Particle production by a spatially homogeneous time-dependent electric field

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We discuss the mechanism of production of positrons and electrons by a spatially homogeneous time-dependent electric field whose amplitude vanishes for large values of time. Using the Hamiltonian diagonalization technique, we compute the density of particles created as a function of time. We show that, as the time parameter goes to infinity, the distribution of pair created by the electric field reduces to the result calculated via the Bogoliubov coefficients.

Keywords: Particle production; Dirac equation.

En este articulo se discute el mecanismo de producción de electrones y positrones debido a un campo eléctrico homogéneo y dependiente del tiempo cuya amplitud se anula para valores grandes del tiempo. Haciendo uso de la técnica de diagonalización del Hamiltoniano calculamos la densidad de partículas creadas como una función del tiempo. Se muestra que, cuando el tiempo tiende a infinito, la distribución de pares creados por el campo eléctrico se reduce al resultado obtenido a través de los coeficientes de Bogoliubov.

Descriptores: Producción de partículas; ecuación de Dirac.

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1. Introduction

In the last years a lot of work has been done on the problem of particle production by strong electromagnetic fields [1–3]. After the publication of the pioneering articles of Heisenberg and Euler [4] and Schwinger [5], spontaneous pair production in the presence of strong electromagnetic fields has been widely discussed in the literature [1, 2, 6–9]. Different methods have been applied in order to compute vacuum effects in strong static and homogeneous electric fields. Among the different approaches and techniques applied in the analysis of quantum effects on stable vacua, we should mention the proper time technique, the imaginary time method [7, 8], the complex multiple reflection approach [10, 11], the diagonalization method [2, 12, 13], and the complex path approach, and tunnelling [14, 15] among others.

Using a proper time technique, Schwinger computed the persistence of the vacuum in the presence of a constant electric field. Using the natural system of units where $\hbar = c = 1$, the imaginary part of effective action takes the form:

$$2Im\mathcal{L}_{eff}^{(1/2)} = \frac{(qE_0)^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-n\pi m^2/eE_0}. \qquad (1)$$

The result (1) shows that pair production in the presence of a constant electric field becomes important when the electric field intensity is comparable to the critical E_{cr} value: $E_{cr} = m^2/e \approx 1.32 \times 10^{16}\mathrm{V/cm}$. The formula (1) has also been used in modelling particle production in the central rapidity region in high-energy nucleus-nucleus collisions. [16]

We are interested in studying the phenomenon of particle creation in the presence of a spatially homogeneous electric field associated with the vector potential (2)

$$A^\mu = A(t)\delta_3^\mu, \qquad (2)$$

where $A(t)$ is given by the expression:

$$A(t) = -(E/k_0)\tanh(k_0 t). \qquad (3)$$

The corresponding electric field $E(t)$ has the form

$$E(t) = E/\cosh^2(k_0 t). \qquad (4)$$

This background permits one to discuss the mechanism of e^+, e^- pair production by a time-varying electric field. In order to compute the dependence on time of the density of particles created by the electric field, we apply the diagonalization technique [2].

2. The diagonalization technique

The proper time technique [1, 2] can be generalized to arbitrary electromagnetic and gravitational backgrounds but is this an uphill task since it is not at all convenient to regularize the effective Lagrangian in all such cases. For to these reasons many workers in this field turn to the Bogoliubov transformation technique [2]. In order to apply the Bogoliubov transformation technique, one needs to solve the wave equation associated with the particle and identify the “positive” and “negative” frequency modes. The adiabatic approach’ as well as the WKB method, in most cases gives a recipe for constructing approximate solutions of the basis; nevertheless this technique neglects those effects associated with multiple reflections and transmission resonances.

In order to study particle production in time-dependent electric fields and to describe how the density of particles created by an electric field evolves trough time, we are going to use the diagonalization method. This method was proposed by Grib *et al.* [2] and developed by different authors, who have shown its usefulness in discussing particle creation processes in the presence of strong electromagnetic and gravitational fields.

Recently Dolby and Gull [12,13] have introduced a modification in order to free it from the problems exhibited by the old diagonalization method. In Ref. [13] the authors show that their new approach is gauge invariant, and therefore time independent vector potentials do not require a barrier-approach interpretation. Among the advantages of the diagonalization method we should mention that it permits one to discuss the evolution of the particle creation process through time and it reproduces the results obtained with the adiabatic approach in those cases where it is possible to define adiabatic asymptotic states [2,9]

The standard diagonalization procedure proposed by Grib *et al.* is equivalent to the particle definition developed by Gull and Dolby, at least in those simple electromagnetic configurations admitting an asymptotic Killing vector. Since the electromagnetic field $E(t)$ (4) is obtained via the time dependent vector potential $A(t)$ (2), we do not have to appeal to any tunnelling interpretation [14,15]. Nevertheless our result should be expected coincide with those of Padmanabhan *et al.* [14,15] in the quasiclassical limit.

