Vacuum stability in a minimal s_3 extension of the standard model

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This work deals with the gauge symmetry breaking and the vacuum stability conditions in a minimal S_3-invariant extension of the Standard Model. In the theory, there are three $SU(2)_L$ Higgs doublets that belong to the singlet and doublet representations of the S_3-flavor group. We find that, as function of the vacuum expectation values of the twelve real Higgs fields, the Higgs potential has three types of minimum: a normal minimum, an electric Charge breaking (CB) minimum and a CP violating minimum (CPB) depending on the vacuum expectation values (vev) of the Higgs fields H_1, H_2 and H_S. Assuming that the non-vanishing vev of the Higgs doublet in the singlet representation of S_3 is real, we obtain that the deepest minimum is the normal minimum. This condition corresponds to the Pakvasa-Sugawara minimum in which we obtain an S_2 residual symmetry. This feature simplifies the structure and therefore the computation of the Higgs mass matrices. We present some results on the mass spectrum of the Higgs bosons in the theory.

Keywords: Higgs bosons non-standard model; electroweak interactions; extensions of Higgs sector; spontaneous symmetry breaking; symmetry in theory of fields and particles; symmetry breaking; gauge field theory.

Este trabajo trata del rompimiento de la simetría de norma y las condiciones de estabilidad del vacío de una extensión mínima invariante de S_3 del Modelo Estándar. En la teoría hay tres bosones de Higgs que son dobletes de $SU(2)_L$ y que pertenecen a las representacions de singlete y doblete del grupo de sabor S_3. Encontramos que, como función de los valores de expectación del vacío (vev) de los doce campos reales de Higgs, el potencial de Higgs tiene tres tipos de mínimo: un mínimo normal, un mínimo que rompe la carga eléctrica (CB) y uno que viola CP (CPB), dependiendo de los valores de los vev de los campos de Higgs H_1, H_2 y H_S. Suponiendo que el vev diferente cero del doblete de Higgs en la representación de singlete de S_3 es real, obtenemos que el mínimo más profundo es el mínimo normal. Esta condición corresponde al mínimo de Pakvasa-Sugawara en el cual obtenemos una simetría S_2 residual. Esta característica simplifica la estructura, y por lo tanto, el cálculo de las matrices de masas de los Higgs. Presentamos algunos resultados en el espectro de masas de los bosones de Higgs en la teoría.

Descriptores: Bosones de Higgs en modelos no-estándar; interacciones electrodébiles; extensiones del sector de Higgs; rompimiento espontáneo de la simetría; simetrías en teorías de campo y partículas; rompimiento de la simetría; teorías de norma.

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1. Introduction

In the Standard Model (SM) each family of fermions enters independently and the masses of the particles are free parameters, whose values are determined experimentally. One possibility to reduce the number of free parameters in the SM and to try to relate the different families is to add a flavour symmetry. Recently interesting progress in this direction has been made by means of a discrete S_3 flavour group [1-3]. This extended model has been used to calculate neutrino masses and mixings [4,5] and flavour changing neutral currents (FCNC) [6,7]. In this model the Higgs sector is extended by two additional electroweak Higgs doublets in order to generate the fermion masses without breaking the flavour symmetry. Thus, the Higgs fields belong to the three-dimensional reducible representation of the flavour permutational group S_3 [3]. In this paper we study the gauge symmetry breaking and the vacuum stability conditions in this minimal S_3-invariant extension of the Standard Model. In this model there are contributions to Charge Breaking (CB) and Charge Parity breaking (CPB) symmetries coming from the Higgs sector. Since the potential and its minimization play a vital part in the successful construction of the model we analyze the S_3-invariant Higgs boson potential. The stationary points can be classified as Normal, CB and CPB minimums according to the vacuum expectation value of the Higgs fields H_1, H_2 and H_S.

We find the conditions under which the potential minimum preserving electric charge and CP symmetries, is the global one, that is, the normal minimum is deepest than the CB and CPB minima. One condition corresponds to the Pakvasa-Sugawara [1] minimum in which an S_2 residual symmetry is obtained. In conclusion, we show that in this model there is no CPB or CB coming from the Higgs sector when the flavour permutational symmetry S_3 is exact.

