

Gowdy Cosmological Models from Stringy Black Holes

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In the framework of 4D Einstein–Maxwell Dilaton–Axion theory we show how to obtain a family of both unpolarized and polarized $S^1 \times S^2$ Gowdy cosmological models endowed with nontrivial axion, dilaton and electromagnetic fields from a solitonic rotating black hole–type solution by interchanging the r and t coordinates in the region located between the horizons of the black hole configuration. We also obtain a family of Kantowski–Sachs cosmologies with topology $R^1 \times S^2$ from the polarized Gowdy cosmological models by decompactifying one of the compact dimensions.

Keywords: Gowdy and Kantowski–Sachs cosmological models; rotating black holes; low–energy string theory; coordinate transformation.

En el marco de la teoría tetradimensional de Einstein–Maxwell con dilatón y axión se muestra cómo obtener una familia de modelos cosmológicos de Gowdy (polarizados y no polarizados) con topología $S^1 \times S^2$. Dichos modelos cosmológicos se obtienen a partir de una solución solitónica de tipo agujero negro rotatorio mediante el intercambio de las coordenadas r y t en la región comprendida entre los horizontes del agujero negro y contempla campos dilatónico, axiónico y electromagnético no triviales. A su vez, a partir de las cosmologías de Gowdy polarizadas se obtiene una subclase de modelos cosmológicos de tipo Kantowski–Sachs con topología $R^1 \times S^2$ mediante la decompactificación de una de las coordenadas compactas.

Descriptores: Cosmologías de tipo Gowdy y Kantowski–Sachs; agujeros negros rotatorios; teoría de cuerdas a bajas energías; transformación de coordenadas.

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1. Introduction

In the last few years there have been many attempts to look at cosmology from a string theory point of view. Moreover, string cosmology is becoming a subject of great interest among both theoreticians and phenomenologists. At the present time there are many cosmological scenarios and the topic itself is experiencing tremendous activity. For useful reviews see, for instance, [1] and the references quoted therein.

In the framework of general relativity, Kantowski and Sachs [2] proposed a method which relates the inner region of a static black hole solutions to a homogeneous cosmological background under the simple coordinate transformation $r \longleftrightarrow t$. This result was further generalized and a relationship between Gowdy cosmologies [3] and the Kerr rotating black hole was established by Obregón, Quevedo and Ryan in Ref. 4. In this latter case, the coordinate change mentioned above relates the region located between the two horizons of the rotating black hole solution to a cosmological model of Gowdy type. Thus, this simple coordinate transformation enables us to obtain straightforwardly cosmological backgrounds from black hole configurations, and viceversa, without solving the Einstein equations, a nontrivial fact which is worth taking into account. Recently, several papers concerning the physics of inhomogeneous cosmologies

have appeared in the literature in the framework of Einstein, Einstein–Maxwell, dilaton gravity, sugra and string/M theories [5–7].

The idea of this brief report consists in extrapolating this method to the realm of the 4D low–energy heterotic string theory which describes gravity coupled to a dilaton, an axion and just one electromagnetic vector fieldⁱ. This generalization becomes possible in the case when the 4D theory possesses two commuting Killing vectors (as occurs within the framework of general relativity) and can be lifted to any dimensions for configurations which possess $D - 2$ commuting Killing vectors.

Thus, in this work we shall perform a straightforward implementation of the coordinate transformation $r \longleftrightarrow t$ in order to obtain several families of cosmological backgrounds from black hole configurations (and viceversa) without tears. Namely, by starting with a rotating field configuration of black hole type possessing two horizons, we just apply such a coordinate interchange in the region located between the horizons, and as a result we get as well several families of inhomogeneous cosmological model of Gowdy type, both polarized and unpolarized. It is interesting to note that a family of Kantowski–Sachs cosmologies arises from the polarized Gowdy cosmological models by decompactifying one of the compact dimensions.

