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Reduction of the Salpeter equation for massless constituents

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Different from the case of the nonrelativistic limit, it is shown in this paper that in the ultrarelativistic limit, the $L = j, j+1$ wave components are large terms for both state with parity $\eta_P = (-1)^j$ and state with parity $\eta_P = (-1)^{j+1}$. Moreover, it is found that the states with parity $\eta_P = (-1)^j$ are degenerate with the states with parity $\eta_P = (-1)^{j+1}$ if the Lorentz structure of the interaction between the massless constituents is four-vector or time-component of four-vector. The scalar interaction violates this degeneracy.

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1. Introduction

The Bethe-Salpeter equation [1] is the appropriate tool to deal with bound-state problems within the framework of relativistic quantum field theory. The nature of the Bethe-Salpeter equation renders it difficult to deal with. It is a four dimensional integral equation, and it requires the full propagators for constituents as well as their interaction kernel from the very beginning. Therefore, simplification is necessary, for example, the ladder approximation and replacing full propagators with free ones. Besides, more simplifications are often used in practice when the Bethe-Salpeter equation is applied to different problems, for instance, massless exchanging boson [2], massless bound state [3], massless constituents [4], large constituent's mass [5] and so on.

Due to various practical reasons, the Salpeter equation [6], one of the three-dimensional reductions of the Bethe-Salpeter equation, is frequently used in practice. From the Salpeter wave function and Salpeter equation, it can be found that the $L = j$ wave components play dominant role in the wave function for state with parity $\eta_P = (-1)^{j+1}$, and the $L = j \pm 1$ wave components are main terms for state with parity $\eta_P = (-1)^j$ [7–9], where L is the relative orbital angular momentum quantum number, j is the spin of the bound state. These features can be shown more obviously in the nonrelativistic limit [10]. In addition, when taking the nonrelativistic limit the $L = j-1$ wave components and $L = j+1$ wave components will decouple for state with parity $(-1)^j$, and for state with parity $(-1)^{j+1}$ the spin singlet and spin triplet will decouple, too. In the limit of vanishing masses of the two bound-state constituents, the Salpeter wave function and Salpeter equation will be simplified in this paper. It maybe be not only of academic interest but also instructive and useful to obtain the features of the Salpeter equation in the ultrarelativistic limit, which are different from the features obtained in the nonrelativistic limit.

This paper is organized as follows. In Sec. 2, the Salpeter equation is briefly reviewed. In Sec. 3, the reduction of the Salpeter equation for the massless constituents is presented. The conclusion is given in Sec. 4.

2. Salpeter equation

Due to the problems in actual applications [4], the reductions of the Bethe-Salpeter equation are highly desirable [1, 6, 11–13]. It has been shown [14] that there exist infinite versions of the reduced Bethe-Salpeter equation. The Salpeter equation is the most famous one, which is based on the instantaneous approximation [6]. In this paper, the Salpeter equation is employed.

Let us briefly review the Salpeter equation in this section. The Salpeter equation for a fermion-antifermion bound state reads in covariant form [10]

$$\psi_P(p_\perp) = \frac{\Lambda_1^+(p_\perp)\ \hat{\not P}\Gamma(p_\perp)\ \hat{\not P}\Lambda_2^-(-p_\perp)}{M-\omega_1-\omega_2} - \frac{\Lambda_1^-(p_\perp)\ \hat{\not P}\Gamma(p_\perp)\ \hat{\not P}\Lambda_2^+(-p_\perp)}{M+\omega_1+\omega_2}, \tag{1}$$

where M is the mass of the bound state, P is the bound-state momentum, and p is the relative momentum. $p_\parallel$ is the longitudinal part of p and parallel to P, $p_\perp$ is the transverse part of p and perpendicular to P [5, 11, 15]

$$\hat{P} = \frac{P}{M}, \quad M = \sqrt{P^2}, \quad p_l = p\cdot\hat{P}, \quad p = p_\parallel + p_\perp,$$
$$p_\parallel = p_l\hat{P}, \quad p_\perp = p - p_l\hat{P}, \quad d^4p = dp_l d^3p_\perp. \tag{2}$$

