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A review on the advances of an effective model of QCD at low energy

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A review on the advances of an effective model for QCD at low energy is given. The basic ingredients of the model are two fermion levels for the quarks, one at positive and one at negative energy. The quarks in these levels are interacting with a boson level, occupied by pairs of gluons with color and spin zero. The meson and baryon spectrum can be described with satisfaction, including the microscopic structure of the nucleon. The Roper resonance is well reproduced and also the position of the pentaquark, though with a possible large width. The vacuum properties are discussed as new partial results. The model was applied to high energy collisions of heavy nuclei, too.

Keywords: Low energy QCD; quark gluon plasma; hadron spectrum.

Se presenta un resumen de un modelo efectivo de la QCD a bajas energías. Los ingredientes básicos del modelo son dos niveles fermiónicos para los cuarks, uno a energía positiva y el otro a energía negativa. Los cuarks interactuan con un nivel bosónico, ocupado por pares de gluones con espín y color zero. El espectro de mesones y bariones se describe satisfactoriamente, incluyendo la estructura microscópico de los nucleones. La resonancia Roper se reproduce bien y también la posición del pentacuark, aunque con una anchura grande. Se discuten las propiedades del vacío como un nuevo resultado parcial. El modelo también se aplica a colisiones de iones pesados.

Descriptores: QCD a bajas energías; plasma de cuarks y gluones; espectro hadrónico.

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1. Introduction

In recent years we presented an effective model of QCD at low and high temperature [1–4]. The main motivation was to construct a model which reproduces non-trivially some aspects of the Hadron spectrum at low energy and afterwards apply approximative many-body techniques. Once the best method is selected it can be applied to real QCD. As it turned out, the model proposed reproduces quite well several properties of QCD at low energy and that it is probably more than just a toy model.

The main requirements for this effective model are, that it should contain the main degrees of freedom (color, spin and flavor), not conserve the particle number (quarks and gluons) because real QCD is relativistic and contains interactions between quarks and gluons. The effective model should also be transparent enough to understand the structure of QCD at low energies and, if possible, has exact solutions (save a numerical diagonalization) such that approximative many-body techniques can be compared to the exact solution. The last point is necessary in order to decide which approximate method works best.

Among the main results of the model, it is worth to mention the following:

i) The gluon content of the nucleon turned out to be 41%,

ii) Only few states appear below 1.5 GeV due to the inclusion of particle mixing interactions.

iii) The pion is lowered in energy due to a strong interaction in the flavor octet, pseudo scalar channel. It produces a quantum phase transition and the results are similar, but not equal, to the language of Goldstone bosons. It is rather related to what in nuclear and solid state physics is called a *collective state*, containing not just a quark and an antiquark but an additional back ground sea of quark-antiquark pairs and gluons.

iv) The Roper resonance was easily reproduced near its correct value, due to the collective nature of this state within the model. It is also a feature not reproduced by standard constituent quark model, except when additional parameters are introduced by hand, allowing a direct fit.

v) The model was also able to describe, without the introduction of additional parameters, exotic states like pentaquarks. However, the coupling of this exotic states with molecular-like configurations of kaons and nucleons may produce a large width.

vi) When temperature was introduced into the model, several aspects of heavy-ion collisions could be reproduced like certain ratios of meson-antimeson and baryons-antibaryons productions.

In what follows, in section 2 we will present shortly the model itself. In section 3 we will resume the main results up to a recent application to the vacuum properties of the model.

2. The model

The main ingredients are: Two fermion levels are positioned at 0.33 GeV and -0.33 GeV, which corresponds to approximately a third of the mass of the nucleon. In this level the quarks are distributed. Each level has the degeneracy of $2\Omega = 2*3*3=18$, where 2 refers to the spin degrees of freedom and the 3 refers to color and flavor degrees of freedom. The state where the lowest level is completely filled corresponds to the perturbative vacuum. Holes in the lowest level are interpreted as anti-quarks while particles in the upper levels are interpreted as quarks. Meson-like particles are described via particle-hole excitations and three quarks in the upper levels, plus particle-hole excitations, represent baryon-like particles. The particle-hole pairs are coupled to a given spin-flavor combination with color zero. There are four possibilities: If by $[S,\lambda]$ we define a pair whose spin is S and flavor (λ,λ), then the possible combinations are [0,0], [0,1], [1,0] and [1,1]. These pairs are mapped to bosons [5].