2. Higgs Boson in the Standard Model

In the SM, one $SU(2)$ doublet Higgs field is introduced to break the the $SU(2)\times U(1)$ symmetry and give masses to the particles. The Higgs potential is given by

$$V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4 \text{ where } \Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}, \quad (1)$$

the parameter λ must be positive to produce a stable vacuum. The parameter μ can have either sign, however it is chosen negative for a non trivial minimum, that is, $-\mu^2<0$. In the SM, the states satisfying the relationship

$$|\phi^+|^2 + |\phi^0|^2 = \mu^2/2\lambda = v^2/2$$

are degenerate minima of the potential and we can choose the vacuum expectation value in the $\langle\phi^0\rangle = v/\sqrt{2}$ direction. One important prediction of the SM is the Higgs Boson, a scalar particle which appears in the physical spectrum. The Higgs gives mass to the quarks and leptons through the Yukawa couplings $m_f = Y_f v/\sqrt{2}$, the W and Z gauge boson masses are given as $m_W = (g/2)\, v$,

$$m_Z = \left(\sqrt{g^2 + g'^2}/2\right) v$$

and the mass of the Higgs boson is given by $m_h = \sqrt{2\lambda} v$. The Higgs boson is the only particle of the SM which has not been discovered yet. Prior to the introduction of the Higgs boson and mass terms, the Lagrangian of the SM is chiral and invariant with respect to any permutations of the left and right quark and lepton fields. For three quarks and lepton families the S_3 flavour symmetry is an exact symmetry of the SM Lagrangian. If we assume that the S_3 permutational symmetry is not broken and the Higgs of the SM is an S_3 singlet, only one fermion in each family can acquire mass. Although the Higgs potential is very simple and sufficient to describe a realistic model of mass generation, it might be that this is not the final form of the theory but rather an effective description of a more fundamental theory.

3. The S_3 flavour symmetry

The ingredients of the extension of the SM are the following:

(*i*) To extend the flavour and family concepts to the Higgs sector,

(*ii*) To associate each family to an irreducible representation of the flavour group and

(*iii*) To construct a Lagrangian invariant under the action of the $SU(3)_c \times SU(2) \times U(1) \times S_3^f$ group.

The group S_3 is a non-Abelian group and has two one dimensional irreducible representations $\mathbf{1}_A$, and $\mathbf{1}_s$, which are an antisymmetric and a symmetric singlet, respectively, and it also has a two-dimensional doublet irreducible representation $\mathbf{2}$. The direct product of two S_3 irreducible representations are:

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A, \mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s,$$
$$\mathbf{1}_s \otimes \mathbf{2} = \mathbf{2}, \mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}, \mathbf{2} \otimes \mathbf{2} = \mathbf{1}_s \oplus \mathbf{1}_A \oplus \mathbf{2}.$$

The direct product of two S_3 doublets

$$\mathbf{p_D} = \begin{pmatrix}p_{D1}\\ p_{D2}\end{pmatrix} \quad \text{and} \quad \mathbf{q}_D = \begin{pmatrix}q_{D1}\\ q_{D2}\end{pmatrix}$$

has two singlets: the symmetric one $r_s = p_{D1}q_{D1} + p_{D2}q_{D2}$ and the antisymmetric one $r_A = p_{D1}q_{D2} - p_{D2}q_{D1}$; and just one doublet r_D^T, with the following form:

$$r_D^T = \begin{pmatrix}p_{D1}q_{D2} + p_{D2}q_{D1}\\ p_{D1}q_{D1} - p_{D2}q_{D2}\end{pmatrix}.$$

With this in mind, the Higgs sector is modified to three $SU(2)$ Higgs field doublets Φ_a, Φ_b, and Φ_c, which enter in a reducible triplet representation of S_3 as follows

$$\Phi \to H = (\Phi_a, \Phi_b, \Phi_c)^T. \quad (2)$$

Since the triplet representation of S_3 decomposes to $\mathbf{1_s} \oplus \mathbf{2}$ we express the three Higgs doublets as

$$H_s = \frac{1}{\sqrt{3}}\Big(\Phi_a + \Phi_b + \Phi_c\Big),$$
$$\begin{pmatrix}H_1\\ H_2\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}(\Phi_a - \Phi_b)\\ \frac{1}{\sqrt{6}}(\Phi_a + \Phi_b - 2\Phi_c)\end{pmatrix}. \quad (3)$$

The quark, lepton and Higgs fields are given by

$$Q^T = (u_L, d_L), u_R, d_R, \qquad L^\dagger = (\nu_L, e_L), e_R, \nu_R, \qquad H.$$

All the fields have three species (flavours) and belong to a reducible representation $\mathbf{1} \oplus \mathbf{2}$ of S_3.

