Plasma Physics Revista Mexicana de Física **67** 061501 1–7 NOVEMBER-DECEMBER 2021

Nonlinear dynamical properties of solitary wave solutions for Zakharov-Kuznetsov equation in a multicomponent plasma

U. M. Abdelsalam

Department of Mathematics, Faculty of Science, Fayoum University, Egypt.
Department of Mathematics, University of Technology and Applied Sciences - Al Rustaq, Oman.
**e-mail: usama.ahmad@rub.de*

Received 22 March 2021; accepted 9 May 2021

Using the reductive perturbation method, we have derived the Zakharov-Kuznetsov equation for a multi-component plasma model consisting of electrons, positrons, and fluid ions with positive and negative charges. The extended homogenous balance method has been applied to obtain the soliton solution in addition to many traveling wave solutions. Various physical parameters have different effects on the profile of the solitary wave pulses, which can show the propagation of the ion, acoustic waves in laboratory plasmas and many astrophysical plasma systems as in Earth's ionosphere.

Keywords: Zakharov-Kuznetsov (ZK) equation; multicomponent plasma; ion-acoustic solitary waves; HB method; nonlinear partial differential equations.

PACS: 52.35.Fp; 52.35.Sb; 52.90.+z; 02.30.Jr

DOI: https://doi.org/10.31349/RevMexFis.67.061501

1. Introduction

Many research efforts have been devoted to positive-negative ion plasma, composed of ions with negative and positive charges and electrons, since it has an important role in studying various plasma science fields. This kind of plasma can be found in low-temperature laboratory experiments and many astrophysical plasma environments [1–8]. The (e-p) plasma, which contains positrons and electrons, can also include positive ions. This (e-p-i) plasma is created in the laboratory and also found in many astrophysical contexts like galactic nuclei and others (see [9–11]). The (e-p-i) plasma model in the presence of negative ions was discussed in many other works as in Ref. [12–16]. The predicted nonlinear excitations that occur in e-p-i plasmas may include electromagnetic solitons, electrostatic oscillations solitons, which depending on the nonlinear plasma in magnetized or unmagnetized cases, including the Zakharov-Kuznetsov (ZK), Korteweg-De Vries (KdV), and the nonlinear Schrodinger (NLS) equations [17, 18].

The ZK equation has great importance in studying the propagation of acoustic waves in the magnetized plasma [19,20]. Various methods were used for obtaining many types of solutions to the nonlinear evolution equations (N.L.E), such as the extended tanh method, (G'/G) method, the general expansion method, and the extended homogeneous balance (ext.HB) method [21–29]. Wang *et al.* had proposed the HB method, which is an effective algebraic method for extract a wide class of analytical solutions [30, 31]. Many N.L.E. had been solved using the extended homogeneous balance. In this paper, the ext. HB method has been used to solve the ordinary differential equation (O.D.E.) reduced from the ZK equation with computer algebra system such as Mathematica to extract various kinds of analytical solutions for ZK equation, the obtained solutions have new solutions and cannot be obtained by many other methods like tanh, extended tanh, G'/G expansion methods.

Many researchers have studied the nonlinear propagation of the ion-acoustic waves for a two-component plasma consisting of classical ions and electrons. Owing to the importance of pair-ion plasma, which is composed of positive and negative ions, as well as the study of the ion-acoustic nonlinear solitary waves for magnetized plasma, it is of paramount interest. In the present work, we have studied the nonlinear properties of the propagation ion-acoustic (IAW) solitary waves in positive-negative ion plasma with positrons and electrons. The ZK equation is derived and has been solved using the ext.HB method to study the small-but-finite amplitude solitary wave. The effects of various physical parameters have been checked on the characteristics of the solitary waves in such plasma, which can be found in many astrophysical plasma systems as in the Earth's ionosphere.

The paper is organized as follows: In the following section, the governing equations for the suggested plasma model are presented, and the ZK equation is derived to describe the system. The extended homogenous balance method is used to solve the ZK equation in Sec. 3; different kinds of solutions are obtained as periodic, rational, blow up, and solitary wave solutions, while we focus on the solitary type solution. The discussion is presented in Sec. 4, and a summary of these results is presented in the last section.

