Bi-dust acoustic waves

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Low frequencies waves in plasmas with two kind of dusty grains have been studied. Each species of dust particle is characterized by the grain radius, which determines its equilibrium charge. Relative velocities between the two kinds of dust grain for the unperturbed plasma is also considered in order to study instabilities and compare with astrophysical and industrial applications. In this analysis, each dust species is handled with a simplified model of kinetic-fluid equations, and the electrons and ions are determined by Boltzmann factors. The low frequency dispersion relation for bi-dust plasma waves with non relative motion between each kind of grain leads to damped waves with two characteristic frequencies. Instabilities are produced by the relative motion between the species. The onset of these instabilities is studied as a function of the plasma dust frequencies and relative velocities among each species.

Keywords: Dusty plasmas.

En el presente trabajo, se estudian ondas de bajas frecuencias en plasmas con dos tipos de granos de polvo. Cada especie está caracterizada por el radio del grano, el cual determina su carga de equilibrio. Tambien se consideran velocidades relativas entre los dos tipos de grano de polvo en el plasma no perturbado para estudiar la inestabilidad y comparar con aplicaciones astrofísicas e industriales. En este análisis, cada especie de polvo es tratada con un modelo simplificado de ecuaciones fluido-cinéticas, y los electrones e iones son determinados por factores de Boltzman. La relación de dispersión para bajas frecuencias, de ondas en plasmas de dos polvos sin movimiento relativo entre cada tipo de grano, conduce a ondas amortiguadas con dos frecuencias características. Las inestabilidades se producen por el movimiento relativo entre las especies. El origen de estas estabilidades es estudiado como una función de las frecuencias de plasma y las velocidades relativas entre cada especie.

Descriptores: Plasmas granulares.

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1. Introduction

The study of dusty plasma with different families of dust grains has become important in understanding different collective phenomena appearing in astrophysics and laboratory plasma, such as cometary tails, planetary rings, ionosphere, low temperature plasma produced in laboratory, and glow discharges [1-6]. The presence of massive dust plasmas moving as a dust streaming current with respect to the surrounding plasma is an usual feature scenario, for instance, the massive dust particles following the planets interacting with the surrounding plasma [7]. In this work, an analysis of the two-stream by bi-dust instability is carried out in the low frequency region characteristic of dust acoustic wave (DAW). An analysis of dust acoustic solitary waves in the presence of dust streaming has been carried out recently [8], and two-stream instability between the solar wind and cometary plasma has been also achieved [9-10]. A general analysis of the conditions for the formation of the two streaming by dust acoustic waves is done using a multi-fluid approach. In our treatment, new variables are introduced in the dispersion relation in order to perform the characteristic analysis of the two-stream plasma instability [11]. Considering the space defined by these new variables, the unstable regions are separated from the stable ones by curves determined here.

2. Theory

Uniform, unmagnetized two-dust plasma is considered here. A low frequency perturbation in this plasma generates a DAW of wave frequency w and wave vector k propagating in the x-direction. In the equilibrium, there is charge neutrality, that is,

$$n_i^{(0)} - n_e^{(0)} - n_{d1}^{(0)} Z_{d1} - n_{d2}^{(0)} Z_{d2} = 0, \tag{1}$$

where $n_i^{(0)}$, $n_e^{(0)}$, $n_{d1}^{(0)}$ and $n_{d2}^{(0)}$ are the equilibrium densities of ion, electron, dust plasma-one and dust plasma-two, respectively. Each of the two plasmas has grains of different sizes and therefore the grain charges Z_{d1} and Z_{d2} are different. Only negative dust grains are considered. The first dust plasma is considered to be at rest and the second grains are impinging on the first one with velocity v_b, which is a short notation for $v_{d2}^{(0)}$. In this analysis,we are mainly interested in the low frequency region and two ion and two electrons Maxwellians can be englobed by the collective densities $n_i^{(0)}$ and $n_e^{(0)}$, respectively. The perturbed electrostatic fluctuations are assumed to vary in 1-D (one dimension) as $\exp[i(kx - wt)]$. For this low frequency analysis, the ions and electrons are treated as being in equilibrium with the perturbed potential ϕ, and their perturbed densities $n_e^{(1)}$ and $n_i^{(1)}$ are described by Maxwellian factors $n_e^{(1)} = n_e^{(0)} \exp[e\phi/k_B T_e]$, $n_i^{(1)} = n_i^{(0)} \exp[-(e\phi/k_B T_i)]$. Here only one kind of ionized ion is considered and k_B is the Boltzman constant. Charge grain fluctuations and friction between grains are neglected. Each species of grain is treated with fluid equations. Thus way the linearized momentum equations for the grains are

$$n_{d1}^{(0)} \frac{\partial v_{d1}}{\partial t} = eZ_{d1} n_{d1}^{(1)} \frac{\partial \phi}{\partial x}, \tag{2}$$

$$n_{d2}^{(0)} \frac{\partial v_{d2}}{\partial t} + n_{d2}^{(0)} v_b \frac{\partial v_{d2}}{\partial x} = eZ_{d2} n_{d2}^{(1)} \frac{\partial \phi}{\partial x}. \tag{3}$$

