A mapping between Lorentz-violating and conventional electrodynamics

Ralf Lehnert
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México,
Apartado Postal 70-543, México, D.F., 04510, Mexico.

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The Chern–Simons-type term in the photon sector of the Lorentz- and CPT-breaking minimal Standard-Model Extension (mSME) is considered. It is argued that under certain circumstances this term can be removed from the mSME. In particular, it is demonstrated that for lightlike Lorentz violation a field redefinition exists that maps the on-shell free Chern–Simons model to conventional on-shell free electrodynamics. A compact explicit expression for an operator implementing such a mapping is constructed. This expression establishes that the field redefinition is non-local.

Keywords: Lorentz violation; CPT violation; Standard-Model Extension; field redefinition.

Se considera el término de tipo Chern–Simons en el sector fotónico de la extensión mínima del modelo estándar con rompimiento de Lorentz y de CPT. Se discuten las circunstancias bajo las cuales este término se puede remover del modelo. En particular, se demuestra que para una violación de Lorentz parametrizada por un cuadrivector tipo luz existe una redefinición de campo que proyecta el modelo de Chern–Simons libre en la capa de masas a la electrodinámica convencional libre en la capa de masas. Se construye una expresión compacta, explícita para un operador implementando tal proyección. Esta expresión establece que la redefinición de campo es no local.

Descriptores: Violación de Lorentz; violación CPT; Modelo Estándar Extendido; redefinición de campo.

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1. Introduction

Despite its phenomenological successes, the present framework for fundamental physics—the Standard Model of particle physics together with the General Theory of Relativity—leaves unanswered various conceptual questions. For this reason, a substantial amount of theoretical work is currently being devoted to the search for an underlying theory that provides a quantum description of gravity. Experimental tests of such ideas face, however, a considerable obstacle of practical nature: most quantum-gravity predictions in virtually every leading candidate model are expected to be extremely small due to the anticipated Planck-scale suppression.

During the last two decades, a variety of theoretical investigations have suggested the possibility of spacetime-symmetry breakdown in leading candidate models for underlying physics. Examples of such investigations involve string field theory [1], realistic field theories on noncommutative spacetimes [2], cosmologically varying scalars [3], various quantum-gravity models [4], four-dimensional spacetimes with a nontrivial topology [5], random-dynamics models [6], multiverses [7], and brane-world scenarios [8]. Although the dynamical structures underlying the above models typically remain Lorentz symmetric, Lorentz and CPT violation can nevertheless occur in the ground state at low energies. These ideas provide a key motivation for Lorentz- and CPT-violation searches in the context of quantum gravity.

At energies that can currently be reached in experimental situations, the effects resulting from Lorentz and CPT violation in underlying physics can be described by the Standard-Model Extension (SME), which is an effective-field-theory framework containing the usual Standard Model [9] and General Relativity [10] as limiting cases. The minimal SME (mSME), which only contains relevant and marginal operators, has provided the basis for numerous experimental investigations of Lorentz- and CPT-symmetry [11]. Specific studies include, for instance, ones with photons [12,13], neutrinos [14], electrons [15], protons and neutrons [16], mesons [17], muons [18], and gravity [19]. Several of the obtained experimental limits can be regarded as testing Planck-scale physics.

Internal consistency and a thorough theoretical understanding are of key importance for test models, such as the SME. For this reason, a number of SME investigations have addressed such questions [20-22]. Some of these studies have shed light on various conceptual questions, but so far none have suggested any internal inconsistencies. It nevertheless remains necessary to keep illuminating the internal structure of the SME, both to gain insight into Lorentz and CPT violation and to solidify further the theoretical basis of the SME. In this context, the theory of free particles is of particular interest: They correspond to external legs in Feynman diagrams and are therefore an important theoretical ingredient in perturbative QFT. Moreover, many experimental studies, such as kinematical cosmic-ray tests of Lorentz symmetry, rely to a large extent on free-particle physics. This work aims at illuminating various aspects of the Chern–Simons-type term contained in the free-photon sector of the mSME. More specifically, we will show that this term can be removed on-shell from this sector under certain conditions.

