Electromagnetic form factors

M. De Sanctis
INFN, Sezione di Roma 1, Piazzale Aldo Moro, Roma, Italy,
Universidad Nacional de Colombia, Bogota, Colombia

M.M. Giannini, E. Santopinto*, and A. Vassallo
INFN and Università di Genova, via Dodecaneso 33, 16146 Genova, Italy.

Recibido el 2 de febrero de 2006; aceptado el 31 de mayo de 2006

The relativized hypercentral Constituent Quark Model is used for the calculation of the elastic electromagnetic form factors of the nucleon. The results are compared with the recent measurements at Jlab.

Keywords: Relativistic quark model; electromagnetic form factors.

Se emplea una versión relativista del modelo hipercentral de cuarks constituyentes para estudiar los factores de forma electromagnéticos del nucleón. Se comparan los resultados con los datos experimentales recién obtenidos en el Laboratorio Jefferson.

Descriptores: Modelo relativista de cuarks; factores de forma electromagnéticos.

PACS: 12.39.Ki; 13.40.Gp

1. Introduction

The interest in the electromagnetic form factors of the nucleon has been again triggered by the recent results of the Jefferson Laboratory on the ratio between the electric and magnetic form factors of the proton [1–3]. At variance with the expectations, the ratio deviates strongly from 1 and, for $Q^2 \geq 1\ (GeV/c)^2$, it decreases with an almost linear behaviour, pointing towards the possible existence of a zero at $Q^2 \approx 8\ (GeV/c)^2$.

This poses the question of the compatibility of the new data obtained from polarization measurements with the traditional ones obtained from a Rosenbluth plot. In this respect much attention has been devoted to the two-photon exchange mechanisms [4] and the recent calculations seem to reduce the discrepancy. Radiative corrections to polarization phenomena [5] have been calculated showing that they are small for the polarization ratio. A re-analysis of the old Rosenbluth procedure is also being performed [6], with promising results, however the situation is still not completely defined [7]. Nevertheless the main problem is the physical picture emerging from the data, that is the origin of the decrease of the ratio and of the eventual presence of a zero in the electric form factor. The first seems to be the manifestation of relativistic effects [8, 9], while the possible presence or not of a zero will help to discriminate among the different models for the nucleon structure [8, 10–12]. For reviews on this subject the readers are referred to [13–15].

The planned experiments at higher Q^2 will clarify these points. From the theoretical point of view a zero in the proton charge form factor is a challenge for most models of the internal proton structure. In this contribution we report the results of recent calculations of the elastic nucleon form factors within a semirelativistic version of the hypercentral Constituent Quark Model (hCQM) [16].

2. The hypercentral constituent quark model

In the hCQM the $SU(6)$ invariant quark potential is assumed to be

$$V(x) = -\frac{\tau}{x} + \alpha x, \tag{1}$$

where $x = \sqrt{\vec{\rho}^{\,2} + \vec{\lambda}^{\,2}}$ is the hyperradius, with $\vec{\rho}$ and $\vec{\lambda}$ being the Jacobi coordinates. Interactions of the type linear plus Coulomb-like have been used since long time for the meson sector, *e.g.* the Cornell potential. This form has been obtained in recent Lattice QCD calculations [17] for $SU(3)$ invariant static quark sources.

The three-quark potential (1), depending on the hyperradius x only, is hypercentral. It can be considered as the lattice two-body interaction within the so called hypercentral approximation, that is averaged over the hyperangle $\xi = arctg(\rho/\lambda)$ (whit ρ and λ respectively the modulus of $\vec{\rho}$ and $\vec{\lambda}$) and the angles Ω_ρ, Ω_λ; this approximation has been shown to be valid, specially for the lower energy states [18]. On the other hand, the hyperradius x depends on the coordinates of all the three quarks, therefore the interaction $V(x)$ contains in general three-body contributions.

The 'hypercoulomb' part $-\tau/x$ of the potential (1) has interesting properties [16]. In particular it leads to a power-law behaviour of the proton form factor and of all the electromagnetic transition amplitudes [16].

