Landau damping in bi-dust ion-acoustic waves

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Ion acoustic dust waves in a bi-dust plasma are analyzed in this paper. In order to model this system, we assume the existence of two different kinds of grains, each characterized by a different radius. Relative velocities between grains and charge fluctuations are neglected. In order to derive the dispersion relation of this system, we use the well known hybrid fluid-kinetic model, in which ions are treated kinetically and other species as fluids. In this plasma, waves with non-relative velocities between species leads to damped waves with frequency modes, defined by the grain radius. The induced damping ratio is studied as a function of the grain and ion densities.

Keywords: Dusty plasmas.

En el presente trabajo, analizamos ondas ionico-acusticas de polvo en un plasma con dos tipos de polvo. Para modelar este sistema, asumimos la existencia de dos tipos diferentes de grano, cada uno caracterizado por un diferente radio. Velocidades relativas entre granos y fluctuaciones de cargas son despreciadas. Para derivar la relacion de dispersion de este sistema, usamos el bien conocido modelo híbrido fluido-cinético, en el cual los iones son tratados con teoría cinética y las otras especies como fluidos. En este plasma, las ondas sin velocidad relativa entre las especies conducen a ondas amortiguadas con modos de frecuencia definidos por el radio del grano. La razón de amortiguamiento inducido es estudiada en funcion de las densidades de los granos y de los iones.

Descriptores: Plasmas granulares.

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1. Introduction

The physics of dusty plasmas is a topic that has been of increasing interest in plasma physics. Dusty plasmas are low-temperature multispecies ionized gases including electrons, ions, and negative (or positive) charged dust grains, typically micrometer or submicrometer. Dusty plasmas are found in space as well as in the laboratory and in industrial processes [1]. The dust particles may be charged due to the collection of electron and ion currents from the background plasma. The presence of charged dust grains can significantly modify the normal plasma modes. Furthermore, new modes arise on a slow time scale involving the motion of dust particles. The ion-acoustic wave (IAW) is one of these modified normal modes, called the dust-ion acoustic wave (DIAW), and has been studied in the last years [2].

In this paper, four component dusty plasmas are considered consisting of electrons, ions and two different species of dust grains (bi-dust plasmas). The effect of Landau damping in a bi-dust, unmagnetized plasma is analyzed, and the grains are defined by the radius (r_{d1}, r_{d2}) of the particules. In order to find the dispersion relation in this complex plasma, a hybrid method is used [1-3], to simplify the way to find the dispersion relation in these plasmas. This bi-dust plasma is being considered with the assumption that most of the free electrons in plasma are attached to the dust grain surface. Charge fluctuations [4] and the drift velocities from ions and dust in the unperturbed states are neglected. Weak Coulomb coupling is also considered, so that we obtain a new form of the Landau damping rate.

2. Theoretical Model

In order to obtain the dispersion relation, it is considered that the electrons are the neutralizing background, and the density perturbation is described by a Boltzman factor, given by the linear approximation

$$n_e^{(1)} = n_e^{(0)} \frac{e\phi^{(1)}}{k_B T_e}. \tag{1}$$

The dusts are treated as fluids [1,2,5,6]. The number density perturbation $n_{d\alpha}^{(1)}$ is determined from the dust continuity equation

$$\frac{\partial n_{d\alpha}}{\partial t} + n_{d\alpha} \nabla.(n_{d\alpha}\vec{v}_{d\alpha}) = 0, \tag{2}$$

where $d_\alpha = d_1, d_2$. From the first order linearized perturbed equation

$$\frac{\partial n_{d\alpha}^{(1)}}{\partial t} + n_{d\alpha}^{(0)} \nabla.(n_{d\alpha}\vec{v}_{d\alpha}^{\,(1)}) = 0, \tag{3}$$

we obtain

$$w\, n_{d\alpha}^{(1)} = k\, n_{d\alpha}^{(0)}\, v_{d\alpha}^{(1)}, \tag{4}$$

and from the dust momentum equation

$$m_{d\alpha} n_{d\alpha} \left(\frac{\partial \vec{v}_{d\alpha}}{\partial t} + (\vec{v}_{d\alpha}.\nabla)\vec{v}_{d\alpha} \right) = q_{d\alpha} n_{d\alpha} \vec{E} - \nabla P_{d\alpha}, \tag{5}$$

we get

$$iw m_{d\alpha} n_{d\alpha}^{(0)} v_{d\alpha}^{(1)} = eZ_{d\alpha} n_{d\alpha}^{(0)} E^{(1)} + 3ikk_B T_{d\alpha} n_{d\alpha}^{(1)}. \tag{6}$$