In the rest frame of the bound state with momentum $P = (M, \mathbf{0})$, $p_l = p^0$, $p_\parallel = (p^0, \mathbf{0})$ and $p_\perp = (0, \mathbf{p})$. $\Gamma(p_\perp)$ is defined as

$$\Gamma(p_\perp) = \int \frac{d^3p'_\perp}{(2\pi)^3} V(p_\perp, p'_\perp)\psi_P(p'_\perp). \tag{3}$$

The projection operators are written in covariant form as

$$\Lambda_i^\pm(p_\perp) = \frac{\omega_i \pm H_i(p_\perp)}{2\omega_i}, \quad H_i(p_\perp) = \hat{\not P}(m_i - \not p_\perp),$$
$$\omega_i = \sqrt{m_i^2+\varpi^2}, \qquad \varpi = \sqrt{-p_\perp^2} \tag{4}$$

with the properties

$$\Lambda_i^{\mp}(p_\perp)\Lambda_i^{\pm}(p_\perp) = 0,$$
$$\Lambda_i^{+}(p_\perp) + \Lambda_i^{-}(p_\perp) = 1,$$
$$\Lambda_i^{\pm}(p_\perp)\Lambda_i^{\pm}(p_\perp) = \Lambda_i^{\pm}(p_\perp),$$
$$H_i(p_\perp)\Lambda_i^{\pm}(p_\perp) = \pm\omega_i\Lambda_i^{\pm}(p_\perp). \quad (5)$$

In case of massless constituents, $m_1 = m_2 = 0$, $\omega_1 = \omega_2 = \varpi$, $\Lambda_1^{\mp}(p_\perp) = \Lambda_2^{\mp}(p_\perp)$.

Applying the energy projectors $\Lambda_1^{\pm}(p_\perp)$ from the left hand side and $\Lambda_2^{\pm}(-p_\perp)$ from the right hand side to the Salpeter equation (1) leads to

$$(M - \omega_1 - \omega_2)\psi_P^{+-}(p_\perp) = \Lambda_1^{+}(p_\perp)\ \hat{\not{P}}\Gamma(p_\perp)\ \hat{\not{P}}\Lambda_2^{-}(-p_\perp),$$
$$(M + \omega_1 + \omega_2)\psi_P^{-+}(p_\perp) = -\Lambda_1^{-}(p_\perp)\ \hat{\not{P}}\Gamma(p_\perp)\ \hat{\not{P}}\Lambda_2^{+}(-p_\perp) \quad (6)$$

together with the constraints on the Salpeter wave function

$$\psi_P^{++}(p_\perp) = \psi_P^{--}(p_\perp) = 0, \quad (7)$$

where $\psi_P^{\pm\pm}(p_\perp) = \Lambda_1^{\pm}(p_\perp)\psi_P(p_\perp)\Lambda_2^{\pm}(-p_\perp)$.

Let the bound state be normalized as $\langle P|P'\rangle = (2\pi)^3 2P_0\delta(\mathbf{P} - \mathbf{P}')$. Then the explicit form of the normalization condition for the Salpeter wave function reads

$$\int \frac{d^3p_\perp}{(2\pi)^3}\mathrm{Tr}\Big\{\ \hat{\not{P}}\overline{\psi}(p_\perp)\ \hat{\not{P}}\Lambda_1^{+}(p_\perp)\psi(p_\perp)\Lambda_2^{-}(-p_\perp) - \hat{\not{P}}\overline{\psi}(p_\perp)\ \hat{\not{P}}\Lambda_1^{-}(p_\perp)\psi(p_\perp)\Lambda_2^{+}(-p_\perp)\Big\} = 2M, \quad (8)$$

where $\overline{\psi}(p_\perp) = \gamma^0\psi^\dagger(p_\perp)\gamma^0$.

3. Salpeter equation for massless constituents

In this section, we will consider the Salpeter wave function and Salpeter equation for bound state with massless constituents. In the ultrarelativistic limit $m_1 = m_2 = 0$, the constraints on the Salpeter wave function as well as the Salpeter equation will be different from that obtained in the nonrelativistic limit.

