

# Carrier heating effects on transport phenomena in intrinsic semiconductor thin films

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The excess of nonequilibrium charge carriers due to heating by electric fields influences substantially the electron heat-diffusion and the carrier current density in thin film semiconductors. With the assumption of hole and phonon thermal equilibrium, the current density for electrons and holes and electron heat flux in the semiconductor thin films are calculated analytically taking into account the contribution of the nonequilibrium of carriers and the electron temperature. By using the continuity equations for the carrier densities and energy balance equation with appropriate boundary conditions at the surfaces of the sample, we find that the current density and electron heat flux depend substantially on the size of the sample.

**Keywords:** Nonequilibrium charge carriers; electron heat diffusion; electron temperature.

Exceso de portadores fuera de equilibrio debido campos eléctricos afecta considerablemente el proceso de difusión de calor electrónico y la densidad de corriente en películas delgadas semiconductoras. En la aproximación de equilibrio térmico entre fonones y huecos la densidad de corriente de electrones y huecos y el flujo de calor asociado al sistema electrónico en películas delgadas semiconductoras son calculados analíticamente considerando la temperatura propia del sistema electrónico fuera de equilibrio. Las propiedades de transporte de los portadores cargados fuera de equilibrio en semiconductores son calculados usando la ecuación de continuidad para electrones y huecos y la ecuación de balance de energía con condiciones a la frontera en la superficie de la muestra. Dentro de estas aproximaciones se demuestra que la densidad de corriente y el flujo de energía asociado al sistema electrónico dependen fuertemente de las dimensiones de la muestra.

**Descriptores:** Portadores cargados fuera de equilibrio; difusión de calor de electrones; temperatura de electrones.

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## 1. Introduction

Strong electric fields produce a great variety of effects in semiconductors. They basically alter the quantum states of carriers and their energy spectrum. This gives rise to the dependence of the macroscopic properties of semiconductors on the applied electric field  $E$ . Examples of such effects are: the dependence of the complex dielectric function on  $E$ , resulting from the possibility of fundamental absorption of photons whose energy is less than the forbidden band gap (Franz-Keldish effect) [1], the tunnel current in a degenerate p-n junction (Esaki effect) [2], etc.

The application of strong electric fields can give rise to states in semiconductors which are far from thermodynamic equilibrium. Hot electrons in semiconductors are a typical example of a non-equilibrium state of carriers in which their average kinetic energy is increased by an external electric field so that it can be described by an effective temperature  $T_n(E)$  which exceeds the lattice temperature  $T_0$ . Such heating of carriers by an electric field considerably alters many physical properties of semiconductors and gives rise to new effects, in particular, to a dependence of the electrical conductivity on the electric field (the deviation from Ohm's law of the current-voltage characteristics); in this situation the carrier mobility begins to increase or decrease with the increase of the electric field, hot electron diffusion and invalidation of the Einstein relation, and non-linear galvanomagnetic effects.

The classical theory of hot carrier transport in semiconductors has been discussed in a number of books [3-7] and review articles [8-11]. Previous calculations have been addressed to nonlinearity caused by a change in mobility [3-5], impact ionization [12,13], carrier lifetime change [10], intervalley redistribution of carriers [14], or by nonparabolic carrier energy dispersion law [15,16]. More recently, high field transport of many-electron systems in the presence of both dc electric and frequency-dependent electric fields has been investigated using Monte Carlo simulations and the coupled force balance and Fokker-Planck equations. Under these conditions, the drift velocity of electrons is found to increase with the amplitude of the ac field due to a suppressed momentum-relaxation process under parallel polarization but decreases with the amplitude due to an enhanced momentum relaxation process under perpendicular polarization [17,18].

On the other hand, experimental optical studies of electric field-induced electron and hole transient transport on optical phonon instability in semiconductor nanostructures have been reviewed by Tsen [19].

In the single-valley semiconductor approximation, neglecting the effects on the nonlinearity of the current density mentioned above, all previous theories usually assume that only the thermal equilibrium density of carriers is subject to heating by electric fields, *i.e.* in the heating process the concentration of non-equilibrium ensemble of high-energy carriers is the same as the thermal equilibrium concentration, *i.e.*

the concentration of carriers remains constant. Therefore, the non-linear current-voltage characteristics are only related to the electron mobility through the electron temperature, which is also electric field dependent.

