REVISTA MEXICANA DE FÍSICA S **53** (2) 120–124 FEBRERO 2007

Noncommutative chiral gravitational anomalies in two dimensions

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Recibido el 18 de julio de 2005; aceptado el 14 de marzo de 2005

Gravitational anomalies in a noncommutative space are examined. The analysis is generic and independent of a particular noncommutative theory of gravity, and it depends only on how gravity is noncommutatively coupled to chiral fermions. Delbourgo-Salam computation of the gravitational correction of the axial ABJ-anomaly is studied in detail in this context. Finally, we show that the two-dimensional gravitational anomaly does not permit noncommutative corrections in the parameter Θ.

Keywords: Gravitational anomalies; Delbourgo-Salam anomaly; noncommutative field theory.

Se examinan las anomalías gravitacionales en un espacio no conmutativo. El análisis es general e independiente de alguna teoría de gravedad no conmutativa específica y depende sólo de cómo la gravedad se acople a los fermiones quirales. El cálculo de Delbourgo-Salam de la correccción gravitacional a la anomalía axial ABJ se estudia en detalle en este contexto. Finalmente se muestra que la anomalía gravitacional en dos dimensiones no admite correcciones no conmutativas en el parámetro Θ.

Descriptores: Anomalias gravitacionales; anomalía de Delbourgo-Salam; teorías de campo nonconmutativas.

PACS: 04.50.+h; 11.30.Rd

1. Introduction

An important effect in quantum field theory are the anomalies. Axial and gauge anomalies in various dimensions, in particular in four dimensions, have been discussed in the context of noncommutative gauge theories by various authors [1–13].

On the other hand, recently, various noncommutative theories of gravity have been proposed by a number of authors, providing different Moyal deformations of Einstein gravity in four dimensions (for a recent review of noncommutative gravity, see [14]). In this context, a noncommutative proposal for a topological gravity generalizing Euler and the signature topological invariants was given in Ref. 15.

However, at the present time there is not a definitive, well-defined, realistic noncommutative theory of gravity. In this note we will not deal with any specific noncommutative theory of gravity. This is because at the end we will not consider a specific theory of pure gravity, but we will be interested only in the interactions of a linearized noncommutative gravitational field with chiral fermions. However, to be concrete, we will briefly review a particular proposal of noncommutative Einstein gravity [16] given by the noncommutative Einstein-Hilbert action:

$$\widehat{I}_{\text{EH}} = -\frac{1}{16\pi G_N}\int\limits_X d^4x(-e) * e^a_\mu(x) * e^b_\nu(x) * R^{\mu\nu}_{ab}(x),$$

where

$$g_{\mu\nu}(x) = e^a_\mu(x) * e^b_\nu(x)\eta_{ab},$$

and

$$R^{ab}_{\mu\nu}(x) = \partial_\mu\omega^{ab}_\nu(x) - \partial_\nu\omega^{ab}_\mu(x) + [\omega_\mu(x),\omega_\nu(x)]^{ab}_*,$$

with $\omega^{ab}_\mu(x)$ being the noncommutative spin connection associated with the tetrad $e^a_\mu(x)$, and $[A,B]_* \equiv A*B - B*A$ is the Moyal bracket. Here the $*$-product is defined by

$$F * G(x) \equiv \exp\left(\frac{i}{2}\Theta^{\mu\nu}\frac{\partial}{\partial y^\mu}\frac{\partial}{\partial z^\nu}\right)F(y)G(z)\bigg|_{y=z=x}.$$

From now on, in order to avoid causality problems, we will take $\theta^{0\nu} = 0$.