3.1. State 0^+

For state 0^+, the Salpeter wave function reads

$$\psi^{0^+}(p_\perp) = g_1 + \hat{\not{P}}g_2 + \hat{\not{p}}_\perp g_5 + \hat{\not{p}}_\perp\ \hat{\not{P}}g_6, \quad (9)$$

where $\hat{p}_\perp^\mu = p_\perp^\mu/\varpi$. When the scalar functions g_i are not to be integrated, $g_i \equiv g_i(\varpi)$, $i = 1,2,5,6$, and when g_i are to be integrated, $g_i \equiv g_i(\varpi')$, $\varpi' = \sqrt{-p'^2_\perp}$. g_1 and g_2 are S wave components, while g_5 and g_6 are P wave components. Applying the constraints (7) on the Salpeter wave function (9) yields

$$g_2 = g_5 = 0. \quad (10)$$

The normalization condition reads

$$\int \frac{d^3p_\perp}{(2\pi)^3}4g_1g_6 = M. \quad (11)$$

For simplicity, taken as an example, the interaction kernel takes the form

$$V(p_\perp - p'_\perp) = V_s + \gamma^0\otimes\gamma^0V_0 + \gamma_\mu\otimes\gamma^\mu V_v, \quad (12)$$

where V_s is scalar function corresponding to interactions of the scalar type, V_0 is the time-component Lorentz-vector part, and V_v is the Lorentz-vector part. Other types of interaction kernels can be treated in similar way. The coupled equations for 0^+ are obtained from Eqs. (6) and (9)

$$Mg_1 = 2\varpi g_6 - \int\frac{d^3p'_\perp}{(2\pi)^3}(V_0 - V_s)\hat{p}_\perp\cdot\hat{p}'_\perp g_6,$$
$$Mg_6 = 2\varpi g_1 + \int\frac{d^3p'_\perp}{(2\pi)^3}(V_0 + V_s + 4V_v)g_1. \quad (13)$$

In the nonrelativistic limit, state 0^+ is P wave state, the P wave components g_5 and g_6 are large terms, while the S wave components g_1 and g_2 are mall and are relativistic corrections to g_5 and g_6. But in the ultrarelativistic limit, $g_2 = g_5 = 0$, g_1 and g_6 are large terms, and the coupled Eqs. (13) are on g_1 and g_6. In the ultrarelativistic limit, the energy projection operator $\Lambda_i^{\pm}(p_\perp) = (1\mp\ \hat{\not{P}}\ \hat{\not{p}}_\perp)/2$ and Eqs. (6), (7) choose the g_1 and g_6 as large terms. In the nonrelativistic limit, we can find that g_1 and g_2 are chosen as main terms by inspecting the energy projection operator $\Lambda_i^{\pm}(p_\perp) = (1\pm\ \hat{\not{P}})/2$, the Salpeter wave function (9) and the Salpeter Eq. (6).

3.2. State 0^-

For state 0^-, the Salpeter wave function reads

$$\psi^{0^-}(p_\perp) = \gamma^5\left[g_1 + \hat{\not{P}}g_2 + \hat{\not{p}}_\perp g_5 + \hat{\not{p}}_\perp\ \hat{\not{P}}g_6\right]. \quad (14)$$

The constraints on the Salpeter wave function (14) are

$$g_2 = g_5 = 0. \quad (15)$$

The normalization condition is

$$\int \frac{d^3p_\perp}{(2\pi)^3}4g_1g_6 = M. \quad (16)$$

The coupled equations read

$$Mg_1 = 2\varpi g_6 - \int\frac{d^3p'_\perp}{(2\pi)^3}(V_0 + V_s)\hat{p}_\perp\cdot\hat{p}'_\perp g_6,$$
$$Mg_6 = 2\varpi g_1 + \int\frac{d^3p'_\perp}{(2\pi)^3}(V_0 - V_s + 4V_v)g_1. \quad (17)$$

In the nonrelativistic limit, state 0^- is S wave state, therefore, the S wave components g_1 and g_2 are large terms, while the P wave components g_5 and g_6 are small and corrections to the S wave components. Different from the case of the nonrelativistic limit, g_1 and g_6 are large terms in the ultrarelativistic limit.