2. The General Relativity Side of the Story

We start this Section by quoting the Schwarzschild black hole solution to the vacuum Einstein's equations

$$ds^2 = -(1 - 2m/r)dt^2 + (1 - 2m/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

where m is a constant parameter which can be interpreted as the mass of the static black hole.

On the other side, we shall recall as well a particular cosmological model which was reported for the first time in [9], but which is quoted in the literature as the Kantowski–Sachs cosmologies [2], namely, a metric of the form

$$ds^2 = -N(t)^2 dt^2 + e^{2\sqrt{3}\beta(t)} dr^2 + e^{-2\sqrt{3}\beta(t)} e^{-2\sqrt{3}\Omega(t)} (d\theta^2 + \sin^2\theta d\varphi^2). \quad (2)$$

Kantowski and Sachs obtained the following one-parameter family of solutions to the vacuum Einstein's equations:

$$N(t)^2 = \left(\frac{\alpha}{t} - 1\right)^{-1}, \quad e^{2\sqrt{3}\beta(t)} = \frac{\alpha}{t} - 1, \\ e^{-2\sqrt{3}\Omega(t)} = t^2 \left(\frac{\alpha}{t} - 1\right). \quad (3)$$

As mentioned above, they also realized that this homogeneous cosmological model is related to the Schwarzschild black hole solution inside the horizon under the coordinate transformation $r \longleftrightarrow t$, i.e. solutions (1) and (2)–(3) map into each other by interchanging the coordinates r and t in the region where $r < 2m$ with the following identification: $\alpha = 2m$.

Taking advantage of this fact, Obregón, Quevedo and Ryan implemented this coordinate map in the Kerr solution and showed that the metric of a spinning black hole between the inner and outer horizons can be reinterpreted as an exact cosmological solution of the Gowdy type with topology $S^1 \times S^2$ (for details see [4]).

In order to see how this fact behaves in this case, let us express the Kerr metric in the Boyer–Lindquist coordinates

$$ds^2 = -f \left(dt + \frac{2m'ra' \sin^2\theta}{\Delta' - a'^2 \sin^2\theta} d\varphi \right)^2 + f^{-1} \left[(\Delta' - a'^2 \sin^2\theta) \left(\frac{dr^2}{\Delta'} + d\theta^2 \right) + \Delta' \sin^2\theta d\varphi^2 \right], \quad (4)$$

where

$$f(r) = \frac{\Delta' - a'^2 \sin^2\theta}{r^2 + a'^2 \cos^2\theta}, \quad (5)$$

$\Delta'(r) = (r - m')^2 - \sigma'^2$, $\sigma'^2 = m'^2 - a'^2$, and m' and a' are two constants that represent the mass and the rotation parameter of the rotating black hole.

This solution possesses two horizons given by the following expressions:

$$r_{\pm} = m' \pm \sqrt{m'^2 - a'^2}, \quad (6)$$

where r_+ and r_- are called outer and inner horizons, respectively.

On the other side, the unpolarized Gowdy cosmological models with topology $S^1 \times S^2$ [3] have the following form:

$$ds^2 = e^{(\tau-\lambda)/2} (-e^{-2\tau} d\tau^2 + d\theta^2) + L \sin(e^{-\tau}) \times e^P \left[(d\delta + Q d\varphi)^2 + e^{-2P} \sin^2\theta d\varphi^2 \right], \quad (7)$$

where λ , P and Q are functions of τ and θ , and L stands for arbitrary constant.

Here we shall say just a few words concerning the topology of the metric. It is clear that the $\theta\varphi$ possesses the topology of the two-sphere S^2 . Thus, in order to get a metric with the topology $S^1 \times S^2$, the δ coordinate must be a compact one. This can be achieved by requiring $0 \leq \delta \leq 2\pi$, with points $\delta = 0$ and $\delta = 2\pi$ identified.