Noncommutative perturbative gravity is defined by a perturbative expansion $I = I^{(0)} + I^{(1)} + I^{(2)} + \mathcal{O}(\kappa^4)$ of the noncommutative Einstein-Hilbert action generated by a perturbative expansion of the metric as follows:

$$g_{\mu\nu} = \eta_{\mu\nu} - \kappa h_{\mu\nu} + \kappa^2 h^\alpha_\mu * h_{\alpha\nu} - \kappa^3 h^\alpha_\mu * h_{\alpha\beta} * h^\beta_\nu + \mathcal{O}(\kappa^4).$$

This note is organized as follows: In Sec. 2 we give the relevant Feynman rules for linear gravity coupled to chiral fermions in a theory $D{=}2k$ dimensions. Sec. 3 discusses the noncommutative correction to the Delbourgo-Salam anomaly. In Sec. 4 we describe the pure noncommutative anomaly in two dimensions.

2. Coupling Gravity to Chiral Fermions

Reference 17 gave the Feynman rules of this pure noncommutative gravity theory. Let us consider the theory in $D = 2k$ dimensions. The coupling of the gravitational field with chiral fermions is given as usual by

$$I_{int} = \int d^{2k}x\ e * e^{\mu a}(x) * \frac{1}{2}\overline{\psi}(x) * i\Gamma_a \overleftrightarrow{D}_\mu\left(\frac{1-\bar{\Gamma}}{2}\right)\psi(x), \qquad (1)$$

where e stands for $\det(e)$ and D_μ is the covariant derivative with respect to the spin connection ω_μ^{ab} given by

$$D_\mu\psi(x){=}\partial_\mu\psi(x){+}\frac{1}{2}\omega_{\mu cd}\sigma^{cd}\psi(x),$$

with

$$\sigma^{cd}{=}\frac{1}{4}[\Gamma^c,\Gamma^d], \qquad \bar{\Gamma}{=}\Gamma_1\ldots\Gamma_{2k}$$

and the Γ's are the Dirac matrices in euclidean $2k$ dimensions.

Expanding e_μ^a around flat space $e_{\mu a}{=}\eta_{\mu a}{+}(1/2)h_{\mu a}$, our noncommutative action splits into two parts, $I_{int}{=}I_1+I_2$, where

$$I_1 = \frac{1}{2}\int dx\; e{*}e^{\mu a}(x){*}\overline{\psi}(x){*}i\Gamma_a \overset{\leftrightarrow}{\partial}_\mu \left(\frac{1-\bar{\Gamma}}{2}\right)\psi(x) \quad (2)$$

and

$$I_2{=}\frac{1}{4}\int dx\; e * e^{\mu a}(x) * \omega_\mu^{cd}(x) \times *i\overline{\psi}(x) * \Gamma_{acd}\left(\frac{1{-}\bar{\Gamma}}{2}\right)\psi(x), \qquad (3)$$

where $\Gamma_{acd} = 1/6(\Gamma_a\Gamma_c\Gamma_d \pm \mathrm{permutations})$. The linearization of our noncommutative action I_{int} given by Eq. (1) leads to the Moyal deformation of linear gravity given by the lagrangians

$$L_1 = -\frac{1}{4}ih^{\mu\nu}(x) * \overline{\psi}(x) * \Gamma_\mu \overset{\leftrightarrow}{\partial}_\nu \left(\frac{1-\bar{\Gamma}}{2}\right)\psi(x), \qquad (4)$$

$$L_2 = -\frac{1}{16}ih_{\lambda\alpha}(x) * \partial_\mu h_{\nu\alpha} * \Gamma^{\mu\lambda\nu}\left(\frac{1-\bar{\Gamma}}{2}\right)\psi(x). \qquad (5)$$