However, in Refs. 20 and 21, it has been shown that there is one more mechanism of nonlinearity associated with the fact that the violation of the energy equilibrium between electrons, holes and phonons (the difference between electron and hole temperatures) inevitably results in the violation of the concentration equilibrium between electron in the conduction band and holes in the valence band and, therefore, the carrier recombination in the semiconductor play an important role by introducing an extra term in the nonlinear component of the carrier current density.

Recently [22,23], it has been shown that the influence of nonequilibrium charge carriers due to heating of the electric field in the recombination process in semiconductors considerably affects the nonlinearity of the charge current density; in particular, the nonlinear behavior of the current density persists even when electron mobility is electron temperature independent. In the classical hot-electron theory, the current density is linear in this approximation.

In this work, the theory of nonlinear charge carrier density and electron heat flux is developed for thin-film semiconductors in the presence of heating electric fields. The nonequilibrium carriers are calculated by solving the continuity equation for electrons and hole simultaneously with the energy balance equation; appropriate boundary conditions are used for the current density and the electron heat flux at the surface of the sample. The size effects on these transport equations are discussed in the text.

## 2. Theoretical model

Let us consider a single-valley nondegenerate semiconductor layer with thickness  $2a$  in the  $z$ -direction and homogeneous in the plane  $x - y$ . We shall assume that a dc electric field  $E_0$  is applied in the  $x$ -direction and the sample is in a reservoir with temperature  $T_0$ . In the presence of a heating electric field, the excess of electrons and holes in an intrinsic semiconductors (no impurity atoms) is essentially due to the electron-hole thermal generation process which depends strongly on the carrier temperature. In addition we also consider that the heating electric field is not so strong that the electron temperature depends on the electric field as  $E_0^2$  and the hole and phonon systems are in thermal equilibrium, *i.e.*  $T_p = T_{ph} = T_0$ . The dimensions of the semiconductor in the plane  $x - y$  are greater than the diffusion length and energy relaxation length so that the heating electric field, nonequilibrium electron and hole concentrations, the electrostatic potential and electron temperature are independent of the coordinates  $x - y$ , *i.e.* the effects of the boundaries in the plane are neglected. Then, under these geometrical conditions, the equations governing the transport of hot carriers in the semiconductor are described by the set of continuity equations for both electrons and holes  $J_n$  and  $J_p$  one-dimensional current

densities [24]:

$$\frac{1}{e} \frac{dJ_n}{dz} - R_n = 0, \quad (1)$$

$$\frac{1}{e} \frac{dJ_p}{dz} + R_p = 0, \quad (2)$$

the Poisson equation [25]

$$\frac{d\delta\varphi}{dz} = \frac{4\pi e}{\varepsilon_0} (\delta n - \delta p) \quad (3)$$

and the energy balance equation [7]

$$\frac{dQ_n}{dz} - J_n E_0 = -n_0 \nu_\varepsilon \theta + \xi \varepsilon_g R_n \quad (4)$$

where  $n = n_0 + \delta n$  and  $p = p_0 + \delta p$  are the electron and hole concentrations,  $n_0$  and  $p_0$  are the equilibrium carrier densities,  $\delta n$  and  $\delta p$  the nonequilibrium electron and hole concentrations, respectively,  $\delta\varphi$  is the contribution to the electrical potential due to  $\delta n$  and  $\delta p$ ,  $R_n$  and  $R_p$  represent the recombination rate for electron and holes,  $e$  the hole charge,  $\varepsilon_0$  the dielectric constant of the sample and  $Q_n$  is the electron heat flux,  $J_n$  the electron carrier density and the term  $J_n E_0$  is the Joule effect. The first term on the right-hand side of Eq. (4) describes the intensity of the electron-phonon (electron-hole) energy exchange with  $\nu_\varepsilon$  the electron energy relaxation frequency;  $T_n = T_0 + \theta$ ,  $\theta$  is the nonequilibrium electron temperature and the last term expresses the thermal power density generated in the electron system due to recombination with efficiency  $\xi$ .

From the continuity equations for the total current  $J = J_n + J_p$ , Volovichev *et al* [21] have shown that the electron and hole recombination are equal and they are given, for an intrinsic semiconductor ( $n_0 = p_0$ ) with  $n_i = n_0 p_0$ , as

$$R_n = R_p = R = \alpha(T_n)(np - n_i^2) = \frac{1}{2\tau}(\delta n + \delta p + \beta\theta) \quad (5)$$

with

$$\tau^{-1} = 2\alpha(T_0)n_0; \quad \beta = 2\tau n_i^2 \left. \frac{d\alpha}{dT_n} \right|_{\theta=0} \quad (6)$$

and  $\alpha(T_n)$  being the electron-hole pair recombination factor which depends on the electron temperature in the conduction band and, as can be observed, an extra term is introduced in Eq. (5) due to the heating of the electron gas by the electric field.

