Investigations into nuclear pairing

R.M. Clark
Nuclear Science Division, Lawrence Berkeley National Laboratory,
Berkeley, CA 94720, U.S.A.

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This paper is divided in two main sections focusing on different aspects of collective nuclear behavior. In the first section, solutions are considered for the collective pairing Hamiltonian. In particular, an approximate solution at the critical point of the pairing transition from harmonic vibration (normal nuclear behavior) to deformed rotation (superconducting behavior) in gauge space is found by analytic solution of the Hamiltonian. The eigenvalues are expressed in terms of the zeros of Bessel functions of integer order. The results are compared to the pairing bands based on the Pb isotopes. The second section focuses on the experimental search for the Giant Pairing Vibration (GPV) in nuclei. After briefly describing the origin of the GPV, and the reasons that the state has remained unidentified, a novel idea for populating this state is presented. A recent experiment has been performed using the LIBERACE+STARS detector system at the 88–Inch Cyclotron of LBNL to test the idea.

Keywords: Collective models; pairing; Pb isotopes.

Este artículo consiste en dos secciones en las cuales se presentan aspectos diferentes del comportamiento colectivo nuclear. En la primera parte, se consideran soluciones para Hamiltonianos describiendo el apareamiento colectivo. En particular, se discute una solución aproximada para el punto crítico de la transición de apareamiento entre vibraciones armónicas (comportamiento regular) y rotaciones deformadas (comportamiento superconductor). Se hace una comparación con las bandas de apareamiento en los isótopos de Pb. La segunda sección se concentra en la búsqueda experimental de la vibración de apareamiento gigante (GPV por sus siglas en inglés) en la física nuclear. Después de haber discutido el origin de la GPV y las razones por las cuales hasta la fecha no ha sido identificada, se presenta una idea novedosa para poblar esta modo de excitación. Recientemiente, se llevó a cabo un experimento con el detector LIBERACE+STARS en el ciclotrón de LBNL para comprobar este método.

Descriptores: Modelos colectivos; apareamiento; isótopos de Pb.

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1. The critical–point description of the transition from vibration to rotation in the pairing phase

Pair correlations in electron motion are directly related to macroscopic phenomena such as superconductivity [1]. The concepts that were developed to describe such correlations found immediate application in nuclear physics and provided a key to understanding the excitation spectra of even–A nuclei, odd–even mass differences, rotational moments of inertia, and a variety of other phenomena [2, 3]. Pair correlations are also of great importance in describing the behavior of other finite Fermion systems such as ^{3}He clusters, Fermi–gas condensates, fullerenes, quantum dots, and metal clusters [4]. Ideas to describe phenomena associated with pair correlations in any one system are likely to find application in the others.

An early approach to describing pair correlations in nuclei was the development of a collective model by Bès and co–workers [5]. The variables in the model are a pair deformation, α (which can be related to the gap parameter), and a gauge angle, ϕ (which is the canonical conjugate to the particle–number operator, N). The collective pairing Hamiltonian was derived in direct analogy to the Bohr collective Hamiltonian which describes the quadrupole degree of freedom for the nuclear shape [6].

Notable benchmarks of nuclear behavior such as the harmonic vibrator [7], the symmetrically deformed rotor [8], and the soft triaxial rotor [9] correspond to analytic solutions of the Bohr Hamiltonian. They also correspond to limits of the Interacting Boson Model (IBM) [10]. An algebraic description of the nature of the transition between these limits has been developed in direct analogy with classical phase transitions [11]. The Bohr Hamiltonian has recently received renewed attention due to the suggestion that simple analytic approximations can be made to describe the critical–point of the transitions between nuclear shapes [12–14]. These can then serve as new benchmarks against which nuclear properties can be compared.

