Enhance the coherence of open two-level system through the superposition of environments

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Dynamics of a two-level system in the superposition of two dephasing environments with Ohmic-like spectral density is studied when considering initial system-environment correlations. The quantum system and one environment are treated as whole thermal equilibrium state, while the other environment is at thermal equilibrium state alone. Which environment the system interacts with is determined by an ancillary two-level system. When the system interacts with mixture of two sub-Ohmic environments, initial correlations can make the mixed dynamics non-Markovian. For two identical sub-Ohmic environments, if performing the projective measurement on the ancillary two-level system at the special time points, whatever the initial state of the system is, the coherence can be enhanced. For two different environments with $\beta\hbar\omega_0/2 \gg 1$, we get the approximate expression about the coherence of the system when measuring the ancillary two-level system.

Keywords: Spin-boson model; superposition of environments; initial system-environment correlations; decoherence.

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1. Introduction

Quantum coherence is a kind of important resource in the field of quantum information [1]. In the opinion of standard quantum mechanics, quantum system is inevitably affected by the environment around [2, 3], so the actual quantum system is open and decoherence is unavoidable. Quantum decoherence includes dephasing and dissipation. The practical implementation of quantum computation and communication needs to effectively resist the quantum decoherence. To work out the problem, many proposals have been put forward. For instance, Quantum error-correcting code tries to encode logic bits with far more qubits to increase reliability to resist noise, and correct errors of a certain number of logic bits [4–6]. Dynamical decoupling imposes a sequence of radio-frequency pulses to repetitively flip the state of the system to suppress the quantum decoherence [7, 8]. Moreover, By embing the qubits in the decoherence-free subspaces, the qubits can resist collective decoherence or dissipation [9–11]. Of course, based on decoherence, some researchers try to introduce other parameters to modify the Schrödinger equation such as Refs [12, 13], we do not consider those conditions.

In traditional quantum Shannon theory, the systems carrying information are in quantum state, while the trajectory of the systems is classical [14, 15]. However, the information carriers can also pass through multiple trajectories simultaneously, such as double slit experiment [16]. Thus, the quantum particle can go through the coherent superpositions of alternative evolutions [17, 18]. Especially, by means of superposition of trajectories, we can create the indefinite causal order [19–21], which is called quantum switch. In addition, by use of beam splitter [20, 22], two coupled cavities [23] or double-well system [24, 25], we can set the particle in the superposition of two environments. That can be achieved in experiment.

As for the dynamics of open system, usually we need to couple the open system to the environment, then acquire the reduced dynamics of composite system. If the system and environment are weakly interacting, initial system-environment correlations can be neglected. However, in the strong system-environment coupling regime, the initial correlations must be considered [26]. By superposing trajectories of the system, we can enhance quantum communication [20, 27]. Further, we research the effect of initial correlations on the dynamics of open quantum system in the superposition of environments, which is useful in quantum computation and quantum memory.

The paper is structured as follows. In Sec. 2, we construct the interaction model of open two-level system with superposition of two dephasing environments, and derive the density matrix of open two-level system and ancillary system over time. In Sec. 3, we find the mixing-induced quantum non-Markovian effect in the model before measuring the ancillary system, the cause is the initial correlations between open system and one of two environments. In Sec. 4, we get the expression about the normalized coherence of open system after measuring the ancillary system in the fixed bases. In Sec. 5 and Sec. 6, we discuss the situation that two environments are identical and different respectively. In Sec. 7, we draw the conclusion.

2. The dynamics with initial correlations

2.1. Correlated initial states

The spin-boson model describes the interaction between a two-level system S with the environment E_j of harmonic oscillators [3, 28, 29]. The Hamiltonian of composite system is

$$\begin{aligned} H_j &= H_S + H_{E_j} + H_{SE_j} \\ &= \frac{1}{2}\hbar\omega_0\sigma_z + \sum_k \hbar\omega_{j,k} a^\dagger_{j,k} a_{j,k} \\ &+ \sum_k \hbar\sigma_z (g^*_{j,k} a_{j,k} + g_{j,k} a^\dagger_{j,k}), \end{aligned} \tag{1}$$

where $a_{j,k}$ and $a^\dagger_{j,k}$ are the annihilation and creation operator of the kth oscillator with the angular frequency $\omega_{j,k}$, $\hbar\omega_0$ is the energy difference of the two-level system (basis states $|e\rangle$, $|g\rangle$). $g_{j,k}$ stands for the coupling strength, and σ_z is Pauli operator.

