Universality of the nuclear matrix element governing the mass sector of the $0\nu\beta\beta$ decay

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We discuss some features of the nuclear structure elements participant in the calculation of the half-life of the neutrinoless double beta decay. Particularly, we discuss the relationship between the matrix elements governing the two neutrino and zero neutrino modes, to study the degree of correspondence between them.

Keywords: Double beta decay; matrix elements; universality.

En esta presentación se discuten diversos aspectos relacionados con la determinación de la masa del neutrino por medio de los límites establecidos para el decaimiento beta doble sin emisión de neutrinos. En particular, se discute la posibilidad de eliminar la dependencia con factores de estructura nuclear de los elementos de matriz involucrados mediante el ajuste de la vida media del modo con emisión de neutrinos.

Descriptores: Decaimiento beta doble; elementos de matriz; universalidad.

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1. Introduction

The neutrino is, perhaps, the most frequently found particle in nuclear processes. Its role is unique, both in participating nuclear single-β decay transitions and in most rare nuclear processes, like double-β decay transitions. The present understanding about the neutrino is based on the oscillation data, extracted from SNO and SK results, as well as from limits extracted from CHOOZ, Kamland and WMAP measurements [1]. The details about the analysis of neutrino mixing angles and phases can be found in [2,3]. The oscillation data show, without ambiguities, that the neutrino is a massive particle. Consequently, the phenomena of neutrino oscillations (between mass eigenstates), clearly demonstrates the existence of physics beyond the Standard Model of weak interactions. The scale (and extent) of the extension depends largely on the determination of the absolute mass scale, something which may eventually be fixed if the neutrinoless double-beta decay process is observed [4,5].

The simultaneous consideration of

a) the nuclear structure aspects, needed to describe the physics of the nuclear double-beta decay, and

b) the neutrino models, needed to specify the details of the adopted mass-mixing mechanism,

is the main ingredient in dealing with the determination of neutrino properties. The interplay between model dependent inputs and experimental results is thus dramatically illustrated in neutrino-nuclear physics, since to extract the information about the neutrino mass from the experimentally determined limits on the non-observability of neutrinoless double-beta decay process one needs also to calculate a set of nuclear matrix elements [4]. This is not an easy task, since the dependence of actual values of the involved nuclear matrix elements upon model parameters may be crucial. The case of the matrix elements governing the two-neutrino double-beta decay mode has been presented elsewhere [4]. In this talk we shall discuss the question of the nuclear structure effects on the matrix elements governing the mass sector of the neutrinoless double beta decay, and, particularly, the notorious g_{pp}-dependence [6, 7], following the point of view advanced in [8], where it is argued that the effective parameters appearing in the nuclear matrix elements of the zero-neutrino mode may be fixed from the adjustment to the observed two-neutrino mode. Also, we shall comment on the possible universality (if any) of the matrix elements.

2. Some useful expressions

For the sake of completeness, and hopefully for the benefit of the reader, we shall present briefly the necessary theoretical background.

2.1. Nuclear Structure Sector

The two-neutrino double-beta decay mode $(2\nu\beta\beta)$, a process which is allowed by the selection rules of the standard model of electro-weak interactions, consists of an uncorrelated pair of virtual, subsequent, single $\beta^{(\mp)}$ decays connecting a (A, N, Z) nucleus with a $(A, N \mp 2, Z \pm 2)$ one. It is not dependent on neutrino properties and its is severely limited by lepton-phase space limitations. It exists only because of the presence of pairing interactions in nuclei, i.e: the ground state energy of a double-odd mass nucleus with

$(A, N \mp 1, Z \pm 1)$ is higher than the ground state energies of the initial (A, N, Z) and final $(A, N \mp 2, Z \pm 2)$ nuclei. The half-life of the $2\nu\beta\beta$ decay is given by the expression [4]

$$\begin{aligned}\left[t^{2\nu}_{1/2}(J_f)\right]^{-1} &= \frac{(g_A G_F \cos\theta_C)^4}{96 \ln 2\pi^7 m_e^2} \\ &\times |\sum_m \langle J_f^+ || \sum_i \sigma(i)\tau^{\pm} || 1_m^+\rangle \langle 1_m^+ || \sum_i \sigma(i)\tau^{\pm} || 0_{gs}^+\rangle|^2 \\ &\times \int F_0(Z_f, \epsilon_2) F_0(Z_f, \epsilon_2) D(K, L) \omega_1^2 \omega_2^2 p_1 p_2 \epsilon_1 \epsilon_2 \delta \\ &\times (\epsilon_1 + \epsilon_2 + \omega_1 + \omega_2 + E_f - M_i) d\omega_1 d\omega_2 d\epsilon_1 d\epsilon_2 \quad (1)\end{aligned}$$

As it was mentioned before, the final leptonic state has four leptons (two neutrinos and two electrons) and there is not a helicity requirement between the first $(e_1, \bar{\nu}_1)$ and second $(e_2, \bar{\nu}_2)$ vertices.