The quarks in these two levels interact with a boson level at 1.6 GeV, whose value was determined in [6]. This boson level is occupied by pairs of gluons coupled to spin and color zero. These pairs are treated as bosons, too.

To resume, the main ingredients are pairs of quarks-antiquarks and gluon pairs, mapped to bosons. The valence quarks are explicitly treated as fermions. In this sense, one can speak of an *Interacting Boson Model for QCD*.

The Pauli-exclusion principle is taken into account (see [1, 3, 4]) comparing the possible states in the boson space to the microscopic content, which is obtained by a complete classification of the many quark-antiquark and gluon system. Programs are available to obtain the content of many quark-antiquark system, up to 18 quark-antiquark pairs minus the number of valence quarks, and of a many gluon system. The observation of the Pauli-exclusion principle is a novel feature, not present in models which treat the meson cloud as in dependent of the valence quarks.

The Hamiltonian of the model is given by

$$\begin{aligned}
\boldsymbol{H} &= 2\omega_f \boldsymbol{n_f} + \omega_b \boldsymbol{n_b} \\
&+ \sum_{\lambda S} V_{\lambda S} \Big\{ \left[(\boldsymbol{b}^\dagger_{\lambda S})^2 + 2\boldsymbol{b}^\dagger_{\lambda S}\boldsymbol{b}_{\lambda S} + (\boldsymbol{b}_{\lambda S})^2 \right] (1 - \frac{\boldsymbol{n_f}}{2\Omega})\boldsymbol{b} \\
&\quad + \boldsymbol{b}^\dagger (1 - \frac{\boldsymbol{n_f}}{2\Omega}) \left[(\boldsymbol{b}^\dagger_{\lambda S})^2 + 2\boldsymbol{b}^\dagger_{\lambda S}\boldsymbol{b}_{\lambda S} + (\boldsymbol{b}_{\lambda S})^2 \right] \Big\} \\
&\quad + \boldsymbol{n}_{(0,1)0} \Big(D_1 \boldsymbol{n_b} + D_2(\boldsymbol{b}^\dagger + \boldsymbol{b}) \Big) \\
&\quad + \boldsymbol{n}_{(2,0)1} \Big(E_1 \boldsymbol{n_b} + E_2(\boldsymbol{b}^\dagger + \boldsymbol{b}) \Big), \qquad (1)
\end{aligned}$$

where $(\boldsymbol{b}^\dagger_{\lambda S})^2 = (\boldsymbol{b}^\dagger_{\lambda S} \cdot \boldsymbol{b}^\dagger_{\lambda S})$ is a short hand notation for the scalar product (Similarly for $(\boldsymbol{b}_{\lambda S})^2$ and $(\boldsymbol{b}^\dagger_{\lambda S}\boldsymbol{b}_{\lambda S})$) and $\boldsymbol{n}_f$ is the total number of fermion pairs while $\boldsymbol{n}_b$ is the number of gluon pairs. The factor $[1-(\boldsymbol{n_f}/2\Omega)]$ simulates the effect of the terms which would appear in the exact boson mapping of the quark-antiquark pairs [5,7]. The operators $\boldsymbol{b}^\dagger$ and $\boldsymbol{b}$ are boson creation and annihilation operators of the gluon pairs with spin and color zero.

The four parameters $V_{\lambda S}$ are obtained through adjusting the meson spectrum. Only selected states of mesons with spin-parity-charge conjugation J^{PC}=0^{PC} and 1^{PC} where used, while the spectrum for 2^{PC} and 3^{PC} is predicted [1]. The parameters D_1 and D_2 where used to adjust the nucleon resonances while the parameters E_1 and E_2 where used to adjust the Δ-resonances [3, 4]. Thus, in total the model has eight parameters.