In the interaction picture, the effective Hamiltonian is

$$\begin{aligned} H^I_{SE_j}(t) &= e^{i(H_S+H_{E_j})t/\hbar} H_{SE_j} e^{-i(H_S+H_{E_j})t/\hbar} \\ &= \hbar\sigma_z \sum_k (g_{j,k} a^\dagger_{j,k} e^{i\omega_{j,k}t} + g^*_{j,k} a_{j,k} e^{-i\omega_{j,k}t}) \\ &= \sigma_z H_{j,e}(t). \end{aligned} \tag{2}$$

Then, $H^I_{SE_j}(t)$ can be decomposed into the direct sum of two operators

$$H^I_{SE_j}(t) = |e\rangle\langle e| \otimes H_{j,e}(t) - |g\rangle\langle g| \otimes H_{j,e}(t). \tag{3}$$

The time-evolution operator is

$$\begin{aligned} U_j(t) &= |e\rangle\langle e| \otimes U_{j,e}(t) + |g\rangle\langle g| \otimes U_{j,g}(t) \\ &= |e\rangle\langle e| \otimes \exp_+\left[-\frac{i}{\hbar}\int_0^t H_{j,e}(\tau)d\tau\right] \\ &+ |g\rangle\langle g| \otimes \exp_+\left[\frac{i}{\hbar}\int_0^t H_{j,e}(\tau)d\tau\right], \end{aligned} \tag{4}$$

where $\exp_+[\ldots]$ is the chronologically ordered exponent.

When the system S is located in the superposition of two dephasing environments E_0 and E_1, the ancillary two-level system A (basis states $|1\rangle$, $|0\rangle$) determines which environment interacts with the system S [27]. The whole Hamiltonian can be expressed in the following form

$$H = |0\rangle\langle 0| \otimes H_0 + |1\rangle\langle 1| \otimes H_1. \tag{5}$$

In the interaction picture, the time-evolution operator of the whole system is

$$U(t) = |0\rangle\langle 0| \otimes U_0(t) + |1\rangle\langle 1| \otimes U_1(t). \tag{6}$$

Before making the quantum state of the system, assume the system S and environment E_0 to be in the thermal equilibrium state at certain temperature T

$$\rho'_{SE}(0) = \frac{e^{-\beta H_0}}{\mathrm{Tr}(e^{-\beta H_0})}, \tag{7}$$

where $\beta = 1/k_B T$. When we prepare the the quantum state of the system in the environment E_0 at time $t = 0$, if considering the initial qubit-environment correlations [29] between system S and environment E_0, the initial quantum state of the environment E_0 is

$$\rho_{E_0}(0) = \frac{\langle\psi_s|e^{-\beta H_0}|\psi_s\rangle}{\mathrm{Tr}\langle\psi_s|e^{-\beta H_0}|\psi_s\rangle}. \tag{8}$$

Here, $|\psi_s\rangle = s_e|e\rangle + s_g|g\rangle$ is the initial normalized state of the system S. Then we set the system S in the superposition of environments E_0 and E_1. At time $t = 0$, there is no initial correlations between system S and environment E_1. Assuming environment E_1 to be in the thermal equilibrium state at the same temperature T, the initial state of environment E_1 is

$$\rho_{E_1}(0) = \frac{e^{-\beta H_{E_1}}}{\mathrm{Tr}[e^{-\beta H_{E_1}}]}. \tag{9}$$

Thus, we can write the the initial quantum state of the composite system in the form of tensor product of density operators

$$\begin{aligned} \rho(0) &= \rho_A(0) \otimes \rho_{SE}(0) \\ &= \rho_A(0) \otimes \rho_S(0) \otimes \rho_E(0) \\ &= \rho_A(0) \otimes \rho_S(0) \otimes \rho_{E_0}(0) \otimes \rho_{E_1}(0), \end{aligned} \tag{10}$$

where $\rho_S(0) = |\psi_s\rangle\langle\psi_s|$, $\rho_A(0) = |\psi_A\rangle\langle\psi_A|$, $|\psi_A\rangle = a_1|1\rangle + a_0|0\rangle$, $|a_1|^2 + |a_0|^2 = 1$.