In this paper similar approximations are applied to obtain an analytic solution of the collective pairing Hamiltonian corresponding to the critical point of the transition from a "normal" to a "superconducting" nucleus. Note, some of this work is described in a recent publication [15]. Nuclei with two identical particles added or removed from a closed–shell configuration should be close to the normal limit, where there is no static deformation of the pair field and the fluctuations of the field give rise to a pairing vibrational spectrum [16]. Pairing vibrational structures have been observed around ^{208}Pb [17], although large anharmonicities must be included in this interpretation. In nuclei with many particles outside of the closed–shell configuration, a static deformation of the pair field arises and rotational behavior results. This corresponds to the superconducting limit. The angular variable in the rotational motion is the gauge angle, ϕ, which describes the orientation in gauge space. This broken symmetry in gauge space results in a pair–rotational band [18] comprising the sequence of ground–states of even–even nuclei, differing by pairs of identical nucleons, and with many nucleons outside a closed shell.

Here, the transition from the pair–vibrational to pair–rotational regimes can be investigated. To do this solutions must be found to the collective pairing Hamiltonian [5]:

$$-\frac{\hbar^2}{2B}\frac{\partial^2\psi}{\partial\alpha^2}-\frac{\hbar^2}{4}\left(\frac{1}{\Im B}\frac{\partial\Im}{\partial\alpha}-\frac{1}{B^2}\frac{\partial B}{\partial\alpha}\right)\frac{\partial\psi}{\partial\alpha} + \left(V\left(\alpha\right)+\frac{\hbar^2M^2}{2\Im}-E\right)\psi=0 \quad (1)$$

where α is the deformation of the pair field, $\Im$ is an inertia parameter, B is a mass parameter, M=A–A_0 (number of particles, A, relative to a reference A_0), and V(α) is the potential. In general, $\Im$ and B are functions of the pair deformation, α. By choosing suitable potentials analytical solutions of equation 1 can be found in the different limits. Consider the potential energy surface as a function of the pair-field deformation parameter, α, as schematically illustrated in Fig. 1. In the vibrational limit the potential is parabolic with a minimum at α=0. The transition to the rotational pairing regime gives rise to a deformed minimum in the potential. At the critical point, these two minima cross and the deformation of the pair–field changes from spherical to deformed. This picture is supported by boson calculations of potential surfaces [19].

In the pair–rotational limit the potential can be approximated by assuming a static deformation of the pair field, $\alpha=\alpha_0$. Under this assumption the derivatives in equation 1 tend to zero implying that:

$$E\propto\left(A-A_0\right)^2 \quad (2)$$

giving the expected parabolic dependence between energy and particle number for pair rotations.

In the case in which the equilibrium deformation is zero and fluctuations of the pair field are small, then B is a constant and $\Im$=4Bα^2 [5]. Equation 1 then becomes:

$$-\frac{\hbar^2}{2B}\frac{\partial^2\psi}{\partial\alpha^2}-\frac{\hbar^2}{2B\alpha}\frac{\partial\psi}{\partial\alpha}+\left(\frac{\hbar^2M^2}{8B\alpha^2}+V\left(\alpha\right)-E\right)\psi=0 \quad (3)$$

Introducing the reduced energy, $\epsilon=(2B/\hbar^2)$E, and reduced potential, u(α)=$(2B/\hbar^2)$V(α), equation 3 can be rewritten as:

$$\frac{\partial^2\psi}{\partial\alpha^2}+\frac{1}{\alpha}\frac{\partial\psi}{\partial\alpha}+\left(\epsilon-u\left(\alpha\right)-\frac{M^2}{4\alpha^2}\right)\psi=0 \quad (4)$$

For pair–vibrations the potential can be taken to be a parabola with a minimum at zero pair deformation (see Fig. 1a). With u(α)=α^2 and m=M/2, equation 4 can be expressed as:

$$\frac{\partial^2\psi}{\partial\alpha^2}+\frac{1}{\alpha}\frac{\partial\psi}{\partial\alpha}+\left(\epsilon-\alpha^2-\frac{m^2}{\alpha^2}\right)\psi=0 \quad (5)$$

Equation 5 has the same form as the radial equation of an isotropic oscillator (see, for example, Ref. 20) and can be solved by using a trial wavefunction of the form:

$$\psi=\alpha^m e^{-\alpha^2/2}W\left(\alpha\right) \quad (6)$$

Solving equation 5 it is found that:

$$E\propto\left(A-A_0\right) \quad (7)$$

which is the expected linear dependence between energy and particle number in the vibrational limit.

