Numerical simulations of liquid flow through restrictors

E. Sira[a], J. Klapp[b], L.Di G. Sigalotti[a], and G. Mendoza[b]
[a]*Centro de Física, Instituto Venezolano de Investigaciones Científicas, IVIC, Apartado 21827, Caracas 1020A, Venezuela*
[b]*Departamento de Física, Instituto Nacional de Investigaciones Nucleares, ININ, Km. 36.5 Carretera México-Toluca, 52045 Estado de México, México*

Recibido el 25 de noviembre de 2003; aceptado el 15 de marzo de 2004

In this paper we describe the results of a two-dimensional numerical simulation of a viscous liquid flow through a wellhead choke of real dimensions using the of Smoothed Particle Hydrodynamics (SPH) method. The study of such flows has a direct application to the oil industry because in oil fields, it is common practice to pass liquid and gas mixtures through chokes to control the flow rates and protect the surface equipment from unusual pressure fluctuations. For the present model calculation, we assume an isothermal flow with a sound speed c of 2.0×10^4 cm s^{-1} and a constant kinematic viscosity coefficient ($\nu = 0.01$ cm^2 s^{-1}). The results predict a pressure drop of about 13% through the choke throat when the flow approaches an approximately stationary pattern. The flow across the choke remains subcritical with velocities of $\sim 0.1c$. These velocities are about 6 and 3.5 times higher than those at the outlet and inlet sections of the choke throat, respectively. Due to the simplifications employed in the present model, the predicted pressure drop is much lower than that obtained experimentally for pure liquid with a velocity of $0.1c$ through the choke.

Keywords: Flows in ducts, channels, nozzles, and conduits; Flow control; computational methods in fluid dynamics; applied fluid mechanics.

En este trabajo se describen los resultados de la simulación numérica del flujo de un líquido viscoso a través de un estrangulador de dimensiones reales usando el método de Hidrodinámica de Partículas Suavizadas (SPH). El estudio de dichos flujos tiene aplicación directa en la industria del petróleo dado que es de uso común en los campos petroleros hacer fluir mezclas de gas y líquido a través de estranguladores con el objeto de controlar las tasas de flujo y protejer los equipos de superficie de eventuales fluctuaciones de presión. Se supone para este cálculo que el flujo es isotérmico con una velocidad del sonido c de 2.0×10^4 cm s^{-1} y un coeficiente de viscosidad cinemática constante ($\nu = 0.01$ cm^2 s^{-1}). Los resultados predicen una caída de presión del 13% a través del estrangulador cuando el flujo alcanza un estado estacionario. El flujo a lo largo del estrangulador permanece subsónico con velocidades del orden de $0.1c$. Estas velocidades son aproximadamente 6 y 3.5 veces mayores que los valores correspondientes en la entrada y salida del estrangulador, respectivamente. Debido a las simplificaciones usadas en este modelo, la caída de presión que se obtiene es mucho menor que el valor medido experimentalmente para un líquido con velocidad de $0.1c$ a través del estrangulador.

Descriptores: Flujo en ductos, canales, inyectores, y tubos; control de flujo; métodos computacionales en dinámica de fluidos; mecánica de fluidos aplicada.

PACS: 47.60.+i; 47.62.+q; 47.11.+j; 47.85.-g

1. Introduction

In the last few decades, the flow of single- and two-phase fluids through restrictors has been the subject of extensive investigation due to its practical significance. In the oil industry, wellhead chokes are installed to limit the flow rates and prevent the formation of unusual pressure fluctuations which could damage the surface equipment. Flow limitation is also useful for addressing a number of safety related design concerns of gas handling systems. For instance, applications where a restrictive flow orifice device would enhance the system's safety may include: limiting the accidental release of hazardous gas and restricting flow from large volume sources as in the case of oil production wells.

Referring to the oil industry, most studies of choked flows are in the form of empirical correlations based on experimental measurements and simplified theoretical models [1-3]. While existing correlations are useful for predicting the dependence of the pressure drop through the choke, measured as the ratio of the outlet to the inlet pressure, on velocity from available experimental data, they usually fail when extrapolated to new conditions. Therefore numerical hydrodynamics simulations aimed at predicting the flow properties through restrictors, such as chokes, orifices and control valves, are of fundamental importance from both a theoretical and practical point of view.

Here we describe the results of a two-dimensional simulation of fluid flow through a wellhead choke device of dimensions similar to those operating in real production tubing. The calculation is made using a Smoothed Particle Hydrodynamics (SPH) based scheme which works equally well for compressible and incompressible flows [4]. In particular, the method has been found to perform well when applied to unsteady plane Poiseuille and Hagen-Poiseuille flows at very low ($Re \ll 1$) and moderate ($Re > 1$) Reynolds numbers [4], and to the formation of a liquid drop for a van der Waals fluid in two dimensions [5]. Recent calculations of flow through wellhead chokes with the same method are also reported in reference [6]. Here we discuss the results for one of these models and compare them with the predictions of existing correlations [1].