In the case when the function Q is set to zero, one obtains the so-called polarized cosmological Gowdy models [10]

$$ds^2 = e^{(\tau-\lambda)/2} (-e^{-2\tau} d\tau^2 + d\theta^2) + L \sin(e^{-\tau}) (e^P d\delta^2 + e^{-P} \sin^2\theta d\varphi^2). \quad (8)$$

By performing the coordinate interchange $t \longleftrightarrow r$ in the region located between the inner and outer horizons $r_- < r < r_+$ of the Kerr solution, one gets the following metric:

$$ds^2 = -f' \left(dt + \frac{2m'ta' \sin^2\theta}{\Delta' - a'^2 \sin^2\theta} d\varphi \right)^2 + f'^{-1} \left[(\Delta' - a'^2 \sin^2\theta) \left(\frac{dt^2}{\Delta'} + d\theta^2 \right) + \Delta' \sin^2\theta d\varphi^2 \right], \quad (9)$$

where we now have

$$f'(t) = \frac{\Delta' - a'^2 \sin^2\theta}{t^2 + a'^2 \cos^2\theta} \quad (10)$$

and $\Delta'(t) = (t - m')^2 - \sigma'^2$.

By performing the following coordinate transformations in this metric (9):

$$t = \alpha \left[1 + \sqrt{1 - \beta^2} \cos(e^{-\tau}) \right], \quad r = \delta, \quad (11)$$

where $a' = \alpha\beta$ and $m' = \alpha$, we effectively compactify the r -coordinate. Thus, we have a metric with the topology $S^1 \times S^2$ of the unpolarized Gowdy cosmological models. It is a straightforward exercise to prove that the metric (9) transforms into the metric (7) under the following identifications:

$$Q = \frac{-2\alpha\beta[1 + \sqrt{1 - \beta^2} \cos(e^{-\tau})] \sin^2\theta}{(1 - \beta^2) \sin^2(e^{-\tau}) + \beta^2 \sin^2\theta}, \quad (12) \\ e^P = \frac{(1 - \beta^2) \sin^2(e^{-\tau}) + \beta^2 \sin^2\theta}{\left\{ \left[1 + \sqrt{1 - \beta^2} \cos(e^{-\tau}) \right]^2 + \beta^2 \cos^2\theta \right\}}$$

$$\times \left[\alpha \sqrt{1 - \beta^2} \sin(e^{-\tau}) \right]^{-1}, \quad (13)$$

$$e^{\frac{(\tau-\lambda)}{2}} = \alpha^2 \left\{ \left[1 + \sqrt{1 - \beta^2} \cos(e^{-\tau}) \right]^2 + \beta^2 \cos^2 \theta \right\}, \quad (14)$$

$$L = \alpha \sqrt{1 - \beta^2} = \sigma'. \quad (15)$$

In the particular case when the rotation parameter a' , in the framework of the spinning black hole configuration, is set to zero, the cosmological parameter β also disappears and, in turn, the function Q vanishes as well. As a result, one obtains a family of polarized Gowdy cosmological models with the topology $S^1 \times S^2$.

Here we would like to make an important remark. The vanishing of the parameter a' or β , which implies the vanishing of the function Q , leads as well to the vanishing of the inner horizon. Thus, when performing the coordinate transformation $r \longleftrightarrow t$, one must take care to map the inner part of the black hole configuration into the polarized Gowdy cosmological background since we no longer have two horizons.

This fact is crucial in order to understand that in this limit, if we decompactify the δ -coordinate, we recover a metric with topology $S^2 \times R^1$ and, hence, another family of Kantowski–Sachs cosmological models with the identifications previously indicated (3).