2. Model equations and ZK equation

The suggested magnetized multi-component plasma model consists of electrons, positrons, and fluid ions with positive and negative charges within a magnetic field $\mathbf{B} = B_0\hat{x}$. We write the system of equations in the normalized form. The fluid equations for the ions are the following continuity and momentum equations

$$\frac{\partial n_{+,-}}{\partial t} + \nabla \cdot (n_{+,-}\mathbf{u}_{+,-}) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_{+} \cdot \nabla\right)\mathbf{u}_{+} = -Q\nabla\phi + \frac{e}{c}(\mathbf{u}_{+} \times B_0\hat{x}), \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_{-} \cdot \nabla\right)\mathbf{u}_{-} = \nabla\phi - \frac{e}{c}(\mathbf{u}_{-} \times B_0\hat{x}). \tag{3}$$

The Poisson equation

$$\nabla^2\phi = n_e - n_p + n_- - n_+, \tag{4}$$

while positrons and electrons distributions are given by

$$n_e = \eta_e \exp(s_p\, \phi), \tag{5}$$

$$n_p = \eta_p \exp(-\phi). \tag{6}$$

Such that $s_p = T_p/T_e$, where T_p and T_e are the positron and electron temperature, respectively. The mass ratio is defined as $Q = m_-/m_+$, where $m_{-,+}$ are the negative and positive ion masses, respectively, while $\eta_e = n_{e0}/n_{+0}$, and $\eta_p = n_{p0}/n_{+0}$. In the last equations, $n_{+,-}$ and $\mathbf{u}_{+,-}$ represent the ions number density and fluid velocity, respectively, while the wave potential is expressed by ϕ. The density n_α (for $\alpha = +,-,p$ and e) is scaled by n_{+0} (unperturbed positive ion density) , $\mathbf{u}_{+,-}$, ϕ and the variables x and t are scaled by $C_{s+} = (k_BT_e/m_+)^{1/2}$, k_BT_e/e and $\lambda_{D+} = \left(k_BT_e/4\pi e^2 n_{+0}\right)^{1/2}$, and $\omega_{p+}^{-1} = (4\pi e^2 n_{+0}/m_+)^{-1/2}$, respectively, and the positive/negative ion cyclotron frequency $\Omega_{c\pm} = eB_0/(m_\pm c)$ is scaled by ω_{p+}^{-1}. The neutrality condition gives

$$1 = \eta_e - \eta_p + \mu, \tag{7}$$

where $\mu = n_{-0}/n_{+0}$. Using the reductive perturbation method, we shall study the nonlinear propagation of the IAWs. According to this method, we use the stretching:

$$\chi = \epsilon^{1/2}(x - \lambda t), \quad Y = \epsilon^{1/2}y, \quad \text{and} \quad \tau = \epsilon^{3/2}t, \tag{8}$$

where λ is the wave velocity and ϵ is a small parameter, while the dependent physical quantities in the model equations are expanded as

$$\Gamma = \Gamma^{(0)} + \sum_{n=1}^{\infty} \epsilon^n \Gamma^{(n)}, \tag{9}$$

where $\Gamma = \{n_+, n_-, n_e, n_p, u_+, u_-, \phi\}^T$ and $\Gamma^{(0)} = \{1, \mu, \eta_e, \eta_p, 0, 0, 0\}^T$. Employing the variable stretching (8) and the expansions (9) in Eqs. (1)-(6), we can isolate distinct orders in ϵ. The lowest-order in ϵ gives

$$n_+^{(1)} = \frac{1}{\lambda^2}\phi^{(1)}, \qquad u_{+,x}^{(1)} = \frac{1}{\lambda}\phi^{(1)},$$

$$u_{+,z}^{(1)} = \frac{1}{\Omega_{c+}}\frac{\partial\phi^{(1)}}{\partial Y}\,, \tag{10}$$

$$n_-^{(1)} = \frac{-\mu Q}{\lambda^2}\phi^{(1)}, \qquad u_{-,x}^{(1)} = \frac{-Q}{\lambda}\phi^{(1)},$$

