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Selective transport in systems with correlated disorder

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We analyze the recent results concerning the anomalous localization properties of one-dimensional and quasi-one-dimensional guiding structures with stratified and surface disorder. The main focus of our study is the role of long-range correlations in random scattering. It is shown that with a proper type of correlations, one can construct a stratification or surface profile that results in a complete transmission/reflection within a prescribed region of the wave number of incoming classical or quantum waves.

Keywords: Anderson localization; correlated disorder; random stratification; rough surface.

Se analizan resultados recientes sobre las propiedades de localización anómala de estructuras de guías de onda uni- y cuasiunidimensionales con desorden estratificado y superficial. El punto principal de estudio es el papel que desempeñan las correlaciones de largo alcance en el esparcimiento aleatorio. Se demuestra que uno puede construir, con el tipo apropiado de correlaciones, una estratificación o perfil superficial que dén una transmisión/reflexión completa en una región prescrita del número de onda de las ondas clásicas o cuánticas entrantes.

Descriptores: Localización de Anderson; desorden correlacionado; estratificación aleatoria; superficie rugosa.

PACS: 42.25.Dd; 72.10.-d; 72.15.Rn; 73.20.Fz

1. Introduction

The study of the properties of disordered complex systems with spatial and/or temporal correlations is one of the hot topics in modern physics. During the last few years much attention has been paid to a study of selective (or, anomalous) ballistic transport in one-dimensional (1D) structures with correlated disorder. The fundamental significance is due to exciting results that revise a commonly accepted belief that any randomness in a 1D long conductor leads to an exponentially small transmission that is caused by the Anderson localization of all eigenstates. In particular, in papers [1–3] it has been shown that specific long-range correlations in a scattering potential give rise to a perfect electron/wave transmission within any given energy/frequency window. This effect was called *selective*, or *anomalous*, *transparency*. The experimental confirmation of the theory has been presented in Ref. 4 where the same disorder was created in a single-mode waveguide by inserting a discrete array of 500 correlated delta-like scatters (screws) with random heights. The experiments clearly displayed the *mobility edges* that separate the region of the perfect transparency from that of localized transport.

As is known, the study of wave propagation through surface-disordered guiding systems is important for a lot of radiophysic, optic and acoustic applications. Also, the waveguiding propagation describes the electron transport in mesoscopic nanostructures. Therefore, we have recently analyzed the anomalous selective transmission in single and multimode waveguides with specific long-range correlations in a random surface profile.

It was shown analytically and by direct numerical simulations [5] that surface-corrugated systems with one open channel can also have any desired selective transparency. A theoretical method was proposed to construct a surface roughness with predefined correlation properties.

Then the study of anomalous transport has been extended to multimode waveguides [6, 7]. It turned out that the effect of correlated surface disorder in many-channel structures is much more sophisticated than in the 1D case. As a result, the total transmittance (the dimensionless conductance) displays the unusual step-wise dependence on the total wave number k.

We have analyzed the anomalous transport and mobility edges in waveguides with stratified disorder [8–11]. This kind of disorder is also quite typical in many applications. Due to the apparent simplicity, this model allows us to reveal the general peculiarities of the anomalous ballistic transport in multimode systems.

In this report we briefly review the corresponding results, paying a special attention to the most important physical conclusions that follow from our theory.

2. 1D Anderson localization: one-parameter scaling and correlated disorder

To start with, let us formulate the main idea of the anomalous selective transport for a 1D disordered system. This idea is based on *two key points* that constitute the phenomenon of Anderson localization.

The first one is *one-parameter scaling*. This principal concept states that transport properties of any 1D disordered structure are governed solely by the ratio L/L_{loc}, between the conductor length L and the only scaling parameter L_{loc}, which is called *localization length* [12]. Such a universal dependence manifests itself, for example, in the expression for the self-average logarithm of the transmittance $T(L/L_{loc})$,

$$\langle \ln T(L/L_{loc})\rangle = -2L/L_{loc}. \quad (1)$$

Here, the angular brackets $\langle\ldots\rangle$ stand for the stochastic average over different realizations of the disorder.