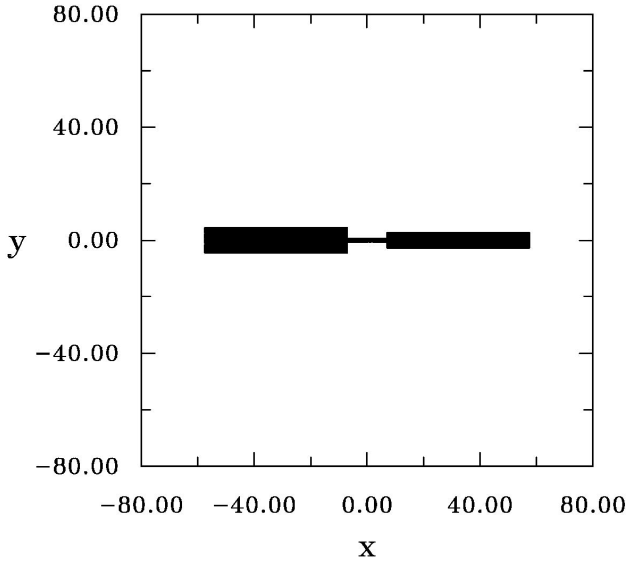

FIGURE 1. Geometry of the wellhead choke model. The flow is in the direction of increasing x and the numbers on the box sides are in centimeters.

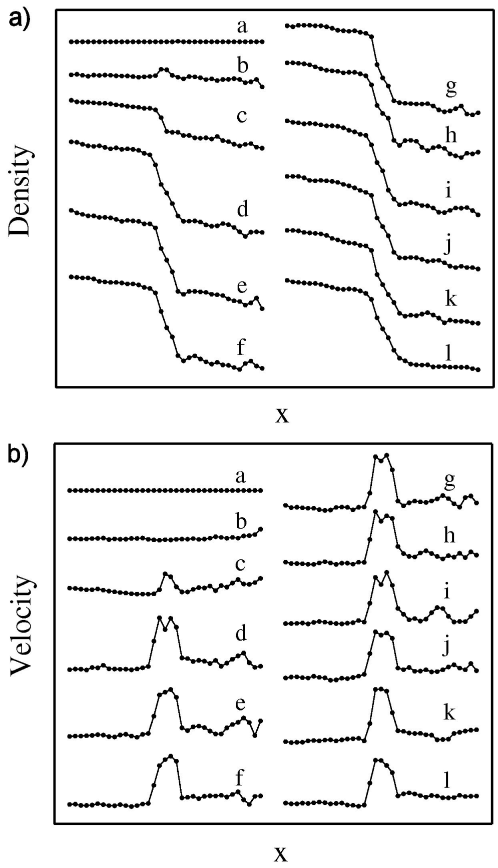

FIGURE 2. Mean density (left panel) and velocity (right panel) profiles along the tube length. The density, velocity and length are in cgs units. A sequence of times is given: 0 s (a), 0.0025 s (b), 0.010 s (c), 0.0325 s (d), 0.070 s (e), 0.085 s (f), 0.1075 s (g), 0.175 s (h), 0.2025 s (i), 0.230 s (j), 0.275 s (k), and 0.295 s (l). An approximate stationary pattern is reached in (d).

2. Wellhead choke model

The geometry of the choke device model used for the present simulation is shown in Fig. 1. The model consists of an upstream and downstream pipe with stationary walls and radii ≈ 4.45 cm and ≈ 2.67 cm respectively, connected by a choke throat of half-length ≈ 6.82 cm and radius ≈ 0.593 cm. The dimensions of the choke throat are designed in accordance with those of typical restrictors employed in real production wells. We choose the (x, y)-plane to represent the flow and the x-axis as the direction of the main flow. In this way, only gradients in the x- and y-directions are considered. We further assume that there is no significant heat exchange between the fluid and the walls of the restrictor so that the pressure and density are related by an isothermal equation of state of the form $p = c^2\rho$, where the speed of sound is fixed at the value of 2.0×10^4 cm s^{-1}. The coefficient of kinematic viscosity is assumed to be constant and equal to $\nu = 0.01$ cm^2 s^{-1}.

With the above assumptions, the flow is completely described by solving the continuity and momentum equations for a viscous fluid. The SPH formalism used for solving these equations is described in full in references [4-6]. Here we shall only briefly comment on salient features of the method and refer to [4] for details. SPH is a fully Lagrangian particle method based on interpolation theory. The conservation laws of continuum dynamics are transformed into integral equations and the integrals are evaluated computationally as sums over neighboring particles by means of an interpolating function, which gives the kernel estimate of the field variables at a point. For instance, the density at each particle's location is simply assigned by

$$\rho_a = \sum_{b=1}^{N} m_b W_{ab}, \tag{1}$$

where the subscripts denote particle labels, m_b is the mass of particle b, $W_{ab} = W(|\mathbf{r}_a - \mathbf{r}_b|, h)$ is a spherically symmetric interpolating kernel (here we adopt a cubic spline kernel [7]), h is the parameter of the kernel or smoothing length which determines the spatial resolution, and the sum is taken over N neighboring particles within a circle of radius $2h$. With the use of Eq. (1), the SPH representation of the momentum equation must be written in symmetrized form to guarantee variational consistency [4,8]. Here, the flow within the choke is represented by a total number of 8785 fluid particles, initially at rest and arranged in a uniformly Cartesian array. With this choice, the interparticle distance is ≈ 0.296 cm along the x- and y-axes, giving an initial $h \approx 0.317$ cm.