An analytical solution for the critical point of the transition from the vibrational to rotational pairing regimes can

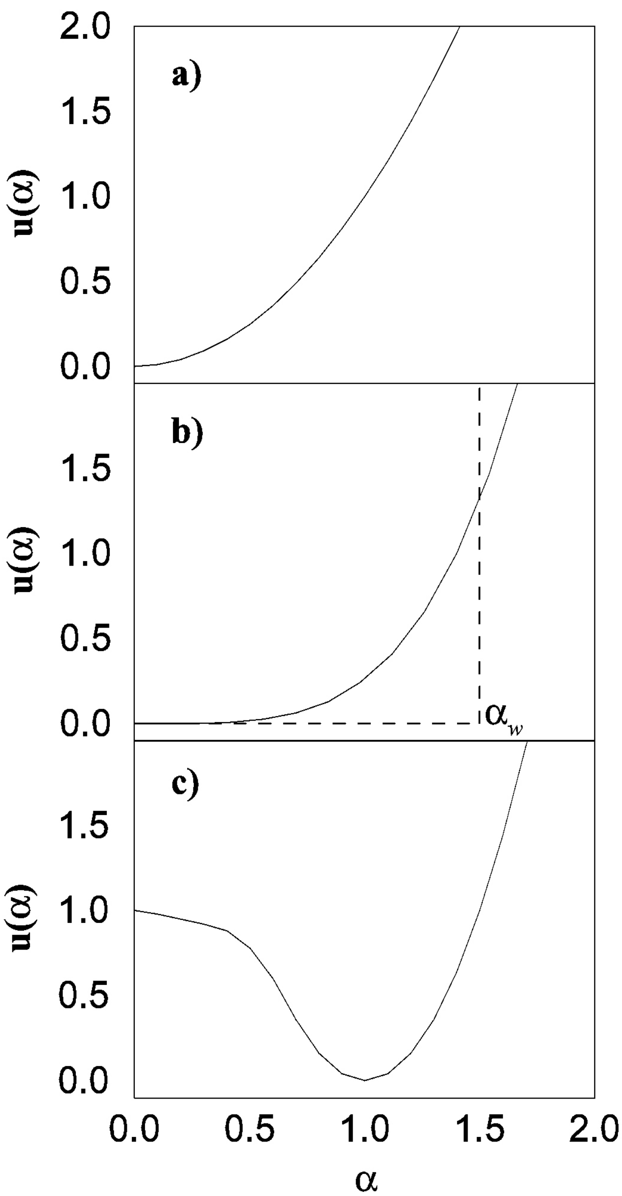

FIGURE 1. Schematic of the potential energy surfaces, u(α), as functions of the deformation of the pair field, α, for the transition from a) spherical vibrations, through b) the critical point (the infinite square well approximation with an outer wall at $\alpha=\alpha_w$ is shown with a dashed line), and to c) deformed rotation.

also be found. As pointed out by Iachello [12] the situation in which a potential has a flat behavior as a function of some coordinate appears typically when the system undergoes a phase transition at a critical point. A simple approximation to the critical–point potential (see Fig. 1b) is an infinite square well:

$$u(\alpha) = 0, \alpha \leq \alpha_w$$
$$u(\alpha) = \infty, \alpha > \alpha_w \tag{8}$$

This approximation to the potential at the critical–point of the pairing phase transition is identical to the assumption of infinite square well potentials used in the critical–point descriptions of nuclear shape transitions [12–14]. Using this potential in Eq. (4) one obtains a Bessel equation:

$$\frac{\partial^2 \psi}{\partial z^2} + \frac{1}{z}\frac{\partial \psi}{\partial z} + \left(1 - \frac{M^2}{4z^2}\right)\psi = 0 \tag{9}$$

where z=αk with k=$\epsilon^{1/2}$. The boundary condition $\psi(\alpha_w)$=0 determines the eigenfunctions to be related to Bessel functions of integer order such that:

$$\psi_{\xi,M}(z) = c_{\xi,M} J_{M/2}(z) \tag{10}$$

where $c_{\xi,M}$ are constants of normalization. The associated eigenvalues are given by:

$$\epsilon_{\xi,M} = k^2_{\xi,M},\ k_{\xi,M} = \frac{x_{\xi,M}}{\alpha_w} \tag{11}$$

where $x_{\xi,M}$ is the ξ^{th} zero of the Bessel function $J_{M/2}$(z).