In accordance with the one-parameter scaling, there are only *two transport regimes* in 1D disordered systems, the regimes of *ballistic* and *localized transport*.

Ballistic transport occurs when the localization length L_{loc} turns out to be much larger than the sample length L. In this case, the conductor is practically transparent since its average transmittance is almost equal to one,

$$\langle T(L/L_{loc})\rangle \approx 1 - 2L/L_{loc} \qquad \text{for} \qquad L_{loc} \gg L. \quad (2)$$

Otherwise, if $L_{loc} \ll L$, the transmittance is exponentially small because of a strong electron/wave localization,

$$\langle T(L/L_{loc})\rangle \approx \exp(-2L/L_{loc}) \qquad \text{for} \quad L_{loc} \ll L. \quad (3)$$

As one can see, in the localization regime a 1D disordered structure almost perfectly (with an exponential accuracy) reflects quantum (electron) or classical waves.

The second key point is that the *localization length* L_{loc} that completely controls the transmission via a 1D random medium is determined by the particular properties of a structure and the *correlation properties* of a disorder. Specifically, for any kind of *weak disorder*, its inverse value

$$L_{loc}^{-1}(k) = W(2k)/8k^2 \quad (4)$$

follows the profile of $2k$-harmonic in the randomness power spectrum $W(k_x)$ [2,11–14] that is nothing but the lengthwise Fourier transform of the binary correlator $\mathcal{W}(x-x')$ for the scattering potential,

$$\mathcal{W}(x) = \int_{-\infty}^{\infty} \frac{dk_x}{2\pi}\, W(k_x)\exp\left(ik_x x\right), \quad (5a)$$

$$W(k_x) = \int_{-\infty}^{\infty} dx\, \mathcal{W}(x)\exp(-ik_x x). \quad (5b)$$

Since the pair correlator $\mathcal{W}(x)$ is a real and even function of the lengthwise coordinate x, its Fourier transform (5b) is an even, real and non-negative function of the wave number k_x.

The above consideration results in a remarkable conclusion: *The binary correlations can suppress the Anderson localization*. This statement can be easily demonstrated for the Gaussian correlator with the power spectrum $W(k_x) = W_0\exp(-k_x^2/4k_c^2)$. Here the localization length reads

$$L_{loc}(k) = (8k^2/W_0)\exp(k^2/k_c^2). \quad (6)$$

One can see that the longer the correlation range k_c^{-1} is, the larger the localization length will be. Therefore, to observe the localized transport, the use of white-noise disorder ($k_c^{-1}\to 0$) is preferable. On the other hand, with a proper value of the correlation parameter k_c, the ballistic regime can be realized even for such a sample length, for which the complete localization takes place in the case of white-noise disorder.

Moreover, if the power spectrum $W(2k)$ *abruptly vanishes* in some interval of wave number k, then the localization length $L_{loc}(k)$ diverges and the structure, even of a large length, turns out to be fully transparent. As an example, let us take a random potential with the *step-wise power spectrum*

$$W(k_x) = W_0\Theta(|k_x| - 2k_c), \quad (7)$$

where $\Theta(x)$ is the Heaviside unit-step function, $\Theta(x{<}0){=}0$ and $\Theta(x>0)=1$. In this case the localization length

$$L_{loc}(k) = 8k^2/W_0\Theta(k-k_c) \quad (8)$$

has infinite value below the transition point $k=k_c$ and falls down to the finite value $8k^2/W_0$ above $k=k_c$. Therefore, the ballistic transport can be changed by a strong localization at this point, *i.e.* a system of sufficiently long length ($L \gg 8k^2/W_0$) is transparent for $k<k_c$ and non-transparent for $k>k_c$.