4. S_3 invariant Yukawa Lagrangian

We can write the S_3 invariant Yukawa Lagrangian as

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_u} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}, \tag{4}$$

where each term is given as

$$\begin{aligned}\mathcal{L}_{Y_D} &= -Y_1^d \overline{Q}_I H_S d_{IR} - Y_3^d \overline{Q}_3 H_S d_{3R} \\ &\quad - Y_2^d \left[\overline{Q}_I \kappa_{IJ} H_1 d_{JR} + \overline{Q}_I \eta_{IJ} H_2 d_{JR} \right] \\ &\quad - Y_4^d \overline{Q}_3 H_I d_{IR} - Y_5^d \overline{Q}_I H_I d_{3R} + h.c,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{Y_U} &= -Y_1^u \overline{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q}_3 (i\sigma_2) H_S^* u_{3R} \\ &\quad - Y_2^u \left[\overline{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \eta \overline{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR} \right] \\ &\quad - Y_4^u \overline{Q}_3 (i\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q}_I (i\sigma_2) H_I^* u_{3R} + h.c.,\\ \mathcal{L}_{Y_E} &= -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} \\ &\quad - Y_2^e \left[\overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \right] \\ &\quad - Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + h.c.,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{Y_\nu} &= -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\ &\quad - Y_2^\nu \left[\overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \right] \\ &\quad - Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + h.c..\end{aligned}$$

Singlets under S_3 carry the index s or 3, and doublets carry indices $I, J = 1, 2$, and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Furthermore, we add a Majorana mass terms for the neutrinos

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R}, \tag{5}$$

where C is the charge matrix.

5. The Higgs sector

The Higgs sector Lagrangian of the S_3-invariant extension of the SM is expressed as

$$\begin{aligned}\mathcal{L}_\Phi &= \left[D_\mu H_S\right]^2 + \left[D_\mu H_1\right]^2 \\ &\quad + \left[D_\mu H_2\right]^2 - V\left(H_1, H_2, H_S\right),\end{aligned} \tag{6}$$

where D_μ is the usual covariant derivative. The gauge boson masses W and Z are given as:

$$m_W^2 = \frac{g^2\left(v_1^2 + v_2^2 + v_3^2\right)}{4},$$

$$m_Z^2 = \frac{\left(g^2 + g'^2\right)\left(v_1^2 + v_2^2 + v_3^2\right)}{4}.$$

In particular, we are interested in the Higgs potential. The phenomenology of the Higgs sector will be presented else where.

5.1. S_3 invariant Higgs Potential

The most general Higgs Potential invariant under

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times S_3$$

can be writen as

$$\begin{aligned}V &= \mu_1^2\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + \mu_0^2\left(H_S^\dagger H_S\right) + a\left(H_S^\dagger H_S\right)^2 \\ &\quad + b\left(H_S^\dagger H_S\right)\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + c\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)^2 \\ &\quad + d\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)^2 + e f_{ijk}\left(\left(H_S^\dagger H_i\right)\left(H_j^\dagger H_k\right)\right) \\ &\quad + f\left\{\left(H_S^\dagger H_1\right)\left(H_1^\dagger H_S\right) + \left(H_S^\dagger H_2\right)\left(H_2^\dagger H_S\right)\right\} \\ &\quad + g\left\{\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1\right)\right\} \\ &\quad + h\Big\{\left(H_S^\dagger H_1\right)\left(H_S^\dagger H_1\right) + \left(H_S^\dagger H_2\right)\left(H_S^\dagger H_2\right) \\ &\quad + \left(H_1^\dagger H_S\right)\left(H_1^\dagger H_S\right) + \left(H_2^\dagger H_S\right)\left(H_2^\dagger H_S\right)\Big\},\end{aligned} \tag{7}$$

where $a, b, c, \ldots, h$ are constants. Also f_{ijk} are constants whose indices run from 1 to 2 and a sum over repeated indices in that term of Eq. (7) is implicit. Their values are

$$f_{112} = f_{121} = f_{211} = -f_{222} = 1,$$

and all the rest are zero. The $SU(2)_L$ Higgs doublets with flavour index $1, 2, S$ are

$$\begin{aligned}H_1 &= \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \\ H_2 &= \begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_8 + i\phi_{11} \end{pmatrix},\ H_S = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}.\end{aligned} \tag{8}$$