Care has to be taken to which experimental data the parameters are fitted. By intention we did not include interactions which distinguish between states within a flavor irreducible representation. These can be added easily using Gel'man type interactions [8]. Also, no flavor mixing interactions are present. These can also be added by using known values of the mixing angles [1, 9]. *The "experimental" data, used to fit the spectrum of mesons, are thus corrected values to the measured ones, having taken into account the Gel'man corrections and those of flavor mixing.* One can also add these corrections to the Hamiltonian, with the disadvantage to obscure the basic structure of the model.

3. Results

3.1. The meson spectrum

In Table I we resume the results of the fit to the meson spectrum.

The last three columns list the expectation value of the quark-antiquark pairs with $[S,\lambda]$ = [1,0], the total number of

TABLE I. Particle content for selected states. In columns we indicate the theoretical energy (E_{theo}), the flavor ((λ_f,μ_f)), spin J and parity (π), expectation value of the boson images of the quark-antiquark pairs in the channel (1,1) 0^-, ($\langle n_{10}\rangle$), expectation value of the total number of quark-antiquark pairs ($\langle n_{q\bar{q}}\rangle$) and the total number of gluon pairs ($\langle n_g\rangle$) with spin 0.

particle	E_{theo}	(λ_f,μ_f)	J^π	$< n_{10} >$	$\langle n_{q\bar{q}}\rangle$	$\langle n_g\rangle$
vacuum	0.0	(0,0)	0^+	3.118	3.177	1.705
f_0(400-1200)	0.656	(0,0)	0^+	0.457	0.471	0.321
f_0(980)	0.797	(1,1)	0^+	3.781	3.832	1.495
f_1(1420)	1.445	(0,0)	1^+	2.392	3.434	0.902
f_2(1270)	1.363	(1,1)	1^+	2.464	3.519	0.993
η'(958)	0.885	(0,0)	0^-	2.509	3.562	1.292
η(1440)	1.379	(0,0)	0^-	0.773	1.790	0.444
η(541)	0.602	(1,1)	0^-	2.711	2.766	1.163
η(1295)	1.428	(1,1)	0^-	1.611	1.638	0.531
η(1760)	1.671	(1,1)	0^-	3.535	4.581	1.254
ω(782)	0.851	(0,0)	1^-	2.563	3.621	1.341
ϕ(1020)	0.943	(1,1)	1^-	2.394	3.438	1.198
ω(1420)	1.389	(1,1)	1^-	0.853	1.870	0.468
ω(1600)	1.639	(1,1)	1^-	3.546	4.597	1.206

quark-antiquark pairs and the average number of gluon pairs. The other possible quark-antiquark pairs are not listed because they give in most cases a small contribution, as can be seen by comparing $\langle n_{1,0} \rangle$ with $\langle n_{q\bar{q}} \rangle$. That the isoscalar pairs with octet flavor dominate the structure is understood by observing that the interaction in this channel is particularly strong, provoking a nearly complete phase transition to a vacuum which is dominated by these pairs.

One important point is that each particle is a complicated mixture of quark-antiquark and gluon pairs. Especially interesting is the content of the vacuum state, for later discussion of the vacuum properties. When we return to it, one has to keep in mind that we are investigating a volume of the size of a hadron and the vacuum state corresponds to the lowest state possible in such a small volume.

Another important point is related to the multiplicity of the meson states at low energy. When the interaction switched off, there is a large multiplicity of states below 1.5 GeV. This multiplicity poses a serious problem to models with a fixed number of quarks (2 for mesons and 3 for baryons). One circumvents this problem enforcing a fixed number of quarks and excluding many quark states. (This is not much different when considering pentaquarks too.) When the interaction is turned on, the multiplicity disappears. The fact that the interaction mixes the number of particles results in a disappearance of the multiplicity problem. *We emphasize that the multiplicity problem always remains when the number of quarks and antiquarks is fixed and many quark-antiquark states are allowed.*