As is well known [26], if the dimensions of the semiconductor are much larger than, the cooling [7] and diffusion [24] lengths for electrons and holes, the inhomogeneities in the nonequilibrium carrier and electron temperature distributions near the boundaries of the sample can be neglected in the average nonlinear current densities across the semiconductor ( $\delta n$ ,  $\delta p$  and  $\theta$  are independent of the coordinates  $x - y$ ). However, the effects of the boundaries of the thin semiconductor in the  $z$ -direction play an important role in the transport properties of the hot electron system. These thermal size

effects are due to the cooling of carriers at the boundaries of the semiconductor because of their interaction with surface energy absorption mechanisms [7], the dependence of the thermal generation on the electron temperature  $T_n$  and the redistribution of nonequilibrium carriers in the sample. In order to take into account the effects of the dimensions of the sample on the electron and hole current densities and the electron heat flux in the  $z$ -axis, we evaluate them from the constitutive relations calculated from Boltzman transport equation [7,27]:

$$J_n(z) = e^2 I_{10}^n E_n^* - e I_{11}^n \frac{1}{T_n} \frac{\partial T_n}{\partial z} \quad (7)$$

$$J_p(z) = e^2 I_{10}^p E_p^* \quad (8)$$

$$Q_n(z) = e I_{11}^n E_n^* - I_{12}^n \frac{1}{T_n} \frac{\partial T_n}{\partial z} \quad (9)$$

where

$$I_{rs}^n = \frac{4n}{3\sqrt{\pi m_n \nu_\varepsilon}} T_n^s \left( \frac{T_n}{T_0} \right)^{rq} \Gamma(rq_n + s + \frac{5}{2}) \quad (10)$$

$m_n$  is the electron effective mass. The effective electric field in terms of the chemical potentials (quasi-Fermi levels) for both types of carriers  $\mu_{np}$  can be written as

$$E_{np}^* = -\frac{d\varphi}{dz} \pm \frac{T_{e,p}}{e} \frac{d}{dz} \left[ \frac{\mu_{np}}{T_e} \right] \quad (11)$$

In Eq. (10),  $q_n$  is a parameter of order of unity which depends on the momentum dissipation mechanisms, *i.e.* the electron-phonon interaction, electron-charged or neutral impurity interaction [7]. The equivalent expression for holes  $I_{rs}^n$  can be obtained by substituting the nonequilibrium electron concentration  $n, q_n, m_n$  by  $p, q_p, m_p$ , respectively. Due to the excess of the carrier densities created in the semiconductor and the increasing of the carrier average energy by the heating electric field, the spatial variation in the chemical potential for electrons and holes are given by the following expressions:

$$\begin{aligned} \frac{d}{dz} \left[ \frac{\mu_n}{T_e} \right] &= \frac{1}{n_0} \frac{d\delta n}{dz} - \frac{3}{2T_0} \frac{d\theta}{dz}, \\ \frac{d}{dz} \left[ \frac{\mu_p}{T_o} \right] &= \frac{1}{n_0} \frac{d\delta p}{dz} \end{aligned} \quad (12)$$

Equations (7)-(12) allow us to express  $J_n, J_p$  and  $Q_n$  in terms of  $\delta n, \delta n, \delta\varphi$  and  $\theta$  in the following form:

$$J_n = \sigma_n^0 \left[ -\frac{d\delta\varphi}{dz} + \frac{T_0}{en_0} \frac{d\delta n}{dz} + \alpha_n \frac{d\theta}{dz} \right] \quad (13)$$

$$J_p = \sigma_p^0 \left[ -\frac{d\delta\varphi}{dz} - \frac{T_0}{ep_0} \frac{d\delta p}{dz} \right] \quad (14)$$

$$\begin{aligned} Q_n = -\kappa_e \frac{d\theta}{dz} + \frac{q_n + 5/2}{e} T_0 \sigma_n^0 \\ \times \left[ -\frac{d\delta\varphi}{dz} + \frac{T_0}{e} \left( \frac{1}{n_0} \frac{d\delta n}{dz} - \frac{3}{2T_0} \frac{d\theta}{dz} \right) \right] \end{aligned} \quad (15)$$