We introduce the following notation :

$$\begin{aligned}x_1 &= H_1^\dagger H_1,\ x_2 = H_2^\dagger H_2,\ x_3 = H_S^\dagger H_S, \\ x_4 &= \mathcal{R}\left(H_1^\dagger H_2\right),\ x_5 = \mathcal{I}\left(H_1^\dagger H_2\right), \\ x_6 &= \mathcal{R}\left(H_1^\dagger H_S\right), x_7 = \mathcal{I}\left(H_1^\dagger H_S\right), \\ x_8 &= \mathcal{R}\left(H_2^\dagger H_S\right),\ x_9 = \mathcal{I}\left(H_2^\dagger H_S\right),\end{aligned} \tag{9}$$

where $\mathcal{R}$ and $\mathcal{I}$ are the real and imaginary parts respectively. Thus, the most general Higgs potential invariant under the exact symmetry $SU(2)_L \times U(1)_Y \times S_3$ can be written as

$$V(\mathbf{X}) = \mathbf{A}^T\mathbf{X} + \frac{1}{2}\mathbf{X}^T\mathbf{B}\mathbf{X}, \tag{10}$$

with $\mathbf{X}$ the vector of fields

$$\mathbf{X}^T = \left(x_1, x_2, x_3, \ldots\ldots\ldots., x_9\right), \tag{11a}$$

$\mathbf{A}$ the vector of mass parameters,

$$\mathbf{A}^T = \left(\mu_1^2, \mu_1^2, \mu_0^2, 0, 0, 0, 0, 0, 0\right), \tag{11b}$$

and $\mathbf{B}$ is a 9×9 real parameter matrix

$$\mathbf{B} = \begin{pmatrix} 2(c+g) & 2(c-g) & b & 0 & 0 & 0 & 0 & 2e & 0 \\ 2(c-g) & 2(c+g) & b & 0 & 0 & 0 & 0 & -2e & 0 \\ b & b & 2a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8g & 0 & 4e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4e & 0 & 2(f+2h) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2(f-2h) & 0 & 0 \\ 2e & -2e & 0 & 0 & 0 & 0 & 0 & 2(f+2h) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2(f-2h) \end{pmatrix}. \tag{11c}$$

6. Stationary Points

This potential has three types of stationary points:

1. The normal minimum with the following field configuration:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_i = 0, \quad i \neq 7, 8, 9$$

2. The stationary point which breaks the electric charge, here two of the charged fields ϕ acquire non zero vev's:

$$\phi_7 = v_1', \phi_8 = v_2', \phi_9 = v_3', \phi_1 = \alpha, \phi_3 = \beta,$$

3. The CP breaking minimum, where two imaginary components of the neutral fields ϕ acquire non zero vev's:

$$\phi_7 = v_1'', \phi_8 = v_2'', \phi_9 = v_3'', \phi_{10} = \delta, \phi_{11} = \gamma,$$

Since we assume H_s to be the SM Higgs, we also assume that it does not break electric charge nor CP. The 2HDM Higgs sector has been studied in Ref. 9. In this extended sector, the tree-level Higgs potential minimum preserving electric charge and CP symmetries, when it exists is the global one.

A. *The normal minimum*

From the definitions above, we obtain $x_i = v_i^2$ for $i=1,2,3$, $x_4 = v_1 v_2$, $x_6 = v_1 v_3$, $x_8 = v_2 v_3$, y $x_5 = x_7 = x_9 = 0$. Then, we can write the minimization conditions as

$$\frac{\partial V}{\partial v_i} = 0 \leftrightarrow \frac{\partial V}{\partial x_j}\frac{\partial x_j}{\partial v_i} = 0, \tag{12}$$

where $i=1,2,3$ and $j=1,2,..,9$; this is a set of three coupled equations.