3. Design of random potentials with predefined correlations: convolution method

In the previous section we have revealed that, in principle, by a proper choice of a disorder one can artificially create the systems with *selective* ballistic transport within a prescribed range of the wave number k. Thus, the important practical problem arises of how to construct a random potential $V(x)$ from a predefined power spectrum $W(k_x)$. This problem can be solved by employing a widely used *convolution method* that was originally proposed by Rice. [15]. The modern applications of this method for the generation of random structures with specific correlations, including the long-range non-exponential correlations, can be found in Refs. [16–21], as well as, in our articles that are the basis of the present review.

The method consists of the following steps. Having a desirable form for the randomness spectrum $W(k_x)$, we derive the *modulation function* $G(x)$ whose Fourier transform is $W^{1/2}(k_x)$,

$$G(x) = \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} W^{1/2}(k_x)\exp\left(ik_x x\right). \quad (9)$$

Then, the random scattering potential $V(x)$ is generated as a convolution of a white-noise $Z(x)$ with the modulation function $G(x)$,

$$V(x) = \int_{-\infty}^{\infty} dx' G(x-x')\, Z(x'). \quad (10)$$

The delta-correlated random process $Z(x)$ is determined by the standard properties,

$$\langle Z(x)\rangle = 0, \qquad \langle Z(x)Z(x')\rangle = \delta(x-x'), \quad (11)$$

and can be easily created with the use of random-number generators.

The important point in this approach is that one can arrange a very *sharp transition* between ballistic and localized transport at any value of the wave number. It is clear that the corresponding power spectrum $W(k_x)$ must abruptly vanish at the prescribed point. This means that the binary correlator $\mathcal{W}(x-x')$ has to be a slowly decaying function of the distance $|x-x'|$. In other words, random scattering potentials $V(x)$ that give rise to the combination of ballistic and localized transport windows should be of specific form with long-range correlations along the structure. Because of an *abrupt* character, the transition point can be regarded as a *mobility edge*.

In particular, for the considered above rectangular power spectrum (7), the corresponding random potential has the following, quite sophisticated, form

$$V(x)=W_0^{1/2}\left[Z(x)-\int\limits_{-\infty}^{\infty} dx' Z(x-x')\frac{\sin(2k_c x')}{\pi x'}\right]. \quad (12)$$

Note that systems with complicated scattering potentials are not exotic. For example, bulk random potentials have been constructed in the experiments [4, 22, 23], while randomly rough surfaces with rectangular power spectrum have been fabricated in experimental study of the backscattering enhancement [17].

4. Multimode stratified waveguide: step-wise non-monotonic transmittance

Now we proceed to a quasi-one-dimensional (quasi-1D) conducting wire (or plane waveguide) of width d with lengthwise random stratification of length L (see Fig. 1).

In such a system the total number N_d of propagating modes (or conducting channels) is equal to the integer part [...] of the ratio kd/π,

$$N_d = [kd/\pi], \quad (13)$$

and is assumed to be large. The total averaged transmittance $\langle T\rangle$ can be expressed as a sum of partial transmittances $\langle T_n\rangle$ for every n-th mode,

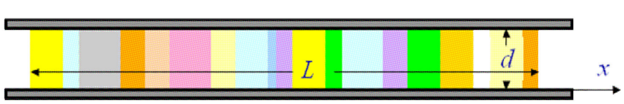

FIGURE 1. (Color online) Waveguiding structure with lengthwise stratification.

$$\langle T\rangle = \sum_{n=1}^{N_d}\langle T_n\rangle. \quad (14)$$

Since the stratified disorder does not depend on the transverse direction, every n-th mode turns out to be independent of the others. Therefore, the stratified structure is reduced to a set of 1D non-interacting channels. Evidently, the partial transport in each channel obeys the one-parameter scaling and is entirely determined by its *mode localization length*,

$$L_{loc}^{-1}(k_n) = W(2k_n)/8k_n^2. \quad (15)$$

This length contains the mode lengthwise wave number

$$k_n = \sqrt{k^2-(\pi n/d)^2} \quad (16)$$

instead of the total wave number k as for the purely 1D case (4).