In this paper, the focus is the use of the eigenvalues given by equation 11 to find the spectrum of states for comparison to experimental data. Transition matrix elements, related to two-nucleon transfer probabilities [5], could also be determined since:

$$\langle \psi_{\xi',M'}|\hat{O}|\psi_{\xi,M}\rangle \propto \int_0^{\alpha_w} \psi^*_{\xi',M'}\alpha^2\psi_{\xi,M}d\alpha \tag{12}$$

where, $\hat{O}$ is the pair transfer operator. An extensive comparison of all data, including pair transfer probabilities, will be the subject of future work.

The energy spectrum of the states can be found from the zeros of the related Bessel functions using equation 11. Normalizing the energies of excited states to that of the first excited state forms a reduced spectrum of states defined as:

$$E_N = \frac{x^2_{\xi,M} - x^2_{1,0}}{x^2_{1,2} - x^2_{1,0}} \tag{13}$$

TABLE I. Excitation energies of the critical–point description.

	ξ=1	ξ=2	ξ=3	ξ=4
|M|=0	0.00	2.77	7.77	14.97
|M|=2	1.00	4.88	10.98	19.30
|M|=4	2.31	7.31	14.52	23.95
|M|=6	3.92	10.06	18.39	28.93

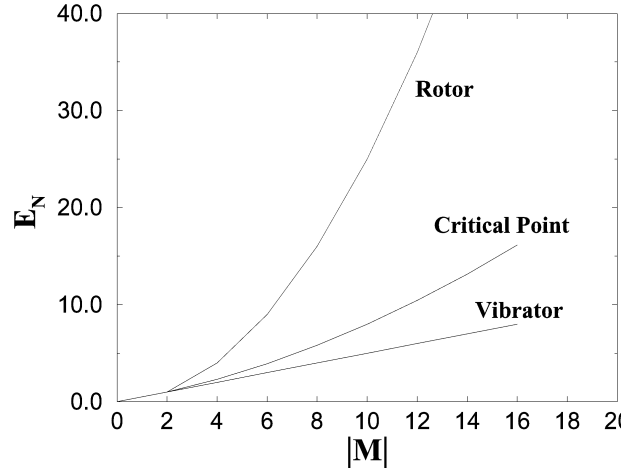

FIGURE 2. Plots of normalized energies for the lowest sequence of states of the rotor, vibrator, and critical–point descriptions.

The normalized excitation energies found in this way for some states are given in Table I. In Fig. 2 it can be seen that the energies for the ξ=1 sequence of states follow a behavior which is between the linear dependence for a pure harmonic vibrator (equation 7) and the parabolic dependence for a deformed rotor (equation 2) as expected for this description of the transition between the two limits. The sequence of states with ξ=1 correspond empirically to the sequence formed by the 0^+ ground–states of neighboring even–even nuclei along an isotopic or isotonic chain. States with ξ >1 correspond to excited 0^+ states formed from pair excitations.

Before proceeding it is worth commenting on the algebraic structure associated with the solutions of the pairing Hamiltonian. Following Iachello [13], the square–well approximation of the critical–point in a generalized phase transition of the form U(n)↔SO(n+1), with n≥2, has the E(n) dynamic symmetry, where E(n) is the n–dimensional Euclidean group. For the pairing phase transition n=2 and the corresponding symmetry at the critical point is E(2). The eigenfunctions of the critical–point solution are Bessel functions of integer order and form a basis for the representations of this group.

Comparison of calculations with experiment can be made by using the known data on the mass excesses [21], ε(A), along an isotopic sequence. Such a comparison for the Pb isotopes is shown in Fig. 3. The empirical neutron pairing energy can be defined, E_{pair}(A), as:

$$E_{pair} = [\varepsilon(A) - \varepsilon(A_0)] - C(A - A_0) \tag{14}$$

where ε(A)–ε(A_0) is the difference between the mass excess for a given isotope with mass number A and the mass excess of the chosen reference nucleus with mass number A_0. A linear term is subtracted and the constant, C, is chosen to make E_{pair}(A_0-2)=E_{pair}(A_0+2) [22]. The values of E_{pair} in Fig. 3 are again normalized to the first excited state.