The outline of this work is as follows. The Chern–Simons limit of the mSME is briefly reviewed in Sec. 2. Section 3 argues that for lightlike Lorentz violation, the Chern–Simons term can be removed on-shell from the free model, and the idea behind the associated mapping is illustrated. Section 4 derives a compact expression for this field-redefinition mapping. A summary and a brief outlook are contained in Sec. 5.

2. Basics

A particularly popular limit of the mSME is the Lorentz- and CPT-violating Chern–Simons extension of electrodynamics. This limit will be the focus of the subsequent discussion in this work, so we begin by reviewing various known results pertaining to Chern–Simons electrodynamics. Adopting natural units $c = \hbar = 1$ and the metric signature $(+,-,-,-)$, the free model Lagrangian is given by [26]

$$\mathcal{L}_{\mathrm{MCS}} = -\frac{1}{4}F^2 + (k_{AF})^{\mu} A^{\nu} \tilde{F}_{\mu\nu}, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{F}^{\mu\nu} = (1/2)\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$, as usual. The nondynamical fixed $(k_{AF})^\mu$ selects a preferred direction in spacetime explicitly breaking Lorentz and CPT symmetry. In what follows, we will drop the AF subscript for brevity. Although this Lagrangian is gauge dependent, the corresponding action integral, and therefore the physics, are invariant under gauge transformations.

The Euler–Lagrange equations associated with the Lagrangian (1) yield the following equations of motion for the potentials $A^\mu = (A^0, \vec{A})$:

$$\left(\Box\eta^{\mu\nu} - \partial^\mu\partial^\nu - 2\,\varepsilon^{\mu\nu\rho\sigma} k_\rho \partial_\sigma\right) A_\nu = 0\,. \tag{2}$$

From Eq. (2), the Chern–Simons modified Maxwell equations

$$\partial_\mu F^{\mu\nu} + 2\,k_\mu \tilde{F}^{\mu\nu} = 0 \tag{3}$$

can be derived, which put into evidence the gauge invariance of the model. For completeness, we also exhibit the modified Coulomb and Ampère laws, which are contained in Eq. (3):

$$\begin{aligned} \vec{\nabla}\cdot\vec{E} - 2\vec{k}\cdot\vec{B} = 0\,, \\ -\dot{\vec{E}} + \vec{\nabla}\times\vec{B} - 2k_0\vec{B} + 2\vec{k}\times\vec{E} = 0\,. \end{aligned} \tag{4}$$

The homogeneous Maxwell equations are left unchanged because the relationship between the fields and potentials is the conventional one.

Paralleling the ordinary Maxwell case, A^0 is nondynamical, and gauge symmetry eliminates another mode of A^μ, so that Eq. (2) contains only two independent degrees of freedom. It is often convenient to fix a gauge, and we will actually do so in the next section. It turns out that any of the usual conditions on A^μ, such as Lorentz or Coulomb gauge, can be imposed. We remark, however, that there are some differences between conventional electrodynamics and the Chern–Simons model regarding the equivalence of certain gauge choices. A more detailed discussion of the degrees of freedom and the gauge-fixing process can be found in the second paper of Ref. [9].

The plane-wave dispersion relation can be obtained with the ansatz $A^\mu(x) = \varepsilon^\mu(\lambda)\exp(-i\lambda\cdot x)$, where $\lambda^\mu \equiv (\omega, \vec{\lambda})$. This ansatz and the equations of motion (2) give

$$\lambda^4 + 4\lambda^2 k^2 - 4(\lambda\cdot k)^2 = 0\,. \tag{5}$$

This equation determines the wave frequency ω as a function of the wave 3-vector $\vec{\lambda}$.

Examples of non-standard effects caused by the inclusion of the Chern–Simons term into electrodynamics are vacuum birefringence [26] and vacuum Cherenkov radiation [13]. We also mention that for a timelike k^μ, the magnitude of the group velocity determined by the dispersion relation (5) can in certain circumstances exceed the light speed c. This is consistent with previous analyses [26,27], which have established theoretical difficulties associated with instabilities and causality violations for $k^2 > 0$. Such issues do not arise for $k^2 \le 0$. In what follows, we focus primarily on the case of a lightlike k^μ. We remark in passing that the lightlike case possesses more residual spacetime symmetries than the timelike and spacelike cases [22].