The mode localization length (15) manifests a quite strong dependence on the mode index n: the larger n is, the smaller $L_{loc}(k_n)$ and, consequently, the stronger the coherent scattering within this mode will be. As a result, we come to the *hierarchy* of mode localization lengths,

$$L_{loc}(k_{N_d})<L_{loc}(k_{N_d-1})<\ldots<L_{loc}(k_2)<L_{loc}(k_1). \quad (17)$$

The smallest mode localization length $L_{loc}(k_{N_d})$ belongs to the highest (last) channel with the mode index $n=N_d$, while the largest mode localization length $L_{loc}(k_1)$ corresponds to the lowest (first) channel with $n=1$.

In contrast to the 1D case, now the interplay between the hierarchy of $L_{loc}(k_n)$ and the one-parameter scaling for every partial transmittances $\langle T_n\rangle$, gives rise to *three transport regimes*.

Ballistic transport: If the *smallest* localization length $L_{loc}(k_{N_d})$ is much *larger* than the waveguide length L, all channels are open. They have unit partial transmittance, $\langle T_n\rangle \approx 1$, and the waveguide is *transparent*. Its total transmittance is equal to the total number of propagating modes,

$$\langle T\rangle \approx N_d \qquad \text{for} \qquad L_{loc}(k_{N_d}) \gg L. \quad (18)$$

Localized transport: If the *largest* localization length $L_{loc}(k_1)$ is much *less* than the waveguide length L, all propagating modes are localized and the waveguide is non-transparent,

$$\langle T\rangle \approx \exp\left[-2L/L_{loc}(k_1)\right] \qquad \text{for} \quad L_{loc}(k_1) \ll L. \quad (19)$$

Coexistence transport: A new phenomenon of the intermediate regime arises when

$$L_{loc}(k_{N_d}) \ll L \ll L_{loc}(k_1). \quad (20)$$

In this case a coexistence of ballistic and localized transport occurs – while the lowest modes are in the ballistic regime, the highest modes are strongly localized.

These regimes are typical when, *e.g.*, the stratification is either a delta-correlated white noise with constant power spectrum, or is a colored noise with Gaussian correlator. The crossover from the ballistic to the localized transport is realized through the successive localization of higher modes. On the contrary, the crossover from the localized to the ballistic transport is realized via the successive opening of lower channels.

The situation fundamentally changes when the stratified medium has specific long-range correlations. To show this, let us take the discussed above step-wise power spectrum (7). Here the explicit expression for mode localization length,

$$L_{loc}(k_n) = 8k_n^2/W_0\Theta(k_n - k_c), \qquad (21)$$

leads to very interesting conclusions that are opposite to the previous ones.

(i) All *low modes* with wave numbers $k_n > k_c$, have finite localization lengths,

$$L_{loc}(k_n > k_c) = 8k_n^2/W_0. \qquad (22)$$

For large enough sample length $L \gg 8k_1^2/W_0$, they are strongly localized. The requirement $k_n > k_c$ implies that the mode indices n must be restricted from above by the condition

$$n \leq N_{loc} = [(kd/\pi)(1-k_c^2/k^2)^{1/2}]\Theta(k-k_c). \qquad (23)$$

So, the integer N_{loc} should be regarded as the total number of localized and non-transparent modes.

(ii) For *high channels* with $k_n < k_c$, the localization length *diverges*,

$$L_{loc}(k_n < k_c) = \infty. \qquad (24)$$

Therefore, each of the modes with $n > N_{loc}$ has a unit partial transmittance, $\langle T_n \rangle = 1$, and exhibits the ballistic transport. Such modes form a subset of completely transparent channels.

(iii) Since the localized modes do not contribute to the total transmittance $\langle T \rangle$, the latter is equal to the total number of completely transparent modes,

$$\langle T \rangle = N_d - N_{loc}$$
$$=[kd/\pi]-[\sqrt{(kd/\pi)^2-(k_c d/\pi)^2}]\Theta(k-k_c). \qquad (25)$$

We remind that square brackets stand for the integer part of the inner expression.