Here

$$\sigma_{n,p}^0 = \frac{4(n,p)e^2}{3\pi^{1/2}m_{n,p}\nu_{n,p}} \Gamma(q_{n,p} + 5/2)$$

is the electrical conductivity for electrons (holes) and

$$\kappa_e = (q_n + 5)(q_n + 5/2) \frac{T_0}{e^2} \sigma_n^0$$

is the electron thermal conductivity,  $\nu_{n,p}$  the electron (hole) momentum relaxation and  $\alpha_n = -(1)/(e)(q_n+4)$  is the Seebeck coefficient. Finally, the continuity equations for electrons and holes, Poisson equation and the thermal balance equation for the nonequilibrium charge carriers, the electrical potential and the nonuniform electron temperature in the sample are:

$$\frac{d^2\delta\varphi}{dz^2} = \frac{4\pi e}{\varepsilon_0} (\delta n - \delta p) \quad (16)$$

$$\begin{aligned} &- \frac{d^2\delta\varphi}{dz^2} + \frac{T_0}{en_0} \frac{d^2\delta n}{dz^2} + \alpha_n \frac{d^2\theta}{dz^2} \\ &= \frac{e}{2\tau\sigma_n^0} (\delta n + \delta p + \beta\theta) \end{aligned} \quad (17)$$

$$-\frac{d^2\delta\varphi}{dz^2} - \frac{T_0}{en_0} \frac{d^2\delta p}{dz^2} = -\frac{e}{2\tau\sigma_p^0} (\delta n + \delta p + \beta\theta) \quad (18)$$

$$\begin{aligned} &- \kappa_e \frac{d^2\theta}{dz^2} + \frac{e\kappa_e}{q_n + 5} \left[ -\frac{d^2\delta\varphi}{dz^2} + \frac{T_0}{en_0} \frac{d^2\delta n}{dz^2} \right] \\ &- \frac{\xi\varepsilon_g}{2\tau} (\delta n + \delta p) - \left( \frac{\xi\varepsilon_g}{2\tau} \beta - n_0 \nu_\varepsilon \right) \theta = \sigma_n^0 E_0^2 \end{aligned} \quad (19)$$

The thermal balance equation should be supplemented by boundary conditions describing the absorption of the carrier energy at the boundaries of the semiconductor. Assuming that the sample is bounded in the  $z$ -axis, we find that according to the physical arguments discussed in the previous section, these can be written

$$Q_n|_{z=\pm a} = \pm \eta \theta|_{z=\pm a} \mp \xi_s \varepsilon_g R_s \quad (20)$$

where  $R_s = S(\delta n + \delta p + \beta\theta)|_{z=\pm a}$  are the carrier recombination rates at the surfaces of the sample with efficiency  $\xi_s$ ,  $S$  is the carrier surface recombination velocity and  $\eta$  is the surface electron heat conductivity;  $\eta=0$  corresponds to the absence of the surface mechanisms. On the other hand for the electron and the hole density currents at the two semiconductor boundaries, we choose

$$J_n|_{z=\pm a} = \pm eR_s, \quad J_p|_{z=\pm a} = \mp eR_s \quad (21)$$

because  $\nabla(J_n + J_p) = 0$  and  $J_n + J_p|_{z=\pm a} = 0$  then  $J_n(z) + J_p(z) = 0$  and only three of the four boundary conditions in Eqs. (21) are independent: the choice of the reference level for  $\delta\varphi$  ( $a=0$ ) acts as a complementary boundary condition [24].

Solutions to Eqs. (16)-(19) with Eqs. (20)-(21) are in general a complicated task even for the simplest intrinsic semiconductor sample. Therefore, in order to gain some insight into the physics of hot electron transport in bounded semiconductors, we restrict ourselves to the case of quasi-neutrality approach *i.e.*  $\delta n = \delta p$  [26] which is valid if the intrinsic Debye length ( $l_D = \sqrt{\varepsilon_0 T_0 / 4\pi e^2 n_0}$ ) is much smaller than the characteristic lengths of the sample. In this approach the Poisson equation is not necessary and the solutions to Eqs. (16)-(19) are

