta had only a formal character, that is to say, they constituted only a part of the demonstration process.

Let's consider Planck's work again. It contains two elements that are incomprehensible from the perspective of traditional methods, and they both have to do with the methodological artifices introduced in order to 'count' the possible system dispositions. The first one, as it has already been mentioned, has to do with not taking the limit to restore the continuum after making the quantization. Planck does so, not because it seems more reasonable to him, but because leaving the energy elements of finite size leads him to the formula he wanted. The second element is the 'counting' method itself. In the previous section we mentioned that Planck treated his resonators in a similar way to what Boltzmann had done with the gas molecules, but the differences were not emphasized and now it is time to do so. Boltzmann considered a distribution of gas as a set of numbers w_0, w_1, w_2, etc., in which w_k represented the number of molecules having energy k_ε. The number of 'complexions' that gave rise to that distribution was N!/($w_o!w_1!w_2!\ldots w_p!$). Then, he calculated the distribution that maximized that expression in order to reach the thermal stability condition. What Planck did, given a total energy E divided in P discrete elements of size ε, *i.e.* $E = P\varepsilon$, was to count the total number of ways that energy could be distributed in N resonators, for which he used the combinatorial expression R = (N+P-1)! / (N-1)! P! and then used this expression to calculate entropy without applying any maximization process [48].

In October 1914, almost two years after Ehrenfest replaced Lorentz in Leiden, he sent the latter the manuscript of a short paper that he wrote together with Professor Kamerlingh Onnes (1853-1926), where he expressed how much fun he had thinking over the problem treated there [49]. The paper's title was *Simplified Deduction of the Formula from the Theory of Combinations Which Planck Uses as the Basis of his Radiation Theory* [50]. Its purpose was, as the title indicates, to give a simplified demonstration of the formula for R described in the previous paragraph and that Planck had used simply by borrowing it from the combinatorial theory. Instead of using a mathematical induction process as it is normally done to demonstrate this kind of formulas, Ehrenfest used a more intuitive procedure that showed clearly the reason of the N-1 term in the equation, but at the same time, he made some notes in an appendix and a footer that are more interesting to our purposes. In his usual eagerness to use examples, analogies and simple conceptual models that could show the essence of a problem [51], Ehrenfest raised the question of the distribution of P energy elements among the N resonators with the following model:

> On a rod, whose length is a multiple P of a given length ε, notches have been cut at distances ε, 2ε, etc. from one of the ends. At each of the notches, and only there, the rod may be broken, the separate pieces may subsequently be joined together in arbitrary numbers and in arbitrary order, the rods thus obtained not being distinguishable from each other otherwise than by a possible difference in length. The question is, in how many different manners the rod may be divided and the pieces distributed over a given number of boxes, to be distinguished from each other as the 1^{st}, 2^{nd},... Nth, when no box may contain more than one rod. If the boxes, which may be thought of as rectangular, are placed side by side in one line, they form together as it were an oblong drawer with (N-1) partitions, formed of two walls each, and these double partitions may be imagined to be mutually exchanged, the boxes themselves remaining where they are [52].

The relevance of all this game is that it guides us to think about the distinguishable nature of the N resonators and the undistinguishable nature of the P energy elements, thus to Ehrenfest, Planck's energy elements were just a formal tool for calculation. This did not happen with quanta conceptualized by Einstein, for whom there had to be N^P possible combinations when distributing each of the P energy elements in N resonators. If the energy quanta must have a physical reality, with Planck's counting those particles would have to present very odd properties, different from any particle considered previously in physics [53]. Ehrenfest recapitulates his speculations saying:

> Einstein's hypothesis leads necessarily to formula (...) for the entropy and thus necessarily to Wien's radiation formula, not Plank's. Plank's *formal device* (distribution of P energy elements ε over N resonators) *cannot be interpreted in the sense of Einsteins's light quanta* [54].

To close this section, and supported by Ehrenfest's analysis, we can assume that, subject to a necessity of theoretically justifying his results, Planck followed strategies that were not entirely convincing for everyone, but that allowed him to set the basis for a whole new interpretation of the physical world even when he was not aware of doing so. Among these strategies is his way of counting, which could have been differently developed by distinguishing, for example, between oscillators or quanta; but it was evidently conducted in the way he knew that it would lead him to the right spectral form, reason enough not to question much more his procedures.

3. The necessity of quanta and Ehrenfest's criticism

The derivation of Planck's law of black body radiation using classical electrodynamics, but at the same time introducing a discontinuity, revealed an inconsistency [55]. Ehrenfest had highlighted the inability of Planck's oscillators, which he used to represent matter, to get radiation to a state of thermal equilibrium, due to the linearity of the equations involved in this model (both Maxwell's equations and those that describe the motion of oscillators). On the other hand, Planck himself tried to limit the role of quantum hypothesis reserving the quantization of energy values to the oscillators and pretending that classical principles could be preserved [56]. This state of things placed the necessity of quanta at a somewhat uncertain level, an issue that Ehrenfest decided to revise and which led to an article in 1911.