The corresponding noncommutative Feynman rules can be obtained from the lagrangians L_1 and L_2, giving

$$-\frac{i}{4}\varepsilon^{\mu\nu}\Gamma_\mu\left(\frac{1-\bar{\Gamma}}{2}\right)(2p+p')_\nu \exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p_\rho p'_\sigma\right), \quad (6)$$

and

$$-\frac{i}{16}\Gamma^{\lambda\mu\nu}\left(\frac{1-\bar{\Gamma}}{2}\right)\varepsilon^{(1)}_{\nu\alpha}\varepsilon^{(2)}_{\lambda\alpha}\exp\left(\frac{i}{2}\Theta^{\rho\sigma}p_\rho p'_\sigma\right) \times\left[k_{1\mu}\exp\left(\frac{i}{2}\Theta^{\rho\sigma}k_{1\rho}k_{2\sigma}\right) - k_{2\mu}\exp\left(\frac{i}{2}\Theta^{\rho\sigma}k_{1\rho}k_{2\sigma}\right)\right], \quad (7)$$

respectively. We have introduced $\varepsilon^{(i)}_{\mu\alpha}$ for the polarization tensors from the graviton field.

3. Noncommutative Delbourgo-Salam Gravitational Anomaly

Gravitational anomalies in four dimensions were studied first by Delbourgo and Salam [18] as a gravitational correction to the violation of a global symmetry responsible for the decay: $\pi^0 \to \gamma\gamma$. This idea was further developed in Refs. 19 and 20. Here we shall discuss the noncommutative counterpart of Delbourgo and Salam work [18], which showed that in addition to the fermion triangle diagram with three currents, the triangle diagram with one current J of a global symmetry and two energy-momentum tensors T is also anomalous. The corresponding contribution from the anomalous Ward identity is given by

$$\frac{1}{384\pi^2}R_{\kappa\lambda\rho\sigma}R^{\rho\sigma}_{\mu\nu}\varepsilon^{\kappa\lambda\mu\nu}. \qquad (8)$$

This is precisely proportional to the signature invariant $\sigma(X)$ (or the first Pontrjagin class) which, with the Euler number $\chi(X)$, are the classical topological invariants of the smooth spacetime manifold X.

Now we shall discuss in detail the derivation of the noncommutative counterpart of Eq.(8). The scattering amplitude of the process in 4 dimensions is given by

$$\mathrm{Tr}\int d^4p\{\Gamma\cdot p,\Gamma_{\kappa\lambda\mu\nu}\}\frac{\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}(p-k_2)_\rho(p+k_1)_\sigma\right)}{[\Gamma\cdot(p+k_1)-M]} \times\varepsilon_{\rho_1\sigma_1}p^{\rho_1}\Gamma^{\sigma_1}\frac{\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}(p+k_1)_\rho p_\sigma\right)}{(\Gamma\cdot p-M)}\varepsilon_{\rho_2\sigma_2}p^{\rho_2}\Gamma^{\sigma_2} \times\frac{\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p_\rho(p-k_2)_\sigma\right)}{[\Gamma\cdot(p-k_2)-M]}, \qquad (9)$$

where we have used the Feynman rule Eq. (6) in each vertex of the triangle diagram and the corresponding fermion propagators. In order to evaluate this amplitude we promote the integral from 4 to 2ℓ dimensions

$$\int d^{2\ell}p\frac{(\Gamma\cdot(p+k_1){+}M)}{\left[(p+k_1)^2{-}M^2\right]}\cdot\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}(p-k_2)_\rho(p+k_1)_\sigma\right)\varepsilon_{\rho_1\sigma_1}p^{\rho_1}\Gamma^{\sigma_1}\frac{(\Gamma\cdot p+M)}{\left[p^2{-}M^2\right]} \times\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}(p+k_1)_\rho p_\sigma\right)\frac{(\Gamma\cdot(p-k_2){+}M)}{\left[(p-k_2)^2{-}M^2\right]}\varepsilon_{\rho_2\sigma_2}p^{\rho_2}\Gamma^{\sigma_2}\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p_\rho(p{-}k_2)_\sigma\right). \qquad (10)$$