Let us define the vector $(\mathbf{V}'_N)_i = (\mathbf{V}' \,|_{\mathbf{x}_N})_i = \partial V/\partial x_i$, evaluated at the minimum. In this notation the minimization conditions read

$$(\mathbf{V}'_N)_1 = -\frac{(\mathbf{V}'_N)_4}{2v_1 v_2} v_2^2 - \frac{(\mathbf{V}'_N)_6}{2v_1 v_3} v_3^2, \tag{13a}$$

$$(\mathbf{V}'_N)_2 = -\frac{(\mathbf{V}'_N)_4}{2v_1 v_2} v_1^2 - \frac{(\mathbf{V}'_N)_8}{2v_2 v_3} v_3^2, \tag{13b}$$

$$(\mathbf{V}'_N)_3 = -\frac{(\mathbf{V}'_N)_6}{2v_1 v_2} v_1^2 - \frac{(\mathbf{V}'_N)_8}{2v_2 v_3} v_2^2. \tag{13c}$$

From here it follows that:

$$\mathbf{V}'_N = \begin{pmatrix} -\left(\frac{\partial V}{\partial x_4} v_2 + \frac{\partial V}{\partial x_6} v_3\right)\frac{1}{2v_1} \\ -\left(\frac{\partial V}{\partial x_4} v_1 + \frac{\partial V}{\partial x_8} v_3\right)\frac{1}{2v_2} \\ -\left(\frac{\partial V}{\partial x_6} v_1 + \frac{\partial V}{\partial x_8} v_2\right)\frac{1}{2v_3} \\ \frac{\partial V}{\partial x_4} \\ 0 \\ \frac{\partial V}{\partial x_6} \\ 0 \\ \frac{\partial V}{\partial x_8} \\ 0 \end{pmatrix} \begin{pmatrix} \frac{-(\mathbf{V}')_4}{2v_1 v_2} v_2^2 + \frac{-(\mathbf{V}')_6}{2v_1 v_3} v_3^2 \\ \frac{-(\mathbf{V}')_4}{2v_1 v_2} v_1^2 + \frac{-(\mathbf{V}')_8}{2v_2 v_3} v_3^2 \\ \frac{-(\mathbf{V}')_6}{2v_1 v_2} v_1^2 + \frac{-(\mathbf{V}')_8}{2v_2 v_3} v_2^2 \\ (\mathbf{V}')_4 \\ 0 \\ (\mathbf{V}')_6 \\ 0 \\ (\mathbf{V}')_8 \\ 0 \end{pmatrix}. \tag{14}$$

It is clear from this expression that the first three entries in $\mathbf{V}'_N$ have the same sign if the ratios

$$\frac{-(\mathbf{V}')_4}{2v_1 v_2}, \frac{-(\mathbf{V}')_6}{2v_1 v_3}, \frac{-(\mathbf{V}')_8}{2v_2 v_3}$$

have equal signs too.

The stationary point is given by the conditions imposed in Eq. (12). Analyzing the second derivatives of the Higgs potential V we obtain the conditions for a minimum, these are given by the matrix of the squared scalar Higss masses. Particularly, for the scalar charged Higgs we have the squared masses $m^2_{H^{\pm}_{1,2}}$ as given in Eq. (37).

From Eq. (14) we can read $(\mathbf{V}'_N)_i$. It is clear from it that the first three entries in $\mathbf{V}'_N$ have the same sign as we mentioned before and the squared masses are positive if the sign of the ratios

$$\frac{-(\mathbf{V}')_4}{2v_1v_2}, \frac{-(\mathbf{V}')_6}{2v_1v_3}, \frac{-(\mathbf{V}')_8}{2v_2v_3}$$

are positive too.

Then, we have that the normal minimum exists if $m^2_{H^{\pm}_{1,2}} > 0$. That is,

$$m^2_{H^{\pm}_{1,2}} = \frac{1}{2}\left[Tr\mathbf{M}^2_C \pm \sqrt{(Tr\mathbf{M}^2_C)^2 - 4\chi^2}\right] > 0\,, \quad (15)$$

which leads to

$$[Tr\mathbf{M}]^2_C > (Tr\mathbf{M}^2_C) > \tfrac{1}{2}\,[Tr\mathbf{M}]^2_C\,. \quad (16)$$

Thus, in the normal minimum we get

$$\mathbf{V}'_N = \mathbf{A} + \mathbf{B}\mathbf{X}_N \qquad \text{and} \qquad \mathbf{X}^T_N\mathbf{V}'_N = 0, \quad (17)$$

where $\mathbf{X}_N = \mathbf{X}\mid_{normal\,min}$. In this notation, the potential evaluated at the normal minimum can be written as:

$$V_N = -\frac{1}{2}\mathbf{X}^T_N\mathbf{B}\mathbf{X}_N = \frac{1}{2}\mathbf{A}^T\mathbf{X}_N. \quad (18)$$