2.2. Time evolution of the composite system

When $t > 0$, the quantum state of the composite system is

$$\rho(t) = U(t)\rho(0)U^\dagger(t) = \begin{pmatrix} |a_1|^2 U_1(t)\rho_{SE}(0)U_1^\dagger(t), & a_1 a_0^* U_1(t)\rho_{SE}(0)U_0^\dagger(t) \\ a_0 a_1^* U_0(t)\rho_{SE}(0)U_1^\dagger(t), & |a_0|^2 U_0(t)\rho_{SE}(0)U_0^\dagger(t) \end{pmatrix} \tag{11}$$

Then, the whole quantum state of system A and system S is

$$\rho_{AS}(t) = \mathrm{Tr}_E\ \rho(t) = \begin{pmatrix} |a_1|^2 \hat{R}_{1,1}(t)\rho_S(0), & a_1 a_0^* \hat{R}_{1,0}(t)\rho_S(0) \\ a_0 a_1^* \hat{R}_{0,1}(t)\rho_S(0), & |a_0|^2 \hat{R}_{0,0}(t)\rho_S(0) \end{pmatrix}, \tag{12}$$

where the super-operator $\hat{R}_{j,k}(t)\bullet = \mathrm{Tr}_E[U_j(t)(\bullet \otimes \rho_E(0))U_k^\dagger(t)]$

3. Mixing-induced quantum non-Markovian effect

If we have no access to the ancilla degrees of freedom about system A, then the system A is equivalent to a mixed state for us. The quantum state of system S is the statistical mixture [30]

$$\begin{aligned}\rho_S(t) &= \mathrm{Tr}_A\ \rho_{AS}(t) \\ &= |a_0|^2\hat{R}_{0,0}(t)\rho_S(0) + |a_1|^2\hat{R}_{1,1}(t)\rho_S(0). \end{aligned} \tag{13}$$

Considering the initial system-environment correlations between system S and environment E_0 [29], the off-diagonal element

$$\begin{aligned}&\langle e|\hat{R}_{0,0}(t)\rho_S(0)|g\rangle = s_e s_g^* \cdot C_0(t) = s_e s_g^* \cdot e^{-\gamma_0(t)} \\ &\cdot [\cos[\Phi_0(t)] - i\cdot\sin[\Phi_0(t)]\cdot S(\omega_0, s_e, s_g, \beta)], \\ &\langle e|\hat{R}_{1,1}(t)\rho_S(0)|g\rangle = s_e s_g^* \cdot C_1(t) = s_e s_g^* \cdot e^{-\gamma_1(t)}, \end{aligned} \tag{14}$$

where

$$\begin{aligned}\gamma_j(t) &= \int_0^\infty d\omega\ J_j(\omega)\coth(\beta\hbar\omega/2)\frac{1-\cos(\omega t)}{\omega^2}, \\ \Phi_0(t) &= \int_0^\infty d\omega\ J_0(\omega)\frac{\sin(\omega t)}{\omega^2}, \\ S(\omega_0, s_e, s_g, \beta) &= \frac{|s_g|^2 e^{\beta\hbar\omega_0/2} - |s_e|^2 e^{-\beta\hbar\omega_0/2}}{|s_g|^2 e^{\beta\hbar\omega_0/2} + |s_e|^2 e^{-\beta\hbar\omega_0/2}}. \end{aligned} \tag{15}$$

Here, $J_j(\omega)$ is the spectral density of environment E_j. Assume the environment E_j has the Ohmic-like spectral density, *i.e*.

$$J_j(\omega) = \lambda_j \omega^{s_j}\Omega_j^{1-s_j}e^{-\omega/\Omega_j}, \tag{16}$$

where λ_j is coupling constant, s_j is Ohmicity parameter of the environment E_j, and Ω_j represents the cutoff frequency.

When $\beta\hbar\omega_0/2 \gg 1$, $S(\omega_0, s_e, s_g, \beta) \approx 1$, then $C_0(t) \approx e^{-\gamma_0(t)}e^{-i\Phi_0(t)}$. Thus, the normalized coherence [1, 31] is $|C_0(t)| = e^{-\gamma_0(t)}$. In this condition, if the system S only interacts with environment E_0, the influence of initial correlations on the coherence can be ignored. Because the function $\gamma_j(t)$ monotonically increases with time t ($s_j < 1$) [29], the evolution of system S caused by environment E_0 is Markovian. Similarly, $\hat{R}_{1,1}(t)$ is also Markovian process.