3. The String Theory Side of the Story

Let us turn our attention to the string theory counterpart of this story. The 4D Einstein–Maxwell theory with dilaton and axion fields (EMDA) is one of the simplest low-energy string gravity models. It arises as the corresponding truncation of the critical heterotic string theory (D=10, with 16 $U(1)$ vector fields) reduced to four dimensions with no moduli fields excited and just one non-vanishing vector field. In the Einstein frame it is described by the action

$$S = \int d^4x |g|^{\frac{1}{2}} \left[-R + 2(\partial\phi)^2 + \frac{1}{2}e^{4\phi}(\partial\kappa)^2 - e^{-2\phi}F^2 - \kappa F\tilde{F} \right],$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \tilde{F}^{\mu\nu} = \frac{1}{2}E^{\mu\nu\lambda\sigma}F_{\lambda\sigma},$$

are the strength of the $U(1)$ Maxwell field and its dual tensor, respectively, ϕ is the dilaton field, κ is the pseudoscalar axion field, and $E^{\mu\nu\lambda\sigma} = (1/\sqrt{|g|})\epsilon^{\mu\nu\lambda\sigma}$. Formally, the EMDA theory can be considered an extension of the Einstein–Maxwell system to the case when one takes into account the (pseudo)scalar dilaton and axion fields.

It turns out that when this theory admits the presence of two commuting Killing vectors, the corresponding field equations can be expressed in a simple chiral form in terms of the so-called matrix Ernst potentials [11] and, hence, the inverse scattering method can be implemented in order to construct

exact solutions. By making use of this method, Yurova has obtained a seven parameter soliton solution to the field equations of this theory [12]. The metric of this solution possesses as well the form of a rotating black hole configuration and is endowed with two horizons.

It is precisely a six parametric subclass of this family of solutions (we shall set to zero the so-called NUT parameter in order to restrict ourself to asymptotically flat field configurations) that we shall use in order to obtain new cosmological backgrounds. Among these solutions we shall encounter inhomogeneous unpolarized and polarized Gowdy cosmological models as well as homogeneous Kantowski–Sachs cosmologies.

Here we shall not derive the Yurova's soliton, but we shall just quote the solution:

$$ds^2 = -\tilde{\mathcal{F}} \left[dt + \frac{2a \sin^2 \theta [mr - (Q_e^2 + Q_m^2)/2]}{\tilde{\Delta} - a^2 \sin^2 \theta} d\varphi \right]^2 + \tilde{\mathcal{F}}^{-1} \left[\left(\tilde{\Delta} - a^2 \sin^2 \theta \right) \left(\frac{dr^2}{\tilde{\Delta}} + d\theta^2 \right) + \tilde{\Delta} \sin^2 \theta d\varphi^2 \right], \quad (16)$$

where

$$\tilde{\mathcal{F}}(r) = \frac{\tilde{\Delta} - a^2 \sin^2 \theta}{r^2 + a^2 \cos^2 \theta - D^2 - K^2}; \quad (17)$$

the function $\tilde{\Delta}$ still possesses the same form

$$\tilde{\Delta} = (r - m)^2 - \tilde{\sigma}^2, \quad (18)$$

but now the constant $\tilde{\sigma}$ consists of a larger quadratic combination of constants

$$\tilde{\sigma}^2 = m^2 + D^2 + K^2 - Q_e^2 - Q_m^2 - a^2, \quad (19)$$

where the constant parameters have the following physical interpretation: m denotes the mass of the gravitational configuration, D is the dilaton charge, K is the axion charge, Q_e and Q_m label the electric and magnetic charges, respectively, and a stands for the angular momentum per unit mass of the rotating soliton.

This solution possesses as well inner and outer horizons defined through the relations

$$r_{\pm} = m \pm \tilde{\sigma}. \quad (20)$$

The expressions for the matter fields in the black hole picture are the following: the dilaton field is

$$e^{2\phi} = \frac{(r + D)^2 - (K + a \cos \theta)^2}{r^2 + a^2 \cos^2 \theta - D^2 - K^2}, \quad (21)$$

and the axion field reads

$$\kappa = \frac{2(Kr - Da \cos \theta)}{(r + D)^2 - (K + a \cos \theta)^2}, \quad (22)$$

whereas the electric and magnetic potentials adopt the form

$$v = \frac{\sqrt{2}(Q_e r + Q_m a \cos \theta + D Q_e + K Q_m)}{r^2 + a^2 \cos^2 \theta - D^2 - K^2}, \quad (23)$$

$$u = \frac{\sqrt{2}(Q_m r - Q_e a \cos \theta + D Q_m - K Q_e)}{r^2 + a^2 \cos^2 \theta - D^2 - K^2}. \quad (24)$$