Einstein, with his more radical conception of light quantum, suggested that it played a role in other phenomena in which light is created or transformed [57]. In the years after his 1905 publication, the theoretical and experimental results regarding these and other physical problems (for example, the subject of specific heats) began to convince the scientific community that the quantum hypothesis would play a very important role in the future development of physics. Walther Nernst (1864-1941) was one of the most enthusiastic advocates of the new theory and convened a conference (the first Solvay conference) in 1911, shortly after the publication of the article by Ehrenfest, which sought to define the new directions that physics should follow.

3.1. Ehrenfest's 1911 article

In 1911, a year before arriving to Leiden to take on Lorentz's position and still living in St. Petersburg [58], Ehrenfest wrote an article that represented a substantial contribution to the clarification of the status of quantum theory. He critically reviewed the role played by the light quanta in thermic radiation theory, trying to obtain clues that lead to the quantization of other systems, beside the Planckian oscillators [59]. Planck's derivation of his radiation law included apparently arbitrary elements that put in question the need of quantization. As indicated in the title he gave to his 1911 article, what Ehrenfest aimed to find out was: *Welche Züge der Lichtquantenhypothese spielen in der Theorie der Wärmestrahlung eine wesentliche Rolle?* (Which features of the hypothesis of light quanta play an essential role in the theory of thermal radiation?) [60]. The relevance of this work has been pointed out and its contribution analyzed by other authors [61], thus we will just briefly highlight some of its main ideas that result significant for the purposes of the present paper.

Ehrenfest's concern regarding Planck's work, since his early articles in 1905 and 1906, was to clarify the way in which the combination of electromagnetism, statistical mechanics, and the new quantum hypothesis fit together and lead to Planck's radiation law. How could one put together all this elements? In Sec. 2.2 we consider the way Ehrenfest tried to follow Planck's ideas, although applying the 'complexions' theory directly to the field (and not to the resonators) using the equipartition principle to normal vibration modes. Proceeding in such way, he was conducted to the Rayleigh-Jeans law. By asking how Planck's theory prevented the infinite growth of the distribution function for high frequencies [62], he realized that it was due to the introduction of the quantization of oscillators. In his new article (the one written in 1911), Ehrenfest attempted to conduct his analysis combining three different elements: the properties that must fulfill the function of heat radiation, the analysis of the normal modes of oscillation according to electromagnetic theory, and the probabilistic approach.

In Sec. 1 of his article, Ehrenfest summarizes the asymptotic properties of black body radiation [63]. According to Wien's displacement law, the spectral distribution function must meet $K(T,\nu) = \nu^3 f(\nu/T)$. This law, however, does not specify the form of the function $f(\nu/T)$, that will vary according to the applied additional considerations [64]. It is, in fact, the restrictions for the function $f(\nu/T)$ what Ehrenfest tries to identify. He refers, among other issues, to the 'red demand' (low frequencies), where $f(\nu/T)$ must match the Rayleigh-Jeans law, the 'violet demand', where $f(\nu/T)$ must be such that the energy remains finite, and the 'augmented red demand', where $f(\nu/T)$ must match Wien's law, which prevents the energy to grow endlessly, and also conforms perfectly with radiation experimental measurements.

In the next section of his article, a certainly brief one, Ehrenfest presents an expression demonstrated by Planck on the density of normal modes of vibration in a cube of lenght l [65]. He also refers to an adiabatic process, although he does not provide this name yet, which will play a very significant role in his further thought on quantum hypothesis, so it is relevant to present it in his own words:

> If we push the sides of the cube, bringing them together, slowly making it smaller, in the same way (at the expenses of the work done against the radiation pressure) the partial energy of all natural oscillations will increase proportionally to the frequency ν (...) [66]

That is, a given frequency ν moves to another frequency ν' and its energy passes from a value E_ν to a value $E'_{\nu'}$, so that the ratio of energy to frequency remains unchanged (E/ν is an adiabatic invariant). He subsequently uses this relation to derivate Wien's displacement law.

Regarding the probabilistic approach, Ehrenfest looks for a clarification of the theory questioning the equipartition principle, by attributing a weight function to the phase space of the normal modes of oscillation (probabilistic density function $\gamma(\nu, E)$ depending on frequency and energy), a point of view overlooked by Planck, since he was not fully aware that his theory implied this departure from the equipartition principle [67]. Ehrenfest's purpose was to find a formulation for that weight function, according to the previously presented background, that is, the theoretical and experimental properties of black body radiation. His arguments are too technical, nevertheless what he developed and found could be briefly stated as follows: He applied the weight function to the basic phase space, and followed Boltzmann's methods [67] more rigorously than Planck, which implied calculating the maximum for the function lnW that represents entropy, and therefore finding Boltzmann's distribution, although modified by its weight factor $\gamma(\nu, E)$. Then he had to compare that expression an make it match with the asymptotic properties of black body radiation, and considering the fact that entropy would not change while going through a process such as the compression of the region containing the radiation, as mentioned in the previous paragraph. Thereby, the weight factor must take the form $\gamma(\nu, E) = Q(\nu)G(E/\nu)$, which could be proven to be equivalent to Wien's displacement law [69]. Ehrenfest also demonstrated that the 'violet demand' could not be satisfied by a continuous domain for G(q). From the generalization of these analyses, Ehrenfest considered discrete weight functions G(q) and concludes after them that in order to prevent the 'ultraviolet catastrophe', it is necessary that each mode of frequency ν requires a finite amount of energy proportional to its frequency, to be set in vibration. Thus, Planck's quantization was established as necessary and sufficient condition for Planck's radiation distribution [70]. In the last section of his article, Ehrenfest highlights these ideas, as a conclusion, stating that "a fast enough decrease in the radiation curve for values of ν that grow infinitely can only be achieved if the resonators present some sort of 'excitation threshold' of proportional value to the frequency of the resonator" [71].