However, when the total dynamics is mixed by two dynamical processes $\hat{R}_{0,0}(t)$ and $\hat{R}_{1,1}(t)$ ($s_0, s_1 < 1$), the off-diagonal element

$$\begin{aligned}\langle e|\rho_S(t)|g\rangle = s_e s_g^* \cdot C_{\mathrm{mix}}(t) = s_e s_g^* \cdot [|a_0|^2 \\ \times e^{-\gamma_0(t)}e^{-i\Phi_0(t)} + |a_1|^2 e^{-\gamma_1(t)}], \end{aligned} \tag{17}$$

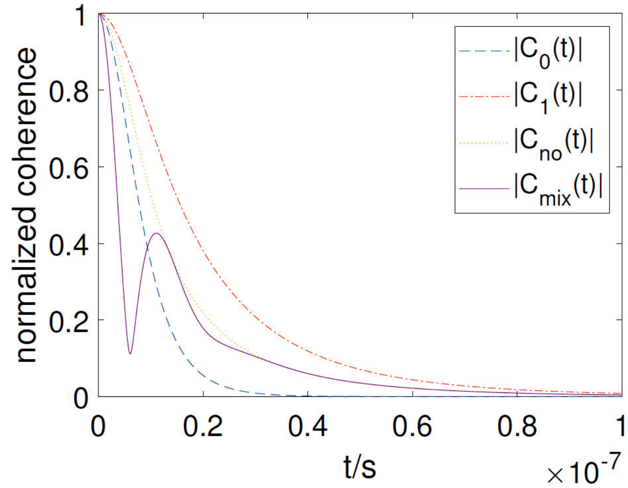

FIGURE 1. The normalized coherence varies with time t. $a_0 = a_1 = 1/\sqrt{2}$, $\omega_0 = 10^8$ Hz, $T = 10^{-6}$ K, $s_e = s_g = 1/\sqrt{2}$, $s_0 = s_1 = 0.5$, $\Omega_0 = \Omega_1 = 10^8$ Hz, $\lambda_0 = 3$, $\lambda_1 = 1$. $|C_0(t)|$ and $|C_1(t)|$ represent normalized coherence in different Markovian processes, $|C_{\mathrm{mix}}(t)|$ varies non-monotonically and appears non-Markovian property. $|C_{\mathrm{no}}(t)|$ is normalized coherence in the mixed process without considering initial correlations, and still shows Markovian property.

the normalized coherence

$$\begin{aligned}|C_{\mathrm{mix}}(t)| = [|a_0|^4 e^{-2\gamma_0(t)} + |a_1|^4 e^{-2\gamma_1(t)} + 2|a_0|^2|a_1|^2 \\ \times e^{-(\gamma_0(t)+\gamma_1(t))}\cos(\Phi_0(t))]^{1/2}. \end{aligned} \tag{18}$$

In the condition, though $\hat{R}_{0,0}(t)$ and $\hat{R}_{1,1}(t)$ are Markovian processes, the coherence of the system S varies non-monotonically. The mixed process is non-Markovian, and caused by the initial system-environment correlations. If not considering the initial correlations ($\Phi_0(t) = 0$), the mixed process is still Markovian, see Fig. 1. The result shows the non-Markovian effect of the mixture of two environments.

4. The interaction with superposition of two dephasing environments

If we perform the measurement on the ancillary system A in the $\{|+\rangle, |-\rangle\}$ basis with $|\pm\rangle = (|0\rangle \pm e^{i\varphi}|1\rangle)/\sqrt{2}$ to get more information, then

$$\begin{aligned}\rho_S^\pm(t) = \frac{1}{Z_\pm(t)}[|a_0|^2\hat{R}_{0,0}(t)\rho_S(0) + |a_1|^2\hat{R}_{1,1}(t)\rho_S(0) \\ \pm e^{i\varphi}a_0 a_1^*\hat{R}_{0,1}(t)\rho_S(0) \\ \pm e^{-i\varphi}a_1 a_0^*\hat{R}_{1,0}(t)\rho_S(0)], \end{aligned} \tag{19}$$

where normalized parameter

$$\begin{aligned}Z_\pm(t) = 1 \pm e^{i\varphi}a_0 a_1^* \cdot \mathrm{Tr}[\hat{R}_{0,1}(t)\rho_S(0)] \\ \pm e^{-i\varphi}a_1 a_0^* \cdot \mathrm{Tr}[\hat{R}_{1,0}(t)\rho_S(0)]. \end{aligned}$$