At the entrance of the upstream pipe, the inlet flow is modelled using a Poiseuille velocity profile given by $v_x = v_{\text{inlet}}(t)(1 - y^2/R^2)$, where t is time, $R \approx 4.45$ cm and $v_{\text{inlet}}(t) = tv_0/\tau$ for $t \leq \tau$ and $v_{\text{inlet}}(t) = v_0$ otherwise, with $\tau = 0.1$ s and $v_0 = 500$ cm s^{-1}. The inlet density is always taken to be 1.0 g cm^{-3}. At the exit of the downstream pipe, the streamwise gradients of the density and velocity are prescribed as equal to zero at the oulet. The presence of the solid walls is modelled through the use of exterior image particles as described in references [4,6,9].

3. Results and discussion

The results of the model evolution are shown in Fig. 2, where the mean density (left panel) and velocity (right panel) profiles across the full length of the tube are displayed for a sequence of discrete times from $t = 0$ (curve a) to $t = 0.295$ s (curve l). The filled dots refer to average values of the density and velocity taken over consecutive tube sections of area $2R\Delta x$, where $\Delta x \approx 3.39$ cm and R may be either the upstream pipe, downstream pipe or choke throat radius.

A flow sets in rapidly across the tube as shown by curve c at 0.0325 s, which pushes the particles downstream making some of them leave the system. At this time a peak in the velocity profile and a small density drop are already present within the choke throat as a result of the much smaller cross-sectional area available for the flow there. Slightly later, at 0.070 s (curve d), a stationary pattern is established which is approximately maintained for the remainder of the evolution (see curves e-l). The stationary state is characterized by a well-marked drop in the mean density within the choke throat, corresponding to an average density ratio of ≈ 0.88. The mean flow velocity increases steeply at the entrance of the choke throat, reaching maximum values in the range 0.098–$0.113c$ across it. At the choke exit, the velocity decreases discontinuously to downstream values that are factors ~ 1.7 times higher than those in the upstream part. Because of the isothermal equation of state, the density drop is equivalent to a pressure drop. The discontinuous behavior of the density and velocity across the choke induces fluctuations in the flow that propagate downstream until they eventually leave the system.

Our results apply to the subcritical flow of an isothermal gas through wellhead chokes. However, a direct comparison with the correlations reported by Fortunati [1] is not possible because of the simplified equation of state employed for the present study. In particular, for pure liquid with a sound speed of 2.0×10^4 cm s^{-1}, we get $0.1c = 2.0 \times 10^3$ cm s^{-1}. For this value, the experimental curve displaying the dependence of the velocity through the choke on the ratio of the outlet to the inlet pressure derived by Fortunati [1] (see his Fig. 2) predicts a pressure ratio of ≈ 0.23, which is much lower than the average ratio of ≈ 0.88 predicted by the present model. This latter value is, however, consistent with the experimental ratio expected for a gas-liquid mixture with gas concentration relative to the mixture of $\beta = 0.4$. A better fit with the experimental data will certainly requiere using a more realistic equation of state, extending the model to three-space dimensions, and following the flow through a wellhead choke with a circular cross-section.

1. F. Fortunati, in The SPE-European Spring Meeting 1972 of the Society of Petroleum Engineering of AIME, Amsterdam, The Netherlands (1972) 1.
2. F.E. Ashford and P.E. Pierce, *J. Petrol. Tech.* (1975) 1145.
3. T.K. Perkins, *SPE Drilling & Completion* (1993) 271.
4. L.Di G. Sigalotti, J. Klapp, E. Sira, Y. Meleán, and A. Hasmy, *J. Comp. Phys.* **191** (2003) 622.
5. Y. Meleán, L.Di G. Sigalotti, and A. Hasmy, *Comp. Phys. Comm.* (2003) in press.
6. J. Klapp, L.Di G. Sigalotti, R. Gabbasov, and E. Sira, *Computers & Fluids* (2003) submitted.
7. J.J. Monaghan and J.C. Lattanzio, *Astron. Astrophys.* **149** (1985) 135.
8. J. Bonet and T.-S.L. Lok, *Comp. Meth. Appl. Mech. Eng.* **180** (1999) 97.
9. H. Takeda, S.M. Miyama, and M. Sekiya, *Prog. Theoret. Phys.* **92(5)** (1994) 939.