3. General idea behind the mapping

Our goal is to establish that a lightlike k^μ can be removed from the (free) equations of motions (2) by an on-shell field redefinition. This section discusses certain features of the solutions to Eq. (2) that give some intuition as to why k^μ is removable, thereby motivating the form of the field redefinition.

For a lightlike k^μ, the dispersion relation (5) can be cast into the form

$$\left[\lambda^\mu + (-1)^a k^\mu\right]^2 = 0\,, \tag{6}$$

where $a = 1, 2$. This equation possesses the roots

$$(\lambda_a^\pm)^\mu = \Big((\lambda_a^\pm)^0(\vec{\lambda}), \vec{\lambda}\Big) \tag{7}$$

with

$$(\lambda_a^\pm)^0(\vec{\lambda}) = \pm\big|\,\vec{\lambda} \pm (-1)^a\vec{k}\,\big| \mp (-1)^a k^0\,. \tag{8}$$

Here, $k^\mu = (k^0, \vec{k})$ is lightlike $|k^0| = |\vec{k}|$, and we have chosen a more convenient labeling of these solutions than that given in Eq. (6). The roots $(\lambda_a^\pm)^\mu$ may alternatively be parametrized as

$$(\lambda_a^\pm)^\mu = p_\pm^\mu \mp (-1)^a k^\mu\,. \tag{9}$$

Here, $p_\pm^\mu \equiv (\pm|\vec{p}|, \vec{p})$ satisfies the conventional photon dispersion relation $(p_\pm)^2 = 0$, where the subscripts $+$ and $-$ label the positive- and negative-frequency solutions, respectively. This result is immediately evident from Eq. (6) when the expression inside the square brackets is identified with the appropriate $p_\pm^\mu$.

Equation (7) determines four roots labeled by $a = 1, 2$ and the subscript $\pm$, which are seemingly independent. This reflects the fact that the dispersion relation (5) is quartic in λ^0. One might then wonder whether this is consistent with the previous statement in Sec. 2 that our Chern–Simons model contains two independent degrees of freedom, just like conventional electrodynamics. To resolve this apparent contradiction, the two negative-frequency roots must be properly interpreted. To this end, recall that in QFT the negative-energy solutions are regarded as positive-energy reversed-momentum states corresponding to antiparticles. Defining $(\lambda_a^{\pm})^0(\vec{\lambda}) \equiv \pm\omega_a(\pm\vec{q})$ leaves unaffected the positive-frequency roots in Eq. (7)

$$\omega_a(\vec{q}) = \big|\,\vec{q} + (-1)^a\vec{k}\,\big| - (-1)^a k^0 \,. \tag{10}$$

The negative-valued roots given by the lower sign in Eq. (7) take the form

$$-\omega_a(\vec{q}) = -\big|\,-\vec{q} - (-1)^a\vec{k}\,\big| + (-1)^a k^0 \,. \tag{11}$$

Inspection shows that with this conventional reinterpretation Eqs. (9) and (10) are identical. Moreover, there are only two independent polarization vectors (A^0 is non-dynamical, and a choice of gauge places an additional constraint on A^μ). This establishes that the physics described by the two negative-energy solutions must be identical to the physics contained in the two positive-energy solutions, as expected. In what follows, we may therefore focus solely on the positive-frequency solutions and omit the subscript $\pm$ from now on. We remark that $a = 1, 2$ labels the two helicity-type polarizations states of plane waves.

The above reparametrization (8) may then be taken to read $(\lambda_a)^\mu = p^\mu - (-1)^a k^\mu$, where $p^\mu \equiv p_+^\mu$ has a positive-valued zeroth component and continues to satisfy $p^\mu p_\mu = 0$. This reparametrization is the key to an intuitive understanding of the field redefinition that removes k^μ from the equations of motion: up to polarization vectors, any plane-wave exponential that solves Eq. (2) is of the form

$$\exp(-i\lambda_a\!\cdot\! x) = \exp(-ip\!\cdot\! x)\exp[+(-1)^a ik\!\cdot\! x]\,. \tag{12}$$