The set of concepts Ehrenfest dealt with in this article represent the basis for the further formulation of his adiabatic hypothesis (whose discussion cannot be addressed in the present paper) which, along with Niels Bohr's correspondence principle, worked as the heuristic principle guiding new physicists in the new quantum physics unraveling process [72].

3.2. An unfortunate circumstance: Solvay conference, Poincaré and Ehrenfest

In the first paragraph of his 1911 article, Ehrenfest refered to the fact that quantum hypothesis had been implemented "to a fast-growing circle of questions that have only a vague connection with the problem of heat radiation" [73], and suggested that the fate of this hypothesis would be decided with the experimental results that were emerging in the new areas of application. The situation in that year was already very different to that of 1905 and 1906, when Ehrenfest first published on the subject, when other than as a mysterious universal constant which characterized all major laws of radiation cavity, quantum was not just in the professional conscience [74]. By mid-1910, there were already many 'converts', Walther Nernst, a professor and director of the Institute of Physical Chemistry at the University of Berlin, standing out among them. Nernst's position was rather pragmatic, but also emphatic, as shown by a lecture he gave that year in which he said:

> At this time, the quantum theory is essentially a computational rule; one may well say a rule with most curious, indeed grotesque, properties. However (...) it has borne such rich fruits in the hands of Planck and Einstein that there is now a scientific obligation to take a stand in its regard and to subject it to experimental test [75].

Nernst himself assumed the responsibility of gathering the more prominent theoretical and experimental physicists better aware of the quantum hypothesis, so that the scientific community defined the new paths to follow, given the moment of uncertainty imposed by the new developments. The meeting was possible thanks to the financial support from the Belgian chemist, and successful businessman, Ernst Solvay. This meeting, known as the first Solvay Conference, was held from October 30 to November 3, 1911, in Brussels. The subject was Radiation and the Quanta, and among the participants who attended by personal invitation, were A. Einstein, M. Planck, H.A. Lorentz, M. Curie, H. Poincaré, E. Rutherford, etc. In the opening conference, Lorentz, who had been given the chairmanship of the event, referred to "the old theories [that] have been proven increasingly powerless to penetrate the darkness around us everywhere" and stated that "in this stage of affairs, the beautiful hypothesis of energy elements (...) has been a wonderful ray of light" [76].

Ehrenfest was not invited to the first Solvay Conference. He had finished his article in July of that same year and it was not published until October, just before the Conference. However, such work was not even taken into account in the discussions that took place in Brussels [77], in spite of the fact that in such work Ehrenfest had clarified several issues that were discussed there, or at least that is what can be deduced from the memories and reports on different topics that were used in the Conference, which make no mention of it [78]. In addition, the fact that Ehrenfest was geographically isolated and did not enjoy of much scientific reputation yet [79], contributed to the situation. Although the latter may be questionable, especially if we consider the invitation Lorentz extended to him a few months later to take his place in Leiden, which speaks of the recognition received from one of the most respected physicists of the time.

During the Conference, several issues were discussed. The secretaries Paul Langevin and Maurice de Broglie were responsible for the publication of the reports and discussions, which included the conferences and debates transcripts [80]. Because the discontinuity implied in the quantum hypothesis suggested the need for a revolution in the available concepts of physics, there was an attempt to analyze during the Conference, among other things, if the quantum effects could be explained by some strange mechanism, but still within the classical principles [81]. However, most of the participants completed the meeting convinced that some of the fundamental principles of the classical description of nature were in danger. The conferences had achieved its goal of showing the full extent of the quantum problem to the experts and persuade them to cooperate more closely to the future development of quantum theory.

One of the participants who arrived to the event with a little knowledge on the subject, but nevertheless had a very active participation and at the end of the meeting was taken up by the issue of quantum hypothesis, was the very well-known and respected French scientist Henri Poincaré [82]. Immediately after the Conference, he dedicated himself to analyze the question of whether quantum theory could be formulated or not in terms of differential equations, leading to the conclusion that any theory from which Planck's law could be derived would necessarily contain an essential discontinuity [83] and proving that it is not possible to explain the quantum phenomenon using a classical description [84]. Poincaré presented his findings to the Academy of Sciences on December 4, 1911 and later in an article that appeared in January 1912 in the Journal de physique [85]. In that article, Poincaré demonstrated what Ehrenfest had already shown some months before [86]. However, given the scientific reputation of Poincaré, his arguments convinced more people to accept the necessity of the quantum hypothesis, including skeptics like the British Jeans [87].