The Dirac equation in the presence of an electromagnetic potential is

$$[i\gamma^\mu(\partial_\mu - ieA_\mu) - m]\,\Psi = 0 \tag{5}$$

where we have adopted the natural system of units $\hbar = c = 1$, and the metric signature $\eta^{\mu\nu} = \mathrm{diag}(+ - - -)$. The Dirac matrices satisfy the anticommutation relations:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{6}$$

In order to apply the diagonalization method, we proceed as follows [2,9]: We introduce the auxiliary spinor χ

$$\Psi = [i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu + m]\,\chi \tag{7}$$

Substituting (7) into the Dirac equation (5), we obtain:

$$[i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu - m]\,[i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu + m]\,\chi = 0 \tag{8}$$

and inserting the electromagnetic potential (2) in (8) we get the second order differential equation:

$$\Big[\partial_0^2 - \partial_i\partial_i + e^2A_3^2(t) - 2ieA_3(t)\partial_3 + ie(\partial_0 A_3(t))\gamma^0\gamma^3 + m^2\Big]\chi = 0 \tag{9}$$

Following Ref. 2, we look for solutions of Eq. (9) in the form

$$\chi_r = e^{ipx} f(p,t) R_r, \tag{10}$$

where R_r are the eigenvectors of the matrix $\gamma^0\gamma^3$, satisfying $\gamma^0\gamma^3 R_r = R_r$, $R_r^+ R_s = 2\delta_{r,s}$, $r,s = 1,2$.

$$R_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \quad R_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{11}$$

Substituting (10) into (9) we obtain that $f^\pm$ satisfies the following second order differential equation

$$\ddot{f}^\pm + [\omega^2 + ie\dot{A}_3)f^\pm = 0, \tag{12}$$

where the frequency ω satisfies the relation

$$\omega^2 = m^2 + p_\perp^2 + (p_3 - eA_3)^2. \tag{13}$$

As $t \to -\infty$ positive and negative frequency modes in Eq. (12) satisfy the asymptotic behavior

$$f^\pm(\mathbf{p},t) \approx \left[4\omega_-(\mathbf{p})(\omega_-(\mathbf{p}) + p^3 - eA_-^3)\right]^{-1/2} \times \exp[\pm i\omega_-(\mathbf{p})t]. \tag{14}$$

A complete system of solutions of the Dirac equation (5) can be expressed as

$$\psi_{pr}^\pm(x) = (2\pi)^{-3/2}\Big[p_\nu\gamma^\nu + i\partial_0\gamma^0 + eA^3(t)\gamma^3 + m\Big]\chi_{pr}^\pm(x) \tag{15}$$

where the spinor solutions $\psi_{pr}^\pm(x)$ are normalized according to the product:

$$\int \psi_{p,r}^\pm(x)\psi_{p'r'}^\pm(x)d^3x = \delta^3(\mathbf{p} - \mathbf{p}')\delta_{r,r'} \tag{16}$$

The spinor field operator $\Psi(x)$ has the form

$$\Psi(x) = \sum_{r=1,2}\int d^3p\left[\Psi_{p,r}^{(-)}(x)a_{p,r}^{(-)} + \Psi_{-p,r}^{(+)}a_{p,r}^{(+)}\right] \tag{17}$$

Following the diagonalization approach [2,9], we obtain that the time dependent Hamiltonian has the form

$$H^{(1/2)}(t) = \sum_{r,s}\int d^3p\Big[A_{r,s}^{(-,-)}(\mathbf{p},t)a_{pr}^{*+}a_{ps}^- + A_{r,s}^{(-,+)}(\mathbf{p},t)a_{pr}^{*+}a_{-ps}^+ + A_{r,s}^{(+,-)}(\mathbf{p},t)a_{-pr}^{*-}a_{ps}^- + A_{r,s}^{(+,+)}(\mathbf{p},t)a_{-pr}^{*-}a_{-ps}^+\Big], \tag{18}$$

where the coefficients $A_{r,s}^{(\delta,\varepsilon)}(\mathbf{p},t)$ of the Hamiltonian $H^{(1/2)}(t)$ can be expressed as:

$$A_{r,s}^{(\delta,\varepsilon)}(\mathbf{p},t) = i\psi_{pr}^{\delta+}\partial_0\psi_{ps}^\varepsilon. \tag{19}$$