$$u_{-,z}^{(1)} = \frac{-Q}{\Omega_{c+}}\frac{\partial\phi^{(1)}}{\partial Y}, \tag{11}$$

$$n_e^{(1)} = \eta_e s_p \phi^{(1)}, \tag{12}$$

$$n_p^{(1)} = \eta_p \phi^{(1)}. \tag{13}$$

The phase velocity relation can be deduced from Poisson equation

$$\lambda = \sqrt{\frac{2[1 + Q\mu]}{\eta_e s_p + \eta_p}}. \tag{14}$$

The next-order in ε gives a system of equations in the second-order perturbed quantities. Eliminating these quantities with the last equations, we can get the Zakharov-Kuznetsov (ZK) equation

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\chi} + B\frac{\partial^3\phi^{(1)}}{\partial\chi^3} + D\frac{\partial^3\phi^{(1)}}{\partial\chi\partial Y^2} = 0. \tag{15}$$

The coefficients of nonlinear and dispersion terms are given as

$$A = B\left[\frac{3}{\lambda^4} - \frac{3\mu Q^2}{\lambda^4} - \eta_e s_e^2 + \eta_p\right], \tag{16}$$

$$B = \left[\frac{\lambda^3}{(2 + 2\mu Q)}\right], \tag{17}$$

$$D = B\left[1 + \frac{\mu Q}{\Omega_{c-}^2} + \frac{1}{\Omega_{c+}^2}\right]. \tag{18}$$

3. Solution of ZK equation

Now, we shall illustrate the approach of the ext.HB method to obtain a class of exact solutions, including the solitary wave kinds for the ZK equation.

Let us first consider the ZK equation

$$u_t + \alpha u u_x + \beta u_{xxx} + \gamma u_{xyy} = 0, \tag{19}$$

we can reduce ZK equation to an O.D.E., using the transformation $u(x,t) = U(\zeta), \zeta = ax + by - \vartheta t$. So we get

$$-\vartheta U'(\zeta) + \beta U^{(3)}(\zeta)a^3 + \alpha U(\zeta)U'(\zeta)a + b^2\gamma U^{(3)}(\zeta)a = 0, \tag{20}$$

where ϑ represent a constant speed and $a^2 + b^2 = 1$. By integrating the last equation, we have

$$-\vartheta U(\zeta) + \frac{1}{2}a\alpha U(\zeta)^2 + U''(\zeta)(a^3\beta + ab^2\gamma) = 0. \tag{21}$$

By balancing the highest order with the highest degree in the last equation, we can see that the solution should be in the form

$$U = a_0 + b_0 + a_1\omega + b_1(1+\omega)^{-1} + a_2\omega^2 + b_2(1+\omega)^{-2}, \qquad (22)$$

with

$$\omega' = k + M\,\omega + P\,\omega^2, \qquad (*)$$

where a_i and b_i are constants, while k, M, and P are the parameters to be determined latter, $\omega = \omega(\zeta)$ and $\omega' = d\omega/d\zeta$.

It is noted that equation (*) is the Riccati equation, which can be solved to give us many forms of solutions depending on the following cases (see [32] for more details):

Case I: when P = 1, M = 0

Cases (**II, III,** and **IV**): when $Pk = (M^2 - p_1^2)/4$.

By substituting Eq. (22) in the integrated O.D.E., we obtain an equation in ω. Then, the coefficients of ω^i ($i = 0, 1, 2, ...$) will be equated to 0 to get a system of algebraic equations that can be solved using MATHEMATICA, to give us:

The first set:

$$a_0 = -\frac{12\left(kP\beta a^2 + b^2kP\gamma\right)}{\alpha}, \quad a_1 = 0, \quad b_1 = 0, \quad a_2 = -\frac{12\left(a^2\beta P^2 + b^2\gamma P^2\right)}{\alpha},$$
$$b_2 = 0, \quad \vartheta = 8kP\beta a^3 + 8b^2kP\gamma a + \alpha a_0 a \qquad (23)$$