When $j \neq k$, and $j, k = 0, 1$, using Eq. (4)

$$\langle e|\hat{R}_{j,k}(t)\rho_S(0)|g\rangle = \langle e|\mathrm{Tr}_E[U_j(t)(\rho_S(0)\otimes\rho_E(0))U_k^\dagger(t)]|g\rangle = s_e s_g^* \cdot \mathrm{Tr}_E[U_{j,e}(t)(\rho_{E_j}(0)\otimes\rho_{E_k}(0))U_{k,g}^\dagger(t)]$$
$$= s_e s_g^* \cdot \mathrm{Tr}_{E_j}[U_{j,e}(t)\rho_{E_j}(0)] \cdot \mathrm{Tr}_{E_k}[\rho_{E_k}(0)U_{k,g}^\dagger(t)] = s_e s_g^* \cdot \langle U_{j,e}(t)\rangle \cdot \langle U_{k,g}^\dagger(t)\rangle, \tag{20}$$

by use of Refs. [27–29], then

$$\langle U_{0,e}(t)\rangle = \langle U_{0,e}^\dagger(t)\rangle^\dagger = e^{-\frac{i}{4}\theta_0(t)}e^{-\frac{1}{4}\gamma_0(t)} \cdot A_0^-(t), \qquad \langle U_{0,g}(t)\rangle = \langle U_{0,g}^\dagger(t)\rangle^\dagger = e^{-\frac{i}{4}\theta_0(t)}e^{-\frac{1}{4}\gamma_0(t)} \cdot A_0^+(t),$$
$$\langle U_{1,e}(t)\rangle = \langle U_{1,g}(t)\rangle = e^{-\frac{i}{4}\theta_1(t)}e^{-\frac{1}{4}\gamma_1(t)}, \qquad \theta_j(t) = \int_0^\infty d\omega\ J_j(\omega)\frac{\omega t - \sin(\omega t)}{\omega^2},$$
$$A_0^\pm(t) = \cos\left(\frac{1}{2}\Phi_0(t)\right) \pm i\cdot\sin\left(\frac{1}{2}\Phi_0(t)\right) S(\omega_0, s_e, s_g, \beta). \tag{21}$$

Thus, from Eq. (14) and Eq. (20), the elements of density matrix

$$\langle e|\rho_S^\pm(t)|g\rangle = \frac{s_e s_g^*}{Z_\pm(t)}\{|a_0|^2 C_0(t) + |a_1|^2 C_1(t) \pm 2\mathrm{Re}[e^{i\varphi}a_0 a_1^* e^{-\frac{i}{4}[\theta_0(t)-\theta_1(t)]}] \cdot e^{-\frac{1}{4}[\gamma_0(t)+\gamma_1(t)]} \cdot A_0^-(t)\},$$
$$\langle e|\rho_S^\pm(t)|e\rangle = \frac{|s_e|^2}{Z_\pm(t)}\{1 \pm 2\mathrm{Re}\{e^{i\varphi}a_0 a_1^* \cdot e^{-\frac{i}{4}[\theta_0(t)-\theta_1(t)]}A_0^-(t)\} \cdot e^{-\frac{1}{4}(\gamma_1(t)+\gamma_0(t))}\},$$
$$\langle g|\rho_S^\pm(t)|g\rangle = \frac{|s_g|^2}{Z_\pm(t)}\{1 \pm 2\mathrm{Re}\{e^{i\varphi}a_0 a_1^* \cdot e^{-\frac{i}{4}[\theta_0(t)-\theta_1(t)]}A_0^+(t)\} \cdot e^{-\frac{1}{4}(\gamma_1(t)+\gamma_0(t))}\}, \tag{22}$$

where

$$Z_\pm(t) = 1 \pm 2\mathrm{Re}[e^{i\varphi}a_0 a_1^* \cdot e^{-\frac{i}{4}[\theta_0(t)-\theta_1(t)]} \cdot (|s_e|^2 A_0^-(t) + |s_g|^2 A_0^+(t))] \cdot e^{-\frac{1}{4}[\gamma_0(t)+\gamma_1(t)]}.$$

From the expressions of Eq. (22) and normalized parameter $Z_\pm(t)$, we can see that, if we let the relevant parameters satisfy the factor $e^{i\varphi}a_0 a_1^* \cdot e^{-\frac{i}{4}[\theta_0(t)-\theta_1(t)]} \in \mathbf{R}$, then diagonal elements of density matrix of open quantum system S remain unchanged before and after the measurement, which accords with the variation feature of dephasing process. Thus, we can compare the effect of measurement on the ancillary system A with condition that the open quantum S only in the environment E_0.