The $SU(6)$ violation is taken into account by adding a standard hyperfine interaction H_{hyp} [19], treated as a perturbation. The three quark hamiltonian for the hCQM is [16]

$$H = \frac{\vec{p}_\rho^{\,2}}{2m} + \frac{\vec{p}_\lambda^{\,2}}{2m} - \frac{\tau}{x} + \alpha x + H_{hyp}, \tag{2}$$

where $\vec{p}_\rho$ and $\vec{p}_\lambda$ are the conjugate momenta of the Jacobi coordinates $\vec{\rho}$ and $\vec{\lambda}$.

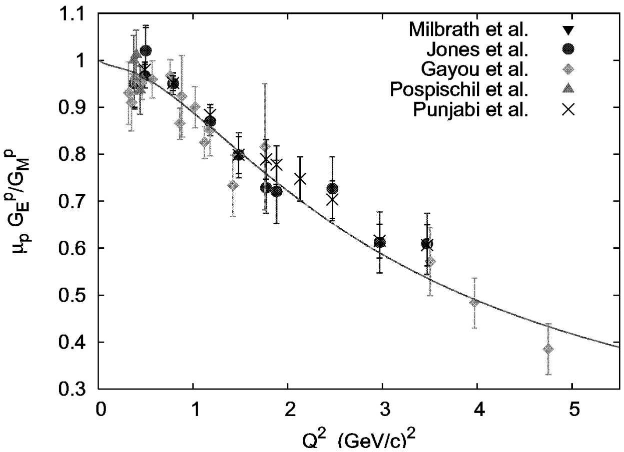

FIGURE 1. (Color online) The ratio $\mu_p G_E^p/G_M^p$ from polarization transfer compared with the semirelativistic hCQM calculation with constituent quark form factors (solid line). The experimental data are taken from [1–3, 24–26].

The spectrum is described with τ=4.59 and α=1.61 fm^{-2} and the standard strength of the hyperfine interaction needed for the $N-\Delta$ mass difference [19].

The model has been used for the prediction of various physical quantities of interest, namely the photocouplings [20], the electromagnetic transition amplitudes [21], the elastic nucleon form factors [22]. The ratio between the electric and magnetic proton form factors [9] has been calculated boosting the three quark nucleon states to the Breit frame and expanding the matrix elements of the three quark current up to the first order in the quark momemtum. The non relativistic calculations predict the value $R = \mu_p\ (G_E^p/G_M^p) = 1$ and introducing the hyperfine interaction makes no difference ($R = 0.99$). However, the first order relativistic corrections [9] give rise to a ratio which significantly deviates from 1.

Relativity is a fundamental ingredient for the description of the elastic nucleon form factors and therefore we have recently reformulated the model and calculated the elastic nucleon form factors. First, we have proposed a semirelativistic constituent quark model that is based on the following Hamiltonian

$$H = \sum_{i=1}^{3} \sqrt{\vec{p}_i^2 + m^2} - \frac{\tau}{x} + \alpha x + H_{hyp} \tag{3}$$

that has been diagonalized in the nucleon rest frame; the $\vec{p}_i$ is the i-th quark 3-momentum and m_i the masses of the constituent quarks. Using the same form of the hypercentral potential and a standard hyperfine $SU_{sf}(6)$ spin-flavour breaking term, we have obtained an equivalently good description of the baryon spectrum with respect to the non relativistic case. However, the semirelativistic wave functions have more high momentum components and therefore we believe they are more realistic. The nucleon current is written in impulse approximation, *i.e.* is chosen to be the sum of one-body quark currents [9, 22]:

$$J_\mu^{(N)} = \sum_{i=1}^{3} e_i\ \gamma_\mu(i) \tag{4}$$

with e_i being the electric charge of the i-th constituent quark. Standard boost operators have been applied to obtain the nucleon initial and final state functions in the Breit frame. With respect to our previous papers [9, 22], both the boosts on the spatial variables and on the Dirac indices are performed without any approximation. Moreover, another important improvement is provided by the use of semirelativistic wave functions.