Since charge fluctuations are not considered in this paper, the continuity equations are the usual ones and they are not written here explicitly. Using the 1-D Poisson equation, it is straightforward to arrive at the dispersion relation

$$\varepsilon(k, w) = 1 + \frac{k_D^2}{k^2} - \frac{w_{d1}^2}{w^2} - \frac{w_{d2}^2}{(w - kv_b)^2} = 0, \tag{4}$$

where the dust plasma frequencies are denoted by w_{d1} and w_{d2} and the total Debye number k_D is given as a function of the Debye number of each species k_{De}, k_{Di}, k_{D1} and k_{D2} by the equation

$$k_D^2 = k_{De}^2 + k_{Di}^2 + k_{D1}^2 + k_{D2}^2. \tag{5}$$

The dust-plasma frequencies w_{pd1} and w_{pd1} are shortened in notation form by w_{d1} and w_{d2}, respectively, and they are given by

$$w_{d1}^2 = \frac{4\pi e^2 Z_{d1}^2 n_{d1}^{(0)}}{m_{d1}} \; ; \; w_{d2}^2 = \frac{4\pi e^2 Z_{d2}^2 n_{d2}^{(0)}}{m_{d2}}, \tag{6}$$

wherem_{d1} and m_{d2} are the mass of the dust grains. Thermal velocities are neglected, which is a good approximation because the temperature of the dust grains is much lower that those of the ions and electrons. In this equation, if the second dust grain is not present then w_{d2} is zero, and the usual dispersion relation for one dusty plasma is recovered. The analysis of this equation is performed in a better way by introducing the dimensionless variables

$$\tilde{w} = \frac{w}{w_{d2}} \; , \; v_0 = \frac{v_b k_D}{w_{d2}} \; , \; \tilde{k} = \frac{k}{k_D}, \tag{7}$$

giving the dispersion relation

$$1 + \tilde{k}^2 = \frac{\delta^2}{\tilde{w}^2/k^2} + \frac{1}{(\tilde{w}/\tilde{k} - v_0)^2}; \delta = \frac{w_{d1}}{w_{d2}}. \tag{8}$$

A further simplification is useful, defining the new variables

$$v_\phi = \frac{w}{\tilde{k}}(1 + \tilde{k}^2)^{1/2}; V = v_0(1 + \tilde{k}^2)^{1/2} \tag{9}$$

to obtain

$$1 = \frac{\delta^2}{v_\phi^2} + \frac{1}{(v_\phi - V)^2}. \tag{10}$$

For our analysis, it is useful to introduce the function $F(v_\phi, V, \delta)$

$$F(v_\phi, V, \delta) = \frac{\delta^2}{v_\phi^2} + \frac{1}{(v_\phi - V)^2}. \tag{11}$$

This dispersion relation now looks like the usual two-stream instability for non-dusty plasmas [11]. A drawing of this function for different values of δ and a fixed value of V is shown in the following figures. Instabilities as well as damping waves can occur, depending on the values of the parameters. Stable waves are shown in Fig. 1, which corresponds to the values 2 and 0.4 for the parameters V and δ, respectively. The v_ϕ roots of the dispersion relation are given for the intersection of the horizontal line through the point (0,1), which is also drawn in the figure. As is clear from this figure, that line intersects the function $F(v_\phi, V, \delta)$ in four points, which means that the four roots are reals and therefore the waves are stable.

It is interesting to analyze what happens when V is kept constant and δ is changed. To keep V constant does not mean that v_b is constant, because we must remember that V depends on v_b, k, k_D and w_{d2}. Furthermore, w_{d2} depends on the plasma density, charge and mass of the second grain. However, these variables are the most suitable for analyzing

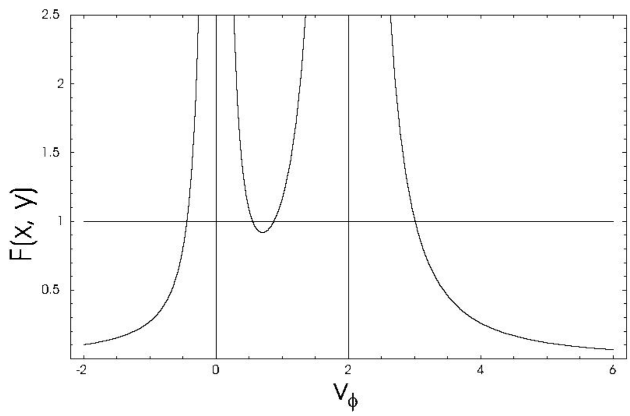

FIGURE 1. F(x,y) vs. v_ϕ, $\delta = 0.4$ and $V = 2$.