This situation was devastating for Ehrenfest. Given the fact that he had not been able to get a permanent job in Russia, in January 1912 he started traveling to different cities visiting his fellow scientists in order to explore the possibility of finding an academic position. While in Leipzig, he heard from the presentation of the first findings of Poincaré in December 1911. "What will become of me?" he wrote in his diary on January 13 [88]. Subsequently, while he was still travelling, he read the most extensive paper of Poincaré, that is, the one that appeared in January 1912. Evidently, Poincaré did not know Ehrenfest's article until the latter sent him a copy of his work. Despite the priority of Ehrenfest, it was Poincaré who received all the credit for demonstrating the adequacy and necessity of the quantum hypothesis for the derivation of the radiation law. By the time Poincaré received Ehrenfest's article, he said that he was pleased to find that someone else had reached the same results by following other paths [89], but he had no time, if any was intended, to give Ehrenfest public credit, because he got sick and died on July 17. Thus Ehrenfest did not receive any credit for his work. This episode has been summarized very well by Klein, who said that

> (...) Poincaré's paper deeply influenced the attitudes of his contemporaries. Whereas Ehrenfest could be ignored and Einstein not accepted, Poincaré's authority could hardly be questioned. His arguments were accepted as the proof that discontinuity in the energy was absolutely necessary for the existence of a finite energy in the black-body radiation [90].

After these episodes, the evidence supporting the quantum hypothesis continued to accumulate. For example, in May 1914 Einstein wrote to Ehrenfest letting him know that

> [James] Franck and [Gustav] Hertz have discovered that electrons will be reflected elastically from mercury atoms, as long as they have velocities [he meant kinetic energies] of up to 4.8 volts. At the latter velocity they lose their entire kinetic energy [when colliding with atoms] and emit monochromatic light, such that the relation, kinetic energy = $h\nu$, is valid to within a few percent (...) wonderful reversal of the photoelectric phenomenon (...) brilliant confirmation of the quantum hypothesis [91].

This kind of experiments, that confirmed Bohr's theory of the atomic structure based on quantum theory, opened a whole new era for physics.

4. Conclusion

The black body radiation's spectral distribution law, presented by Planck in the very beginning of the 20th century, was considered correct among the scientific community, due to its accordance with experimental observations. Nevertheless, its theoretical demonstration was found unclear to his contemporaries, especially because it introduced statistical elements, following a similar approach to that used in kinetic theory of gases but without providing any support for the validity of the extension or extrapolation of concepts implied in doing so. In fact, even when those statistical methods were applied to the study of gases safeguarding classical mechanics, classical electromagnetism could not be preserved in the same way when such methods were applied to the study of radiation. The introduction of energy quanta by Planck was merely a formal approach, due to the profound unawareness of the interaction among matter and radiation. Introducing the quantum postulate along with other *ad hoc* assumptions made his work rather an exercise of invention, even though he pretended to conduct his works attached to classical physics principles. Yet, it represented the first step in setting new fundamental principles that would generate a new science.

We consider the black body problem as an illustrative example of the way an anomaly serves as the seed that, under certain circumstances, can germinate and lead to scientific innovation and to a new paradigm. As we attempted to demonstrate in this paper, the history of the black body radiation problem shows the simultaneous presence of different physical conceptions, including some in clear confrontation with others. Those conceptions coexisted and interacted in such a way that a new vision emerged from that struggle. The new paradigms arise through certain continuity of ideas, thus the historical analysis presents Planck not as a revolutionary character, but rather as a transitional one who, while not aiming to do so, opened the gate for modern physics. Our interpretation probably diverges in some degree from Kuhn's more linear analysis, as it states that there was not a unique paradigm in mature 19th century physics orienting 'normal science' development, but an amalgam of competing paradigms (Mechanism, Field Theory, Atomism, Phenomenalism) that, in its articulation, and with the audacity of some scientists to take risks and imagine other possibilities, generated new conceptions that gradually proved to be very promising. In this sense, the study of this specific case exemplifies Gerald Holton's ideas as the one presented in his following statement:

> we can understand why scientists need not hold substantially the same set of beliefs to communicate meaningfully with one another, in either agreement or disagreement, while they contribute to the cumulative, generally evolutionary improvement of the state of science (...) [92]

But it must also be recognized that this process, in which scientists take risks and appeal to their imagination attempting to untie the knot in which their science is, demands to be restrained by critical analysis to prevent its rambling into fantasy. Paul Ehrenfest stands as a significant figure playing this critical role in the quantum revolution case. For years he sought for the meaning of the quantum hypotheses introduced by Planck and Einstein, pondering the differences, assumptions and implications these hypotheses supposed. Although he probably lacked the recognition he deserved in his time, his work offered clues to understand the new concepts and suggested new interrogations to be posed in his discipline. Particularly his 1911 article appeared in a crucial moment, when important decisions had to be made regarding the orientation physics should follow when the quantum concept began to prove fruitful, and it demanded to be reformulated in broader general terms so it could be applied to different situations besides Planck's simple harmonic resonators. The concept of adiabatic invariants that Ehrenfest began to work in 1911 would have further significant consequences and would contribute in the settlement of novel principles for the new physics.