Finally, considering that constituent quark have a finite size [23], we have introduced constituent quark form factors. The free parameters in the quark form factors have been fitted to the ratio R, the proton magnetic form factor G_M^p, the neutron electric G_E^n and magnetic G_M^n form factors [12]. The results for the ratio R are shown in Fig. 1. The free parameters provided by the quark form factors are not sufficient by themselves to obtain a good fit, it is necessary that already the pointlike calculations provide a realistic description.

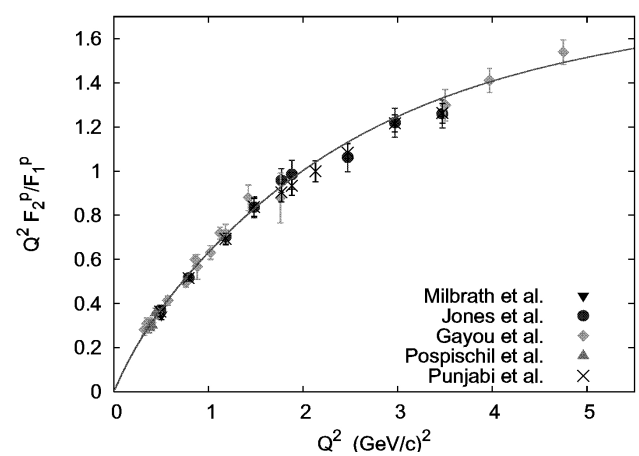

FIGURE 2. The ratio $Q^2(F_{2p}/F_{1p})$ calculated with the relativized hCQM (solid line). The experimental data are taken from [1–3, 24–26].

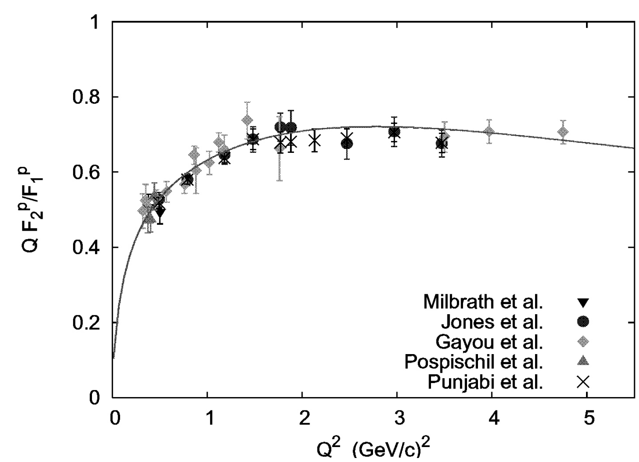

FIGURE 3. The ratio $Q(F_{2p}/F_{1p})$ calculated with the relativized hCQM (solid line). The experimental data are taken from [1–3, 24–26].

In Fig. 2 the results of the hCQM are compared with the data for the ratio $Q^2\frac{F_{2p}}{F_{1p}}$. The perturbative QCD (pQCD) predictions are that the asymptotic behavior of the helicity conserving Dirac form factor $F_{1p} \propto 1/Q^4$ and of the helicity-flipping Pauli form factor is $F_{2p} \propto 1/Q^6$ [27], so that $Q^2(F_{2p}/F_{1p})$ would reach a constant value at high enough Q^2. The asymptotic regime has not been reached yet (see the experimental data). Ralston *et al.* showed that if one takes into account the contributions from the non zero orbital angular momentum in the proton wave function, F_{2p}/F_{1p} goes as $1/Q$ [28] and this kind of scaling behavior is reported in Fig.3. A more or less constant values is reached starting at $Q^2 \sim 2\ GeV^2$, even if before giving any kind of conclusion an extension to higher values of Q^2 of the theoretical calculations is necessary. We observe that the experimental data for the ratios of Fig. 2 and Fig. 3 are quite well reproduced by the theoretical calculations.