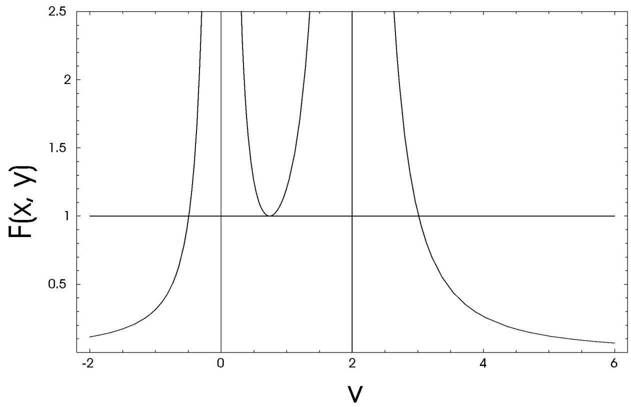

FIGURE 2. F(x,y) vs. v_ϕ, $\delta = 0.45$ and $V = 2$.

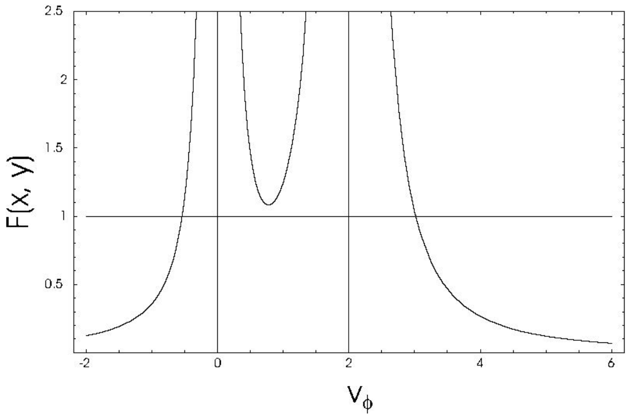

FIGURE 3. F(x,y) vs. v_ϕ, $\delta = 0.5$ and $V = 2$

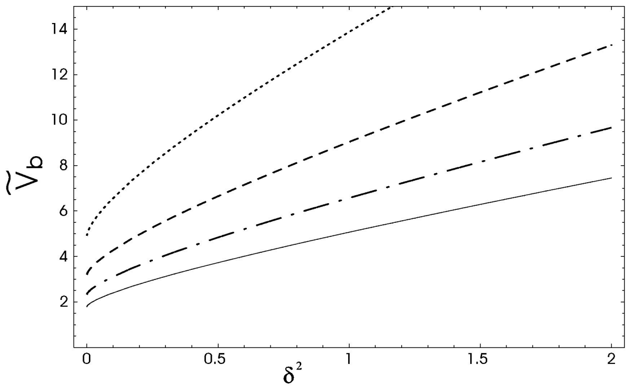

FIGURE 4. $\tilde{v}_b$ vs. δ^2; $\tilde{k}$ = 0.2, 0.3, 0.4 and 0.5 .

the onset of instability. To vary δ means that the ratio between the two frequency grains is changed. Figure 2 shows how the pattern of the function $F(v_\phi, V, \delta)$ is modified due to changes in δ, when V is fixed. In that figure, $\delta = 0.45$ and $V = 2$, and it correspond' to the limit case where two real roots will become imaginary roots, as will be clear in the next figure.

In Fig. 3, $\delta = 0.5$ and $V = 2$, one wave is unstable. As is shown, there are two real roots and, because of the four degree dispersion relation, the other two roots are complex conjugate numbers. Therefore, one of the roots has a positive imaginary part, which corresponds to an unstable wave.

In order to define the regions of stability and instability, the position of the minimum of the function $F(v_\phi, V, \delta)$ must be analyzed. This minimum is given by the equation

$$\frac{\partial F(v_{\phi 0}, V, \delta)}{\partial v_\phi} = -\frac{2\delta^2}{v_{\phi 0}^3} - \frac{2}{(v_{\phi 0} - V_0)^3}, \qquad (12)$$

$$v_{\phi 0} = \frac{\delta^{2/3}}{(1+\delta^{2/3})} V_0. \qquad (13)$$

Looking now for the limit case shown in Fig. 2, this minimum must verify also that $F(v_{\phi 0}, V_0, \delta) = 1$, which means that

$$V_0 = (1 + \delta^{2/3})^3. \qquad (14)$$

This value of V_0 corresponds to a critical, dimensionless beam velocity $\tilde{v}_0$, given by

$$\tilde{v}_0^2 = \frac{V_0}{(1 + \tilde{k}^2)} = \frac{(1 + \delta^{2/3})^3}{(1 + \tilde{k}^2)}. \qquad (15)$$

This result is shown in Fig.4, where $\tilde{v}_{b0}$ is shown as a function of δ^2 for four values of the dimensionless wave number $\tilde{k}$: $\tilde{k} = 0.2\ (\ldots)$; $0.3\ (- - -)$; $0.4\ (-. - .-)$ and 0.5 (__). The unstable region is below each curve and the stable region above each critical curve.

3. Conclusion

An instability analysis for bi-dust plasmas with dust streaming velocity has been performed in this paper. The onset of the instability appears for some critical values of densities, beam velocities and wave numbers. Equations relating these critical values have been found. Further analysis leads to maps, showing the regions of stable and unstable waves.

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