To calculate this integral, we introduce Feynman's parameters x, y and z, in the usual way, integrating out the variable z, keeping only the divergent terms and integrating out the momentum variable p. We finally obtain

$$2^{\ell+1}(\ell-2)k_2^{\rho_1}k_1^{\rho_2}\varepsilon^{\sigma_1\sigma_2\alpha\beta}\varepsilon_{\kappa\lambda\alpha\beta}k_{1\alpha}k_{2\beta} \times \exp\left(-\frac{i}{2}\Theta^{\rho\sigma}k_{1\rho}k_{2\sigma}\right)(4\pi)^{-\ell}\Gamma(2-\ell) \times \int (k_3^2xy-M^2)^{\ell-2}ixy\,\theta(1-x-y)dxdy+\dots, \quad (11)$$

where $\theta(z)$ is the usual Heaviside function. In the last expression we have used the trace identity given by:

$$\mathrm{Tr}\big(\Gamma_{\kappa\lambda\mu\nu}\Gamma^{\sigma_1}\Gamma^{\sigma_2}\Gamma^{\alpha}\Gamma^{\beta}\big)=2^{\ell}\delta^{[\sigma_1}_{[\kappa}\delta^{\sigma_2}_{\lambda}\delta^{\alpha}_{\mu}\delta^{\beta]}_{\nu]}=2^{\ell}\varepsilon^{\sigma_1\sigma_2\alpha\beta}\varepsilon_{\kappa\lambda\mu\nu}.$$

Performing the expansion of the gamma function $\Gamma(\varepsilon)$ for small values of ε with $\varepsilon=2-\ell$, taking the limit $\ell\to 2$ and evaluating the integral in x and y, we finally get

$$-i\frac{k_2^{\rho_1}k_1^{\rho_2}}{12\pi^2}\varepsilon^{\sigma_1\sigma_2\alpha\beta}\varepsilon_{\kappa\lambda\alpha\beta}k_{1\alpha}k_{2\beta} \times \exp\big(-\frac{i}{2}\Theta^{\rho\sigma}k_{1\rho}k_{2\sigma}\big). \quad (12)$$

Now, taking into account the most general Lorentz invariant amplitude, the last expression becomes

$$-\frac{i}{192\pi^2}\varepsilon_{\rho_1\sigma_1}(k_1)\varepsilon_{\rho_2\sigma_2}(k_2)k_{1\alpha}k_{2\beta}\varepsilon^{\sigma_1\sigma_2\alpha\beta}\big(\eta^{\rho_1\rho_2}k_1\cdot k_2 -k_1^{\rho_2}k_2^{\rho_1}\big)\varepsilon_{\kappa\lambda\alpha\beta}\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}k_{2\rho}k_{1\sigma}\right). \quad (13)$$

In the coordinate space, this equation can be rewritten as

$$\varepsilon^{\sigma_1\sigma_2\alpha\beta}(\partial_\alpha\partial_\gamma h_{\rho_1\sigma_1}*\partial_\beta\partial^\gamma h^{\rho_1}_{\sigma_2}-\partial_\alpha\partial^{\rho_2}h_{\rho_1\sigma_1}*\partial_\beta\partial^{\rho_1}h_{\rho_2\sigma_2}) \times\varepsilon_{\kappa\lambda\alpha\beta}, \quad (14)$$

which finally we recognize as the invariant

$$\frac{1}{384\pi^2}R_{\kappa\lambda\rho\sigma}*R^{\rho\sigma}_{\mu\nu}\varepsilon^{\kappa\lambda\mu\nu}. \quad (15)$$

This is precisely the noncommutative signature invariant

$$\widehat{\tau}(X)=\int R*\widetilde{R}\,d^4x,$$

where the tilde over R stands for the Hodge dual with respect to the tangent space indices. Compare this with the noncommutative signature $\widehat{\sigma}(X)$ of Ref. 15, where the Hodge duality was associated with the tetrad indices.

4. Noncommutative Pure Gravitational Anomaly in Two Dimensions

In Sec. 2, we introduced Feynman rules for noncommutative perturbative quantum gravity relevant to computing the chiral gravitational anomalies. In this section, we are going into the details of the computation of the pure gravitational anomaly in two dimensions. We shall follow the notation and conventions of Ref. [21] (for further work, see [22]). We shall not consider global gravitational anomalies [23] here.

