Non-Abelian Born-Infeld action

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We introduce a realization of the algebra of diffeomorfisms in terms of the fields of a non-abelian Born-Infeld theory(NBI), which generalizes the abelian case. We argue that the non-abelian Born Infeld action which describes the interaction of D-branes in the low energy regime, may be constructed from the Hamiltonian obtained by taking the symmetry generators as gauge constraints. We explicitly obtain the interacting terms to the sixth order in the curvature of the Yang-Mills fields. Our results for the NBI action agree up to the fourth order with the ones obtained from the interacting string theory.

Keywords: NBI, Diffeomorfisms, D-branes

Introducimos una realización del álgebra de los difeomorfismos en en términos de los campos de un modelo de Born-Infeld no abeliano. Argumentamos que la acción del modelo de Born Infeld no abeliano que describe la interacción de D-branas en el límite de bajas energías puede ser construido a partir del hamiltoniano, que se obtiene de tomar los generadores de la simetria como vínculos de calibre. Obtenemos explícitamente los términos hasta el orden seis en el campo de Yang-Mills. Nuestros resultados concuerdan hasta el orden cuarto con los obtenidos de la teoría de cuerdas.

Descriptores: NBI, Difeomorfismos , D-branas

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1. Introduction

D-brane actions describe the effective theory of interacting strings with Dirichlet boundary conditions. For a single D-brane, the low energy limit is given by the Born-Infeld (BI) action [1, 2]. In D=10 dimensions, the D2-brane is dual to the D=11 supermembrane over a compactified target space $M_{10} \times S^1$. The compactified direction of the supermembrane which is wrapped on the S^1 is manifested as a $U(1)$ gauge vector field living in the world-volume of the D2-brane immersed in M_{10}. The D=11 supermembrane, together with the D=11 super 5-brane and supergravity are relevant ingredients of the M-theory. One important problem to understand is the interaction of D=11 supermembranes, which is equivalent to the description of the interactions of the dual D2-branes in D=10. Some years ago, Witten [3] observed that the interacting D-brane theory in the overlapping limit should be described by a non-abelian Born-Infeld (NBI) action generalizing the abelian Born-Infeld [2] . The construction of this supersymmetric NBI action to all orders, in a systematic way, is still an open problem in spite of various efforts [4–6] which include recent contributions [7, 8]. To address this problem in an alternative way, we propose to start with an explicit realization of the algebra of diffeomorfisms as a set of first class constraints imposed on the fields of the NBI. Since the action describing the overlapping D-branes must be invariant under diffeomorfisms on the world-volume, the Hamiltonian must be a linear combination of the first class constraints of our construction. In what follows, we describe the realization of the algebra, and pursue the construction of the Hamiltonian for the bosonic NBI theory as a first step towards the construction of the supersymmetric NBI. The construction of the Hamiltonian for the abelian Born-Infeld theory has been discussed by several authors, see for example [9].

2. Generalized non-abelian algebra. The NBI model

In this section, we generalize the diffeomorfisms symmetry algebra of the BI model associated to the $D2$-brane or Dirichlet membrane in D=10 to the non abelian case. The NBI model related to the interacting $D2$-branes, should be written in terms of the intrinsic metric β_{ab} of the world volume, the space-time coordinates X^m, a non-abelian gauge field A^I_a, and their corresponding momenta $P_{ab} = 0$, P_m and Π^{Ia}. Here, I is the non-abelian index, the index $m = 0, \dots 9$ labels the target dimensions and $a = 1, 2$, the spatial dimensions of the D2-brane. We define:

$$\Pi^2 \equiv \beta_{ab}\Pi^{Ia}\Pi^{Ib} , \quad (1)$$

$$F^I_{ab} = \partial_a A^I_b - \partial_b A^I_a + f^{IJK} A^J_a A^K_b \equiv \varepsilon_{ab} f^I , \quad (2)$$

$$\mathcal{D}^{IJ}_a = \partial_a \delta^{IJ} + f^{IKJ} A^K_a . \quad (3)$$

The hamiltonian analysis for the abelian case in which we base our computations was done in Refs. 10 and 11. In order to realize the non abelian symmetry of the system, one should modify the constraints which appear in the abelian case. It is natural to define the constraints