The transmittance (25) reveals a quite unexpected non-monotonic step-wise dependence on the mode parameter $\alpha = kd/\pi$ that is controlled by the normalized correlation parameter $\alpha_c = k_c d/\pi$. An example of this dependence is depicted in Fig. 2.

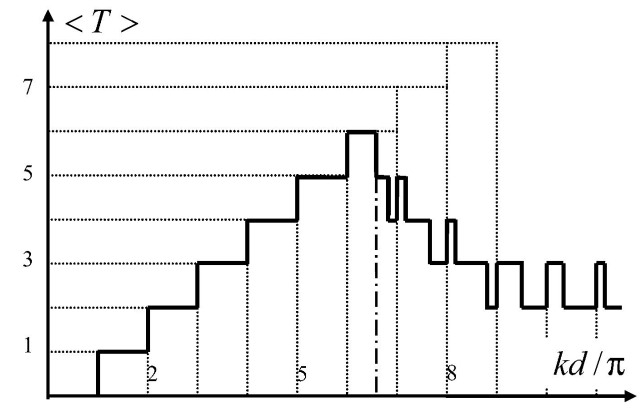

FIGURE 2. Non-monotonic step-wise transmittance (25) versus kd/π for $k_c d/\pi = 6.5$.

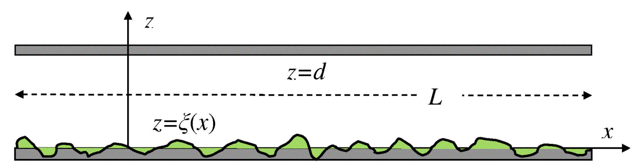

FIGURE 3. (Color online) Plane waveguide with lower corrugated edge.

Within the region where $k_1 < k_c$ ($\alpha < \sqrt{\alpha_c^2+1}$), the second term in Eq. (25) is zero and all propagating modes are transparent. Here the average transmittance exhibits a *ballistic step-wise increase* with an increase of α. Each step up arises for an integer α when a new conducting channel emerges in the waveguide.

When $k_1 \geqslant k_c$ ($\alpha \geqslant \sqrt{\alpha_c^2+1}$), the transmittance shows not only standard steps up associated with the first term, but also *steps down* due to the second term. Every step down is caused by a successive abrupt localization of low modes, and arises at the corresponding *mobility edges* of these modes. The first step down stands at the mobility edge $k_1 = k_c$ of the first mode, the second step down is due to the mobility edge $k_2 = k_c$ of the second mode, etc. The interplay between steps up and down results in a new kind of step-wise oscillating dependence for the total average transmittance.

5. Guiding Systems with Surface Disorder

Now we address the structures with surface scattering. To this end let us consider a plane waveguide (or quasi-1D electron wire) of length L and average width d. Its upper edge $z = d$ is, for simplicity, a straight line. The profile of the lower edge $z = \xi(x)$ is randomly corrugated.

The boundary roughness is described by a random function $\xi(x)$ with standard statistical properties,

$$\langle \xi(x) \rangle = 0, \qquad \langle \xi(x)\xi(x') \rangle = \sigma^2 \mathcal{W}(x-x'). \qquad (26)$$

As usual, we assume the rough surface be flat on average, σ is r.m.s. height of surface corrugations, the binary correlator $\mathcal{W}(x)$ has the unit amplitude, $\mathcal{W}(0) = 1$, and decreases with increase of $|x|$. The angular brackets stand for the ensemble averaging over realizations of $\xi(x)$.

The boundary roughness is supposed to be small in height, $\sigma \ll d$. In the waveguiding direction (x-axis) the system is open. In the transverse direction (z-axis) the zero Dirichlet boundary conditions are applied at both walls of the waveguide.