After the measurement on the ancillary system A, the normalized coherence of open quantum system S

$$|C_\pm(t)| = \left|\frac{\langle e|\rho_S^\pm(t)|g\rangle}{s_e s_g^*}\right|. \tag{23}$$

5. The interaction with superposition of two identical environments

If two environments are identical, then $\theta_0(t) = \theta_1(t), \gamma_0(t) = \gamma_1(t) = \gamma(t)$. Equation (23) becomes

$$|C_\pm(t)| = \left|\frac{|a_0|^2 C_0(t) + |a_1|^2 C_1(t) \pm 2\mathrm{Re}[e^{i\varphi}a_0 a_1^*] \cdot e^{-\frac{1}{2}\gamma(t)} \cdot A_0^-(t)}{1 \pm 2\mathrm{Re}[e^{i\varphi}a_0 a_1^*(|s_e|^2 A_0^-(t) + |s_g|^2 A_0^+(t))] \cdot e^{-\frac{1}{2}\gamma(t)}}\right|. \tag{24}$$

Let $a_0 = a_1 = 1/\sqrt{2}$, to make diagonal elements of density matrix of open quantum system S unchanged before and after the measurement, we can choose $\varphi = 0$. Then

$$|C_\pm(t)| = \left|\frac{\frac{1}{2}C_0(t) + \frac{1}{2}C_1(t) \pm e^{-\frac{1}{2}\gamma(t)} \cdot A_0^-(t)}{1 \pm \cos(\frac{1}{2}\Phi_0(t)) \cdot e^{-\frac{1}{2}\gamma(t)}}\right|. \tag{25}$$

Simplify the Eq. (25) further, we can get

$$|C_\pm(t)| = e^{-\frac{1}{2}\gamma(t)}\sqrt{\cos^2(\frac{\Phi_0(t)}{2}) + \sin^2(\frac{\Phi_0(t)}{2})S^2(\omega_0, s_e, s_g, \beta)}. \tag{26}$$

On the other hand, from Eq. (14)

$$|C_0(t)| = e^{-\gamma(t)}\sqrt{\cos^2(\Phi_0(t)) + \sin^2(\Phi_0(t))S^2(\omega_0, s_e, s_g, \beta)}, \tag{27}$$

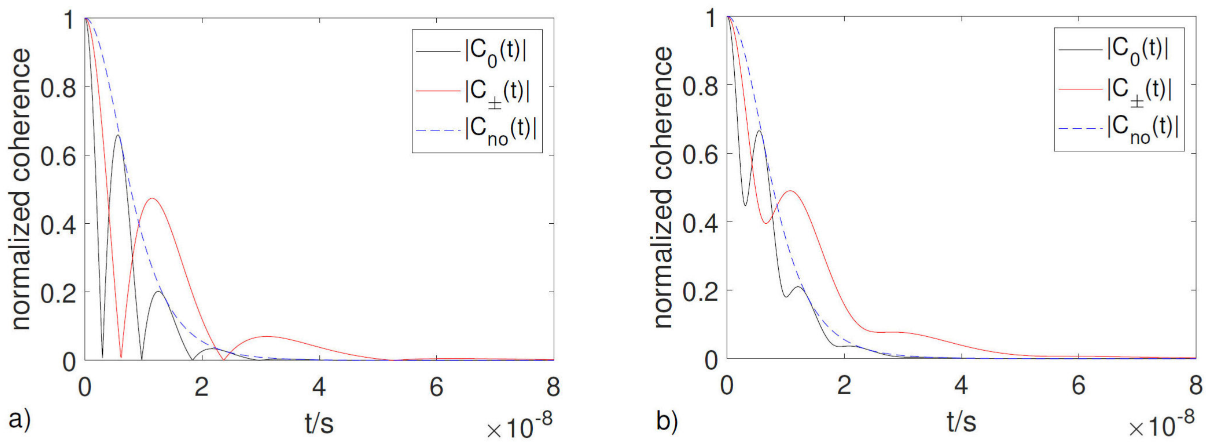

FIGURE 2. Influence of measurement on the coherence of system in the $\{|+\rangle,|-\rangle\}$ basis when considering initial correlations. $a_0 = a_1 = 1/\sqrt{2}$, $\omega_0 = 10^3$ Hz, $T = 10^{-6}$ K, $s_0 = s_1 = 0.5$, $\Omega_0 = \Omega_1 = 10^8$ Hz, $\lambda_0 = \lambda_1 = 3$. $|C_\pm(t)|$ corresponds to measurement result in the $\{|+\rangle,|-\rangle\}$ basis. $|C_0(t)|$ is normalized coherence when the system only interacts with environment E_0 with initial correlations, while $|C_{no}(t)|$ without initial correlations. a) $s_e = s_g = 1/\sqrt{2}$, b) $s_e = 1/2$, $s_g = \sqrt{3}/2$.