The zero-neutrino mode $(0\nu\beta\beta)$ mode is a process where the virtual decays are connected, i.e: the anti-neutrino emitted in the first vertex is absorbed as a neutrino in the second vertex. It is not allowed in the Standard Model, where neutrinos are massless, purely left-handed Dirac particles, because of the helicity mismatch. If neutrinos are massive, and / or if the weak interaction has left-right terms, the processes is allowed and it will reveal the nature of the neutrino as a Majorana particle.

If we focus our attention on the mass sector of the half-life we obtain [4]

$$\begin{aligned}\left[t^{2\nu}_{1/2}(J_f)\right]^{-1} &= C^{(0\nu)}_{mm} \frac{\langle m_\nu \rangle^2}{m_e^2} C^{(0\nu)}_{mm} \\ &= G_1^{(0\nu)} \left[(M^{(0\nu)}_{GT})(1 - \chi_F)\right]^2 \quad (2)\end{aligned}$$

where

$$\chi_F = \frac{M_F^{(0\nu)}}{M_{GT}^{(0\nu)}} \quad (3)$$

is the ratio between the matrix elements of the double Fermi and Gamow-Teller operators:

$$\begin{aligned}M^{(0\nu)}_{GT} &= \sum_a \langle 0_F^+ || h_+(r_{mn}, E_a) \sigma_n . \sigma_n || 0_I^+ \rangle \\ M^{(0\nu)}_{F} &= \sum_a \langle 0_F^+ || h_+(r_{mn}, E_a) || 0_I^+ \rangle \quad (4)\end{aligned}$$

To calculate these matrix elements one needs to compute the nuclear charge density

$$\rho = \langle J^\pi, n \mid (Y_k \sigma)_{\lambda,\mu} f_k(r) \tau^- \mid 0^+_{I(F)} \rangle \quad (5)$$

for the multipole operators resulting from the partial wave expansion of the neutrino potential $h_+(r_{mn}, E_a)$. The nuclear states participant in the decay path are calculated in the quasiparticle random phase approximation [7], which consists of the diagonalization of the residual two-quasiparticle interactions in the configuration space which includes zero and four quasiparticles in the vacuum, namely

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \Omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} \quad (6)$$

where

$$\begin{aligned}A_{ab} &= E_a \delta_{ab} - g_{pp} G(ab, J) - g_{ph} F(ab, J) \\ B_{ab} &= g_{pp} G(ab, J) - g_{ph} F(ab, J) \quad (7)\end{aligned}$$

The parameters of the model are the coupling strenghts g_{ph}, which may be fixed by the energetics of giant resonances (although few multipoles are known) and g_{pp}, which is unknown, except for symmetry considerations [6, 7].

The model dependence may be minimized by adopting either one of the following procedures: a)g_{pp} fixed by single-beta decay data, or b)g_{pp} fixed by $2\nu\beta\beta$ decay data. We shall return to the discussion of both procedures later on.

2.2. Neutrino Sector

The relation between the flavor and mass neutrinos eigenstates is given by the mixing-matrix (only for light-mass neutrinos and assuming CP invariance)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \otimes \begin{pmatrix} \nu_1 \\ \nu_1 \\ \nu_3 \end{pmatrix} \quad (8)$$

The matrix elements of the mixing matrix U are determined from the oscillation data (SNO,SK,K2K,Kamland,CHOOZ) [1], while the average mass scale may be fixed from the limits on neutrino densities determined by WMAP [1].

$$U = \begin{pmatrix} 2\sqrt{\frac{2}{11}} & \sqrt{\frac{3}{11}} & 0 \\ -\sqrt{\frac{3}{22}} & \frac{2}{\sqrt{11}} & \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{22}} & -\frac{2}{\sqrt{11}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (9)$$