3.2. The baryon spectrum

In Fig. 1 we show the results of the fit to the nucleon- and Δ-resonances. Important to note is the correct position of the Roper resonance in energy. This is usually very hard to obtain in models with a fixed number of quarks, except when additional parameters are introduced which permit to shift this state by hand to the correct position. The reason why in this effective model the Roper resonance is lowest in energy is due to its collective character. Observe the total number of quarks plus antiquarks and gluons, given in parenthesis, which is particularly large for the Roper resonance. Another feature is the appearance of penta- and heptaquarks, containing at least 5 and 7 quarks respectively. We give a prediction of their spin-parity content. In fact, in our model one can not talk about a fixed number of quarks, say 5 for the pentaquark, because many more contribute. However, the pentaquark is defined here as a state which at least contains 5 quarks and in addition is couples to flavor irreps which can not be reached by five quarks alone.

The case of the existence or not of the pentaquark [10] is more complicated here. Our model can not distinguish between a state which has 5 quarks and a state which is just the *sum* of a two (meson) and a three quark state (baryons), *i.e.* the pentaquark state can very well be just the sum of a meson plus a nucleon. In order to illustrate that, in Table II we list the sum of the content, in terms of quark-antiquark and gluon pairs, of a nucleon and a meson. If we compare the sum of these states with the content of the lowest pentaquark state, it is almost the same. This suggests that probably the pentaquark can be at most a molecular state with a large width. From this observation we cannot conclude if the pentaquark can be observed or not, it may very well be just a molecular state of a meson (Kaon) and a nucleon. Still, it can exist and a search for it is worthwhile.

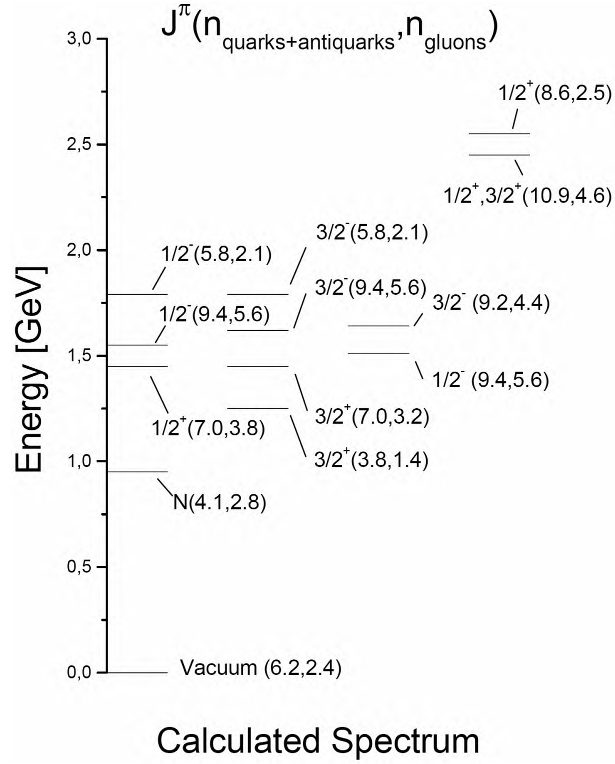

FIGURE 1. Nucleon resonances (first group of levels), Δ resonances (second group), pentaquarks (third group) and heptaquarks (fourth group). On the right side of each level are given the assigned spin and parity (J^π), and the total quark and antiquark ($n_q + n_{\bar{q}}$) and gluon ($2 * n_g$) contents (see the text) The list is not complete. Only the most important states are listed.

TABLE II. Content of the Kaon the nucleon and the pentaquark. The numbers in the square parenthesis refer to the spin and flavor content. For details, see the text.

particle	$[0,0]$	$[0,1]$	$[1,0]$	$[1,1]$	$\langle n_{q\bar{q}} \rangle$	$\langle n_g \rangle$
Kaon	0.005	0.0024	2.710	0.026	2.766	1.167
nucleon	0.007	0.029	0.466	0.047	3.549	1.395
sum	0.012	0.053	3.176	0.073	6.315	2.562
pentaquark	0.006	0.032	3.140	0.034	6.213	2.805

3.3. Including finite temperature

One can introduce finite temperature into the model . For that one has to define the partition function

$$Z = \sum_i e^{-\beta(E_i - \mu_B B - \mu_s s - \mu_T T_z)}, \qquad (2)$$

where μ_B, μ_s and μ_T are the baryon, strange and isospin chemical potentials, respectively. The sum over the index i refers to all possible meson and baryon states. Their energy is given by E_i and it is calculated using the model Hamiltonian. (In the cases discussed here the parameters D_k and E_k were put to zero. This is not a severe restriction because the dominant contribution comes from the light meson states.)