It is interesting that for both psudoscalar state 0^- and scalar state 0^+, the constraints and the normalization conditions are the same, see Eqs. (10), (11), (15) and (16). In case of the nonrelativistic limit, state 0^+ is P wave state and state 0^- is S wave state. But in case of the massless constituents, the wave components are the same for both state 0^- and state 0^+, the S wave component g_1 and the P wave component g_6 are large. Moreover, if the interaction is vector or time component of vector, Eqs. (13) and (17) imply the existence of the degeneracy of the spectra, but the scalar interaction V_s will destroy the degeneracy.

3.3. State with parity $\eta_P = (-1)^j$

For state with parity $\eta_P = (-1)^j$ ($j > 0$), the general form of the Salpeter wave function reads [8, 9]

$$\begin{aligned}\psi^j(p_\perp) &= \epsilon_{\mu_1\cdots\mu_j}\hat{p}_\perp^{\mu_2}\cdots\hat{p}_\perp^{\mu_j}\Big[\hat{p}_\perp^{\mu_1}(g_1+\hat{\not{P}}g_2)\\ &+\gamma_\perp^{\mu_1}(g_3+\hat{\not{P}}g_4)+\left(\hat{p}_\perp^{\mu_1}\ \hat{\not{p}}_\perp+\frac{j}{2j+1}\gamma^{\mu_1}\right)\\ &\times(g_5+\hat{\not{P}}g_6)+\sigma^{\mu_1\nu}\hat{p}_{\perp\nu}(g_7+\hat{\not{P}}g_8)\Big],\end{aligned}\tag{18}$$

where $\gamma_\parallel^\mu = \hat{\not{P}}\hat{P}^\mu$, $\gamma_\perp^\mu = \gamma^\mu - \gamma_\parallel^\mu$, $\sigma^{\mu\nu} = [\gamma_\perp^\mu, \gamma_\perp^\nu]$, $g_i \equiv g_i(\varpi)$, $i = 1, 2, 3, 4, 5, 6, 7, 8$. g_3, g_4 are pure $L = j-1$ wave components, and g_5, g_6 are pure $L = j+1$ states. g_1, g_2, g_7 and g_8 are $L = j$ wave components. In the nonrelativistic limit, g_3, g_4, g_5 and g_6 are main terms [9], while g_1, g_2, g_7, and g_8 are small terms, which are relativistic corrections in wave function.

The constraints on the Salpeter wave function (18) read in the ultrarelativistic limit

$$g_2 = g_7 = 0, \quad g_4 = -\frac{j}{2j+1}g_6, \quad g_3 = \frac{j+1}{2j+1}g_5. \tag{19}$$

In the nonrelativistic limit, for state with parity $(-1)^j$, the $L = j-1$ wave components g_3, g_4 and the $L = j+1$ wave components g_5, g_6 are large. But in case of massless constituents, $L = j$ wave components g_1 and g_8 are large which are small in the nonrelativistic limit.

Using Eqs. (8) and (18), the normalization condition can be obtained

$$\int \frac{d^3p_\perp}{(2\pi)^3}4\left[S_1^j g_1 g_6 - (S_1^j + S_2^j)g_5 g_8\right] = M, \tag{20}$$

where [16]

$$\begin{aligned}S_1^j &= \sum \mathcal{P}_{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}\hat{p}_\perp^{\mu_1}\cdots\hat{p}_\perp^{\mu_j}\hat{p}_\perp^{\nu_1}\cdots\hat{p}_\perp^{\nu_j},\\ S_2^j &= \sum \mathcal{P}_{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}g^{\mu_1\nu_1}\hat{p}_\perp^{\mu_2}\cdots\hat{p}_\perp^{\mu_j}\hat{p}_\perp^{\nu_2}\cdots\hat{p}_\perp^{\nu_j}.\end{aligned}\tag{21}$$

