Legendre polynomial in space charge potentials for velocity analysers with spherical geometry

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The analysis of space-charge effect in spherical geometries has been performed using kinetic theory. The current collected by central electrodes immersed in plasmas with a discriminant grid has been determined through the Poisson equation and truncated Maxwellian Distribution functions for the ions. We have assumed that the electrons are repelled by an entrance grid hold at a negative potential. This analysis leads to a strongly non linear dimensionless equation, where the lapalacian operator depends only of r and θ. The case of plane geometry was already published [8], and a preliminary work in cylindrical geometry has been reported [4]. This equation has been solved approximately by expanding the above functions around the potential ϕ_p, and the solution is given by the spherical Bessel functions and the Legendre polynomials.

Keywords: Space charge effects.

El análisis del efecto de carga espacial en geometría esférica ha sido elaborado con teoría cinética. La corriente colectada por electrodos inmerso en un plasma con rejilla discriminante ha sido determinada usando la ecuación de Poisson y funciones de distribución Maxwelliana truncada para los iones. Hemos asumido que los electrones son repelidos por la rejilla de entrada mantenida a potencial negativo. El análisis nos lleva a una ecuación adimensional fuertemente no lineal, donde el operador Laplaciano depende de r y θ. El caso de geometría plana fue publicado[8], y trabajos preliminares en geometría cilíndrica han sido reportados[4]. Esta ecuación ha sido resuelta en una forma aproximada por expansión de la función alrededor del potencial ϕ_p; la solución esta dada en término de las funciones de Bessel y los polinomios de Legendre.

Descriptores: Efectos de carga espacial.

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1. Introduction

The space-charge effect for planar electrodes has been extensively studied, using both fluid equations and Kinetic theory (Page and Adams, 1958; Braun *et al.*,1973)for planar velocity analyzers, thermoionic diodes,and other engineering problems. For cold plasmas and simple boundary conditions, the fluid equations can be integrated and the result is known as the Langmuir-Chid equation (Langmuir,1923)[1,7]. This equations do not include temperature effects and they can be considered only as a first approximation of the Martìn-Donoso's papers [8]. The analysis using kinetic theory in spherical geometry leads to a non linear, second-order differential equation which can be solved using a linear expansion around the point of zero potential [5,8–12]. In kinetic theory treatment, there are two possibilities of truncated Maxwellians depending on whether the trapped charged particles in the entrance grid are included or not. Experiments using velocity analyzers of variable interelectrodic distance seem to confirm that the trapped particles participate in the space-charge phenomena, and the they should be included Refs. 8, and 10, thus the analysis done here in detail seems to be the most appropriate one.

2. Theory

The study of the electric potential curves and its comparison in presence of spatial charge effects is of great relevance. To illustrate this, the potential curve is studied in spherical geometry. In the interelectrodic region, between the maximum potential V_p and the discriminating grid G_1at $\widetilde{r} = \widetilde{r}_c$, there are no reflected particles because it is assumed that all the ions arriving at the discriminating grid are collected by the collector G_0, at the origin. The ion distribution function $f(\rho, \dot{\rho}, \dot{\theta}, \dot{\varphi})$ for a homogeneous spherical symmetric ion plasma with potential $V(\widetilde{r})$ is a Maxwellian distribution function:

$$f(\rho, \dot{\rho}, \dot{\theta}, \dot{\varphi}) = n_0 \left(\frac{m}{2\pi}\right)^{\frac{1}{2}} \times \exp\left\{\frac{-\frac{1}{2}m\left[\left(\dot{\rho}^2+\rho^2\dot{\theta}^2+\rho^2\dot{\varphi}^2\sin^2\varphi\right)+qV(\widetilde{r})\right]}{T}\right\}, \quad (1)$$

where the temperature T of the ions is given in electron volts eV, and the total velocity is $v^2 = v_\rho^2 + v_\theta^2 + v_\varphi^2$. The ions with radial kinetic energy large than qV_p will be reflected. Therefore, the distribution function in the interval (a, r_p), where r_p is the maximum, will be

$$f(\rho, \dot{\rho}, \dot{\theta}, \dot{\varphi}) = \begin{cases} f_0(\rho, v(\widetilde{r})), & v_{\widetilde{r}} > -v_p(\widetilde{r}) \\ 0, & v_{\widetilde{r}} < -v_p(\widetilde{r}) \end{cases} ;$$
$$\widetilde{r}_a < \widetilde{r} < \widetilde{r}_p, \tag{2}$$

where $f_0(\rho, v(\widetilde{r}))$ is the truncated Maxwellian distribution function, and V_p is defined by the equation

$$\frac{1}{2} m [v_p(\widetilde{r})]^2 + qV(\widetilde{r}) = qV_p. \tag{3}$$

We have asumed that $\widetilde{r}$=(ρ, θ), therefore $v(\widetilde{r})$=(v_ρ, v_θ); in such a way that $\phi(\widetilde{r}_p) = 0$ for $\widetilde{r} = \widetilde{r}_p$, and both distribution functions are coincident. In this way, the continuity of the current is secured. Using the Debye length $\lambda_D^2 = T/4\pi n_0 e^2$, and the dimensionless variables

$$r = \frac{\rho}{\lambda_D}; \qquad r_p = \frac{\rho_p}{\lambda_D};$$
$$\phi(\widetilde{r}) = \frac{q[V_p - V(\widetilde{r})]}{T}; \quad \phi_P = \frac{qV(\widetilde{r}_p)}{T}, \tag{4}$$

the Poisson equation can be written as

$$\nabla^2 \phi = \frac{1}{2} e^{(\phi - \phi_p)} \left[1 \pm \; erf \left(\phi^{\frac{1}{2}} \right) \right]. \tag{5}$$