where $S^2(\omega_0, s_e, s_g, \beta) \leq 1$. Comparing Eq. (26) and Eq. (27), we can see, when the system S initially corelated with environment E_0 interacts with superposition of two environments E_0 and E_1, whatever the parameters of environments are, if the function $\Phi_0(t)$ can come to the value $2k\pi(k \in Z)$ with time t, we can enhance the coherence of system S at the special time point. Thus, the conclusion does not restrict the range of parameter ω_0 and initial state of open quantum system S, and it is suitable for all kinds of open quantum system. If $\beta\hbar\omega_0/2 \gg 1$, $S(\omega_0, s_e, s_g, \beta) \approx 1$. Obviously, $|C_0(t)| \approx e^{-\gamma(t)}$,$|C_\pm(t)| \approx e^{(-1/2)\gamma(t)}$, $|C_0(t)| < |C_\pm(t)|$. Hence, influence of initial correlations can be neglected. In reality, it is easy to put the open quantum system in the superposition of two identical environments, so it is reasonable to consider two identical environments.

(i) If two identical environments are both sub-Ohmic (Ohmicity parameters $s_0 = s_1 < 1$), the function $\Phi_0(t)$ increases with time t [29]. On the other hand, the function $\gamma(t)$ always increases with t. When the function $\Phi_0(t)$ satisfies $\Phi_0(t) = 2\pi$, the coherence of system S can be enhanced obviously by measurement.

If $\beta\hbar\omega_0/2 \ll 1$, $S(\omega_0, s_e, s_g, \beta) \approx |s_g|^2 - |s_e|^2 = -\langle\sigma_z\rangle$. Let $s_e = s_g = 1/\sqrt{2}$, $S(\omega_0, s_e, s_g, \beta) \approx 0$, we can get

$$|C_\pm(t)| \approx e^{-\frac{1}{2}\gamma(t)} \left|\cos\left(\frac{1}{2}\Phi_0(t)\right)\right|. \tag{28}$$

Then $t > 0$, we can get $|C_\pm(t)| \in [0, e^{-\gamma(t)/2}]$, in addition, $|C_+(t)|$ and $|C_-(t)|$ have the same curve. In Eq. (15) and Eq. (16), $\lim_{t\to+\infty}\Phi_0(t) = +\infty(s_0 < 1)$ [29]. If we set Ohmicity parameter $s_0 = s_1 = 0.5$, seeing Fig. 2a). Curves of $|C_0(t)|$ and $|C_\pm(t)|$ show the property of damped oscillation, and the period of $|C_\pm(t)|$ is twice period of $|C_0(t)|$. At the particular time point (*i.e.* $\Phi_0(t) = 2k\pi, k \in Z$), $|C_\pm(t)| > |C_0(t)|$. When $\Phi_0(t) = 2\pi$, if carrying out the measurement on the ancillary system A at the time, the decoherence process of system S can be suppressed effectively.

Moreover, even if the initial state of the system S is any superposition state in the Hilbert space spanned by the bases $\{|e\rangle, |g\rangle\}$, the coherence still can be enhanced. For example, $s_e = 1/2$, $s_g = \sqrt{3}/2$, seeing Fig. 2b). Although curves are a little different from Fig. 2a), when time t satisfies condition $\Phi_0(t) = 2\pi$, the coherence of quantum system S can be still enhanced effectively.

(ii) If the environments are Ohmic $(s_0 = s_1 = 1)$ or super-Ohmic $(s_0 = s_1 > 1)$, the situation is different. From Ref. [29], $\lim_{t\to+\infty}\Phi_0(t) = \lambda_0\pi/2\,(s_0 = 1)$ and $\lim_{t\to+\infty}\Phi_0(t) = 0\,(s_0 > 1)$. Therefore, we can not guarantee that the function $\Phi_0(t)$ must get to the value $2k\pi\ (k \in Z)$ over time t. The time points may not exist in some conditions. Besides, when $\Phi_0(t) = 0$, apparently, $|C_0(t)| < |C_\pm(t)|$.

6. The interaction with superposition of two different environments

If two environments are different, then Eq. (23) is extremely cumbersome and complicated. For this reason, we just discuss the problems with $\beta\hbar\omega_0/2 \gg 1$. Set $a_0 = a_1 = 1/\sqrt{2}$, from the Sec. 4, to make the diagonal elements of density matrix of open quantum system S remain unchanged before and after the measurement, we should choose proper φ to satisfy that factor $e^{i\varphi}a_0a_1^* \cdot e^{-(i/4)[\theta_0(t)-\theta_1(t)]}$ is real number when measuring the ancillary system A in the $\{|+\rangle,|-\rangle\}$ basis at time t. In the condition, it requires $\varphi - (1/4)[\theta_0(t) - \theta_1(t)] = k\pi(k \in Z)$, of course we need to know the variation rule about $\theta_0(t) - \theta_1(t)$ in advance. When k is any integer, the results after measurement are same. Thus, let $\varphi - (1/4)[\theta_0(t) - \theta_1(t)] = 0$, Eq. (23) becomes