The parameters of the theory are

i) the elementary volume, *i.e.* the size of a hadron,

ii) the number of participants (the number of nucleons which fuse into the QGP) and

iii) the baryon quemical potential.

The elementary volume was fitted to the bag pressure, the number of participants to the absolute production number of pions and the quemical potential μ_B to the production ratio of K^-/K^+. For details, please see Ref. [2].

Observables, like the average number of a certain particle species $\langle n_\alpha \rangle$ are calculated in the usual way [2]. Other observables can be the eigenvalue of the second order Casimir operator of the color and of its variation. They are all calculated with respect to a finite volume of the size of a hadron. One can look at it as if a special part of the whole volume is cut out and the content of particles, or others like the average color, in this volume is investigated.

Particularly interesting is the average color and its variation. In Fig. 2 we plot these values as a function of the temperature. For low temperature the color is essentially zero, while at higher temperature it increases. The variation is also small for low temperature. It is small but larger than the average color at low temperature. It is of the same order as the average color at about 0.17 GeV and afterwards it is large but smaller than the average color. A possible interpretation comes into the mind: At large temperature a volume of the size of a hadron has an average color different from zero. There is a variation but smaller than the average color, *i.e.* in general a given volume of the size of a hadron has color. The Quark-Gluon-Plasma (QGP) is then formed. It can not hadronize because all volumes have color and cannot be separated, due to confinement. At about 0.170 GeV the the color is as large variation of as the average color. Thus, in some occasions a volume of the size of a hadron has color zero and it can escape from the QGP. The hadronization process starts! At lower temperature the probability to have color zero increases, because the variation of color is much larger than the average color, *i.e.* the QGP hadronizes.

This gives a nice intuitive picture on the hadronization process. The temperature at which the hadronization process starts coincides with the estimated temperature of more realistic models.

The next question, which one can pose, it related to the number of particles produced in a given heavy ion collision at a certain energy. Also this can be treated in our model. In Fig. 3 several particle ratios are plotted for the case of a collision of two gold nuclei at $\sqrt{s}$ = 200 GeV/nucleon. It includes meson/anti-meson and baryon/anti-baryon ratios. The K^-/K^+ ratio was fitted, while the others come out as predicted by the model. As one can see, the agreement is very good and of the same quality as gas models [11]. Note, however, that *within our model the interaction between the particles are strong* and not as in [11] where a negligible interaction was assumed.

We cannot go further into the detail. For that, please consult Ref. 2.

3.4. Vacuum properties

Finally we give the results for vacuum expectation values as obtained within the model. Consider a finite volume of the size of a hadron. The vacuum in this model is then the lowest state possible. The content of particles in this vacuum state is

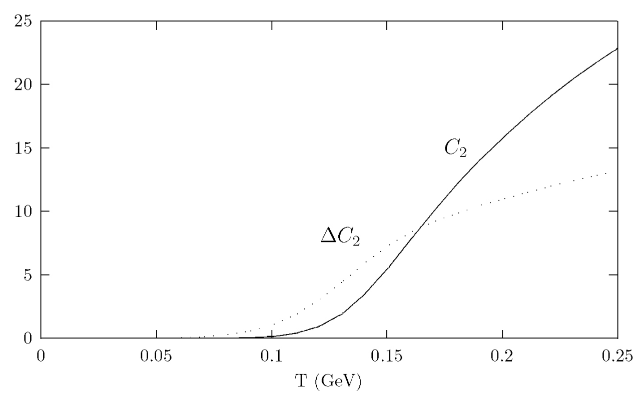

FIGURE 2. The average number of color as a function of temperature. The average color is measured via the eigenvalue of the second order Casimir operator. When (λ_C, μ_C) is the color irrep, the eigenvalue of C_2 is given by $(\lambda_C^2 + \lambda_C\mu_C + \mu_C^2 + 3\lambda_C + 3\mu_C)$. Also the variation ΔC_2 is plotted.