From Eqs. (4) and (6), we can writte

$$n_{d\alpha}^{(1)} = \frac{-iekZ_{d\alpha}n_{d\alpha}^{(0)}E^{(1)}}{(w^2 m_{d\alpha} - 3k^2 k_B T_{d\alpha})}. \tag{7}$$

The linearized Poisson equation looks like

$$ikE^{(1)} = -\frac{e}{\varepsilon_0}\left(n_e^{(1)} - n_i^{(1)} + Z_{d1}\, n_{d1}^{(1)} + Z_{d2}\, n_{d2}^{(1)}\right). \tag{8}$$

This set of equations is closed with the first order quasineutrality condition,

$$n_i^{(0)} = n_e^{(0)} + Z_{d1}\, n_{d1}^{(0)} + Z_{d2}\, n_{d2}^{(0)}, \tag{9}$$

and

$$E^{(1)} = -\nabla\phi^{(1)}. \tag{10}$$

The idea of the hybrid model is to handle the most important species for Landau Damping using kinetic theory and to handle the other species using fluid equations or Boltzman Factors. For instance in dust-acoustic waves, the dust grains will be handled using kinetic theory, but for ion-acoustic waves the ions will be handled kinetically and, for Langmuir waves, kinetic theory will be used for electrons. The species hauled using kinetic theory will be that in which the number of trapped particles becomes important.

Now, using the kinetic approach for the ions, the perturbed ion density is obtained from the linearized Vlasov equation,

$$\frac{\partial f_i^{(1)}}{\partial t} + \vec{v}.\nabla f_i^{(1)} + \frac{eZ_i}{m_i}E^{(1)}\frac{\partial f_i^{(1)}}{\partial v} = 0\ . \tag{11}$$

Collisions between ions and dust charged grains are neglected. From Eq. (11), using Fourier analysis $n_i^{(1)}$ is given by

$$n_i^{(1)} = \frac{-2\,i\,e\,n_i^{(0)}\,E^{(1)}}{k\,m_i\,(v_i^{th})^2} \tag{12}$$

Introducing the Eqs. (7), (10) and (12) in (8), the dimensionless dispersion relation is

$$1 + \frac{1}{K^2} - \frac{N_1\theta_i Z_{d1}^2}{(\Omega^2\theta_i M_1 - 3K^2\theta_1)} - \frac{N_2\theta_i Z_{d2}^2}{(\Omega^2\theta_i M_2 - 3K^2\theta_2)} + \frac{N_i\theta_i Z_i}{K^2}\,(1+\zeta Z(\zeta)) \;=\; 0 \tag{13}$$

Where the dimensionless parameters are defined as

$$K = \frac{k}{k_{De}}; \qquad \Omega = \frac{w}{w_{pi}}; \qquad U_\alpha = \frac{v_\alpha}{C_\alpha},$$

$$N_i = \frac{n_i^{(0)}}{n_e^{(0)}}; \qquad N_1 = \frac{n_{d1}^{(0)}}{n_e^{(0)}}; \qquad N_2 = \frac{n_{d1}^{(0)}}{n_e^{(0)}},$$

$$M_i = \frac{m_i}{m_e}; \qquad M_1 = \frac{m_{d1}}{m_i}; \qquad M_2 = \frac{m_{d2}}{m_i},$$

$$\vartheta_i = \frac{T_e}{T_i}; \qquad \vartheta_1 = \frac{T_{d1}}{T_i}; \qquad \vartheta_2 = \frac{T_{d2}}{T_i},$$

with

$$k_{De}^2 = \frac{n^{(0)}e^2}{\epsilon_0 k_B T_e}; \quad w_{pi}^2 = \frac{n^{(0)}e^2}{\epsilon_0 m_i}, \quad C_{si}^2 = \frac{T_e}{m_i}; \quad w_{pi} = C_{si}k_{De}.$$

In DIA waves limit [4] we have

$$kv_i^{th}, w_{pd\alpha} \ll w \ll kv_e^{th}, \tag{14}$$

and therefore

$$\Omega^2\theta_i M_1 \gg 3K^2\theta_1; \Omega^2\theta_i M_2 \gg 3K^2\theta_2, \tag{15}$$

and $\zeta = w/kv_i^{th} = (\sqrt{\theta_i}\Omega)/(\sqrt{2}K) \gg 1$. For this reason the asymptotic expansion for the plasma dispersion function can be used:

$$Z(\zeta) = i\sigma\pi^{1/2}\exp[-\zeta^2] - \frac{1}{\zeta}\left(1 + \frac{1}{2\zeta^2} + \frac{3}{4\zeta^2} + \dots\right). \tag{16}$$