Substituting the spinor $\psi_{pr}^\pm(x)$ into (19), it is not difficult to verify that $A_{r,s}^{(\delta,\varepsilon)}(\mathbf{p},t) = 0$ for $r \neq s$. The non-zero coefficients in Eq. (19) are

$$A_{11}^{(-,-)} = A_{22}^{(-,-)} \equiv \omega E,$$
$$A_{11}^{(-,+)} = A_{22}^{(-,+)} \equiv \omega F,$$
$$A_{11}^{(+,+)} = A_{22}^{(+,+)}. \tag{20}$$

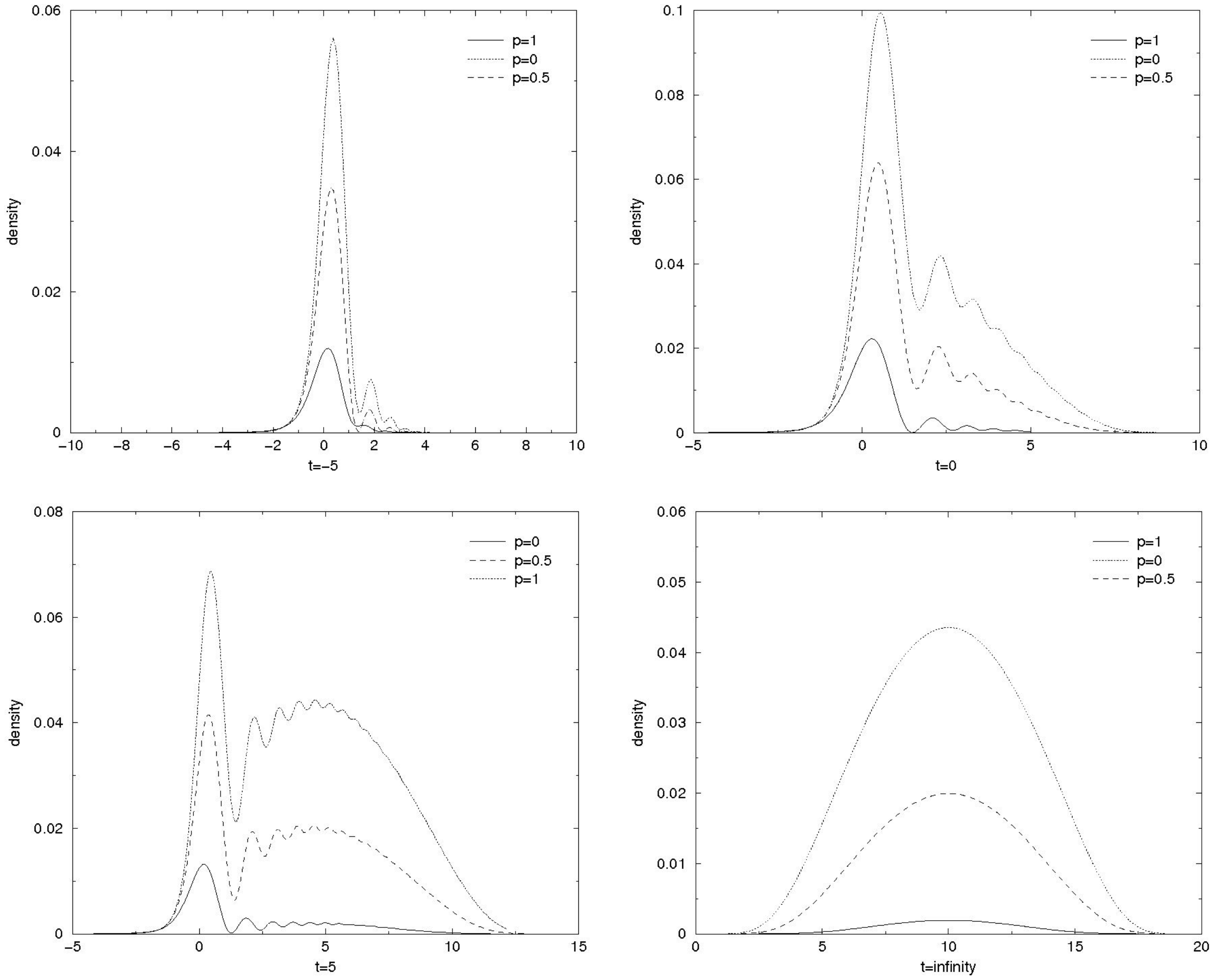

FIGURE 1. Particle density as a function of p^z_{out}/m, for $E/E_{cr} = 1$, $k_0 = 0.1$ $p = 0$, 0.5 and 1, and for tm=-5, 0, 5, and ∞.