Table I shows a compilation of the adopted values extracted from the experiments quoted in [1]. Concerning the mass-eigenvalues, the following cases, known as *mass hierarchies* may be present:

$$m_1 = f m_2, \qquad m_2 = g m_3, \quad (10)$$

$(m_1 \approx m_2 < m_3$, Normal)

$$m_1 = f m_2, \qquad m_3 = g m_1, \quad (11)$$

$(m_1 \approx m_2 > m_3$, Inverse). These relationships are used to write the equations

$$\frac{1}{1 - g^2} - \frac{r}{1 - f^2} = 1, \quad (12)$$

(for the normal hierarchy)

$$\frac{r}{1 - f^2} - \frac{1}{1 - g^2} = r, \quad (13)$$

TABLE I. Current limits on oscillation parameters.

$\delta m^2_{12} = \delta m^2_{\rm solar}$	5×10^{-5} eV^2 $\rightarrow 1.1 \times 10^{-4}$ eV^2
$\delta m^2_{23} = \delta m^2_{\rm atm}$	10^{-3} eV^2 $\rightarrow 5 \times 10^{-3}$ eV^2
$\sin^2 2\theta_{\rm solar}$	≈ 0.86
$\sin^2 2\theta_{\rm atm}$	≈ 1.0
Ω_ν	< 0.71 eV

(for the inverse mass hierarchy), where

$$r = \frac{\delta m^2}{\Delta m^2}, \tag{14}$$

is the ratio between the small (solar) and large (atmospheric) mass differences [1]. The variation of f and g under the constraints given by the above equations determines the region of allowed values of m_i ($i = 1, 2, 3$).

Table II shows the value of the average mass value $\langle m_\nu \rangle$

$$\langle m_\nu \rangle = \sum_{i=1}^{3} m_i U^2_{ei} = m_1 U^2_{e1} \pm m_2 U^2_{e2}, \tag{15}$$

relevant for the nuclear double-beta decay. A detailed analysis of these results is presented in [3]. The fact that nearly vanishing average masses may be obtained from definite (and very different scenarios) is one of the main results of the present calculations. It means that even after the massive character of the neutrino has been conclusively established by the oscillation data, the positive signals of neutrinoless double beta decay may be hampered by the cancellation of $\langle m_\nu \rangle$, which is operative if CP is conserved. This is an important result, considering the planned sensitivity of future double-beta decay experiments [9].

2.3. Joint Nuclear Structure and Neutrino Data

The experimental limits on the half-life of the neutrinoless double beta decay are fixed by the analysis of events (emission of two-electrons with a sum energy equals to the Q-value of the decay) and it is expressed by the ratio

$$F_N = t^{(0\nu)}_{1/2} C^{(0\nu)}_{mm} = (\langle m_\nu \rangle / m_e)^{-2}, \tag{16}$$

TABLE II. Average electron-neutrino mass $\langle m_\nu \rangle$. The results shown in the column labelled $U(a)$ have been obtained by using the matrix U extracted from the best fit of the oscillation data, the results labelled $U(b)$ have been obtained by using a maximal mixing between mass eigensates.

5Hierarchy		$\langle m_\nu \rangle$	U(a)	U(b)
Normal	$(m_1 = 0)$	$\langle m_\nu \rangle_-$	-0.010	-0.012
		$\langle m_\nu \rangle_+$	0.011	0.012
Inverse	$(m_3 = 0)$	$\langle m_\nu \rangle_-$	0.105	0.087
		$\langle m_\nu \rangle_+$	0.234	0.235
Degenerate	(extreme)	$\langle m_\nu \rangle_-$	0.107	0.088
		$\langle m_\nu \rangle_+$	0.237	0.237

Clearly, if one aims at the extraction of the value of the average electron-neutrino mass, $(\langle m_\nu \rangle / m_e)^{-2}$, from the data, $t^{(0\nu)}_{1/2}$, the nuclear structure factor $C^{(0\nu)}_{mm}$ must be known accurately enough. Table III shows actual F_N factors for the considered nuclear systems. The range of values shows the variation due to the use of different nuclear models.

The phase space factors used in the calculations are shown in Table IV, together with the matrix elements extracted from compilation of the published results and the ones of the present calculations [3, 10].

To illustrate the degree of dependence of the nuclear matrix elements we shall focuss on the case of the decay of ^{76}Ge.

TABLE III. Results of $0\nu\beta\beta$. The second column shows the values of the nuclear structure factor $C^{(0\nu)}_{mm}$ and the third one shows the product of it with the experimental half-life limit.