$$\theta(z) = \theta_1(z) + \theta_2(z) + \theta_c \quad (22)$$

$$\delta n(z) = N_1(z) + N_2(z) + N_c \quad (23)$$

$$\delta\varphi(z) = \varphi_1(z) + \varphi_2(z) + \varphi_c \quad (24)$$

Here

$$\theta_1(z) = b_0 e^{m_1 z} + b_1 e^{-m_1 z}; \quad \theta_2 = b_2 e^{m_2 z} + b_3 e^{-m_2 z}; \quad \theta_c = \frac{\sigma_n^0}{n_0 \nu_\varepsilon} E_0^2 \quad (25)$$

$$N_i(z) = N_i \theta_i(z) \quad (26)$$

$$\varphi_i(z) = \varphi_i \theta_i(z) i = 1, 2 \quad (27)$$

$$N_i = -\frac{\alpha_n s_i - e\beta/2\tau\sigma}{2T_0 s_i / en_0 - e/\tau\sigma}, \quad N_c = -\frac{1}{2}\beta\theta_c \quad (28)$$

$$\varphi_i = \frac{1}{s_i} \left[ \frac{e\beta}{2\tau\sigma_p^0} - N_i \left( \frac{T_0 s_i}{en_0} - \frac{e}{\tau\sigma_p^0} \right) \right]; \quad \varphi_c = \varphi_0^* + \varphi_1^* z \quad (29)$$

$$b_0 + b_1 = \frac{\chi_2 \frac{t_1}{m_2} + \frac{t_2}{m_1} (\tanh m_2 a - \xi_2)}{(s_1 t_2 - s_2 t_1) \cosh m_1 a [(\chi_1 + \tanh m_1 a)(\tanh m_2 a - \xi_2) + \chi_2 \xi_1]} \eta \theta_c \quad (30)$$

$$b_2 + b_3 = \frac{\xi_1 \frac{t_2}{m_1} - \frac{t_1}{m_2} (\tanh m_1 a - \chi_1)}{(s_1 t_2 - s_2 t_1) \cosh m_2 a [(\chi_1 + \tanh m_1 a)(\tanh m_2 a - \xi_2) + \chi_2 \xi_1]} \eta \theta_c \quad (31)$$

$$b_2 - b_3 = -\frac{\xi_1 \frac{t_2}{m_1} \left( \frac{s_2}{t_2} - \frac{e\kappa_e}{q_n + 5} \right) - \frac{t_1}{m_2} \left( \frac{s_1}{t_1} - \frac{e\kappa_e}{q_n + 5} \right) (\coth m_1 + \chi_1)}{(s_1 t_2 - s_2 t_1) \sinh m_1 a [(\chi_1 + \cot m_1 a)(\coth m_2 a - \xi_2) + \chi_2 \xi_1]} \varphi_1^* \quad (32)$$

$$b_0 - b_1 = -\frac{\chi_1 \frac{t_1}{m_2} \left( \frac{s_1}{t_1} - \frac{e\kappa_e}{q_n + 5} \right) + \frac{t_2}{m_1} \left( \frac{s_2}{t_2} - \frac{e\kappa_e}{q_n + 5} \right) (\coth m_2 - \xi_2)}{(s_1 t_2 - s_2 t_1) \sinh m_1 a [(\chi_1 + \coth m_1 a)(\coth m_2 a - \xi_2) + \chi_2 \xi_1]} \varphi_1^* \quad (33)$$

$$\xi_{1,2} = \frac{t_1}{m_2 (s_1 t_2 - s_2 t_1)} \left[ S(2N_{1,2} + \beta) \left( \frac{e}{\sigma_n^0} \frac{s_1}{t_1} + \xi_s \varepsilon_g \right) - \eta \right] \quad (34)$$

$$\chi_{1,2} = \frac{t_2}{m_1 (s_1 t_2 - s_2 t_1)} \left[ S(2N_{1,2} + \beta) \left( \frac{e}{\sigma_n^0} \frac{s_2}{t_2} + \xi_s \varepsilon_g \right) - \eta \right] \quad (35)$$

$$s_i = -\kappa_e - \frac{e\kappa_e}{q_n + 5} \varphi_i + \frac{T_0}{en_0} N_i; \quad t_i = -\varphi_i + \frac{T_0}{en_0} N_i + \alpha_n \quad (36)$$

$$m_i = \sqrt{\psi_i}; \quad \psi_i = \frac{-B \pm \sqrt{B^2 + 4AC}}{2A} \quad (37)$$