The Landau damping rates of the DIA waves can be obtained by letting $\Omega = \Omega_r + i\Omega_i$, and by using the expression

$$\Omega_i = -\frac{\varepsilon_i(K,\Omega_r)}{[\partial(\varepsilon_r(\Omega))/(\partial\Omega)]_{\Omega=\Omega_r}} \tag{17}$$

to obtain the imaginary part of the frequency. Where $\varepsilon_r(\Omega)$ and $\varepsilon_i(\Omega)$ are the real and imaginary parts of the dielectric function, respectively. The real part of the frequency Ω_r is obtained considering that $\varepsilon_r(\Omega_r) = 0$, and is giving by

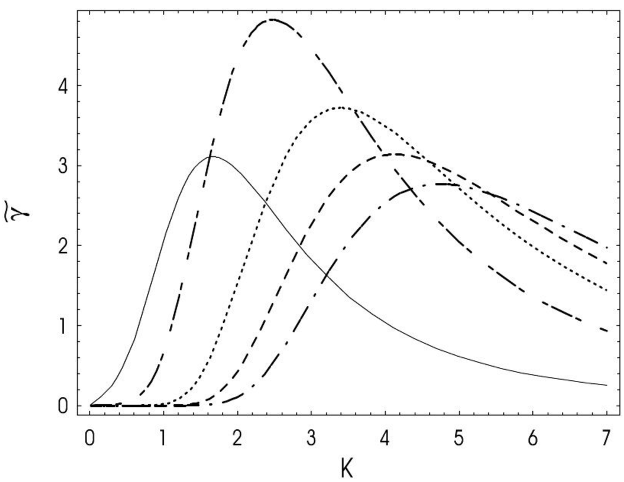

FIGURE 1. Dimensionless Landau damping rate versus K: electron-ion (continuous line), electron-ion and dusty d1 (two point dash line), electron-ion and bi-dust with $N_{d2} = 0.05N_{d1}$(point line), electron-ion and bi-dust with $N_{d2} = 0.1N_{d1}$(dashed line), and electron-ion and bi-dust with $N_{d2} = 0.25N_{d1}$(dashed-point line) .

$$\Omega_r = \frac{\sqrt{\delta}\, K}{\sqrt{\theta_i\,(1+K^2)}}, \tag{18}$$

where

$$\delta = \alpha_1\theta_i + \alpha_2, \tag{19}$$

and

$$\alpha_1 = \frac{N_1\, Z_{d1}^2}{M_1} + \frac{N_2\, Z_{d2}^2}{M_2}, \qquad \alpha_2 = Z_i N_i \theta_i. \tag{20}$$

Then, the growth rate in the dimensionless form is giving by

$$\tilde{\gamma} = -\frac{2\,\sqrt{\theta_i}\,\Omega}{\sqrt{2\,\pi}} = \frac{\alpha_2 K\delta}{(1+K^2)^2}\exp\left[\frac{\delta}{2(1+K^2)}\right], \tag{21}$$

3. Results and Conclusions

To illustrate the previous result, the following experimental values are used: $n_e^{(0)}$=1.10$^{10}cm^{-3}$, $n_i^{(0)}$=5.10^{9}cm^{-3}, $n_{d1}^{(0)}$=1.10^{6}cm^{-3}, Z_{d1}=1000, Z_{d2}=5000, M_{d2}=10$^{-3}M_{d1}$ and M_{d1}=1.5210^{10}.

Figure 1 shows the Landau damping in a bi-dust plasma in comparison with the normal Landau damping, in a non dusty and single dusty plasma. Continuous curves correspond to the damping rate when dusty grains are not present. The maximun of this curve is lower than that of the curve with only one dust grain (two points-dustline). This is reasonable due to the massive effect of the particules in our expression for the Landau damping rates. This seems to be a generalization of the growth rate in comparison with other formulas, where the damping rate is independent of the grain mass, density and charge. In a bi-dust plasma, a shift of the damping rate in the direction of large K is observed, when the concentration of the dust (N$_{d2}$) increases. To the best of our knowledge, this is a new effect not published before in the literature [1,2,7] in which the effect of Landau damped rate dust is neglected.

1. P.K. Shukla, *Phys. Plasmas* **8** (2001) 1791.
2. P.K. Shukla and A.A. Mamun, *Introduction to Dusty Plasma Physics* (IOP Publishing Ltd, London 2002) Chap. 4.
3. J. Puerta, J. Silva, and C. Cereceda, *ICPP 2002. AIP Conf. Prod* **669** (2002) 540.
4. J. Vranjes, B.P. Pandey, and S. Poedts, *Phys. Plasmas* **9** (2002) 1464.
5. C. Cereceda, J. Puerta, and P. Martín, *Physica Scripta* **T84** (2000) 206.
6. J. Puerta, C. Cereceda, *Astr. and Space Sci.* **256** (1998) 349.
7. S.K. El-Labany and W.M. Moslem, *Phys. Plasmas* **10** (2003) 4217.