In the same spirit as in the Kerr solution, we perform the coordinate transformation $t \longleftrightarrow r$ in the region located between the two horizons and obtain the following gravitational field configuration

$$ds^2 = -\mathcal{F} \left[dr + \frac{2a \sin^2 \theta [mt - (Q_e^2 + Q_m^2)/2]}{\Delta - a^2 \sin^2 \theta} d\varphi \right]^2 + \mathcal{F}^{-1} \left[(\Delta - a^2 \sin^2 \theta) \left(\frac{dt^2}{\Delta} + d\theta^2 \right) + \Delta \sin^2 \theta d\varphi^2 \right], \quad (25)$$

where now

$$\mathcal{F}(t) = \frac{\Delta - a^2 \sin^2 \theta}{t^2 + a^2 \cos^2 \theta - D^2 - K^2}, \quad (26)$$

and

$$\Delta(t) = (t - m)^2 - \tilde{\sigma}^2. \quad (27)$$

In order to interpret this metric as an inhomogeneous unpolarized Gowdy cosmological model with topology $S^1 \times S^2$, we again perform a coordinate transformation of the form (11)

$$t = \alpha + \sigma \cos(e^{-\tau}), \quad r \equiv \delta, \quad m = \alpha, \quad (28)$$

thus compactifying the r -coordinate, and we make the following identifications for the functions Q , P and λ :

$$Q = \frac{-2\gamma \sin^2 \theta \{ \alpha [\alpha + \sigma \cos(e^{-\tau})] - (q_e^2 + q_m^2)/2 \}}{\sigma^2 \sin^2(e^{-\tau}) + \gamma^2 \sin^2 \theta}, \quad (29)$$

$$e^P = \frac{\sigma^2 \sin^2(e^{-\tau}) + \gamma^2 \sin^2 \theta}{\{ [\alpha + \sigma \cos(e^{-\tau})]^2 + \gamma^2 \cos^2 \theta - d^2 - k^2 \}} \times [\sigma \sin(e^{-\tau})]^{-1}, \quad (30)$$

$$e^{(\tau-\lambda)/2} = [\alpha + \sigma \cos(e^{-\tau})]^2 + \gamma^2 \cos^2 \theta - d^2 - k^2, \quad (31)$$

and for the constant parameter L

$$L = \sigma = \sqrt{\alpha^2 + d^2 + k^2 - q_e^2 - q_m^2 - \gamma^2}, \quad (32)$$

where we have introduced the new charges of the system $D = d$, $K = k$, $Q_e = q_e$, $Q_m = q_m$, and, finally, the new parameter $a = \gamma$.

The corresponding expressions for the matter fields in the cosmological framework are the following:

the dilaton field reads

$$e^{2\phi} = \frac{\{ [\alpha + \sigma \cos(e^{-\tau})] + d \}^2 - (k + \gamma \cos \theta)^2}{[\alpha + \sigma \cos(e^{-\tau})]^2 + \gamma^2 \cos^2 \theta - d^2 - k^2}, \quad (33)$$

and the axion field is

$$\kappa = \frac{2 \{ k [\alpha + \sigma \cos(e^{-\tau})] - d\gamma \cos \theta \}}{\{ [\alpha + \sigma \cos(e^{-\tau})] + d \}^2 - (k + \gamma \cos \theta)^2}, \quad (34)$$

whereas the electric and magnetic potentials are given by the following relations:

$$v = \frac{\sqrt{2} q_e [\alpha + \sigma \cos(e^{-\tau})]}{[\alpha + \sigma \cos(e^{-\tau})]^2 + \gamma^2 \cos^2 \theta - d^2 - k^2} + \frac{\sqrt{2} (q_m \gamma \cos \theta + d q_e + k q_m)}{[\alpha + \sigma \cos(e^{-\tau})]^2 + \gamma^2 \cos^2 \theta - d^2 - k^2}, \quad (35)$$