$$|C_{\pm}(t)| = \left| \frac{\frac{1}{2}e^{-\gamma_0(t)}e^{-ix} + \frac{1}{2}e^{-\gamma_1(t)}e^{ix} \pm e^{-\frac{1}{4}[\gamma_0(t)+\gamma_1(t)]}}{1 \pm \cos(x) \cdot e^{-\frac{1}{4}[\gamma_0(t)+\gamma_1(t)]}} \right|, \tag{29}$$

where $x = (1/2)\Phi_0(t)$, then

$$C_{\pm}(t) = \frac{\frac{1}{2}\cos(x)(e^{-\gamma_0(t)} + e^{-\gamma_1(t)}) \pm e^{-\frac{1}{4}[\gamma_0(t)+\gamma_1(t)]}}{1 \pm \cos(x) \cdot e^{-\frac{1}{4}[\gamma_0(t)+\gamma_1(t)]}} + i \cdot \frac{\frac{1}{2}\sin(x)(e^{-\gamma_1(t)} - e^{-\gamma_0(t)})}{1 \pm \cos(x) \cdot e^{-\frac{1}{4}[\gamma_0(t)+\gamma_1(t)]}}. \tag{30}$$

Ignoring the imaginary part of $C_{\pm}(t)$, then

$$|C_{\pm}(t)| \approx \left| \frac{\frac{1}{2}\cos(x)(e^{-\gamma_0(t)} + e^{-\gamma_1(t)}) \pm e^{-\frac{1}{4}[\gamma_0(t)+\gamma_1(t)]}}{1 \pm \cos(x) \cdot e^{-\frac{1}{4}[\gamma_0(t)+\gamma_1(t)]}} \right|. \tag{31}$$

Not considering the initial correlations between system S and environment $E_0(\Phi_0(t) = 0)$, then $x = 0$. If there are initial correlations between the system S and environment E_0, correlation factor will appear in expression.

If two sub-Ohmic environments are different only in the coupling constants, let $\lambda_1 = k\lambda_0 (k \geq 0)$, when $x = (2k+1)\pi/2, k \in Z$. From Eq. (31), then

$$|C_{\pm}(t)| \approx e^{-\frac{1}{4}\gamma_0(t)(1+k)}. \tag{32}$$

Requiring $|C_{\pm}(t)| \geq e^{-\gamma_0(t)}$, then $0 \leq k \leq 3$. From the example we can see, to enhance the coherence of open quantum system in the superposition of environments, the coupling constants need to be restricted to some range.

7. Conclusion

In this paper, we research the open system S interacting with superposition of two dephasing environments when considering the initial system-environment correlations between the system and one environment. The initial state of environment E_0 depends on the initial quantum state of system via interaction, the other environment E_1 is at thermal equilibrium state. Then we can get the evolution form of composite system.

With regard to the whole system composed of system S and ancillary two-level system A, if we do not perform any operation on the system A, then the dynamics evolution is the mixture of two dephasing processes. As for two sub-Ohmic environments, without initial correlations, two processes are Markovian and the mixed process too. When considering the initial correlations with $\beta\hbar\omega_0/2 \gg 1$, two processes are still Markovian. However, the mixed process is non-Markovian. Thus, initial system-environment correlations play a very important part in the memory effects of environment.

If we perform measurement on the ancillary system A in $\{|+\rangle, |-\rangle\}$ basis, as for two identical sub-Ohmic environments, we can enhance the coherence of system S by measuring the ancillary system at the special time points. When $\beta\hbar\omega_0/2 \gg 1$, influence of initial correlations can be neglected, and measurement can enhance the coherence of system S. For two different environments, the correlation term appears in the coherence expression. Besides we discuss the approximate range of coupling constants between system S and two different environments.

In the realistic quantum information process, if we operate on the quantum system in one environment and conserve the quantum state in the superposition of two or more environments, maybe we can prolong the coherence time of open quantum system. In the time-evolution process, the open quantum system and ancillary system establish the correlations, perhaps we can enhance the coherence of open quantum system by measuring the ancillary system in the orthogonal bases. In the paper, we choose a set of fixed measurement bases. However, the optimal bases choice may be different over time. If we can find the optimal measurement in any time, the effect of measurement will be better. The setup may be useful in the field of quantum memory and quantum register.

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