3. Conclusions

As a conclusion we can say that an extension of the measurement of the form factors to higher Q^2 values is important in order to test the pQCD scaling predictions for the Dirac and the Pauli form factors F_{1p} and F_{2p} and in particular to understand at which scale this behavior starts, moreover a presence or not of a zero in the ratio of the electric and magnetic form factors of the proton, will help to discriminate among the different models of the nucleon.

∗ Corresponding author; e-mail: santopin@ge.infn.it.

1. M.K. Jones *et al.*, *Phys. Rev. Lett.* **84** (2000) 1398.
2. O. Gayou *et al.*, *Phys. Rev. C* **64** (2001) 038202.
3. O. Gayou *et al.*, *Phys Rev. Lett.* **88** (2002) 092301.
4. P.G. Blunden, W. Melnitchouk, and J.A. Tjon, *Phys. Rev. C* **72** (2005) 034612; S. Kondratyuk, P.G. Blunden, W. Melnitchouk, and J.A. Tjon, *Phys. Rev. Lett.* **95** (2005) 172503; P.G. Blunden, W. Melnitchouk and J.A. Tjon, *Phys. Rev. Lett.* **91**(2003) 142304; P.A.M. Guichon and M. Vanderhaegen, *Phys. Rev. Lett.* **91** (2003) 142303; M.P. Rekalo and E. Tomasi-Gustafsson, *Eur. Phys. J.* **A22** (2004) 331; M. P. Rekalo and E. Tomasi-Gustafsson, *Nucl. Phys. A* **742** 322 (2004); M.P. Rekalo and E. Tomasi-Gustafsson, *Nucl. Phys. A* **740** (2004) 271; E. Tomasi-Gustafsson, *Nucl. Phys. A* **755** (2005) 277; E. Tomasi-Gustafsson and G.I. Gakh, *Phys. Rev. C* **72** (2005) 015209;
5. A. Afanasev, I. Akushevich, and N. Merenkov, *Phys. Rev. D* **64** (2001) 113009; A.V. Afanasev, I. Akushevich, A. Ilyichev, and N. P. Merenkov, *Phys. Lett. B* **514** (2001) 269.
6. J. Arrington, *Phys. Rev. C* **69** (2004) 022201; *Phys. Rev. C* **71** (2005) 015202.
7. E. Tomasi-Gustafsson, F. Lacroix, C. Duterte, and G.I. Gakh, *Eur. Phys. J. A* **24** (2005) 419; E. Tomasi-Gustafsson, *Nuovo Cim. C* **27** (2004) 413.
8. M.R. Frank, B.K. Jennings, and G.A. Miller, *Phys. Rev. C* **54** (1996) 920; G.A. Miller, *Phys. Rev. C* **66** (2002) 032201(R); F. Cardarelli and S. Simula, *Phys. Rev. C* **62** (2000) 065201; R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici, *Phys. Lett. B* **511** (2001) 33; S. Boffi, L.Y. Glozman, W. Klink, W. Plessas, M. Radici, and R.F. Wagenbrunn, *Eur. Phys. J. A* **14** (2002) 17.
9. M. De Sanctis, M.M. Giannini, L. Repetto, and E. Santopinto, *Phys. Rev. C* **62** (2000) 025208.
10. F. Iachello, A.D. Jackson, and A. Lande, *Phys. Lett. B* **43** (1973) 191; F. Iachello and Q. Wan, *Phys. Rev. C* **69** (2004) 055204; R. Bijker and F. Iachello, *Phys. Rev. C* **69** (2004) 068201.