$$A = -\frac{2T_0}{en_0} \left( \kappa_e + \frac{e\kappa_e}{q_n + 5} \alpha_n \right) \quad (38)$$

$$B = \frac{2T_0}{en_0} \left( \frac{1}{2} \beta \gamma_n + n_0 \nu_\varepsilon \right) + \kappa_e \frac{e}{\tau\sigma} + \alpha_n \gamma_p \quad (39)$$

$$C = \frac{e}{\tau\sigma} n_0 \nu_\varepsilon \quad (40)$$

$$\frac{1}{\sigma} = \frac{1}{\sigma_n^0} + \frac{1}{\sigma_p^0}; \quad \gamma_{n,p} = \frac{e\kappa_e}{q_n + 5} \frac{e}{\tau\sigma_{n,p}^0} \mp \frac{\xi \varepsilon_g}{\tau} \quad (41)$$

The evaluation of the unknown parameters  $\varphi_1^*$  and  $\varphi_0^*$  is straightforward; it follows from the boundary conditions  $J_n(z) + J_p(z) = 0$  and  $\delta\varphi(a)=0$ , and presents a lengthy algebraic expression proportional to  $E_0^2$  so it will be omitted here.

As can be observed, the finite size of the semiconductor introduces a spatial dependence on the nonequilibrium carrier current densities, electron temperature and electrostatic potential; of course this dependence disappears for homogeneous sample *i.e.* in the limit  $m_i a \gg 1$ .

Setting the electron temperature  $T_n(z, E_0)$  and the nonequilibrium charge densities  $n(z, E_0), p(z, E_0)$ , the total electrical conductivity for an intrinsic semiconductor and in the quasineutrality approximation in the  $x$ -direction leads to

$$\begin{aligned}\sigma &= \sigma_n(z, n, p, T_n) + \sigma_p(z, p) = \frac{ne^2\tau_n(T_n)}{m_n} + \frac{pe^2\tau_p(T_p)}{m_p} \\ &= \sigma_0 \left[ 1 + q_n \frac{\sigma_n^0}{\sigma_0} \frac{\theta}{T_0} + \frac{\delta n}{n_0} \right] \quad (42)\end{aligned}$$

with  $\sigma_0 = \sigma_n^0 + \sigma_p^0$  the total electrical conductivity in thermal equilibrium. Therefore the average electric current is

$$j_x = \frac{1}{2a} \int_{-a}^a \sigma(z, n, p, T_n, n) E_0 dz = \sigma_0 [1 + \lambda\theta_c] E_0 \quad (43)$$

where

$$\begin{aligned}\lambda &= \left[ \frac{q_n}{T_0} \frac{\sigma_n^0}{\sigma_0} - \frac{\beta}{2n_0} \right] + \left[ \frac{q_n}{T_0} \frac{\sigma_n^0}{\sigma_0} + \frac{N_1}{n_0} \right] \frac{b_0 + b_1}{\theta_c} \frac{\sinh m_1 a}{m_1 a} \\ &+ \left[ \frac{q_n}{T_0} \frac{\sigma_n^0}{\sigma_0} + \frac{N_2}{n_0} \right] \frac{b_2 + b_3}{\theta_c} \frac{\sinh m_2 a}{m_2 a} \quad (44)\end{aligned}$$

At this point, we believe that it is important to compare our results with previous theories on electron transport in the presence of heating electric fields, in particular bounded semiconductors. The gist of standard theory of hot electron transport in semiconductors is that nonlinear effects which arise when current carriers move in heating electric fields and the electron current density is related to the electric field as

$$J_x = \sigma_0 [1 + \lambda_0 (1 - \gamma^*) E_0^2] E_0 \quad (45)$$

where  $\gamma^*$  is a factor including size and thermal surface effects as well as the electron-phonon energy interaction and is defined as

$$\gamma^* = \frac{F \sinh ka}{G} \frac{1}{ka},$$

where

$$\begin{aligned}F &= \zeta k + \zeta^2 \tanh ka, \\ G &= [(k^2 + \zeta^2) \tanh ka + k\zeta(1 + \tanh^2 ka)] \cosh ka \quad (46)\end{aligned}$$

and

$$\zeta = (\eta/\kappa_e(T_0))(q_n + 2), k^2 = n_0 \nu_\varepsilon (q_n + 2)/\kappa_e(T_0).$$

The parameter  $k^{-1}$  is the length scale of the variation in the electron temperature, *i.e.* the energy relaxation length (or cooling length). As can be observed, in the limit of uniform semiconductor,  $ka \gg 1$ , the conventional theory of hot electron transport in homogeneous sample is recovered, *i.e.*  $\gamma^* = 0$ .