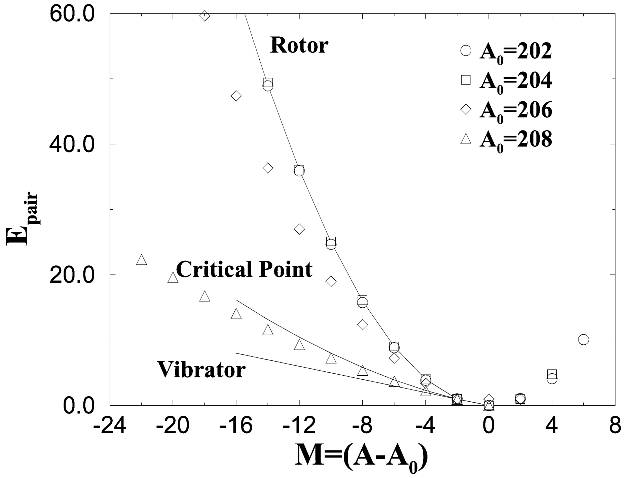

FIGURE 3. Plots of the empirical neutron pair energies for the sequence formed by the 0^+ ground–states of the Pb isotopes using as a reference ^{202}Pb (open circles), ^{204}Pb (open squares), ^{206}Pb (open diamonds), and ^{208}Pb (open triangles). For comparison are shown the expectations of the pure vibrator, pure deformed rotor, and the critical–point description (solid lines).

For A_0=202 or 204 it is found that the sequence follows the parabolic dependence of the rotational pairing regime. This indicates a large static deformation of the pair field (associated with a superconducting phase). With A_0=206, deviations from the rotational parabolic dependence are seen. With A_0=208 the sequence is closer to the vibrational (normal) phase. Empirically, the transition to the rotational regime requires only a few pairs outside of the closed shell configuration. This result is closely related to the fact that only a few nucleon pairs contribute to the pairing gap [23].

The isotopes around ^{208}Pb have been used as the textbook example of pair vibrations in nuclei [22]. Deviations from the pure vibrational spectrum were described in terms of large anharmonicities. These deviations in the energies are clearly seen in Fig. 3 and the sequence around A_0=208 lies much closer to the transitional description. The sequences of Ni and Sn isotopes using the doubly magic nuclei ^{56}Ni and ^{132}Sn as references have also been examined. Again, the spectra of neutron pair energies lie closer to the transitional description than to the vibrational description. Comparison to the proton–pair sequence formed by the N=82 isotones based on the ^{132}Sn doubly–magic nucleus also shows a similar behavior. These observations suggest a general phenomenon (see Fig. 4). In using the collective pairing Hamiltonian, a square–well potential provides a simple analytic approach that can naturally account for the observed anharmonicities associated with the harmonic oscillator solution. The measurement of properties of new doubly magic nuclei such as ^{100}Sn [24] and ^{78}Ni [25], and their even–even neighbors, will be of great interest in testing this idea.

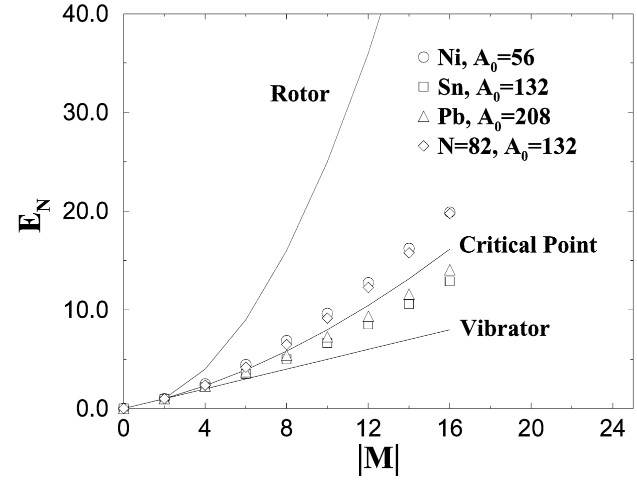

FIGURE 4. Plots of the empirical pair energies for the sequences formed by the 0^+ ground–states of the Ni, Sn, and Pb isotopes and the N=82 isotonic chain using as references ^{56}Ni (open circles), ^{132}Sn (open squares), ^{208}Pb (open triangles), and ^{132}Sn (open diamonds), respectively. For comparison are shown the expectations of the pure vibrator, pure deformed rotor, and the critical–point description (solid lines).