System	$C^{(0\nu)}_{mm}$	F_N
^{48}Ca	(1.55-4.91) 10^{-14} (a)	(1.47-4.66) 10^{8}
	(9.35-363) 10^{-15} (b)	(8.88-345) 10^{7}
^{76}Ge	(1.42-28.8) 10^{-14} (d)	(3.55-72.0) 10^{11}
^{82}Se	(9.38-43.3) 10^{-14} (d)	(2.53-11.7) 10^{9}
^{96}Zr	(9.48-428)10^{-15} (b)	(9.48-428) 10^{6}
^{100}Mo	(0.07-2490) 10^{-15} (b)	(0.38-13700) 10^{7}
^{116}Cd	(5.57-66.1) 10^{-14} (b)	(3.90-46.3) 10^{9}
^{124}Sn	(2.29-5.70) 10^{-13} (b)	(5.50-13.7) 10^{4}
^{128}Te	(1.71-33.6) 10^{-15} (b)	(1.47-28.9) 10^{8}
^{130}Te	(1.24-5.34) 10^{-13} (b)	1 (1.74-7.48) 10^{10}
^{136}Xe	(2.48-15.7) 10^{-14} (a,b)	(1.09-6.91) 10^{10}
^{150}Nd	(4.78-77.4) 10^{-13} (b,c)	(8.13-132) 10^{8}

TABLE IV. Phase-space factors $G^{(0\nu)}_1$ and calculated nuclear matrix elements M_{GT}. The values shown in the third column have been extracted from the literature and the ones shown in the fourth column are the results of the present calculations. The values of the average electron-neutrino mass are shown in the last column, in units of eV.

System	$G^{(0\nu)}_1 \times 10^{14}$	N.M.E.	N.M.E. (this work)	$\langle m_\nu \rangle$
^{48}Ca	6.43	1.08-2.38		8.70-19.0
^{76}Ge	0.63	2.98-4.33	3.33	0.30-0.43
^{82}Se	2.73	2.53-3.98	3.44	4.73-7.44
^{96}Zr	5.70	2.74	3.55	19.1-24.7
^{100}Mo	11.30	0.77-4.67	2.97	1.38-8.42
^{116}Cd	4.68	1.09-3.46	3.75	2.37-8.18
^{128}Te	0.16	2.51-4.58		9.51-17.4
^{130}Te	4.14	2.10-3.59	3.49	1.87-3.20
^{136}Xe	4.37	1.61-1.90	4.64	0.79-2.29

3. Results

In view of the large spreading exhibited by the results (see Table IV) it is, therefore, desirable to estimate the degree of uncertainty introduced by model assumptions. We have considered two aspects, namely:

a) the suppression of the matrix elements governing the $2\nu\beta\beta$ mode, and,

b) the relevance of this suppression for the $0\nu\beta\beta$ mode.

Figure 1 shows the dependence of the double Gamow Teller matrix elements, for the case of the decay of ^{76}Ge, as a function of g_{pp}. The horizontal lines indicate the range of values which are allowed by the experimental results. Since the sign of the matrix element cannot be determined by the data, both positive and negative matrix elements should be considered. Then, we may adopt any of these values for g_{pp} to perform the calculation of the matrix element governing the $0\nu\beta\beta$ decay mode. It has been argued that this may be a procedure to eliminate the degree of freedom associated to this parameter

The results shown in Fig. 2 correspond to calculations of the matrix element of the $0\nu\beta\beta$ decay which have been performed by adopting values of g_{pp} inside the region determined by the fit to the observed $2\nu\beta\beta$ mode (see Fig. 1).