5.1. Single-mode Structure

First, we shall study a single-mode waveguide. Here the transmission is provided by the *lowest normal mode* that propagates with the wave number k_1 ($0 < k_1 d/\pi < \sqrt{3}$). All other modes are evanescent. In this case, the transport problem is reduced to a purely 1D model with the surface-scattering potential [13]

$$V(x) = (2/\pi)\,(\pi/d)^3\,\xi(x), \tag{27}$$

which is entirely determined by the rough surface profile $\xi(x)$. Thus, the surface scattering in a single-mode structure is relevant to the 1D Anderson localization with the localization length

$$L_{loc}^{-1}(k_1) = \frac{2\sigma^2}{\pi^2}\left(\frac{\pi}{d}\right)^6 \frac{W(2k_1)}{(2k_1)^2}. \tag{28}$$

Equation (28) directly follows from the general definition (4) with the use of the explicit form (27) for the scattering potential.

The surface-scattering localization length (28) is specified by the roughness power spectrum $W(k_x)$. Therefore, with a proper fabrication of randomly corrugated surface we can arrange a desirable selective transport through a single-mode waveguide. The direct generalization of the discussed above *convolution method* gives the expression

$$\xi(x) = \sigma \int_{-\infty}^{\infty} dx' G(x - x') Z(x') \tag{29}$$

with $G(x)$, as before, being the generation function whose Fourier transform is $W^{1/2}(k_x)$. This expression solves the inverse scattering problem of the reconstruction of rough surface from its power spectrum.

Below we demonstrate a possibility of constructing the surface-disordered structures with anomalous selective transport by considering two complimentary examples of a long-range correlated surface profile.

(a) The waveguide is non-transparent when $0 < k_1 < k_c$ and transparent if $k_c < k_1 < \pi\sqrt{3}/d$. For this case we get the following roughness power spectrum and the corresponding surface profile

$$W_a(k_x) = \frac{\pi}{2k_c}\,\Theta(2k_c - |k_x|), \tag{30a}$$

$$\xi_a(x) = \frac{\sigma}{\sqrt{2\pi k_c}} \times \int_{-\infty}^{\infty} dx' Z(x - x') \frac{\sin(2k_c x')}{x'}. \tag{30b}$$

The inverse localization length has a *step-down* form,

$$L_{loc}^{-1}(k_1) \propto \Theta(k_c - k_1). \tag{31}$$

Therefore, as k_1 increases the localization length also smoothly increases and after, goes abruptly to infinity at the transition point $k_1 = k_c$. In this case, for $k_1 < k_c$ the transport is expected to be exponentially small due to the strong localization. Then, the ballistic regime occurs with a perfect transparency.

(b) The waveguide is transparent if $0 < k_1 < k_c$ and non-transparent for $k_c < k_1 < \pi\sqrt{3}/d$. The corresponding roughness power spectrum and the random surface profile are

$$W_b(k_x) = \frac{\pi}{2k_c}\,\Theta(|k_x| - 2k_c), \tag{32a}$$

$$\xi_b(x) = \frac{\sigma}{\sqrt{2\pi k_c}}\left[\pi Z(x) - \int_{-\infty}^{\infty} dx' Z(x - x') \frac{\sin(2k_c x')}{x'}\right]. \tag{32b}$$

It is remarkable that in this case the corrugated surface is described by a superposition of a white noise and the roughness of the first type. Therefore, the inverse localization length has a *step-up* form,

$$L_{loc}^{-1}(k_1) \propto \Theta(k_1 - k_c). \tag{33}$$

Here the localization length is equal to infinity below the mobility edge $k_1 = k_c$ where it abruptly falls down to a finite value. In contrast to the first case, the ballistic transport is changed by a strong localization.

Figure 4 displays the above predictions by direct numerical simulations. In a few details, we computed the discrete analogue of the 1D Schrödinger equation with a random surface potential (27) by the method described in Ref. [2].

The surface profile $\xi(x)$ was generated with the use of the discrete versions of the expressions (30b) and (32b) from two previous examples with the step-wise localization length. The plots clearly exhibit the sharp dependence at the mobility edge $K = k_1/k_c = 1$. Thus, we have arranged numerically the selective transport in surface-disordered single-mode waveguides as was predicted by the asymptotical theory.