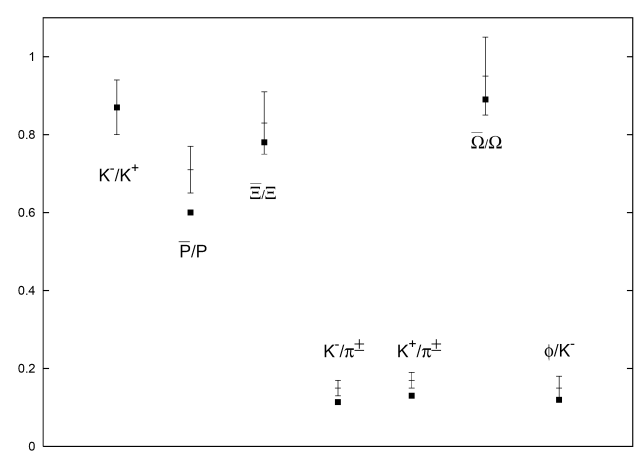

FIGURE 3. Production rations of some particle. The value for the K^-/K^+ ratio was fitted.

listed in Table I. These values will be used to obtain the quark and gluon condensate *without further parameters*.

The vacuum expectation value of the gluon sector of the QCD Lagrangian has the value [12, 13]

$$\langle vac \mid \frac{\alpha_s}{\pi} F^a_{\mu\nu} F^{\mu\nu}_a \mid vac\rangle = (0.360\, GeV)^4. \qquad (3)$$

In this expression $\alpha_s = (g^2/4\pi)$ is the strong coupling constant and depends on the energy of the particle and its radius. The number on the right hand side is a first estimate on the structure of the vacuum state and serves as a test of the effective model. Again, this number should be different from zero and can not be reproduced by constituent quark model, except when additional phenomenological operators, other than the Hamiltonian, with parameters are introduced. In our model, it will be a consequence of the non-vanishing expectation values of quark-antiquark and gluon pairs. These values results in our model due to particle mixing interactions. For the gluons we consider constant modes in the time gauge $A^a_0 = 0$, *i.e.* we have also $\partial_i A^a_j = 0$, and therefore $F_{ij} = gf_{abc}A^b_i A^c_j$. After some coupling and recoupling manipulations one arrives at $16g^2(1/4\omega_g^2 V^2)[4n_g + 9]$.

Using $V{=}(4\pi/3)r_0^3$ and the average values of gluon pairs (see Table I), we arrive at the result $(\alpha_s^2/r_0^6)0.923{\cdot}10^{-4}$ [GeV]^4 for the gluon condensate of Eq. (3). To extract the value of α_s we have taken r_0=0.875 fm to be the value of the proton radius [14] and replaced ω_g by the glue-ball value [6], leading to $\alpha_s = 13.49\ r_0^3 = 9.937$. This is in excellent agreement to the estimations given in [12].

Also the quark-antiquark condensate can be estimated. Using constant functions (as an approximation) for the orbital part of the quark-antiquark pairs we arrive at

$$\langle vac \mid \bar{\Psi}_f \Psi_f \mid vac\rangle = \frac{1}{V}\left(n_{f,q} + n_{f,\bar{q}} - 6\right). \qquad (4)$$

where the index f refers to the flavor. Also here the numbers for $n_{f,q}$ and $n_{f,\bar{q}}$ are taken from Table I. The final result is

$$\langle vac \mid \bar{\Psi}_u \Psi_u \mid vac\rangle = -\left(0.225\left(\frac{0.867}{r_0}\right) GeV\right)^3,$$

where we used that all flavors appear in equal numbers, *i.e.*,

$$(n_{f,q} + n_{f,\bar{q}}) = \frac{1}{3}\left(n_q + n_{\bar{q}}\right) = \frac{2n_{q\bar{q}}}{3}.$$

Again it is in excellent agreement to the estimation given in Ref. [12].