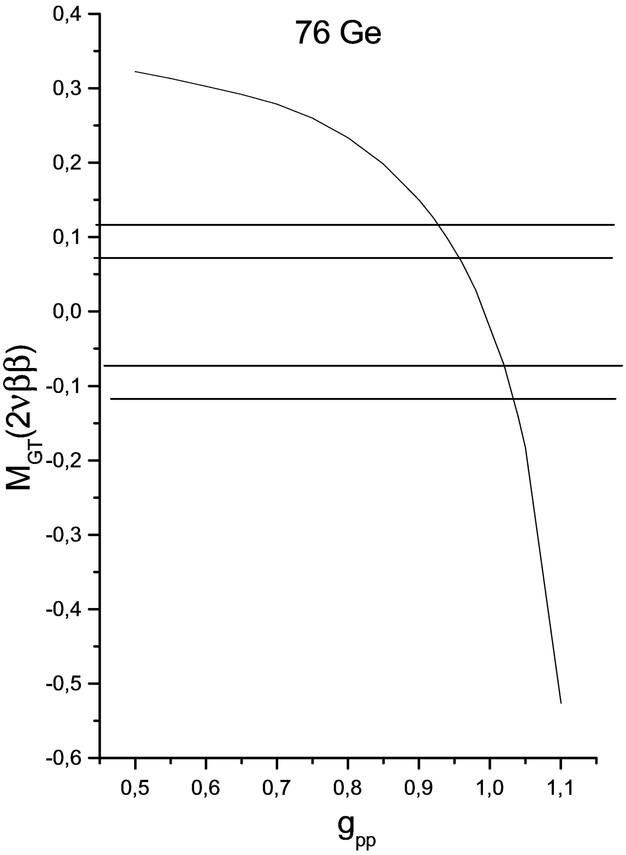

FIGURE 1. Matrix element for the $2\nu\beta\beta$ decay mode of ^{76}Ge.

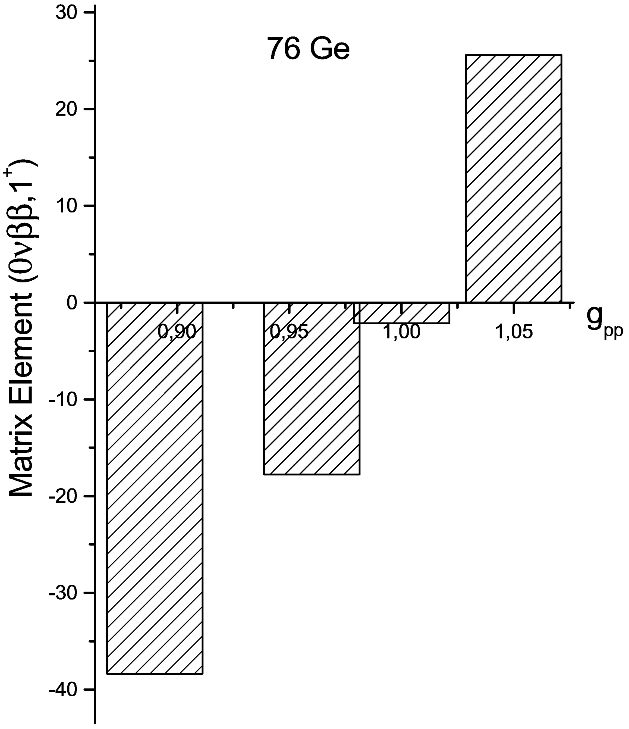

FIGURE 2. Contribution of the $J^\pi = 1^+$ multipole to the matrix element of the $0\nu\beta\beta$ decay mode of ^{76}Ge, for values of g_{pp} around the value which best reproduces the observed $2\nu\beta\beta$ mode.

The suppression induced by the particle-particle channel of the two-body interaction is rather evident. However, it is operative on one multipole ($J^\pi = 1^+$), only. Since in principle all multipoles are allowed, we may investigate the effect of this suppression relatively to other multipole contributions to the matrix element. The results are shown in Table V.

TABLE I. Contributions to the matrix element $M^{(0\nu)}_{\rm GT}$ as a function of g_{pp}. The third column shows the total (sum on all multipolarities) matrix element. The last column shows the dependence on g_{pp} of the contribution due to $J^\pi = 1^+$ states. The scaling factor $(m_{\rm e}R)^{-2}$ is included in the definition of the matrix element $M^{(0\nu)}_{\rm GT}$.

Case	$g_{\rm PP}$	$M^{(0\nu)}_{\rm GT}({\rm all})$	$M^{(0\nu)}_{\rm GT}(1^+)$
^{76}Ge	0.89	162.35	19.18
	0.96	148.31	8.89
	1.00	137.98	1.06
	1.05	120.39	-12.79
^{82}Se	0.98	114.83	12.23
	1.10	103.39	3.07
	1.17	95.16	-3.69
	1.23	86.70	-10.82
^{100}Mo	1.16	142.30	20.44
^{116}Cd	1.44	66.12	6.80
	1.50	62.77	4.37
	1.55	59.06	1.55
	1.58	56.01	-0.98