In Eq. (45)  $\lambda_0 = q_n \sigma_0 \nu_\varepsilon / n_0 T_0$  is the nonlinear parameter for a homogeneous sample which describes the influences of the carrier heating on the electrical conductivity for  $q_n \neq 0$ . This coefficient vanishes for the case where  $q_n = 0$ , *i.e.* the carrier transport is a linear function of the electric field.

As is well known, recombination is a key feature when describing carrier transport in semiconductors, since it strongly affects the electrical response of the semiconductor at all levels of external excitation. It has been shown in Ref. 22 that recombination of hot electrons plays a major role in the nonlinear transport of hot electrons when the temperatures of the electron gas and phonons differ. This effect results from the dependence of the capture coefficient on the electron temperature ( $\beta \neq 0$ ). As can be seen, when recombination of carriers and  $\beta \neq 0$  are taken into account in hot carrier transport in bounded semiconductors, two special cases are important to emphasize in our theory:

One of them is related to when the recombination of carriers vanishes, *i.e.*  $\tau \rightarrow \infty$ ,  $R=0$ : the nonequilibrium electrons and holes distributions remain in the conduction and valence band, respectively, and in this case,  $\delta n \neq 0$  and  $\delta p \neq 0$ , *i.e.* they reach their maximum value.

The second one is concerned with the strong recombination limit, *i.e.*  $\tau \rightarrow 0$ , and in order to keep  $R=constant$ , the following relationship, see Eq. (5),  $\delta n + \delta p + \beta\theta = 0$ . Therefore, if  $\beta \neq 0$  (the capture coefficient depends on the electron temperature),  $\delta n \neq 0$  and  $\delta p \neq 0$  and the solutions can be obtained by solving the set of Eqs. (16)-(19) without the bulk recombination term in Eq. (4), the recombination of carriers in this limit becomes significant only at the surface of the sample since the diffusion length is extremely small as compared with all the characteristic lengths involved in the hot carrier transport phenomenon. It is worth mentioning that, for this special case, if in addition the capture factor is electron temperature independent, *i.e.*  $\beta=0$ , then  $\delta n, \delta p=0$  and Eq. (45) is recovered.

Finally, it is important to note that in our theory, the cooling and diffusion lengths  $m_1^{-1}, m_2^{-1}$  respectively, depend besides on the momentum ( $\nu_n$ ) and energy relaxation frequency ( $\nu_\varepsilon$ ), on the recombination time of carriers and the energy gap of the semiconductor [Eq. (37)]. Unlike previous theories, where these parameters are well defined, in our theory they are slightly different as a consequence of allowing the simultaneous recombination of carriers and energy exchange between electrons and phonons. In addition when  $q_n = 0$ , the nonlinear behavior in the carrier current density is still present, see Eqs. (43), and (44), in other words,

when the latter parameters are considered in the theory, additional terms appear in the nonlinear coefficient in the current-voltage characteristic curve even when the electron (hole) mobility is independent of the electron temperature. These additional nonlinear terms in the current density have never been considered in the standard classical theory of hot electron transport in semiconductors.

### 3. Conclusions

Nonequilibrium electron and hole distributions generated by the presence of heating electric fields in bounded semiconductors provide an interesting topic for study. When the average carrier energy exceeds the equilibrium thermal energy (hot electrons and holes), the electron (hole) populations will build up preferentially above the minimum (maximum) conduction (valence) band, and as a consequence the recombination rate will be strongly affected; in particular the electron-hole pair capture factor depends on the carrier temperature.

The calculations in this work describe the nonlinearity of the electron current density in heating electric fields considering the effects of the boundaries of the sample, the recombination mechanisms and electron temperature dependence on the capture coefficient in the quasi-neutrality approximation. Under these conditions we obtain results not found in standard theories on hot electron transport in bounded semiconductors, *e.g.* it is shown that the nonequilibrium carriers are different from zero in both strong and weak electron-hole recombination limits, the cooling and diffusion lengths differ substantially from those calculated in Ref. 28 and in addition, the behavior of the nonlinear current-voltage characteristics persists even for  $q_n = 0$ .

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