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1. G. Holton, *The advancement of Science, and its Burdens* (Cambridge, Harvard University Press, 1998), p. 230.
2. P. Ehrenfest and T. Ehrenfest, *The Conceptual Foundations of the Statistical Approach in Mechanics* (New York, Dover Publications Inc., 1959), pp. 67-70. The *Encyklopädie* was a project led by Felix Klein, Franz Meyer and Heinrich Weber whose aim was to present the achievements, methods and main applications of the mathematical sciences of the nineteenth century, in order to offer a picture of the place they had in the culture of the time. Paul Ehrenfest and his wife were given the task of writing an article on the principles of statistical mechanics, which became an invaluable reference for anyone interested in the subject.
3. Although he had been working on the subject for several years, his first formal publication came out during this year.
4. T.S. Kuhn, *The Structure of Scientific Revolutions* (Chicago, University of Chicago Press, 1962/1970).
5. T.S. Kuhn, *Black Body Theory and the Quantum Discontinuity 1894-1912* (Chicago, University of Chicago Press, 1987). Some authors have pointed out an apparent discrepancy between the concepts developed in The Structure of Scientific Revolutions and its application in this case study carried out by Kuhn, where his ideas seem more hidden. We remain on the sideline at this point, but more information is available in W. Sharrock and R. Read, *Kuhn: Philosopher of Scientific Revolution* (Malden, Blackwell Publishers, Inc., 2002) and in A. Roy, *Theory and Science* **3** (2002) 1.
6. P. Stehle, *Order, Chaos, Order: The Transition from Classical to Quantum Physics* (New York, Oxford University Press, 1994), p. 54.
7. P. Ehrenfest, *Collected Scientific Papers* (M.J. Klein, editor, Amsterdam, North-Holland Publishing Company, 1959), p. 88.
8. This idea emerges from the application of analogy and metaphor, for as Thomas Young himself expressed: "it [the idea that light is the propagation of an impulse in ether] is strongly confirmed by the analogy between the colors of a thin plate and the sounds of a series of organ pipes". The quotation is taken and explained fully in G. Holton (1998), op. cit., pp. 232-234.

9. Most of the secondary sources that mention the genesis of quantum physics start talking about Planck, but in S. Stehle (1994), op. cit., pp. 47-54 we can find a more or less extensive discussion on these Kirchhoff's theoretical considerations.

10. This feature of universality was the one that attracted Planck's attention in his search for "absolutes", which is what the goal of a unified image of the world meant for him.

11. Black body radiation then took different names. Planck called it "natural radiation". It was also called "cavity radiation", because if there is electromagnetic radiation in a cavity and it is allowed to reach a state of equilibrium with its own walls, the spectrum of this radiation will correspond to the black body's, and this is the way in which that result was intended to be recreated experimentally. S. Sthele (1994), op. cit., explains in detail what would have been Kirchhoff's indications to create an ideal black body: "One should form a cavity in a good conductor of heat and drill a small hole in the wall. The walls of the cavity are maintained at the desired temperature. The interior of the cavity should be shaped so that any radiation entering the hole is necessarily reflected away from the entrance so that it will have to undergo many reflections, each with the possibility of some being absorbed, before it can possibly reach the hole and escape from the cavity. Such a source, if well designed and constructed and with a well-controlled temperature, could be taken to be a source of blackbody radiation. It was necessarily large and massive. It had to be used in an environment much cooler than itself so that any radiation present assuredly came from the blackbody and not from something else in the neighborhood" (p. 115).

12. Some experimental physicists who had a very important role in this story were Paschen, Lummer, Pringsheim and Rubens, among others. The connections between theoretical and experimental aspects related to black body radiation are well explained in E. Segré, *From X-rays to Quarks: Modern Physicists and their Discoveries* (Mineola, New York, Dover publications, 2007), pp. 61-77.

13. Boltzmann's theoretical work had the vision to assume that electromagnetic radiation in a cavity behaves like an ideal gas in equilibrium. R.D. Purrington, *Physics in the Nineteenth Century* (London, Rutgers University Press, 1997), p. 152). Planck's continuity on this analogy would be crucial in the genesis of quantum discontinuity.

14. H.W. De Regt, *Philosophica* **58** (1996) 125.

15. M. Planck, *Scientific autobiography and other papers* (New York, Philosophical Library, 1949), p. 13.

16. M. Planck, *Nobel Lecture: The Genesis and Present State of Development of Quantum Theory* (1920). This is the lecture he gave when he received the Nobel Prize "in recognition of the services he rendered to the advancement of Physicis by his discovery of energy quanta" http://nobelprize.org/nobel_prizes/physics/laureates/1918/.

17. The two articles published by Ehrenfest in 1905 and 1906 to which we will refer in this section are entitled *Über die Physikalische Voraussetzungen der Theorie der irreversibility Planckschen Strahlungsvorgänge* (On the Physical Assumptions of Planck's Radiation Theory of Irreversible Processes) presented at the Wiener Berichte 114, and *Zur Planckschen Strahlungstheorie* (On Planck's Radiation Theory) presented in the Physikalische Zeitschrift 7. Both are located in P. Ehrenfest (1959), op. cit. pp. 88-101 and 120-124.