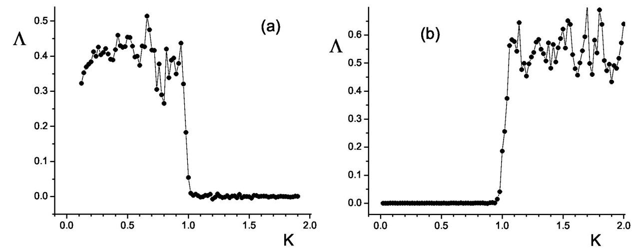

FIGURE 4. Selective dependence of the rescaled inverse localization length $\Lambda \propto L_{loc}^{-1}$ on the normalized wave number $K = k_1/k_c$ for two discussed realizations of a random surface.

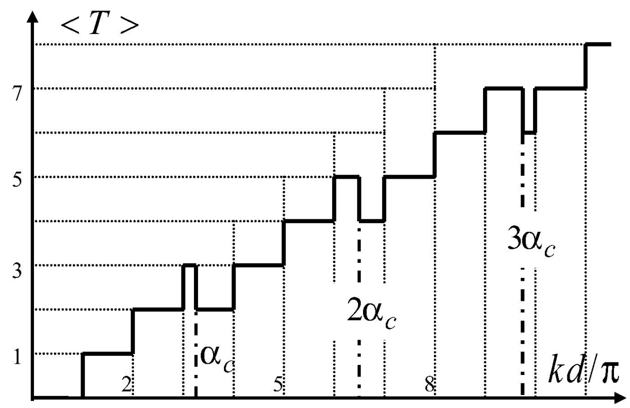

FIGURE 5. Step-wise transmittance (40) versus kd/π at the normalized correlation parameter $k_c d/\pi = 0.32$.

5.2. Surface-Corrugated Multimode Structure

Unlike the single-mode case, in the multimode waveguide the transport properties of any n-th conducting channel are determined by two attenuation lengths, the *length of the forward scattering* $L_n^{(f)}$ and the *backscattering length* $L_n^{(b)}$,

$$\frac{1}{L_n^{(f)}} = \sigma^2 \frac{(\pi n/d)^2}{k_n d} \sum_{n'=1}^{N_d} \frac{(\pi n'/d)^2}{k_{n'} d} W(k_n - k_{n'}), \quad (34a)$$

$$\frac{1}{L_n^{(b)}} = \sigma^2 \frac{(\pi n/d)^2}{k_n d} \sum_{n'=1}^{N_d} \frac{(\pi n'/d)^2}{k_{n'} d} W(k_n + k_{n'}). \quad (34b)$$

Because of the strong dependence on the channel index n, there is the hierarchy of surface-scattering lengths (34),

$$L_{N_d}^{(f,b)} < L_{N_d-1}^{(f,b)} < ... < L_2^{(f,b)} < L_1^{(f,b)}. \quad (35)$$

The smallest mode attenuation lengths, $L_{N_d}^{(f)}$ and $L_{N_d}^{(b)}$, belong to the highest (last) channel with the mode index $n = N_d$, while the largest scattering lengths, $L_1^{(f)}$ and $L_1^{(b)}$, corresponds to the lowest (first) channel with $n = 1$.

As is known, the quasi-1D systems with isotropic volume disorder reveal three typical transport regimes, the regimes of a *ballistic*, *diffusive* (metallic), and *localized* transport. In contrast to this conventional picture, it was shown in Ref. 24 that in the case of surface disorder, due to hierarchy (35) of the scattering lengths, a very important phenomenon of the *coexistence* of ballistic, diffusive, and localized transport arises. Specifically, while the lowest modes can be in the ballistic regime, the intermediate and highest modes can exhibit the diffusive and localized behavior, respectively. Therefore, the concept of one-parameter scaling is no more valid for the transport in multimode surface-disordered systems. Indeed, expressions (34) manifest that, in general, both attenuation lengths are contributed by the scattering of a given n-th propagating mode into all others. As a result, all conducting channels turn out to be mixed.