Note that $\exp(-ip\cdot x)$ corresponds to Lorentz-symmetric massless plane waves; the Lorentz-violating contribution $\exp[+(-1)^a ik\cdot x]$ can be removed via a field redefinition

$$\text{(plane wave)} \;\to\; \text{(plane wave)}\,\exp\left[-(-1)^a ik\!\cdot\! x\right]. \tag{13}$$

We remark that this field redefinition depends only on the plane-wave label a; it is independent of the plane-wave momentum. In other words, any superpositions of plane-wave exponentials with label $a = 1$ can be redefined by removing the common $\exp(-ik\cdot x)$ factor, and superpositions of plane-wave exponentials with labels $a = 2$ can be redefined by removing the common $\exp(+ik\cdot x)$ factor.

To make this idea more precise, consider the general explicit solution to the free equations of motions:

$$A^\mu(x) = \int \frac{d^3\vec{\lambda}}{(2\pi)^3} \times \sum_{a=1,2}\left[\epsilon_a^\mu(\vec{\lambda})e^{-i\lambda_a\cdot x} + \epsilon_a^{\mu *}(\vec{\lambda})e^{+i\lambda_a\cdot x}\right]. \tag{14}$$

The polarization vectors $\epsilon_a^\mu(\vec{\lambda})$ are constrained by the equations of motion, the gauge choice, and—in cases of degeneracy—by the requirement of linear independence. We have absorbed the relativistic normalization factor of the integration measure into the definition of the $\epsilon_a^\mu(\vec{\lambda})$, so that they do not transform as 4-vectors. In what follows, we will nevertheless continue to refer to these quantities as polarization vectors.

With Eq. (8) at hand, we may change integration variables from $\vec{\lambda}$ to $\vec{p}$ in Eq. (13). Note that this is just a linear shift, so that the Jacobian is trivial. The exponentials will now contain a $\vec{p}$-independent piece, which can be pulled out of the integral:

$$A^\mu(x) = \mathcal{A}^\mu(x)\exp(-ik\!\cdot\! x) + \mathcal{A}^{\mu *}(x)\exp(+ik\!\cdot\! x)\,, \tag{15}$$

where

$$\mathcal{A}^\mu(x) \equiv \int \frac{d^3p}{(2\pi)^3}\left[\xi_1^\mu(\vec{p})e^{-ip\cdot x} + \xi_2^{\mu *}(\vec{p})e^{+ip\cdot x}\right]. \tag{16}$$

In this expression, the new polarization vectors $\xi_a^\mu(\vec{p})$ are given in terms of the old polarization vectors $\epsilon_a^\mu(\vec{\lambda})$ simply by a shift in the argument $\xi_a^\mu(\vec{p}) = \epsilon_a^\mu(\vec{p} - [-1]^a\vec{k})$.

The Definition (15) reveals that the fields $\mathcal{A}^\mu(x)$ are superpositions of plane waves with Lorentz-symmetric dispersion relation $p^\mu p_\mu = 0$. This fact implies that $\Box\mathcal{A}^\mu(x) = 0$. Note that this equation resembles the conventional Maxwell equations in Lorentz gauge. As advertised above, this field is obtained from the original solution A^μ by first splitting off all exponentials with the common factor $\exp(-ik\cdot x)$ to find $\mathcal{A}^\mu(x)$ and subsequently removing this factor. An analogous procedure must be performed for $\mathcal{A}^{\mu *}(x)$.

The complex-valued $\mathcal{A}^\mu(x)$ field gives rise to a real-valued vector field $\underline{A}^\mu(x)$ in a natural way:

$$\begin{aligned}\underline{A}^\mu(x) &= \mathcal{A}^\mu(x) + \mathcal{A}^{\mu *}(x)\\ &= \int \frac{d^3\vec{p}}{(2\pi)^3}\sum_{a=1,2}\left[\xi_a^\mu(\vec{p})e^{-ip\cdot x} + \xi_a^{\mu *}(\vec{p})e^{+ip\cdot x}\right].\end{aligned} \tag{17}$$

This field also obeys an equation that is consistent with the conventional Maxwell theory in Lorentz gauge:

$$\Box\underline{A}^\mu(x) = 0\,. \tag{18}$$

We therefore see that a given solution $A^\mu(x)$ of the Chern–Simons modified electrodynamics leads to a field $\underline{A}^\mu(x)$ that obeys a Klein–Gordon-type equation in each component, so at least the plane-wave exponentials are Lorentz symmetric.