18. The literature on the subject is obviously very broad. Here we will give only a brief presentation in order to generate the context where we find the Ehrenfest's analysis and criticism. For more depth, you can see, for example, J.M. Sánchez Ron, *Historia de la Física Cuántica* (Barcelona, Editorial Crítica, 2001); P. Sthele (1994), op. cit.; J. Mehra and H. Rechenberg, *The Historical Development of Quantum Theory* (New York, Springer-Verlag, 1982); C. Jungnickel and R. McCormmach, *Intellectual Mastery of Nature* (Chicago, The University of Chicago Press, 1990); S.G. Brush, *The Kind of Motion We Call Heat* (Amsterdam, North Holland Personal Library, 1994); etc.

19. T.S. Kuhn (1987), op. cit., p. 12.

20. M. Planck (1920), op. cit.

21. "Natural radiation" is introduced as a constraint on the theory, assuming that it is the only type of radiation that can be found in nature. A randomness criterion, with a lack of correlation between different harmonic radiation, is also assumed.

22. Interestingly, if only he had used the quadratic term, it would have led him to the Rayleigh's law, which unlike that of Wien, performs well for low frequencies and fails for high ones, *i.e.*, Planck's procedure is a simple interpolation between the formulas of Rayleigh and Wien. Planck himself refers to this part of his analysis as follows: "a happily chosen interpolation formula". M. Planck (1920), op. cit.

23. It was Boltzmann himself who made him see the need to use probabilistic arguments and show him that the equations of Maxwell (like mechanical laws) were symmetrical in time and that a unidirectional process in time (irreversible) could not be derived from them. Boltzmann had undergone the same process with Loschmidt's criticism on his attempt to base in mechanical terms the laws of thermodynamics.

24. M. Planck, *Annalen der Physik* **4** (1901) 553.

25. *e.g.*, molecule 1 has 3ε of energy, molecule 2 has 5ε of energy, etc.

26. *e.g.*, N_1 molecules have ε energy, N_2 molecules have 2ε energy, N_3 molecules have 3ε energy, etc.

27. In this formula, the number of 'complexions' R that correspond to a gas state is a measure of the probability W, and to Planck, the expression $S_N = k \log W$ indeed "serves as a definition of the probability W, since in the basic assumptions of electromagnetic theory there is no definite evidence for such a probability". On the other hand, if the probability W is proportional to the number of 'complexions' R it means that all the 'complexions' are equally likely, which Planck admits has no physical justification and "whether this actually occurs in nature one can, in the last analysis, prove only by experience". L. Navarro and E. Pérez, *Dynamis* **22** (2002) 386, rescue these Planck's quotes in order to emphasize that he was fully aware of the arbitrariness of his notion of probability.

28. According to T.S. Kuhn (1987), op. cit., p. 170, Einstein and Ehrenfest noticed that Planck's procedure certainly required an energy discontinuity, while Planck himself did not notice it until several years later. Yet in his *Vorlesungen über die Theorie der Wärmestrahlung* (Lectures on the Theory of Heat Radiation), published in the spring of 1906, he made no explicit mention of a discontinuity, although he did refer to the constant h as

the element or unit of action. M.J. Klein, *Paul Ehrenfest: The Making of a Theoretical Physicist* (Amsterdam, North-Holland Physics Publishing, 1985), p. 238.