Using (11), (10), and (15), one obtains

$$E(\mathbf{p}, t) = 4\frac{(m^2 + p_\perp^2)}{\omega} Im\left[f^{(+)*} f^{(+)}\right] - \frac{p_3 - eA_3}{\omega} \quad (21)$$

and

$$E^2(p, t) + |F(p, t)|^2 = 1 \quad (22)$$

Taking into account Eq. (20) and Eq. (21), we obtain that the Hamiltonian (18) takes the form

$$H^{(1/2)}(t) = \sum_{r=1,2} \int d^3p\omega(\mathbf{p}, t)\Big[E(\mathbf{p}, t)(a^{*+}_{pr} a^-_{pr} - a^{*-}_{-pr} a^+_{-pr}) + F(\mathbf{p}, t) a^{*+}_{pr} a^+_{-pr} + F^*(\mathbf{p}, t) a^{*-}_{-pr} a^-_{pr}\Big]. \quad (23)$$

The diagonalization of the Hamiltonian (23) can be carried out with the help of the Bogoliubov transformations

$$a^-_{pr} = \alpha^*_p(t) b^-_{pr}(t) - \beta_p(t) b^+_{-pr}(t) \quad (24)$$

$$a^{*-}_{pr} = \alpha^*_{-p}(t) b^{*-}_{pr}(t) - \beta_{-p}(t) b^{*+}_{-pr}(t), \quad (25)$$

where the Bogoliubov coefficients α_p, β_p satisfy the relation:

$$|\alpha_p(t)|^2 + |\beta_p(t)|^2 = 1. \quad (26)$$

In terms of the creation and annihilation operators $b^+_{pr}(t)$ and $b^-_{pr}(t)$, the Hamiltonian (23) reduces to the form

$$H^{(1/2)} = \sum_{r=1,2} \int d^3p\omega(p, t)\left[b^{*(+)}_{p,r} b^{(-)}_{p,r} - b^{*(-)}_{-p,r} b^{(+)}_{-p,r}\right] \quad (27)$$

where the coefficients $\alpha_p(t)$ and $\beta_p(t)$ satisfy the relations:

$$\frac{\beta_p(t)}{\alpha_p(t)} = [1 - E(p, t)]/F^*(p, t), \quad (28)$$

$$|\beta_p(t)|^2 = (1 - E(p, t))/2. \quad (29)$$

Expression (29) gives the density of particles created by the electric field. The density of particles created per unit of volume is

$$n^{1/2} = \frac{2}{(2\pi)^3} \int d^3p\, |\beta_p(t)|^2 \quad (30)$$

The exact solution of Eq. (12) with the vector potential (3), having an asymptotic behavior associated with positive frequency modes can be written in terms of the Gauss hypergeometric function $F(\mu, \nu, \gamma, z)$ [17] as:

$$f_{<}(t) = Ne^{i\omega_- t}(1 + e^{2k_0 t})^{-i\alpha} F(\mu, \nu, \gamma, -e^{2k_0 t}). \quad (31)$$

Requiring that $f_{<}(t)$ have the asymptotic behavior given by Eq. (14), we determine the value of the constant N

$$N = [4\omega_-(p)(\omega_-(p) + p^3 - eA_-^3)]^{-1/2}, \quad (32)$$

and

$$\mu = i\left(\frac{\omega_+ + \omega_-}{2k_0} - \alpha\right), \quad \nu = i\left(\frac{\omega_- - \omega_+}{2k_0} + \alpha\right) \quad (33)$$

$$n = \frac{\sinh\left[\pi\left(\alpha - \frac{\omega_+ + \omega_-}{2k_0}\right)\right] \sinh\left[\pi\left(\alpha + \frac{\omega_+ + \omega_-}{2k_0}\right)\right]}{\sinh(\frac{\pi\omega_+}{k_0}) \sinh(\frac{\pi\omega_+}{k_0})} \quad (34)$$

The asymptotic limit of $|\beta_p(t)|^2$, as $t \to +\infty$, reduces to the expression (34). This result shows that, for the vector potential (3), the density of particles created obtained via the diagonalization method reduces to that derived with the help of the standard adiabatic approach.

Figure 1 shows the evolution of the particle creation process. The particles are created with small momentum, and they are accelerated by the electric field. The peak at the origin corresponds to particles created at late times and therefore with less linear momentum than those produced at early times.

3. Concluding remarks

We have applied the standard diagonalization technique approach in order to compute pair production by a hyperbolic field (3). The computation of the time dependent particle density with the help of the adiabatic asymptotic positive and negative frequency modes shows that, the Hamiltonian diagonalization gives reliable results if one chooses asymptotic modes computed via the the adiabatic method. The dependence on time of the quasiparticle distribution created by the hyperbolic field (3) shows that the results obtained in the adiabatic field case can be extended to other electric field configurations.

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