The second set:

$$M = 2P, \quad a_0 = -\frac{12\left(-P^2\beta a^2 + kP\beta a^2 - b^2P^2\gamma + b^2kP\gamma\right)}{\alpha}, \quad a_1 = 0, \quad b_1 = 0, \quad a_2 = 0,$$
$$b_2 = -\frac{12\left(k^2\beta a^2 + P^2\beta a^2 - 2kP\beta a^2 + b^2k^2\gamma + b^2P^2\gamma - 2b^2kP\gamma\right)}{\alpha},$$
$$\vartheta = -8P^2\beta a^3 + 8kP\beta a^3 - 8b^2P^2\gamma a + 8b^2kP\gamma a + \alpha a_0 a. \qquad (24)$$

In Ref. [32], Abdelsalam et al., the method was explained in detail, and four cases regarding the Riccati equation were discussed. Here, these cases are not discussed as full details of these four cases are already published. Hence, for the first set, we can obtain case I solutions with $M = 0$, $P = 1$, these solutions (for ZK equation) are: for $k > 0$,

$$u_1(\zeta) = \frac{3\vartheta}{\alpha a}\sec^2\left(\zeta/\sqrt{[-4a\{\beta a^2 + \gamma b^2\}/\vartheta]}\right), \qquad (25)$$

and

$$u_2(\zeta) = \frac{3\vartheta}{\alpha a}\csc^2\left(\zeta/\sqrt{[-4a\{\beta a^2 + \gamma b^2\}/\vartheta]}\right), \qquad (26)$$

while for $k < 0$

$$u_3(\zeta) = \frac{3\vartheta}{\alpha a}\text{sech}^2\left(\zeta/\sqrt{[4a\{\beta a^2 + \gamma b^2\}/\vartheta]}\right), \qquad (27)$$

$$u_4(\zeta) = \frac{3\vartheta}{\alpha a}\cosh\left(2\zeta/\sqrt{[4a\{\beta a^2 + \gamma b^2\}/\vartheta]}\right)\text{csch}^2\left(\zeta/\sqrt{[4a\{\beta a^2 + \gamma b^2\}/\vartheta]}\right), \qquad (28)$$

for $k = 0$

$$u_5(\zeta) = \frac{3\vartheta}{\alpha a}\alpha\zeta^2. \qquad (29)$$

Now, the compatibility condition for these solutions in the three other cases (II & III & IV) is

$$Pk = \frac{M^2 - p_1^2}{4}. \qquad (30)$$

Substituting from the values in the first set in the last equation to get a relation for p_1:

$$p_1 = 2\sqrt{\frac{M^2}{4} + \frac{\alpha a_0}{12\left(\beta a^2 + b^2\gamma\right)}}, \qquad (31)$$

which helps us to obtain the following solutions:

For case II:

$$u_6(\zeta) = \frac{3\vartheta}{\alpha a}\left(M + 2\tanh\left[\zeta/\sqrt{\{4a(\beta a^2+\gamma b^2)/\vartheta\}}\right]\right)^2, \tag{32}$$

and

$$u_7(\zeta) = \frac{3\vartheta}{\alpha a}\left(M + 2\coth\left[\zeta/\sqrt{\{4a(\beta a^2+\gamma b^2)/\vartheta\}}\right]\right)^2. \tag{33}$$

For case III, the solution will be

$$u_8(\zeta) = \frac{3\vartheta}{\alpha a}\left(M + \frac{\sinh\left[\zeta/\sqrt{\{4a(\beta a^2+\gamma b^2)/\vartheta\}}\right] + \sqrt{r^2-1}}{r + \cosh\left[\zeta/\sqrt{\{4a(\beta a^2+\gamma b^2)/\vartheta\}}\right]}\right)^2, \tag{34}$$

with the condition that $p_1 = 1$.