From this analysis one can conclude that for multimode structures with surface disorder the role of specific long-range correlations should be much more sophisticated in comparison with that discussed above for single-mode waveguides. To show this, let us take the random surface with the power spectrum as a "window function",

$$W(k_x) = \frac{\pi}{k_c} \Theta(k_c - |k_x|), \qquad k_c > 0. \quad (36)$$

It is clear that now the number of modes into which the n-th mode is scattered, is determined by the width k_c of the rectangular spectrum. If the distance between neighboring k_n is larger than k_c,

$$|k_n - k_{n\pm1}| > k_c\,, \quad (37)$$

then all inter-mode transitions are forbidden. As a consequence, for the forward-scattering length $L_n^{(f)}$, the sum in Eq. (34a) contains only one term with $n' = n$, which describes the direct intra-mode scattering. At the same time, every term in the sum (34b) for the backscattering length $L_n^{(b)}$ is equal to zero. This consideration gives rise to the following conclusions.

(i) All *high modes* that satisfy the condition (37), are independent of the others and form a subset of 1D non-interacting channels with finite $L_n^{(f)}$ and infinite $L_n^{(b)}$,

$$L_n^{(f)} = (k_c/\pi\sigma^2)(k_n d)^2 (\pi n/d)^{-4}, \quad L_n^{(b)} = \infty. \quad (38)$$

(ii) As is well known [12,14], the transport via any 1D system is provided only by the backscattering length $L_n^{(b)}$ that is twice as small than the corresponding localization length. Since $L_n^{(b)}$ diverges for every independent mode, all of them are completely transparent with the partial transmittance $\langle T_n \rangle = 1$.

(iii) *Low modes* with the indices n contradicting to Eq. (37), remain to be mixed and non-transparent. Therefore, for large enough waveguide length L, they do not contribute to the total transmittance which, in this case, is equal to the total number of the independent modes.

For the waveguide with many conducting channels, $N_d = [kd/\pi] \approx kd/\pi \gg 1$, the number of mixed modes can be estimated as

$$N_{mix} = [kd/\pi\alpha_c]\,, \quad \alpha_c = \sqrt{1 + (k_c d/\pi)^{-2}}, \quad (39)$$

and the total average transmittance reads

$$\langle T\rangle = N_d - N_{mix} = [kd/\pi] - [kd/\pi\alpha_c]\,. \qquad (40)$$

Equations(40) exhibits a step-wise dependence on the mode parameter kd/π with both steps up and steps down, see Fig. 5. Each *step up* arises when kd/π increases by one and a new propagating mode emerges. The *steps down* are located at the integer values of the ratio $kd/\pi\alpha_c$ which, in general, do not coincide with the integer values of kd/π. The steps down arise due to successive mobility edges of low modes.

Thus, the long-range correlations in rough surface suppress the scattering between different conducting channels. This reduces a system of mixed modes to the subset of independent modes with purely 1D transport. Then, the same correlations provide a perfect transparency of each independent mode.

6. Summary

Within the unified approach based on the theory of Anderson localization, we have analyzed the effect of correlated disorder on the transport properties in 1D and quasi-1D systems.

For 1D guiding structures, the perfect transparency can be arranged by a proper construction of both bulk and surface random scattering potentials.

In multimode waveguides, a long-range stratified or surface disorder can suppress the mode scattering. This reduces the system to a subset of 1D transparent channels whose number can be equal even to the total number of propagating modes. As a result, the transmission can be significantly enhanced in comparison with the case of uncorrelated white-noise disorder.

A typical manifestation of the correlated disorder is the step-wise dependence of the total transmittance on the mode parameter kd/π with both steps up and down. A ballistic step up arises when kd/π becomes an integer and a new propagating mode occurs in the waveguide. Otherwise, a step down emerges at fractional values of kd/π, due to successive mobility edges of low modes.

Our results may be used for the fabrication of a new class of electron nanodevices, electromagnetic or acoustic waveguides, optic fibers with selective transmission and/or reflection. Due to the abrupt behavior of the disorder power spectrum, the mobility edges are expected to be sharp enough in order to observe them experimentally.

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