B. *Charge breaking minimum.*

In this case, the S_3 CB doublet of the Higgs field takes the values $\phi_7{=}v'_1$, $\phi_8{=}v'_2$, $\phi_9{=}v'_3$ and $\phi_1{=}\alpha$, $\phi_3{=}\beta$. Then, the vector $\mathbf{X}_{CB}$ can be written as:

$$\mathbf{X}_{CB} = \begin{pmatrix} \alpha^2 + v'^2_1 \\ \beta^2 + v'^2_2 \\ v'^2_3 \\ \alpha\beta + v'_1v'_2 \\ 0 \\ v'_1v'_3 \\ 0 \\ v'_2v'_3 \\ 0 \end{pmatrix}. \quad (19)$$

Direct analysis of the potential for this stationary point gives:

$$\mathbf{V}'_{CB} = \mathbf{A} + \mathbf{B}\mathbf{X}_{CB}, \quad (20)$$

and so the potential evaluated at the CB minimum can be written as:

$$V_{CB} = -\frac{1}{2}\mathbf{X}^T_{CB}\mathbf{B}\mathbf{X}_{CB} = \frac{1}{2}\mathbf{A}^T\mathbf{X}_{CB}. \quad (21)$$

From this equation and Eq. (18) for the normal minimum, we can compare the potential evaluated at the normal and CB breaking different minima,

$$V_{CB} - V_N = \frac{1}{2}\left(\mathbf{X}^T_{CB}\mathbf{V}'_N - \mathbf{X}^T_N\mathbf{V}'_{CB}\right). \quad (22)$$

If the signs of the ratios

$$\frac{-(\mathbf{V}')_4}{2v_1v_2}, \frac{-(\mathbf{V}')_6}{2v_1v_3}, \frac{-(\mathbf{V}')_8}{2v_2v_3}$$

are positive, the normal minimum exists and the following product is also positive:

$$\begin{aligned}\mathbf{X}^T_{CB}\mathbf{V}'_N = &-\frac{(\mathbf{V}'_N)_6}{2v_1v_3}\left[\alpha^2v_3^2 + (v'_1v_3 - v'_3v_1)^2\right] \\ &-\frac{(\mathbf{V}'_N)_8}{2v_2v_3}\left[\beta^2v_3^2 + (v'_2v_3 - v'_3v_2)^2\right] \\ &-\frac{(\mathbf{V}'_N)_4}{2v_1v_2}\left[(\alpha v_2 - \beta v_1)^2 + (v'_1v_2 - v'_2v_1)^2\right]. \quad (23)\end{aligned}$$

If the product:

$$\mathbf{X}^T_N\mathbf{V}'_{CB} = \frac{(\mathbf{V}'_{CB})_4}{2\alpha\beta}\left\{(v_2\alpha - v_1\beta) + \frac{v_3}{v'_3}\left(v'_1\beta - v'_2\alpha\right)\right\}^2 \quad (24)$$

vanishes, the normal minimum is the deepest one. That is:

$$\frac{v'_2}{v'_3}\left(\frac{v'_1}{v'_2} - \frac{\alpha}{\beta}\right) + \frac{v_2}{v_3}\left(\frac{\alpha}{\beta} - \frac{v_1}{v_2}\right) = 0. \quad (25)$$

One possible solution to Eq. (25) is that two S_3 doublet Higgs fields H_1 and H_2 acquire equal vevs $v_1{=}v_2$, $v'_1{=}v'_2$ and $\alpha{=}\beta$. Then, we can see that S_3 extended Higgs potential has an accidental S_2 symmetry in the normal and the electric charge violating minima. This solution corresponds to the Pakvasa-Sugawara minimum [1], and is the condition for the normal minimum to be the deepest one. Other possible solution is realized when $(\mathbf{V}'_{CB})_4$ vanishes, that is,

$$2g\left(\alpha\beta + v'_1v'_2\right) + ev'_1v'_3 = 0, \quad (26)$$

which implies one parametric relationship.