However, the case IV solutions are

$$u_9(\zeta) = \frac{3\vartheta}{\alpha a}\left(M + \coth\left[\zeta/\sqrt{\{4a(\beta a^2+\gamma b^2)/\vartheta\}}\right] + \text{csch}(\zeta)\right)^2, \tag{35}$$

with the condition that $p_1 = 1$, and

$$u_{10}(\zeta) = -\frac{3\vartheta\,\text{csch}^2\left(\zeta/\sqrt{4a[\beta a^2+\gamma b^2]/\vartheta}\right)\left(-M^2 + 4\sinh\left[2\zeta/\sqrt{4a\{\beta a^2+\gamma b^2\}/\vartheta}\right]M - 4kP\right)}{2\alpha}, \tag{36}$$

with the condition that $p_1 = 2$.

For the second set, there no solutions are satisfying case I. Similarly, we can obtain for the second set

$$p_1 = 2\sqrt{-\frac{a^2\beta M^2}{4\left(\beta a^2+b^2\gamma\right)} - \frac{b^2\gamma M^2}{4\left(\beta a^2+b^2\gamma\right)} + \frac{M^2}{4} + \frac{\alpha a_0}{12\left(\beta a^2+b^2\gamma\right)}}. \tag{37}$$

Hence, the obtained solutions for ZK Eqn., will be

$$u_{11}(\zeta) = \frac{b^2M^2}{\left(M - p_1\left[M + 2\tanh\left\{\zeta/\sqrt{4a(\beta a^2+\gamma b^2)/\vartheta}\right\}\right]p_1\right)^2} + a_0, \tag{38}$$

$$u_{12}(\zeta) = \frac{b^2M^2}{\left(M - \left[M + 2\coth\left\{\zeta/\sqrt{4a(\beta a^2+\gamma b^2)/\vartheta}\right\}\right]p_1\right)^2} + a_0, \tag{39}$$

$$u_{13}(x,t) = \frac{M^2b^2\left(r + \cosh\left[\zeta/\sqrt{4a\{\beta a^2+\gamma b^2\}/\vartheta}\right]\right)^2}{\left(\sinh\left[\zeta/\sqrt{4a\{\beta a^2+\gamma b^2\}/\vartheta}\right] + \sqrt{r^2-1}\right)^2} + a_0, \tag{40}$$

with condition $p_1 = 1$,

$$u_{14}(\zeta) = a_0 + \frac{b^2}{\left(\coth\left[\zeta/\sqrt{4a\{\beta a^2+\gamma b^2\}/\vartheta}\right]\right) + \text{csch}\left(\zeta/\sqrt{4a[\beta a^2+\gamma b^2]/\vartheta}\right)^2}, \tag{41}$$

for $p_1 = 1$, and

$$u_{15}(\zeta) = \frac{1}{4}M^2b^2\tanh^2\left(\zeta/\sqrt{4a[\beta a^2+\gamma b^2]/\vartheta}\right] + a_0, \tag{42}$$

for $p_1 = 2$.

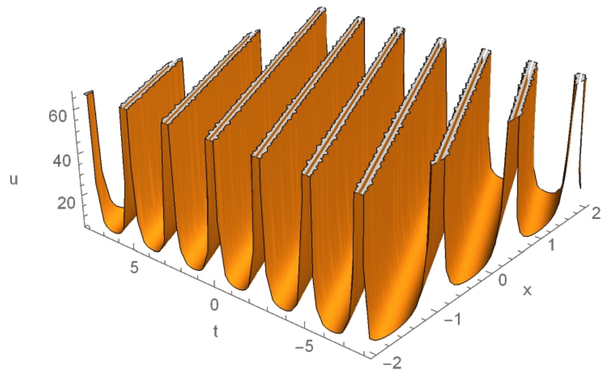

FIGURE 1. The periodic solution profile [given by Eq. (25)].

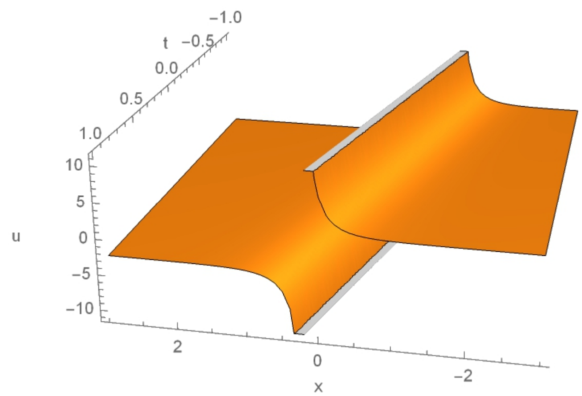

FIGURE 2. The explosive/blowup pulse profile [given by Eq. (33)].