Using the laplacian operator in spherical coordinates, and assuming an axis of symmetry (*i.e.* $\partial / \partial \varphi = 0$); as well as the new variables

$$\psi = R(r) Y(\theta, 0); \quad \psi = (\phi - \phi_p);$$
$$R_\pm = \alpha_{0\pm} \pm \beta_0^2 \psi, \tag{6}$$

then the following dimensionless equation is obtained

$$\frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} - \left(l\,(l+1) \mp \beta_0^2 \right) R = 0 \tag{7}$$

where

$$R(r) = A j_l(\beta_0 r) + B n_l(\beta_0 r). \tag{8}$$

The solution of the differential equation in terms of the Legendre polynomials in the interval $r_p \le r \le r_c$, is

$$\phi(r) = \sum c_l \left(A j_l(\beta_0 r) + B n_l(\beta_0 r) + C^+ \right) \times P_l(\cos\theta). \tag{9}$$

The solution of the differential equation in terms of the Legendre polynomials in the interval $r_a \le r \le r_p$, is

$$\phi(r) = \sum c_l (C i_l(\beta_0 r) + D k_l(\beta_0 r) + C^-) \times P_l(\cos\theta). \tag{10}$$

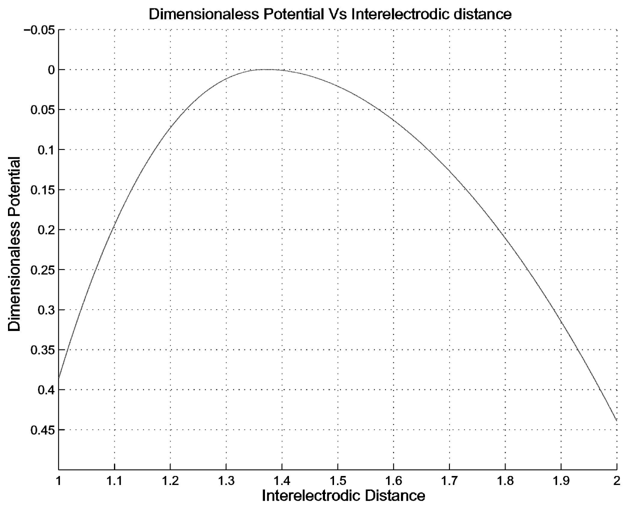

FIGURE 1. In figure is shown the space charge potential including temperature effects when the symmetry is spherical, the dimensionless potentials are $\phi_a = 0.44$ and $\phi_c = 0.39$,the dimensionless interelectrodic distance is one.

Now, the boundary conditions of the solution are

$$\phi_a = \phi_p - \frac{V_a}{T}; \quad \phi_c = \phi_p - \frac{V_c}{T}; \quad \left. \frac{d\phi}{dr} \right|_{r=r_p} = 0 \tag{11}$$

where $V_a = V(\widetilde{r}_a)$, and $V_c = V(\widetilde{r}_c)$.

Therefore, we have the system of equations

$$c_l (A j_l(\beta_0 c) + B n_l(\beta_0 c) + C^+) P_l(\cos\theta_c) = \phi(\widetilde{r}_c) = \phi_c, \tag{12}$$
$$c_l (C j_l(\beta_0 a) + D n_l(\beta_0 a) + C^-) P_l(\cos\theta_a) = \phi(\widetilde{r}_a) = \phi_a, \tag{13}$$

$$0 = (A j_l(x_p) + B n_l(x_p) + C^+) P_l(\cos\theta_p) \tag{14}$$
$$= (C i_l(x_p) + D k_l(x_p) + C^-) P_l(\cos\theta_p). \tag{15}$$

$$A j_l'(x_p) + B n_l'(x_p) = 0 \tag{16}$$
$$C i_l'(x_p) + D k_l'(x_p) = 0 \tag{17}$$

where $x_p = \beta_0 r_p$, and the solution of this system of equations is given by

$$A = C^- x_p^2 n_l(x_p), \qquad B = -C^- x_p^2 j_l(x_p), \tag{18}$$
$$C = C^+ x_p^2 k_l(x_p), \qquad D = -C^+ x_p^2 i_l(x_p). \tag{19}$$

Finally, we obtain the pair of equation for intervals $r_p \leq r \leq r_c$, and $r_a \leq r \leq r_p$, respectively

$$\phi(r) = \sum_{l=0} c_l \big(C^+ x_p^2 k_l(x_p) i_l(\beta_0 r) - C^+ x_p^2 k_l(x_p) k_l(\beta_0 r) + C^+ \big) P_l(\cos\theta), \quad (20)$$

$$\phi(r) = \sum_{l=0} c_l \big(C^- x_p^2 n_l(x_p) j_l(\beta_0 r) - C^- x_p^2 j_l(x_p) n_l(\beta_0 r) + C^- \big) P_l(\cos\theta). \quad (21)$$

This it is the potential function of the depending on the variable position and between the electrodes.

For instance, when $l = 0$, in such a way that if $\phi_p = 2$ and $r_p = 3.1$, the graphic on Fig. 1, is a representation obtained.

3. Conclusion

The space charge analysis performed here in detail for spherical electrodes leads to a non-linear differential equation, which is solved when the laplacian operator and the potential depend on r and θ. The advantage of studying this phenomenon with θ is very important because it enables precision in the space charge effect. The curve of the potential in spherical probes was drawn for certain values of l, r_p, ϕ_p. It is for that reason that the range of validity is limited to some range of parameters.

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