In the future, it should be possible to apply many of the modifications and ideas that have arisen as a consequence of the introduction of the critical–point descriptions of shape transitions to the description of the pairing–phase transition. For instance, modifying the infinite–square well potential to a finite square–well potential [26], varying the stiffness of the wall of the potential [27], and changing the softness of the deformation [28], are all important problems open to analytical solution. An advantage of the collective model is that the role of fluctuations of the pair gap are naturally accounted for by the choice of potential. For instance, by varying the softness one can investigate the extent to which these fluctuations might alter the nature of the phase transition.

The comparison with the experimental data can be expanded to include energies of excited states (corresponding to $\xi > 1$ in the description) and transfer strengths between the different states. In the case of excited 0^+ states in ^{206}Pb and ^{208}Pb, suggested as pair excitations, the energies lie close to the vibrational limit. It would be interesting to see if the first-excited pairing band continues this trend over a longer sequence of states.

2. Searching for the Giant Pairing Vibration

It has long been predicted that there should be a concentration of strength, with L=0 character, in the high–energy region ($\sim$10 MeV) of the pair–transfer spectrum [29]. This is called the Giant Pairing Vibration (GPV) and is understood microscopically as the coherent superposition of 2–particle (or 2–hole) states in the second major shell above the Fermi surface. It is analogous to the giant resonances of nuclear shapes which involve the coherent superposition of ph excitations. The GPV should be populated through pair–transfer

TABLE II. Calculated cross–sections (in mb) for ground–state and GPV transitions (based on Ref. 31).

	$^{14}C \rightarrow ^{12}C$	$^{6}He \rightarrow ^{4}He$
$^{116}Sn \rightarrow ^{118}Sn_{g.s.}$	19.4	0.4
$^{208}Pb \rightarrow ^{210}Pb_{g.s.}$	15.3	1.8
$^{116}Sn \rightarrow ^{118}Sn_{GPV}$	0.14	2.4
$^{208}Pb \rightarrow ^{210}Pb_{GPV}$	0.04	3.1

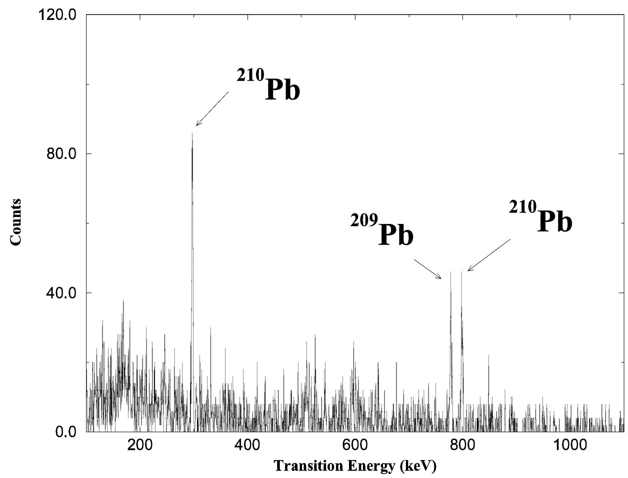

FIGURE 5. Spectrum of gamma rays in coincidence with both protons and alpha particles from the ^{7}Li+^{208}Pb reaction. Transitions in ^{210}Pb populated via two–neutron transfer from fragments of the ^{7}Li break–up are marked.

reactions. Despite efforts using conventional transfer reactions, such as (p,t) [30], the GPV has never been identified.