$$\Phi_a \equiv \left(\partial_a X^m P_m + \Pi^{Ib} F^I_{ab}\right) \quad (4)$$

$$\Phi \equiv \left(P^m P_m + \Pi^2 + \beta\beta^{ab}\partial_a X^m \partial_b X^m - \beta + f^I f^I\right) \quad (5)$$

$$\varphi^I \equiv -\left(\mathcal{D}^{IJ}_a \Pi^{Ja}\right) \quad (6)$$

where $\beta \equiv det\,(\beta_{ab})$, Φ_a is the generator of spatial diffeomorfisms and is a topological constraint that does not depend on the target metric, and Φ is the generator of temporal diffeomorfisms and is metric dependent. φ^I is the Gauss constraint. The algebra of these constraints is given by

$$\{\Phi(\xi),\Phi(xi')\}{=}\left(C^{ab}\Phi^a(\xi){+}C'^{ab}\Phi_a(\xi')\right)\partial_b\delta^2(\xi{-}\xi').$$
$$\{\varphi^I(\xi),\varphi^{'J}(\xi')\}{=}f^{IJK}\varphi_K\delta^2(\xi{-}\xi')$$
$$\{\Phi a(\xi'),\Phi_b(\xi')\}{=}\Phi_a(\xi')\partial_b\delta^2(\xi{-}\xi'){+}\Phi_b(\xi)\partial_a\delta^2(\xi{-}\xi')$$
$$-\,\varepsilon_{ab}f^J\varphi^J(\xi)\delta^2(\xi{-}\xi').$$
$$\{Phi(\xi),\varphi^I(\xi')\}{=}0, \qquad \{Phi_a(\xi),\varphi^I(\xi')\}{=}0.$$
$$\{\Phi_a(\xi),\Phi(xi')\}{=}\left(\Phi(\xi){+}\Phi(\xi){+}\Phi(\xi')\right)\partial_a\delta$$
$$+C^I_a\varphi^I\delta^2(\xi{-}\xi'). \quad (7)$$

where $C^{ab} = 4\beta\beta^{ab}$ and C^I_a depends on the solution for β. This algebra closes, but the determination of some of the structure constants depends explicitly on the solution for β.

3. Determination of β_{ab}

To continue, we have to determine the solution for β, order by order. Here, we perform the computation up to the fourth order to compare with the results obtained from the interacting string theory [5]. First, we consider the variations of the constraint Φ under changes of β_{ab}. Since they must vanish, we obtain,

$$\Pi^{Ia}\Pi^{Ib} + \beta\beta^{ab}\beta^{cd}\partial_c X^m \partial_d X^m$$
$$-\,\beta\beta^{ab} - \beta\beta^{ac}\beta^{bd}\partial_c X^m \partial_d X^m = 0 \quad (8)$$

After contracting with β_{ab} we have,

$$\Pi^{Ia}\Pi^{Ib} + \beta\beta^{ab} - \beta\beta^{ab} - \beta^{ab}\Pi^2$$
$$-\,\beta\beta^{ac}\beta^{bd}\partial_c X^m \partial_d X^m = 0. \quad (9)$$

From the determinant of Eq. (9), we obtain, if $\beta \neq 0$

$$det\left(\Pi^{I\bullet}\Pi^{I\bullet}\right) + \beta - \Pi^2 = det\left(\partial_\bullet X^m \partial_\bullet X^m\right), \quad (10)$$

where the $\bullet$ means a target space index $a = 1, 2$. After some algebra using Eqs. (9)and (10) , and still providing that $\beta \neq 0$

$$\Pi^{Jc}\Pi^{Jd}\Pi^{Ia}\Pi^{Ib}\beta_{ac}\beta_{bd}$$
$$+\Pi^2\left[det\left(\partial_\bullet X^m \partial_\bullet X^m\right) - det\left(\Pi^{I\bullet}\Pi^{I\bullet}\right)\right]$$
$$-beta\left(\Pi^{Jc}\partial_c X^m\right)\left(\Pi^{Jd}\partial_d X^m\right) = 0. \quad (11)$$