4. Numerical results and discussion

To investigate the nonlinear dynamics of IAWs, the reductive perturbation technique is used to obtain the ZK equation for a collisionless, four-component magnetized plasma model composed of negative and positive ions with electrons and positrons. The ext.HB method has been used to obtain various types of traveling wave solutions for ZK equation as periodical, singular, rational, and solitary wave solutions. For example, Eq. (25) represents a periodical solution as depicted in Fig. 1, and it is clear that the solution (29) is a rational-type solution. In Fig. 2, we can notice that the solution (33) is an explosive/blowup solution, while the balance between the dispersion term and the nonlinear term in the PDE gives the soliton solution represented in Eq. (27), the balance between nonlinearity and dispersion may be disturbed by plasma quantities variations (*e.g.* density, pressure, temperature, etc).

Now, to study the properties of the ion-acoustic waves IAWs, we should focus on the soliton wave pulses represented in (27),

$$\phi = \phi_0 \text{sech}^2\left(\zeta/W\right), \tag{43}$$

where the amplitude of the soliton pulse is expressed as $\phi_0 = 3\vartheta/\alpha a$ and the width is $W = \sqrt{4a(\beta a^2 + \gamma b^2)/\vartheta}$.

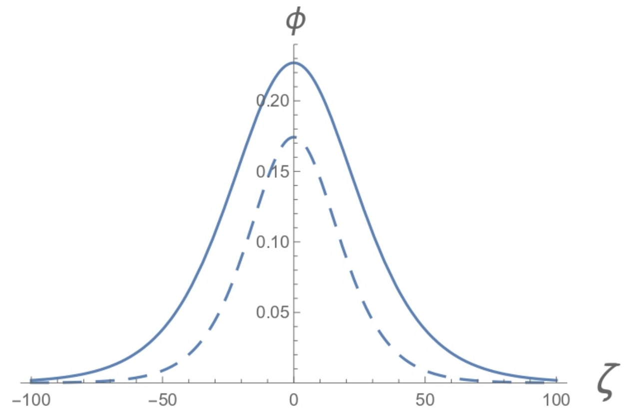

FIGURE 3. The variation of solitary wave profile for $\eta_e = 0.4$ (solid line) and $\eta_e = 0.6$ (dashes line).

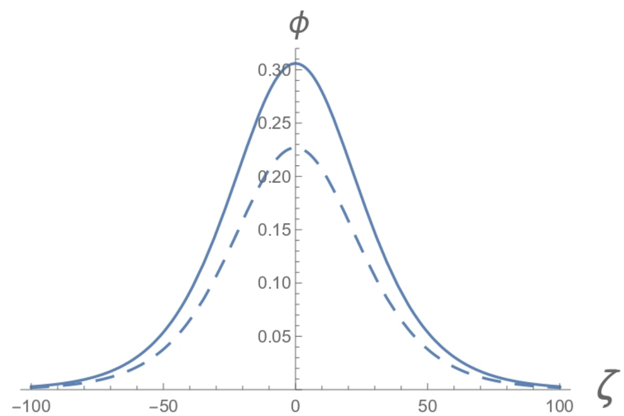

FIGURE 4. The variation of solitary wave profile for $s_p = 0.8$ (solid line) and $s_p = 0.84$ (dashes line).