A recent paper [31] has studied the problem of exciting high–energy collective pairing modes in two–neutron transfer reactions and pointed out that, using conventional reactions with strongly–bound beam nuclei, one is faced with a large energy mismatch that favors the transition to the ground state over the population of high–lying states. Instead, the Q–values in a stripping reaction involving the weakly bound ^{6}He nucleus are much closer to the optimum for the transition to excited states in the 10–15 MeV range.

Particle–particle RPA calculations on ^{208}Pb and BCS+RPA calculations on ^{116}Sn were performed as examples of the response to the pairing operator in a closed–shell and open–shell nucleus, respectively. Two–neutron transfer form factors were then constructed via a collective model and used in a DWBA calculation to estimate cross–sections for the ground–state and GPV transitions for the (^{14}C,^{12}C) and (^{6}He,^{4}He) reactions. The results, which are shown in Table II, indicate a large enhancement of the population of the GPV when using the (^{6}He,^{4}He) reaction.

The low intensity of available beams of radioactive ^{6}He makes the experiment difficult. However, recent studies [32] of the break–up of a ^{7}Li beam indicate that there may be a large ($>$10 mb) cross–section for the p+^{6}He channel. The idea is that the ^{6}He fragment may then transfer two neutrons to the target nucleus, as discussed above, and populate the GPV. By looking at the coincidence of remnant protons and alpha particles one should be able to identify states, including the GPV, populated in the neutron–pair transfer from fragments of the initial ^{7}Li beam.

An experiment, was performed to test this idea. The ^{7}Li+^{208}Pb reaction was used at a beam energy of 49 MeV. The beam, accelerated by the 88–Inch Cyclotron of the Lawrence Berkeley National Laboratory, was incident on a target comprising a 900 μg/cm^2 self–supporting foil of enriched ^{208}Pb.

Charged–particles were detected with the STARS (Silicon Telescope Array for Reaction Studies) Si ΔE–E telescope system which consisted of two annular silicon strip detectors with inner radius 11 mm and outer radius 59 mm. The detectors were electrically segmented into 24 concentric rings on the front face and 8 wedge–shaped sectors on the back face. The ΔE detector was of $\approx$140 μm thickness while the E detector was $\approx$1000 μm in thickness. The detectors were placed at forward angles with respect to the beam direction. The configuration had a target–to–detector distance of $\approx$3 cm to the ΔE detector which was separated from the E detector by $\approx$1 cm. This gave an angular coverage from $\approx$20° to $\approx$55° with respect to the beam direction.

Gamma rays were detected with the new LIBERACE (Livermore Berkeley Array for Collaborative Experiments) Ge–detector array which consists of up to six Compton–suppressed clover detectors situated in the horizontal plane around the target chamber with two detectors each at $\pm$45° and two at 90°. The distance between the target and the front of each Ge detector was $\approx$17.25 cm.

The data is still under analysis but as a first step the γ rays in coincidence with both a proton and an alpha particle were examined. The resultant spectrum is shown in Fig. 5. Transitions in ^{210}Pb populated via two-neutron transfer from fragments of the ^{7}Li beam can be clearly seen. The next stage in the analysis is to examine the particle spectra of coincident protons and alpha particles in order to reconstruct which fragments are involved in the transfer process and to attempt to identify the GPV.

3. Summary

In the first section of this paper, I have presented analytical solutions of the collective pairing Hamiltonian [5] by using simple approximations to the potential in the limits of harmonic vibrations (zero deformation of the pair field corresponding to normal behavior), deformed rotation (static deformation of the pair field corresponding to superconducting behavior), and at an intermediate transitional point. In the latter situation the potential is approximated as an infinite square well. The eigenvalues are expressed in terms of the zeros of Bessel functions of integer order. Comparison to the pairing bands based on the Pb isotopes suggests that this description may provide a simple approach to explaining the observed anharmonicities of the pairing vibrational structure around ^{208}Pb. In the second section of this paper, I discussed some issues related to the experimental search for the Giant Pairing Vibration (GPV) in nuclei. In particular, a technique is proposed to use a two–step process of break–up followed by two–neutron transfer of a ^{7}Li primary beam that may result in enhanced population of the GPV. Results from a preliminary analysis of data taken with the new LIBERACE+STARS detector system at LBNL suggest that we see two–neutron transfer from fragments of the ^{7}Li break–up.

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