The definition of $\mathcal{P}_{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}$ in Eq. (21) is in appendix. Using Eqs. (6) and (18), the coupled equations can be obtained

$$\begin{aligned}Mg_1 &= 2\varpi g_6 - \int\frac{d^3p'_\perp}{(2\pi)^3}\frac{T_1^j}{S_1^j}(V_0 - V_s)\hat{p}_\perp\cdot\hat{p}'_\perp g_6,\\ Mg_6 &= 2\varpi g_1 + \int\frac{d^3p'_\perp}{(2\pi)^3}\frac{T_1^j}{S_1^j}(V_0 + V_s + 4V_v)g_1,\\ Mg_5 &= 2\varpi g_8 - \int\frac{d^3p'_\perp}{(2\pi)^3}\frac{(\hat{p}_\perp\cdot\hat{p}'_\perp T_3^j - T_5^j)}{(S_1^j + S_2^j)}\\ &\quad\times(V_0 + V_s + 2V_v)\,g_8,\\ Mg_8 &= 2\varpi g_5 + \int\frac{d^3p'_\perp}{(2\pi)^3}\frac{(\hat{p}_\perp\cdot\hat{p}'_\perp T_1^j + T_2^j + T_3^j + T_4^j)}{(S_1^j + S_2^j)}\\ &\quad\times(V_0 - V_s + 2V_v)g_5,\end{aligned}\tag{22}$$

where

$$\begin{aligned}T_1^j &= \sum \mathcal{P}_{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}\hat{p}_\perp^{\mu_1}\cdots\hat{p}_\perp^{\mu_j}\hat{p}'^{\nu_1}_\perp\cdots\hat{p}'^{\nu_j}_\perp,\\ T_2^j &= \sum \mathcal{P}_{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}\hat{p}'^{\mu_1}_\perp\hat{p}'^{\nu_1}_\perp\hat{p}_\perp^{\mu_2}\cdots\hat{p}_\perp^{\mu_j}\hat{p}'^{\nu_2}_\perp\cdots\hat{p}'^{\nu_j}_\perp,\\ T_3^j &= \sum \mathcal{P}_{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}g^{\mu_1\nu_1}\hat{p}_\perp^{\mu_2}\cdots\hat{p}_\perp^{\mu_j}\hat{p}'^{\nu_2}_\perp\cdots\hat{p}'^{\nu_j}_\perp.\\ T_4^j &= \sum \mathcal{P}_{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}\hat{p}_\perp^{\mu_1}\hat{p}_\perp^{\nu_1}\hat{p}_\perp^{\mu_2}\cdots\hat{p}_\perp^{\mu_j}\hat{p}'^{\nu_2}_\perp\cdots\hat{p}'^{\nu_j}_\perp,\\ T_5^j &= \sum \mathcal{P}_{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}\hat{p}'^{\mu_1}_\perp\hat{p}_\perp^{\nu_1}\hat{p}_\perp^{\mu_2}\cdots\hat{p}_\perp^{\mu_j}\hat{p}'^{\nu_2}_\perp\cdots\hat{p}'^{\nu_j}_\perp.\end{aligned}\tag{23}$$

In Eq. (22), there are two sets of coupled equations in which g_1 and g_6 are coupled, and g_5 and g_8 are coupled, but g_1, g_6 and g_5, g_8 are decoupled. When not considering the nonrelativistic limit or ultrarelativistic limit, the obtained coupled equations are on g_3, g_4, g_5 and g_6, which are coupled to each other, *i.e.*, the $L = j-1$ wave components g_3 and g_4 and the $L = j+1$ wave components g_5 and g_6 are coupled. In the nonrelativistic limit, the $L = j-1$ wave components and the $L = j+1$ wave components are decoupled, and the $L = j$ wave components are small terms. But in the ultrarelativistic limit, the $L = j$ wave components g_1 and g_8 become large.

The constraints (19) rule out the exotic states with parity $\eta_P = (-1)^j$ and charge-conjugation parity $\eta_P = (-1)^{j+1}$

[8, 9], *i.e.*, such states cannot be constructed by the Salpeter equation, which is consistent with the results obtained by L-S coupling analysis in the nonrelativistic limit.