$$u = \frac{\sqrt{2} q_m [\alpha + \sigma \cos(e^{-\tau})]}{[\alpha + \sigma \cos(e^{-\tau})]^2 + \gamma^2 \cos^2 \theta - d^2 - k^2} - \frac{\sqrt{2} (q_e \gamma \cos \theta - d q_m + k q_e)}{[\alpha + \sigma \cos(e^{-\tau})]^2 + \gamma^2 \cos^2 \theta - d^2 - k^2}, \quad (36)$$

respectively.

As occurs in the case of the Kerr metric, if we set to zero the rotation parameter a or, equivalently, the cosmological parameter γ , which in turn yields to a vanishing function Q , we are led to a family of polarized Gowdy cosmological models with the topology $S^1 \times S^2$.

However, it is worth noticing that, within the framework of string theory, the vanishing of the function Q does not yield to the vanishing of the inner horizon. This fact is due to the presence of the matter fields, which make their contribution to the size of the region located between the horizons. Thus, by looking at the expression of the horizons (20), we see that the region located between them gets larger because of the minus sign of the γ^2 term in the definition (32) of the constant σ .

If we indeed decompactify the δ -coordinate, we obtain a manifold with topology $R^1 \times S^2$ and, hence, a new family of Kantowski-Sachs cosmological models with the following identifications for the components of the metric tensor:

$$N(t)^2 = \left(\frac{(t - \alpha)^2 - \sigma^2}{d^2 + k^2 - t^2} \right)^{-1},$$

$$e^{2\sqrt{3}\beta(t)} = \frac{(t - \alpha)^2 - \sigma^2}{d^2 + k^2 - t^2},$$

$$e^{-2\sqrt{3}\Omega(t)} = -[(t - \alpha)^2 - \sigma^2]. \quad (37)$$

On the other side, the dilaton field is given by

$$e^{2\phi(t)} = \frac{(t + d)^2 - k^2}{t^2 - d^2 - k^2}, \quad (38)$$

and the axion field reads

$$\kappa(t) = \frac{2kt}{(t+d)^2 - k^2}, \quad (39)$$

whereas the electric and magnetic potentials adopt the form

$$v(t) = \frac{\sqrt{2}(q_e t + dq_e + kq_m)}{t^2 - d^2 - k^2}, \quad (40)$$

$$u(t) = \frac{\sqrt{2}(q_m t + dq_m - kq_e)}{t^2 - d^2 - k^2}, \quad (41)$$

A remarkable feature of this latter cosmological Kantowski–Sachs solution is that all the matter fields decay to zero as we approach the limit $t \rightarrow \infty$. Thus, their physical relevance takes place in an interval where t is finite and becomes crucial as we approach the singularity $t \rightarrow 0$, *i.e.* in the early universe.

4. Discussion

In this work we have presented an implementation of the coordinate transformation $t \longleftrightarrow r$ in order to obtain new (unpolarized and polarized) Gowdy and Kantowski–Sachs cosmological models in the framework of the $4D$ low-energy effective field theory of the heterotic string, the so-called EMDA theory. These stringy cosmological solutions display a different structure with respect to its general relativity counterparts. In particular, the behaviour of the horizons of the

backgrounds generated turns out to be very different for the field solutions of the string cosmologies. This is an interesting subject which deserves further investigation and will be pursued elsewhere.

There are more directions in which the coordinate exchange presented in this work can be exploited. For instance, this idea can be easily generalized to models which involve more than four space–time dimensions with the aid of the so called matrix Ernst potentials [11]; for instance, one could take as a starting solution the field configuration obtained in Ref. 10, apply the coordinate transformation $r \longleftrightarrow t$ in the region located between the horizons and get multi-dimensional ($D > 4$) inhomogeneous cosmological models of a Gowdy type. Another issue concerns the inclusion of the moduli fields that come from the extra dimensions. We hope to develop some research along these lines in the near future.

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