Equation (17) essentially governs the spacetime dependence of the redefined field $\underline{A}^\mu(x)$ via the plane-wave exponentials, but it leaves undetermined the polarizations vectors. This is consistent with the gauge invariance of the Chern–Simons model: we have not yet selected a gauge for A^μ, but the field redefinition gives a field $\underline{A}^\mu$ satisfying Eq. (17), which looks gauge fixed (Lorentz gauge). This is, of course, not the case precisely because of the above issue that the polarizations vectors are still undetermined. For $\underline{A}^\mu$ to obey the usual Maxwell equations in Lorentz gauge, we not only need Eq. (17), but also the additional Lorentz condition $\partial_\mu \underline{A}^\mu = 0$. This condition constrains the polarization vectors $\xi_a^\mu(\vec{p})$ to be transverse.

Suppose we select Lorentz gauge in the Chern–Simons model $\partial_\mu A^\mu = 0$. Then, the question arises as to whether our field redefinition leaves unchanged this gauge condition. This is, in fact, not the case. We obtain

$$\partial_\mu A^\mu = 0 \quad \to \quad \partial_\mu \underline{A}^\mu = -2\,\mathrm{Im}\,(k\cdot\mathcal{A}) \tag{19}$$

for the redefined condition. Since $\mathrm{Im}\,(\mathcal{A}^\mu)$ cannot be freely chosen (it is determined by $\mathrm{Re}\,(\mathcal{A}^\mu)$ to yield plane-wave exponentials), the redefined field $\underline{A}^\mu$ fails to obey the Lorentz condition. Let us instead select the gauge

$$\partial_\mu A^\mu(x) = 2\,\mathrm{Im}\,[\,k\cdot\mathcal{A}(x)\,\exp(-ik\cdot x)\,] \tag{20}$$

for the solution of the Chern–Simons model. Substituting Eq. (15) on the left-hand side of Eq. (20) then gives $\mathrm{Re}\,[\,\exp(-ik\cdot x)\,\partial\cdot\mathcal{A}(x)\,] = 0$. Using the plane-wave expansion of $\mathcal{A}^\mu(x)$, one can verify[i] that this essentially implies $\partial\cdot\mathcal{A}(x) = \partial\cdot\mathcal{A}^*(x) = 0$, and therefore

$$\partial_\mu \underline{A}^\mu(x) = 0\,. \tag{21}$$

This result establishes that with a carefully selected gauge for solutions A^μ in the Chern–Simons model, the field redefinition discussed above yields a solution $\underline{A}^\mu$ of conventional electrodynamics in Lorentz gauge. It follows that such a mapping, defined on-shell, removes Lorentz and CPT violation from the Chern–Simons model.

4. Compact expression for the mapping

In the previous section, we have discussed the possibility of removing a lightlike Lorentz- and CPT-violating k^μ from Chern–Simons electrodynamics. We have illustrated why and how this can be achieved. The basic idea has been the following. The first step is to decompose an arbitrary solution A^μ of the Chern–Simons model into two pieces, one containing the plane waves with dispersion relation shifted by $+k^\mu$ and the other containing those with the opposite shift $-k^\mu$. In the second step, the Lorentz-violating shift is undone with a simple multiplicative field redefinition involving $\exp(+ik\cdot x)$ in one of these pieces and $\exp(-ik\cdot x)$ in the other.

We now set out to find a more compact form for such an on-shell mapping from the set of solutions in the Chern–Simons model to the set of solutions in ordinary electrodynamics. Clearly, the more challenging step in the field redefinition is the first one, which decomposes a given solution A^μ according to the shift direction in k^μ yielding both $\mathcal{A}^\mu$ and $\mathcal{A}^{\mu *}$. In principle, this task can be performed with Fourier methods that simply project out the desired pieces. This section gives closed-form expressions for suitable projectors.