3.4. State with parity $\eta_P = (-1)^{j+1}$

For state with parity $\eta_P = (-1)^{j+1}$ ($j > 0$), the general form of the Salpeter wave function reads

$$\psi^j(p_\perp) = \gamma^5 \epsilon_{\mu_1\cdots\mu_j} \hat{p}_\perp^{\mu_2} \cdots \hat{p}_\perp^{\mu_j} \Big[\hat{p}_\perp^{\mu_1}(g_1 + \hat{\not{P}} g_2) + \gamma^{\mu_1}(g_3 + \hat{\not{P}} g_4) + \left(\hat{p}_\perp^{\mu_1} \hat{\not{p}}_\perp + \frac{j}{2j+1}\gamma^{\mu_1} \right) \times (g_5 + \hat{\not{P}} g_6) + \sigma^{\mu_1\nu}\hat{p}_{\perp\nu}(g_7 + \hat{\not{P}} g_8) \Big], \tag{24}$$

where g_1, g_2, g_7, and g_8 are main terms which are pure $L = j$ wave component [9]. While g_3, g_4, g_5 and g_6 are small terms, which are relativistic corrections in wave function.

The constraints on the Salpeter wave function (24) read

$$g_2 = g_7 = 0, \quad g_4 = -\frac{j}{2j+1} g_6, \quad g_3 = \frac{j+1}{2j+1} g_5. \tag{25}$$

which are the same as the constraints for state with parity $(-1)^j$, see Eq. (19).

In case of the ultrarelativistic limit, the normalization condition reads

$$\int \frac{d^3 p_\perp}{(2\pi)^3} 4 \left[S_1^j g_1 g_6 - (S_1^j + S_2^j) g_5 g_8 \right] = M, \tag{26}$$

which is the same as Eq. (20).

The coupled equations read

$$\begin{aligned}
Mg_1 &= 2\varpi g_6 - \int \frac{d^3 p'_\perp}{(2\pi)^3} \frac{T_1^j}{S_1^j} (V_0 + V_s) \hat{p}_\perp \cdot \hat{p}'_\perp g_6, \\
Mg_6 &= 2\varpi g_1 + \int \frac{d^3 p'_\perp}{(2\pi)^3} \frac{T_1^j}{S_1^j} (V_0 - V_s + 4V_v) g_1, \\
Mg_5 &= 2\varpi g_8 - \int \frac{d^3 p'_\perp}{(2\pi)^3} \frac{(\hat{p}_\perp \cdot \hat{p}'_\perp T_3^j - T_5^j)}{(S_1^j + S_2^j)} \\
&\quad \times (V_0 - V_s + 2V_v) g_8, \\
Mg_8 &= 2\varpi g_5 + \int \frac{d^3 p'_\perp}{(2\pi)^3} \frac{(\hat{p}_\perp \cdot \hat{p}'_\perp T_1^j + T_2^j + T_3^j + T_4^j)}{(S_1^j + S_2^j)} \\
&\quad \times (V_0 + V_s + 2V_v) g_5.
\end{aligned} \tag{27}$$

Eqs. (22) and (27) are different only in the sign of V_s term.

In the nonrelativistic limit, the constraints on the Salpeter wave functions, the normalization conditions and the spectra for the states with parity $(-1)^j$ and $(-1)^{j+1}$ are different [10]. But in the ultrarelativistic limit, the constraints on the Salpeter wave functions and the normalization conditions are the same for states with different parity, see Eqs. (19), (20), (25) and (26). Moreover, we can obtain from Eqs. (13), (17), (22) and (27) that there are degenerate doubles with the same spin but with different parity if the interaction is vector or time-component of vector. And the scalar interaction will destroy this degeneracy. These results maybe be only of academic interest. Nevertheless, it is instructive to pursue the insight of the bound states in the ultrarelativistic limit.

4. Conclusion

In this paper, we have presented the reduction of the Salpeter equation for the massless constituents. It is shown that $L = j$ and $L = j + 1$ wave components play main roles for both states with different parity in the ultrarelativistic limit while in the nonrelativistic limit, $L = j$ wave components are large terms for $\eta_P = (-1)^{j+1}$ state, and $L = j \pm 1$ wave components are main terms for $\eta_P = (-1)^j$ state. And in the ultrarelativistic limit, there exists degeneracy of the spectra of the states with the same spin but with different parity if the interaction is vector or time component of vector. However, the scalar interaction will destroy the degeneracy.