11. E.L. Lomon, *Phys. Rev. C* **66** (2002) 045501; G. Holzwarth, *Z. Phys. A* **356** (1996) 339; G. Holzwarth,hep-ph/0201138; D. Merten, U. Loring, K. Kretzschmar, B. Metsch, and H.R. Petry, *Eur. Phys. J. A* **14** (2002) 477; U. Loring, B.C. Metsch, and H.R. Petry, *Eur. Phys. J. A* **10** (2001) 395; D.H. Lu, A.W. Thomas, and A.G. Williams, *Phys. Rev. C* **57** (1998) 2628; D.H. Lu, S.N. Yang, and A.W. Thomas, *J. Phys. G* **26** (2000) L75; D.H. Lu, S.N. Yang, and A.W. Thomas, *Nucl. Phys. A* **684** (2001) 296; J.D. Ashley, D.B. Leinweber, A.W. Thomas, and R.D. Young, *Eur. Phys. J. A* **19** (2004) 9; B.Q.Ma, D.Qing and I. Schmidt, *Phys. Rev. C* **65** (2002) 035205; *Phys. Rev. C* **66** (2002) 048201.
12. M. De Sanctis, M. M. Giannini, E. Santopinto, and A. Vassallo, nucl-th/0506033.
13. H.Y. Gao, *Int. J. Mod. Phys. E* **12** (2003) 1 [Erratum-ibid. E **12** (2003) 567].
14. K. De Jager and C. Hyde-Wright, *Ann. Rev. Nucl. Part. Phys.* **54** (2004) 1.
15. B. Plaster *et al.* [Jefferson Laboratory E93-038 Collaboration], nucl-ex/0511025.
16. M. Ferraris, M.M. Giannini, M. Pizzo, E. Santopinto, and L. Tiator, *Phys. Lett. B* **364** (1995) 231; E. Santopinto, F. Iachello, and M.M. Giannini, *Nucl. Phys.* **A623** (1997) 100c; E. Santopinto, F. Iachello, and M. M. Giannini, *Eur. Phys. J. A* **1** (1998) 307; R. Bijker, F. Iachello, and E. Santopinto, *J. Phys. A* **31** (1998) 9041.
17. G. Bali *et al.*, *Phys. Rev. D* **51** (1995) 5165; G. Bali, *Phys. Rept.* **343** (2001) 1; C. Alexandrou, P. de Forcrand, and O. Jahn, *Nucl. Phys. Proc. Suppl.* **119** (2003) 667; H. Suganuma, T.T. Takahashi, F. Okiharu, and H. Ichie, *Nucl. Phys. Proc. Suppl.* **141** (2005) 92.
18. M. Fabre de la Ripelle and J. Navarro, *Ann. Phys. (N.Y.)* **123** (1979) 185.
19. N. Isgur and G. Karl, *Phys. Rev. D* **18** (1978) 4187; *Phys. Rev. D* **19** (1979) 2653.
20. M. Aiello, M. Ferraris, M.M. Giannini, M. Pizzo, and E. Santopinto, *Phys. Lett. B* **387** (1996) 215.
21. M. Aiello, M.M. Giannini, and E. Santopinto, *J. Phys. G: Nucl. Part. Phys.* **24** (1998) 753.
22. M. De Sanctis, E. Santopinto, and M.M. Giannini, *Eur. Phys. J. A* **1** (1998) 187.

23. R. Petronzio, S. Simula, and G. Ricco, *Phys. Rev. D* **67** (2003) 094004.
24. B. D. Milbrath *et al.* [Bates FPP collaboration], *Phys. Rev. Lett.* **80** (1998) 452 [Erratum-ibid. **82** (1999) 2221]
25. T. Pospischil *et al.* [A1 Collaboration], *Eur. Phys. J. A* **12** (2001) 125.
26. V. Punjabi *et al.*, *Phys. Rev. C* **71** (2005) 055202 [Erratum-ibid. C **71** (2005) 069902].
27. S.J. Brodsky and G.R. Farrar, *Phys. Rev. Lett.* **31** (1973) 1153; S.J. Brodsky and G.R. Farrar, *Phys. Rev. D* **11** (1975) 1309.
28. J.P. Ralston and P. Jain, *Phys. Rev. D* **69** (2004) 053008.