Note, that in the final values enter the expectation numbers of the average value of quark-antiquark and gluons pairs. These numbers are the result of adjusting the meson spectrum, *i.e.* the values of the gluon and quark-antiquark condensate is a direct consequence of these numbers and are not adjusted! Details of this section will be published in [15].

4. Conclusions

We have given a review on the advances of an effective model of QCD. The original motivation was to construct a model sufficiently complex to mimic QCD. However, it turned out that the model describes quite well the cross structure of QCD at low energy. It is part of a surprise because it does not include orbital excitations.

One answer to that might be that the model has all relevant degrees of freedom, like color flavor and spin. The model space is equivalent to the microscopic space of QCD. An additional possible reason is that orbital excitations might be possible to simulate via particular quark-antiquark pairs with open spin. We will continue to test the model further.

Decay properties have yet to be calculated. We expect to see in the decay properties some deviations which explains the simple nature of our model. Also in the case of the spin of the nucleon we do not expect a good agreement. One reason is that the gluon pairs have spin 0 and thus do not contribute to the spin. In an earlier calculation (unpublished) we introduced pairs gluons with spin two. These are the energetically the next objects to contribute. However, their contribution to the low energy structure turned out to be negligible and, thus, do not contribute to the spin of the nucleon. Only if the contribution of the gluons turn out to be small, our model still would represent some reality. However, if the dominant contribution to the spin of the nucleon comes from orbital excitations of the quarks and antiquarks, our model will be able to simulate it.

Acknowledgments

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1. S. Lerma, S. Jesgarz, P.O. Hess, O. Civitarese, and M. Reboiro, *Phys. Rev. C* **67** (2003) 055209.
2. S. Jesgarz, S. Lerma, P.O. Hess, O. Civitarese, and M. Reboiro, *Phys. Rev. C* **67** (2003) 055210.
3. M. Nuñez *et al.*, *Phys. Rev. C* **70** (2004) 025201.
4. M. Nuñez, S. Lerma H., P.O. Hess, O. Civitarese, and M. Reboiro, *Phys. Rev. C* **70** (2004) 035208.
5. A. Klein and E.R. Marshalek, *Rev. Mod. Phys.* **63** (1991) 375.
6. P.O. Hess, A. Weber, C.R. Stephens, S.A. Lerma H., and J.C. López, *Eur. Phys. Jour. C* **9** (1999) 121.
7. O. Civitarese and M. Reboiro, *Phys. Rev. C* **57** (1998), 3055; *ibid.* **58** (1998), 2787.
8. B. Müller and W. Greiner, *Quantum Mechanics: Symmetries*, (Springer, Heidelberg, 1989).

9. Fl. Stancu, *Group Theory in Subnuclear Physics* (Oxford University Press, Oxford, 1996).

10. T. Nakano *et al.*, *Phys. Rev. Lett.* **91** (2003), 012002; V.V. Barmin *et al.* (DIANA collaboration), preprint hep-ex/0304040, Yed. Fis. (in press);
S. Stepanyan *et al.* (CLAS collaboration), preprint hep-ex/0307018; J. Barth *et al.* (SAPHIR collaboration), preprint hep-ex/307083, *Nucl. Phys. B* (in press); A.E. Asratyan, A.G. Dolgolenko, and M.A. Kubantsev, preprint hep-ex/0309042, submitted to Yad. Fis. (2003).

11. P. Braun-Munzinger, I. Heppe, and J. Stachel, *Phys. Lett. B* **465** (1999) 15; P. Braun-Munzinger and J. Stachel, *J. Phys. G* **28** (2002) 1971.

12. L.J. Reinders, H. Rubinstein, and S. Yazaki, *Phys. Rep.* **127** (1985), 1.

13. W. Greiner and A. Schäfer, *Quantum Chromodynamics*, (Springer, Heidelberg, 1994).

14. S. Eidelman *et al.*, *Phys. Lett. B* **592** (2004) 1.

15. P.O. Hess and O. Civitarese, *Int. J. Mod. Phys. E* **15** (2006) 1233.