Appendix

A. Polarization tensor

For completeness, we list the formulas which are useful in this paper. It is known that the polarization tensor is totally symmetric, transverse, and traceless, *i.e.*,

$$\epsilon_{\mu_1\mu_2\cdots} = \epsilon_{\mu_2\mu_1\cdots}, \quad P^{\mu_1}\epsilon_{\mu_1\mu_2\cdots} = 0, \quad \epsilon^{\mu}_{\mu\nu\cdots} = 0. \tag{A.1}$$

The usual spin-1 polarization vector $\epsilon^\mu(j_m)$ obeys the relations [17]

$$P^\mu \epsilon_\mu(j_m) = 0, \quad \sum_{j_m} \epsilon^*_\mu(j_m)\epsilon_\nu(j_m) = \mathcal{P}_{\mu\nu}, \quad \mathcal{P}_{\mu\nu} \equiv -g_{\mu\nu} + \frac{P_\mu P_\nu}{M^2}. \tag{A.2}$$

The polarization tensor $\epsilon^{\mu\nu}(j_m)$ is for a particle of spin-2 and it obeys

$$P^\mu \epsilon_{\mu\nu}(j_m) = 0, \qquad \epsilon^{\mu\nu} = \epsilon^{\nu\mu}, \qquad \epsilon^\mu_\mu = 0,$$
$$\sum_{j_m} \epsilon^*_{\mu\nu}(j_m)\epsilon_{\alpha\beta}(j_m) = \frac{1}{2}\left(\mathcal{P}_{\mu\alpha}\mathcal{P}_{\nu\beta} + \mathcal{P}_{\nu\alpha}\mathcal{P}_{\mu\beta}\right) - \frac{1}{3}\mathcal{P}_{\mu\nu}\mathcal{P}_{\alpha\beta}. \tag{A.3}$$

For integer spin, the expression of $\mathcal{P}^{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}(j, P)$ reads [16]

$$\mathcal{P}^{\mu_1\cdots\mu_j\nu_1\cdots\nu_j}(j,P)=\sum_{j_z}\epsilon^{*\mu_1\cdots\mu_j}\epsilon^{\nu_1\cdots\nu_j}=\left(\frac{1}{j!}\right)^2\sum_{\substack{P(\mu)\\P(\nu)}}\left[\prod_{i=1}^{j}\mathcal{P}^{\mu_i\nu_i}+a_1^j\mathcal{P}^{\mu_1\mu_2}\mathcal{P}^{\nu_1\nu_2}\prod_{i=3}^{j}\mathcal{P}^{\mu_i\nu_i}+\cdots\right.$$
$$+a_r^j\mathcal{P}^{\mu_1\mu_2}\mathcal{P}^{\nu_1\nu_2}\ldots\mathcal{P}^{\mu_{2r-1}\mu_{2r}}\mathcal{P}^{\nu_{2r-1}\nu_{2r}}\prod_{i=2r+1}^{j}\mathcal{P}^{\mu_i\nu_i}+\cdots$$
$$\left.+\begin{cases}a_{j/2}^j\mathcal{P}^{\mu_1\mu_2}\mathcal{P}^{\nu_1\nu_2}\ldots\mathcal{P}^{\mu_{j-1}\mu_j}\mathcal{P}^{\nu_{j-1}\nu_j}, & \text{for even j}\\ a_{(j-1)/2}^j\mathcal{P}^{\mu_1\mu_2}\mathcal{P}^{\nu_1\nu_2}\ldots\mathcal{P}^{\mu_{j-2}\mu_{j-1}}\mathcal{P}^{\nu_{j-2}\nu_{j-1}}\mathcal{P}^{\mu_j\nu_j}, & \text{for odd j}\end{cases}\right], \tag{A.4}$$

where the sum is over all permutations of μ and ν, and

$$a_r^j=\left(-\frac{1}{2}\right)^r\frac{j!}{r!(j-2r)!}\frac{(2j-2r-1)!!}{(2j-1)!!}. \tag{A.5}$$

In the above formula, $n!$ gives the factorial of n, $n!=n(n-1)\cdots$, and $n!!$ gives the double factorial of n, $n!!=n(n-2)\cdots$.

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