C. *CP breaking.*

The CP breaking stationary point is given by the following vevs: $\phi_7{=}v'_1$, $\phi_8{=}v'_2$, $\phi_9{=}v'_3$, $\phi_{10}{=}\delta$ and $\phi_{11}{=}\gamma$. Then,

$$\mathbf{X}_{CP} = \begin{pmatrix} \delta^2 + v''^2_1 \\ \gamma^2 + v''^2_2 \\ v''^2_3 \\ \delta\gamma + v''_1v''_2 \\ v''_1\gamma - v''_2\delta \\ v''_2v''_3 \\ -\delta v''_3 \\ v''_2v''_3 \\ -\gamma v''_3 \end{pmatrix}. \quad (27)$$

We obtain

$$\mathbf{X}_{CP}^{T}\mathbf{V'}_{CP} = 0, \tag{28}$$

and

$$V_{CP} = \mathbf{A}^{T}\mathbf{X}_{CP} + \frac{1}{2}\mathbf{X}_{CP}^{T}\mathbf{B}_{CP}\mathbf{X}_{CP}, \tag{29a}$$

$$\mathbf{V}'_{CP} = \mathbf{A} + \mathbf{B}_{CP}\mathbf{X}_{CP}. \tag{29b}$$

In this case, the potential evaluated at the normal minimum and at the CP violating minimum can be compared as follows:

$$\mathbf{V}_{CP} - \mathbf{V}_{N} = (1/2)[\mathbf{X}_{CP}^{T}\mathbf{V}'_{N} - \mathbf{X}_{N}^{T}\mathbf{V}'_{CP}] \tag{30}$$

That is, the normal minimum is the deepest one if $\mathbf{X}_{CP}^{T}\mathbf{V}'$ is positive.

$$\begin{aligned}\mathbf{X}_{CP}^{T}\mathbf{V}'_{N} = &-\frac{(\mathbf{V}'_{N})_6}{2v_1v_3}\left[\delta^2 v_3^2 + (v_1''v_3 - v_3''v_1)^2\right]\\ &-\frac{(\mathbf{V}'_{N})_8}{2v_2v_3}\left[\gamma^2 v_3^2 + (v_2''v_3 - v_3''v_2)^2\right]\\ &-\frac{(\mathbf{V}'_{N})_4}{2v_1v_2}\left[(\delta v_2 - \gamma v_1)^2 + (v_1''v_2 - v_2''v_1)^2\right].\end{aligned}$$

The signs of the ratios

$$\frac{-(\mathbf{V}')_4}{2v_1v_2}, \frac{-(\mathbf{V}')_6}{2v_1v_3}, \frac{-(\mathbf{V}')_8}{2v_2v_3}$$

are positive, and the normal minimum is the deepest one when the second term in the right hand side of Eq. (30) vanishes.

7. Higgs mass matrix

We need to know the nature of the stationary points, thus it is necessary to compute the second derivatives of the Higgs potential. These are given by

$$\frac{\partial^2 V}{\partial\phi_i\partial\phi_j} = \frac{\partial V}{\partial x_l}\frac{\partial^2 x_l}{\partial\phi_i\partial\phi_j} + \frac{\partial^2 V}{\partial x_l\partial x_m}\frac{\partial x_l}{\partial\phi_i}\frac{\partial x_m}{\partial\phi_j}. \tag{31}$$

Defining:

$$\begin{aligned}&(\mathbf{V}')_i = \frac{\partial V}{\partial X_i},\ [\mathbf{B}_{lm}] = \frac{\partial^2 V}{\partial X_l\partial X_m},\\ &l, m = 1, 2, ..., 9\\ &[\mathbf{C}]_{ij} = \frac{\partial X_i}{\partial\phi_j}, \quad i = 1, 2, ..., 9;\\ &\quad j = 1, 2, ..., 12.\end{aligned} \tag{32}$$

The corresponding mass matrix has the form

$$[\mathbf{M}^2] = \frac{1}{2}\left([\mathbf{M}_I^2] + \mathbf{C}^T\mathbf{B}\mathbf{C}\right). \tag{33}$$

The first term in Eq. (33) is

$$[\mathbf{M}_I^2]_{ij} = V_l'\frac{\partial^2 X_l}{\partial\phi_i\partial\phi_j}. \tag{34}$$