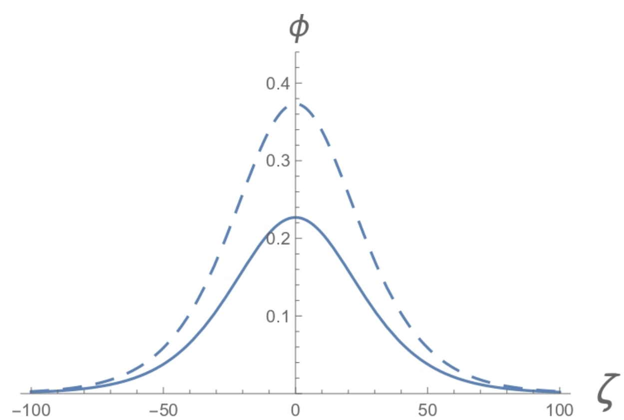

FIGURE 5. The variation of solitary wave profile for $\mu = 0.88$ (solid line) and $\mu = 0.9$ (dashes line).

In Fig. 3, we notice that by increasing the electron concentration (η_e), the values of the soliton pulse amplitude and width will decrease, while in Fig. 4, we can see the role of the electron and positron temperature in variation the profile of the soliton pulse, it is clear that increasing s_p makes the soliton pulse shorter but has no effect on the width of the

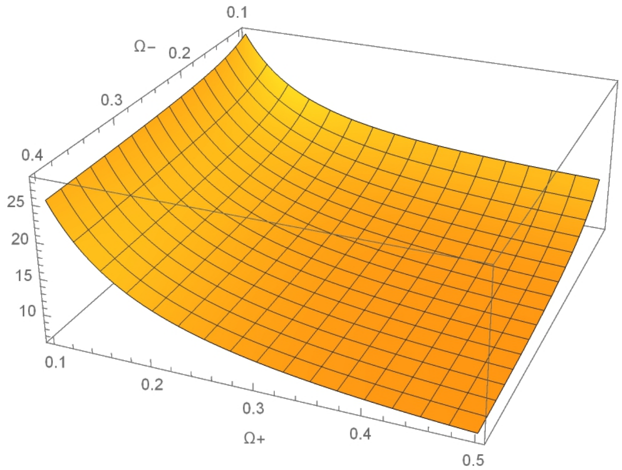

FIGURE 6. The variation of the soliton width W vs Ω_{c+} and Ω_{c-}.

wave shape. In Fig. 5, it is clear that the soliton shape is taller and wider by increasing the negative ion concentration μ. However, increasing the values of the magnetic field Ω_{c+} and Ω_{c-} will decrease the value of the width but does not affect the amplitude value. As depicted in Fig. 6, we can see the effect of the magnetic field on the width W of the soliton wave. At the lowest values of Ω_{c-}, W is found to be high in value. With the increasing values of Ω_{c-}, W has been found to decrease. Again, at the lowest values of Ω_{c+}, W is found to be high in value. With the increasing values of Ω_{c+}, W has been found to decrease in values very sharply more than found for Ω_{c-}.

Physically, when the amplitude increases, then the nonlinearity increases and vis versa. When the physical parameter increases, the amplitude then more energy is pumped into the plasma system, and the nonlinearity increases. We can notice that when the negative ion concentration μ increases, more energy is pumped into the plasma, which leads to enhance the amplitude.

5. Summary

In this study, the properties of the IAWs have been investigated for four-component magnetized plasma contains fluid ions with negative and positive charges in addition to electrons and positrons. The solitary pulses have been described by the ZK equation, which is solved analytically using the ext.HB method. The used method extracts many types of traveling solutions as periodical, singular, rational, and solitary wave solutions. The ZK equation has new solutions that cannot be obtained by many other methods like tanh, extended tanh, G'/G expansion methods. By the numerical analysis of the obtained solitary wave solution, the effects of some physical parameters have been checked to the propagation and the shape of the produced acoustic waves. The increase of the concentration of electrons makes the pulse shape shorter and narrower, while it will be taller and wider by increasing the negative ions, and we can see that the magnetic field affects only the width of the solitary pulse. However, the effect of the electron-to-positron temperature ratio will be on the amplitude of soliton pulse shape. The present study should be helpful to understand the properties of the nonlinear IAWs in laboratory plasmas and many astrophysical plasma systems as in Earth's ionosphere.

Acknowledgement

The author would like to thank the referees for their valuable comments, which helped to improve the manuscript.

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