In two dimensions, the noncommutative action for a Majorana-Weyl fermion in a gravitational field is given by $I=\int d^2x\; e*e^{\mu a}(x)*\frac{1}{2}\bar{\psi}(x)*i\Gamma_a\partial_\mu\psi(x)$. At the linearized level, the lagrangian is

$$L_{int}=-\frac{1}{4}h^{\mu\nu}(x)*i\bar{\psi}(x)*\Gamma_\mu\partial_\nu\psi(x). \quad (16)$$

The corresponding energy-momentum tensor is given by

$$T_{\mu\nu}(x)=\frac{1}{2}i\bar{\psi}(x)*\Gamma_\mu\partial_\nu\psi(x). \quad (17)$$

In order to facilitate the computation, as usual, we introduce light-cone coordinates

$$x^{\pm}=\frac{1}{\sqrt{2}}(x^0\pm x^1).$$

Dirac matrices are decomposed into

$$\Gamma^{\pm}=\frac{1}{\sqrt{2}}(\Gamma^0\pm\Gamma^1),$$

with $(\Gamma^{\pm})^2=0$ and $\Gamma^+\Gamma^-+\Gamma^-\Gamma^+=2$. In these coordinates, the energy-momentum tensor takes the form

$$T_{++}(x)=\frac{1}{2}i\bar{\psi}(x)*\Gamma_+\partial_+\psi(x), \quad (18)$$

while the interaction action (16) of the gravitational field with fermions in the light-cone coordinates reduces to

$$L_{int}=-\frac{1}{4}ih_{--}(x)*\bar{\psi}(x)*\Gamma_+\partial_+\psi(x). \quad (19)$$

Only the component $h_{--}(x)$ of the graviton is coupled to chiral matter described by the component $T_{++}(x)$ of the energy-momentum tensor. The effective action to the second order in the metric perturbation h is encoded in the two-point correlation function

$$U(p)=\int d^2x\exp\big(ip{\cdot}x\big)\langle\Omega|T(T_{++}(x)*T_{++}(0)|\Omega\rangle. \quad (20)$$

The naive Ward identity is given by $p_-U(p)=0$. This should imply $U(p)=0$ for all p_-; thus, it should be an anomaly. We are going to compute $U(p)$ by evaluating the corresponding one-loop diagram with two external gravitons; this yields

$$U(p)=-\frac{1}{4}\int\frac{dk_+dk_-}{(2\pi)^2}(2k+p)_+^2\frac{1}{k_-+i\varepsilon/k_+} \times\frac{\exp\big(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\big)}{(k+p)_-+i\varepsilon/(k+p)_+}\delta(p+p') \times\exp\big(i(p+p')x\big), \quad (21)$$

where we have again used the Feynman rule Eq. (6) to compute $U(p)$. In light-cone coordinates, the Moyal product is given by

$$\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right) = \exp\left(-\frac{1}{2}\Theta^{+-}(p'_+p_- - p'_-p_+)\right).$$

Thus, by analytic methods, the computation of the integrals gives

$$U(p) = \frac{i}{24\pi}\frac{p_+^3}{p_-}\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right) \times \exp\big(i(p+p')x\big)\delta(p+p'). \tag{22}$$

Then the anomalous gravitational Ward identity is given by

$$p_-U(p) = \frac{i}{24\pi}p_+^3\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right) \times \exp\big(i(p+p')x\big)\delta(p+p'). \tag{23}$$

The computation of the two-graviton diagram coupled with chiral fermions in the noncommutative theory is given by the effective action

$$L_+^{eff}(h_{\mu\nu}) = -\frac{1}{192\pi}\int d^2pd^2p'\frac{p_+^3}{p_-}h_{--}(p) \times \exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)h_{--}(p') \times \exp\big(i(p+p')x\big)\delta(p+p'). \tag{24}$$

Similarly to the usual commutative case, there is no way to add generic counterterms ΔL_+^{eff} such that $L_+^{eff}+\Delta L_+^{eff}$ is invariant under general coordinate transformations.