In the normal minimum this matrix takes the form:

$$[\mathbf{M}_I^2] = Diag\left(\mathbf{M}_{11}^2,\ \mathbf{M}_{12}^2\right)$$

where $\mathbf{M}_{11}^2$ y $\mathbf{M}_{12}^2$ are the following 6×6 matrices:

$$[\mathbf{M}_{11}^2] = \begin{pmatrix} 2V_1' & V_4' & V_6' & 0 & 0 & 0\\ V_6' & V_8' & 2V_3' & 0 & 0 & 0\\ V_4' & 2V_2' & V_8' & 0 & 0 & 0\\ 0 & 0 & 0 & 2V_1' & V_4' & V_6'\\ 0 & 0 & 0 & V_6' & V_8' & 2V_3'\\ 0 & 0 & 0 & V_4' & 2V_2' & V_8' \end{pmatrix},$$

$$[\mathbf{M}_{12}^2] = \begin{pmatrix} 2V_1' & V_4' & V_6' & 0 & 0 & 0\\ V_4' & 2V_2' & 2V_8' & 0 & 0 & 0\\ V_6' & V_8' & 2V_3' & 0 & 0 & 0\\ 0 & 0 & 0 & 2V_1' & V_4' & V_6'\\ 0 & 0 & 0 & V_4' & 2V_2' & V_8'\\ 0 & 0 & 0 & V_6' & V_8' & 2V_3' \end{pmatrix}. \tag{35}$$

The entries in matrix $\mathbf{B}$ are given in Eq. (11c) by the second derivatives of the Higgs potential. Defining the 9×12 matrix $\mathbf{C}_{ij} = \partial x_i/\partial\phi_j$ as:

$$[\mathbf{C}] = \begin{pmatrix} 2\phi_1 & 0 & 0 & 0 & 0 & 0 & 2\phi_7 & 0 & 0 & 2\phi_{10} & 0 & 0\\ 0 & 0 & 2\phi_3 & 0 & 0 & 0 & 0 & 2\phi_8 & 0 & 0 & 2\phi_{11} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\phi_9 & 0 & 0 & 0\\ \phi_3 & 0 & \phi_1 & 0 & 0 & 0 & \phi_8 & \phi_7 & 0 & \phi_{11} & \phi_{10} & 0\\ 0 & -\phi_3 & 0 & \phi_1 & 0 & 0 & \phi_{11} & -\phi_{10} & 0 & -\phi_8 & \phi_7 & 0\\ 0 & 0 & 0 & 0 & \phi_1 & 0 & \phi_9 & 0 & \phi_7 & 0 & 0 & \phi_{10}\\ 0 & 0 & 0 & 0 & 0 & \phi_1 & 0 & 0 & -\phi_{10} & -\phi_9 & 0 & \phi_7\\ 0 & 0 & 0 & 0 & \phi_3 & 0 & 0 & \phi_9 & \phi_8 & 0 & 0 & \phi_{11}\\ 0 & 0 & 0 & 0 & 0 & \phi_3 & 0 & 0 & -\phi_{11} & 0 & -\phi_9 & \phi_8 \end{pmatrix}. \tag{36}$$

Evaluated at each of the different stationary points, only the fields ϕ_7, ϕ_8, ϕ_9, $\phi_1, \phi_3, \phi_{10}$ and ϕ_{11} appear, the remaining fields are zero at the stationary points. Hence, the mass matrix of the squared masses can be computed from (33) to (36) it takes the following form diag$(\mathbf{M}_C^2, \mathbf{M}_C^2, \mathbf{M}_S^2, \mathbf{M}_P^2)$.

The mass of the physical charged Higgs can be expressed as

$$
\begin{aligned}
m^2_{H^{\pm}_{1,2}} =& \frac{1}{2} Tr\mathbf{M}^2_C \pm \sqrt{(Tr\mathbf{M}^2_C)^2 - 4\chi^2} \\
=& V'_1 + V'_2 + V'_3 \pm \sqrt{(V'_1 + V'_2 + V'_3)^2 - (4V'_1V'_2 + 4V'_1V'_4 + 4V'_2V'_3 + V'^2_4 + V'^2_6 + V'^2_8)}.
\end{aligned} \tag{37}
$$

The mass matrices of the scalar and pseudoscalar Higgs fields are given by $\mathbf{M}^2_S$ and $\mathbf{M}^2_P$ respectively, and are both block diagonal.

8. Conclusions

We have analyzed the most general Higgs potential invariant under the non-Abelian flavour symmetry S_3 of the extended SM. In particular, we studied the nature of the critical points in the Higgs potential: The normal one, the charge violating and the CP breaking one. We have found that the normal minimum is stable and it is the deepest one when the flavour S_3 symmetry is unbroken. We also found an S_2 accidental symmetry at the normal minimum.

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