We begin by characterizing the set of solutions A^μ to Eq. (2), which we take as the domain for our field-redefinition mapping. As before, we fix k^μ to be lightlike $k^\mu k_\mu = 0$, and we consider all well-behaved fields of the form displayed in Eq. (13). Note that all plane-wave momenta in the exponentials satisfy the dispersion relation (6). In particular, any field A^μ obeying Eq. (13) therefore also satisfies

$$\left[\Box^2 + 4(k\cdot\partial)^2\right] A^\mu(x) = 0\,. \tag{22}$$

We remark that all other solutions to Eq. (2) can differ from Eq. (13) only by a total derivative, a quantity that leaves unaffected the physics. Note that we are not committing ourselves to a definite gauge because of the remaining freedom in the choice of polarization vectors in Eq. (13).

Consider the operators P_+ and P_- defined by

$$P_\pm \equiv \frac{1}{2}\left(1 \pm 2i\,\frac{k\cdot\partial}{\Box}\right). \tag{23}$$

When these operators act on our set of solutions A^μ characterized above, we find the following: the operators are complete in the sense that $P_+ + P_- = 1$, they are orthogonal in the sense $P_\pm P_\mp = 0$, and they are idempotent $P_\pm^2 = P_\pm$. To arrive at these results, we have employed Eq. (21). It is apparent that P_+ and P_- are operators that project onto two disjoint subsets of solutions. Moreover, the union of these subsets is equal to our full set of solutions.

Applying $P_\pm$ to the plane-wave exponentials occurring in the general solution (13) yields

$$\begin{aligned} P_+\,e^{\pm i\lambda_a\cdot x} &= \frac{1}{2}\,[1 \pm (-1)^a]\,e^{\pm i\lambda_a\cdot x},\\ P_-\,e^{\pm i\lambda_a\cdot x} &= \frac{1}{2}\,[1 \mp (-1)^a]\,e^{\pm i\lambda_a\cdot x}. \end{aligned} \tag{24}$$

Employing these relations, one can then show that

$$\begin{aligned} P_+\,A^\mu(x) &= \mathcal{A}^\mu(x),\\ P_-\,A^\mu(x) &= \mathcal{A}^{\mu *}(x). \end{aligned} \tag{25}$$

With the above results at hand, we are now in the position to give a more concise form of our field-redefinition mapping:

$$\underline{A}^\mu(x) = e^{-ik\cdot x}\,P_+\,A^\mu(x) + e^{+ik\cdot x}\,P_-\,A^\mu(x)\,. \tag{26}$$

We mention that with the field redefinition (25) and the equations of motion (21) one can verify Eq. (17) directly without using plane-wave expansions. We also remark that a similar field redefinition exists for the closely related Lorentz- and CPT-violating b^μ parameter for SME fermions [21].

5. Summary and outlook

The electrodynamics sector of the SME contains a Chern–Simons-type operator contracted with a Lorentz- and CPT-violating four-vector coupling k^μ. Such a coupling can, for example, arise through a nontrivial spacetime topology or in certain cosmological supergravity models as a result of varying scalar fields. If k^μ is lightlike, the free solutions of this model can be mapped to the solutions of conventional Maxwell electrodynamics. The specific form of this field redefinition is determined by Eq. (25). This mapping is nonlocal, and it applies on-shell. The existence of such a mapping does *not* imply that the Chern–Simons term is unphysical and cannot be measured: both off-shell physics and interactions will typically lead to observable effects.

There are still a few open questions regarding this field redefinition that need to be addressed in the future. It is, for instance, interesting to determine whether this mapping is one-to-one and onto. If so, there would be a direct correspondence between the Lorentz- and CPT-violating Chern–Simons model and ordinary Lorentz- and CPT-symmetric electrodynamics. Certain known results in the conventional Maxwell model, could then simply be translated to the more complex Chern–Simons model via the field-redefinition map. Other open issues concern the question as to whether such types of field redefinitions can also be found for timelike and spacelike k^μ or for interacting models. An additional important aspect we have left largely unaddressed is gauge symmetry. Although we have discussed the example of Lorentz gauge, a more detailed analysis of how gauge conditions are affected by the mapping could yield valuable insight into the mathematical structure of the Chern–Simons model and the field redefinition.

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i. We exclude special cases, such as solutions containing a single plane wave with $p^\mu = \pm k^\mu$ and specially chosen phases or solutions of two plane waves with $p^\mu = 0$.

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