Combining Eqs. (9) and (10) we can state that,

$$\beta^{ab}\left[det\left(\partial_{bullet}X^m \partial_{bullet}X^m\right) - det\left(\Pi^{I\bullet}\Pi^{I\bullet}\right)\right]$$
$$+\Pi^{Ia}\Pi^{Ib} - \beta\beta^{ac}\beta^{bd}\partial_c X^m \partial_d X^m = 0 \quad (12)$$

allowing us to solve for β_{ab}, in terms of powers of $\Pi^I\Pi^I$, using an iterative procedure. We also realize that the constraint Φ only depends on β_{ab} through its dependence in β, which is of the form,

$$\Phi = P^m P_m + f^I f^I + \beta. \quad (13)$$

In the abelian case $det\,(\Pi^\bullet\Pi^\bullet) = 0$, obtaining the exact solution for β:

$$\beta = det\left(\partial_\bullet X^m \partial_\bullet X^m\right) + \Pi^c\Pi^d\partial_c X^m \partial_d X^m \quad (14)$$

The corresponding exact expression for the constraint Φ is that case:

$$\Phi \equiv P^m P_m + ff + det\left(\partial_\bullet X^m \partial_{bullet}X^m\right)$$
$$+\,\Pi^c\Pi^d\partial_c X^m \partial_d X^m. \quad (15)$$

Returning to the non-abelian case we are interested in, we introduce a power expansion for β_{ab} in the form

$$\beta_{ab} = g_{ab} + O_{1ab} + O_{2ab} + \ldots, \quad (16)$$

where the metric $g_{ab} \equiv \partial_a X^m \partial_b X^m$, and O_{iab} is a term of order i in $\Pi^I\Pi^I$. For β^{ab} we can write then:

$$\beta^{ab} = g^{ab} - g^{ac}O_{1cd}g^{db} - g^{ac}O_{2cd}g^{db}$$
$$+\,g^{ac}O_{1ce}g^{ef}O_{1fd}g^{db} + \ldots, \quad (17)$$

and for β we have

$$\beta = g + g_{ab}\Pi^{Ia}\Pi^{Ib} - det\left(\Pi^{I\bullet}\Pi^{I\bullet}\right)$$
$$+\,O_{1ab}\Pi^{Ia}\Pi^{Ib} + O_{2ab}\Pi^{Ia}\Pi^{Ib} + \ldots \quad (18)$$

where $g = det\,(g_{ab})$. Using these expressions in our equations, we obtain after some calculations the solution for the O_{iab} to order $i = 2$. They are given by,

$$gO_{1ab} = g_{ab}\left(\Pi\right)^2 - g_{ac}g_{bc}\Pi^{Ia}\Pi^{Ib}\,, \quad (19)$$

$$gO_{2ab} = g_{ab}O_{1cd}\Pi^{Ic}\Pi^{Id} + 2gO_{1ac}g^{cd}O_{1bd}$$
$$-\,2g_{cd}\Pi^{Ic}\Pi^{Id}O_{1ab}. \quad (20)$$

As can be seen from Eqs. (19) and (20), in the abelian case $O_{1ab} = 0$ and $O_{2ab} = 0$, thus recovering from (18), the exact result for this case.

4. The Hamiltonian and the action of NBI

As stated above, since the action describing the overlapping D-branes must be invariant under diffeomorfisms on the world-volume, the Hamiltonian must be a linear combination of the first-class constraints of our construction. The Hamiltonian associated to the bosonic NBI action is then given by

$$H = \Lambda\Phi + \Lambda^a \Phi_a + \Lambda_I \varphi^I, \qquad (21)$$

where Λ, Λ^a and Λ_I are the Lagrange multipliers. The action is given by

$$S = \left\langle P_m \dot{X}^m + \Pi^{Ia} \dot{A}^I_a - H \right\rangle. \qquad (22)$$

It can be shown, after some calculations, that the flat limit of this action is in agreement with the interacting terms computed from the interacting string theory, which have been explicitly obtained up to α'^2 order [12]. The agreement is obtained, as usual, in the terms without derivatives of the field strength.

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