29. Lorentz had been the first one to submit written views on the work of Planck, in which he termed the finite 'units of energy' as an essential part of the theory. Ibid., p. 232. Ehrenfest, incidentally, got to be Lorentz's successor in his chair at Leiden, many years later, in 1912.
30. P. Ehrenfest (1959), op. cit., p. 88.
31. Ibid., p. 90.
32. Ibid., p. 91.
33. Ibid., p. 96.
34. Ibid., p. 100. M.J. Klein (1985), op. cit., p. 234 says that in this phrase Ehrenfest is misinterpreting Planck's thinking, as Planck's discretization, in any case, related to the energy of oscillators and in no way to free radiation. In the same vein, T.S. Kuhn (1987), op. cit., p. 143 believes that Ehrenfest, just like Einstein, contributed at different times (perhaps this is one of them) to "prepare the way for a new attitude towards the significance of Planck's work". In other words, Planck may have been responsible for the emergence of a belief among his colleagues that he did not share.
35. P. Ehrenfest (1959), op. cit., p. 100.
36. Ibid., p. 121.
37. T.S. Kuhn (1987), op. cit., pp. 157-158 explains that this model had been worked at least five months before, giving evidence of this by means of notes that Ehrenfest had in his workbooks.
38. P. Ehrenfest (1959), op. cit., p. 122.
39. T.S. Kuhn (1987), op. cit., p. 168 characterizes it as an application of the theory of 'complexion' (combinatorial) directly to the field.
40. The problem here is that the number of molecules is finite, while the number of possible frequencies of vibration is not limited. If we imagine a cube (the cube of Jeans) and vibration along one of its sides, stationary vibrations containing multiples of half the wavelength can be accommodated to infinity. G. Gamow, *Thirty Years that Shook Physics* (New York, Dover Publications, 1966/1985), pp. 12-15.
41. P. Ehrenfest (1959), op. cit., p. 124.
42. A. Pais, *Subtle is the Lord: The Science and the Life of Albert Einstein* (Oxford and New York, Oxford University Press, 1982), p. 374 says that "from 1900 to 1905, Plank's radiation formula was generally considered to be neither more nor less than a successful representation of the data". We may add that the agreement was maintained throughout the century in different scenarios and different ranges of temperatures and frequencies. For example, in relatively recent times, in 1990, measurements of background radiation in space, presumably resulting from the Big Bang, and taken by a satellite called the *Cosmic Background Explorer* (COBE) show that this radiation follows Planck's law in a very precise way for very low temperatures, close to absolute 0.
43. A. Einstein, in *Albert Einstein: Philosopher-Scientist*, edited by P.A. Schilpp (New York, Library of Living Philosophers, MJF Books, 1970), p. 45.
44. Planck particularly refers to the decision of introducing the statistical approach in his demonstration as "an act of desperation (...) I had to obtain a positive result, under any circumstances and at whatever cost". Cited in A. Pais (1982), op. cit., p. 370.
45. The article is titled *Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt* (On a Heuristic Point of View Concerning the Production and Transformation of Light). Included in J. Stachel, *Einstein's Miraculous Year: Five Papers that Changed the Face of Physics* (Princeton and Oxford, Princeton University Press, 2005), pp. 177-198. For his interpretation of the photoelectric effect presented in this work Einstein was awarded with Nobel prize in 1921.
46. Ibid., p. 177. The 'profound difference' lies in the fact that gases are considered to be formed by discrete particles, or molecules, while the field is continuous. For Einstein's view it represents an affront to the unity of nature. Einstein's unifying program stands out especially in his 1905 main works, as in his famous article *Zur Elektrodynamik Bewegter Körper* (*On the Electrodynamics of Moving Bodies*), which gave rise to the special theory of relativity. There, he stated that "when Maxwell's electrodynamics, as it is now generally understood, is applied to bodies in motion, it leads to asymmetries that does not seem inherent to phenomena", so he aimed to suppress these asymmetries by starting from new assumptions that would prove the concept of luminous ether to be superfluous. The general structure of his articles is as follows: to demonstrate the unfinished status of the theories accepted by the time, to propose a new theory, and finally to suggest the experimental situations that would validate the ideas just introduced.
47. Ibid., p. 178.
48. T.S. Kuhn (1987), op. cit., clarifies on this by stating that "for Planck's problem, unlike Boltzmann's, (...) any set of the wk's that satisfy these constraints [that the sum of the wk's equals N and the sum of the kwk's equals P] corresponds *to the same distribution* of the total energy" (p. 105). That is so, explains Kuhn, because Planck "assumes from the beginning that he is dealing with resonators already in equilibrium with the radiation field", therefore "there is no chance for new maximizations" (p. 108); hence he calculates considering single frequency resonators.
49. M.J. Klein (1985), op. cit., p. 255.
50. Originally published in *Proc. Academ. Amsterdam* **17** (1914) 870. Included in P. Ehrenfest (1959), op. cit. pp. 353-356.
51. That is the way in which Ehrenfest is described by one of his disciples, H. Casimir, in the Introduction of his *Collected Scientific Papers*. Ibid., pp. XI-XII.
52. Ibid., p. 355, note 1.
53. A. Pais (1982), op. cit., considers that this way of counting indistinguishable objects, adopted by Planck, "prefigures the Bose-Einstein counting of a quarter century later [and] cannot be justified by any stretch of the classical imagination". That does not happened with Boltzmann's counting, because for him "the question was to determine the most probable way in which a fixed number of *distinguishable* gas molecules with fixed total energy are distributed over cells in phase space" (p. 371).
54. P. Ehrenfest (1959), op. cit., p. 356.
55. L. García-Colín, *La Naturaleza Estadística de la Teoría de los Cuantos* (México, El Colegio Nacional, 2004), p. 66.

56. In the sixth from a series of eight lectures he dictated in 1909 in New York, Planck said: "I am of the opinion (...) that at present it is not necessary to proceed in so revolutionary a manner, and that one may come successfully through by seeking the significance of the energy quantum $h\nu$ solely in the mutual actions with which the resonators influence one another". M. Planck, *Eight Lectures on Theoretical Physics* (New York, Dover Publications, 1915/1998), pp. 95-96.

57. In his 1905 article, Einstein mentioned particularly the photoelectric effect (in which the incident ultraviolet radiation energy is used to release electrons in a metal and provide them with kinetic energy) and fluorescence (which consisted in the absorption of short wavelengths light by some substances, that later reemit longer wavelengths).

58. Ehrenfest and his wife moved from Göttingen to St. Petersburg in 1907 and lived there for 5 years. There he started to organize weekly informal meetings with local physicists, a practice he continued successfully in Leiden beginning in 1912 ('Ehrenfest colloquium'). In this Russian city he met one of his closest friends, Abram Fedorovich Joffe, whom would share his personal interests in Physics. Regarding Ehrenfest settlement in St. Petersburg and his scientific relationships developed there see M.J. Klein (1985), op. cit., pp. 83-93 and P. Huijnen and A. J. Kox, *Physics in Perspective* **9** (2007) 186. Both Ehrenest and Joffe focused, among other issues, on revising the status of the quantum hypothesis and justifying its need to match experimental data.