Thus, let us consider a Dirac fermion in $1+1$ dimensions. Then we have the corresponding action L_D^{eff}, which is the superposition of L_+^{eff}, and its corresponding parity conjugate L_-^{eff} resulting in

$$L_D^{eff}(h_{\mu\nu}) = -\frac{1}{192\pi}\int d^2pd^2p' \times\left[\frac{p_+^3}{p_-}h_{--}(p)\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)h_{--}(p') + \frac{p_-^3}{p_+}h_{++}(p)\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)h_{++}(p')\right] \times \exp\big(i(p+p')x\big)\delta(p+p'). \tag{25}$$

This action is not invariant under infinitesimal general coordinate transformations $\delta x^\mu=\varepsilon^\mu$, $h_{\mu\nu}$ transforms as $\delta h_{\mu\nu}(x)=-\partial_\mu\varepsilon_\nu(x)-\partial_\nu\varepsilon_\mu(x)$, or in the momentum space

$$\delta h_{++}(p) = -2ip_+\varepsilon_+,\;\; \delta h_{+-}(p) = -ip_-\varepsilon_+ - ip_+\varepsilon_-,$$

$$\delta h_{--}(p) = -2ip_-\varepsilon_-.$$

However, in this case there exists a counterterm ΔL_D^{eff} which can be added to L_D^{eff} so that it becomes invariant under general coordinate transformations

$$\begin{aligned}\Delta L_D^{eff} = &-\frac{1}{192\pi}\int d^2pd^2p'\left[\frac{p_+^3}{p_-}h_{--}(p)\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)h_{--}(p')\right.\\ &+\frac{p_-^3}{p_+}h_{++}(p)\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)h_{++}(p') + 2p_+p_-h_{++}(p)\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)h_{--}(p')\\ &-4p_+^2h_{--}(p)\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)h_{+-}(p') - 4p_-^2h_{++}(p)\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)h_{+-}(p')\\ &\left.+4p_+p_-h_{+-}(p)\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)h_{+-}(p')\right]\delta(p+p'). \end{aligned}\tag{26}$$

It is easy to see that this action can be rewritten in a compact form as the following:

$$\Delta L_D^{eff} = -\frac{1}{192\pi}\int d^2pd^2p'\frac{R(p)\exp\left(-\frac{i}{2}\Theta^{\rho\sigma}p'_\rho p_\sigma\right)R(p')}{p_+p_-}\delta(p+p'), \tag{27}$$

which after integration in the variable p' gives precisely the usual correction to the commutative counterpart of [21],

$$\Delta L_D^{eff} = -\frac{1}{192\pi}\int d^2p\frac{R(p)R(-p)}{p_+p_-}, \tag{28}$$

where $R(p)$ is the linearized term of the noncommutative scalar cuvature, which is given by

$$R(p) = p_+^2h_{--} + p_-^2h_{++} - 2p_+p_-h_{+-}.$$

There is a quantum correction to $T_{+-}(p)=0$ which holds classically due to the introduction of h_{+-} in the counterterm lagrangian ΔL_D^{eff} and we have an expectation value of T_{+-} different form zero, which gives rise to the gravitational anomaly

$$\langle 2T_{+-}(p)\rangle = -2\frac{\delta\Delta L_D^{eff}}{\delta h_{+-}(-p)} = -\frac{1}{24\pi}R(p). \tag{29}$$

By momentum conservation, we have in the above analysis that $p' = -p$ through the $\delta(p+p')$, and the phase factor $\exp\big(-i\Theta^{\rho\sigma}p'_\rho p_\sigma\big)$ is equal to one, and therefore there is no modification to the gravitational anomaly in two dimensions in a noncommutative space.

Acknowledgments

This work was supported in part by CONACyT México grant No. 33951E. C.S.-C. is supported by a CONACyT (México) graduate fellowship.

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