59. M.J. Klein (1985), op. cit., p. 12.

60. Originally published in the *Annalen der Physik*, **36** (1911) 91. Included in P. Ehrenfest (1959), op. cit., pp. 185-212.

61. M.J. Klein (1985) op. cit., pp. 245-254 and L. Navarro and E. Pérez, *Archives for History of Exact Sciences* **58** (2004) 97.

62. In his 1911 article, Ehrenfest coined the expression 'ultraviolet catastrophe', widely adopted later within the scientific community, for this unlimited growth of the distribution function. The expression clearly exemplifies the sense of drama that urged for a reformulation of physical science' principles.

63. As mentioned in Sec. 2.1, Rayleigh-Jeans law matched the experimental results for low frequencies, while Wien's law matched the ones for high frequencies.

64. *e.g.*, Wien's law, which has an heuristic nature, and mentioned in previous note (63), uses a decreasing exponential form for the function f(ν/T).

65. See note 40. This density depends on ν^2 since a higher frequency implies a higher number of normal oscillations for a given range of frequencies. Hence, in order to prevent the 'ultraviolet catastrophe', Ehrenfest assigned a probabilistic weight factor.

66. P. Ehrenfest (1959), op. cit., p. 188.

67. P. Stehle (1994), op. cit., p. 124, comments indeed, that it is certainly too strange that Planck did not consider the implications of the equipartition of energy principle for the black body radiation while adopting Boltzmann's methods; though, the omission proved successful, since its consideration would have interrupted or spoiled his argumentation.

68. Following the analogy, of equating the number of the different frequencies' normal modes of oscillation to the number of atoms in a gas.

69. Q(ν) is an irrelevant factor, since it does not have any influence in the spectral energy distribution.

70. In other words, revisiting the question he posed in 1906, that is, how Planck's theory prevents the infinite growth of the distribution function for high frequencies; the answer achieves now a greater mathematical rigor by introducing the weight function: at extremely high frequencies the probability for the energy to distribute in the corresponding normal modes diminishes. L. Navarro and E. Pérez (2004), op. cit., p. 114.

71. P. Ehrenfest (1959), op. cit., p. 204.

72. B.L. Waerden, *Sources of Quantum Mechanics* (New York, Dover Publications, 1968), p. 4.

73. P. Ehrenfest (1959), op. cit., p. 185.

74. T.S. Kuhn (1987), op. cit., pp. 228-229.

75. Cited in A. Pais (1982), op. cit., p. 399.

76. Cited in J.M. Sánchez Ron (2001), op. cit., p. 204.

77. Considering the proximity in dates, it is almost certain that most participants were unaware of Ehrenfest's work or had no opportunity of considering its relevance regarding the Conference subject. It is known, however, that Einstein had read the article since he refers to it in a letter to his friend M. Besso dated October 21st, 1911. A. Einstein, *Correspondencia con Michele Besso: 1903-1955 / Albert Einstein* (Barcelona, Tusquets Editores, 1994), p. 96.

78. M.S. Klein (1985), op. cit., p. 251.

79. R. McCormmach, *Isis* **58** (1967) 1. refers to Ehrenfest indicating that during the Solvay Conference "Ehrenfest was a relatively unknown physicist living in Russia" (p. 51).

80. J. M. Sánchez Ron (2001), op. cit., p. 204.

81. J. Mehra and H. Rechenberg (1982), op. cit., p. 135.

82. M.J. Klein (1985), op. cit., mentions that "Poincaré came to Brussels ignorant of the quantum theory" (p. 252), while J.M. Sánchez Ron (2001), op. cit., says that through the analysis of the Conference's acts "it is evident that he had minimum knowledge of the quantum world" (p. 206).

83. J. Mehra and H. Rechenberg (1982), op. cit., p. 152.

84. J.M. Sánchez Ron (2001), op. cit., correctly highlights that "strictly, it is not completely true Poincarés association between quantum discontinuity and the validity of differential equations". In fact, it is now known that Schrödinger equation describing quantum phenomena is indeed a differential equation in which "quantum discontinuity follows from other theoretical elements: Born's probabilistic interpretation of the wave function, and from postulates such as the wave function collapse" (p. 210).

85. R. McCormmach (1967), op. cit., pp. 43-50 offers a description of Poincaré's article.

86. L. Navarro and E. Pérez (2004), op. cit., p. 136, though, warm against the inaccuracy of considering Ehrenfest and PoincarÃ© reaching the same results independently, since their approaches and initial assumptions were very different. They consider the rigorous analysis of those differences as an issue yet to be done.

87. Jeans, in fact, firmly opposed quantum theory during the Solvay Conference. R. McCormmach (1967) op. cit., p. 53 states that his 'conversion' was, with no doubt, due to the reading of Poincaré's article.

88. Cited in M.J. Klein (1985), op. cit., p. 174.

89. R. McCormmach (1967), op. cit., p. 51.

90. M.J. Klein (1985) op. cit., p. 253.

91. Einstein to Ehrenfest, May 25th, 1914, cited in J. Mehra and H. Rechenberg (1982), op. cit., p. 153.

92. G. Holton, *Victory and Vexation in Science: Einstein, Bohr, Heisenberg, and Others* (Cambridge, Harvard University Press, 2005), p. 145.