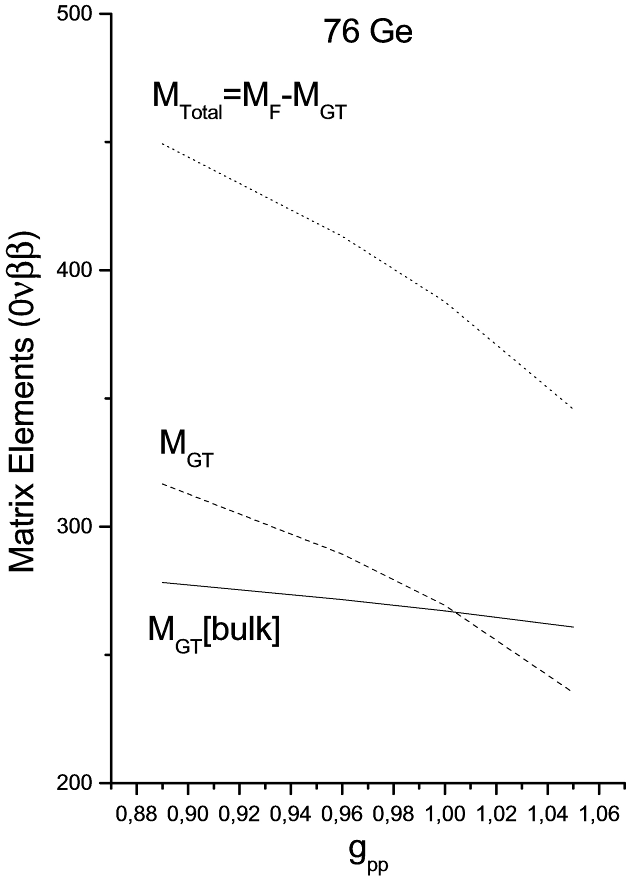

FIGURE 3. Dependence of the total matrix element due to Gamow-Teller and Fermi transitions, on the parameter g_{pp}.

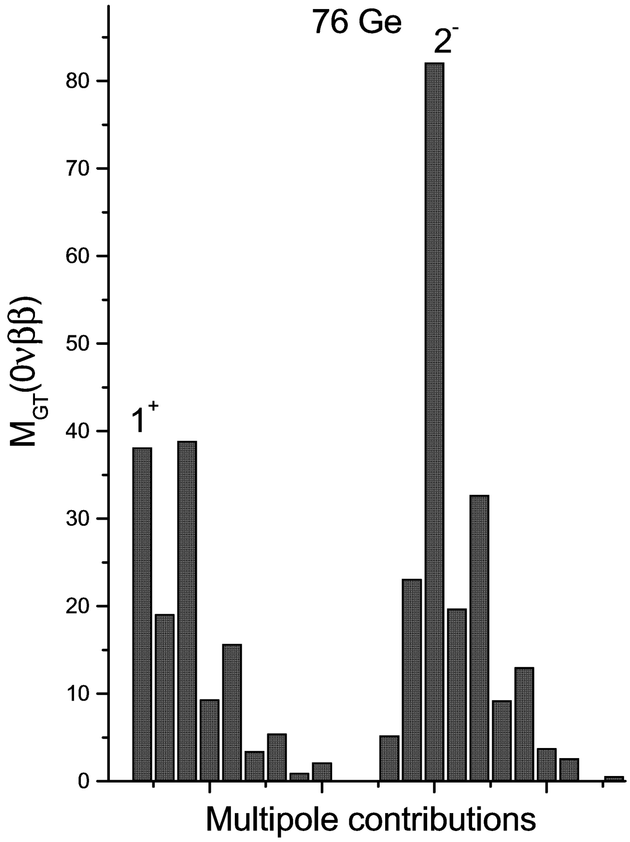

FIGURE 4. Multipole decomposition of the nuclear matrix element $M_{\rm GT}^{(0\nu)}$. The values shown in the figure have been obtained by adopting a g_{pp} value near suppression.

It is evident, from the results shown in Table V, that even if the contribution of the $J^{\pi} = 1^{+}$ multipole is completely suppressed it represents a minor effect on the total matrix element. Thus, it seems that the procedure of [8] may not necessarily lead to a phenomenological value of g_{pp}.

The fact that matrix elements of the neutrinoless double beta decay mode may be dominated by few multipole contributions to the charge density is illustrated by the results shown in Fig. 4. The calculations include multipole contributions of normal and abnormal parity, up to $10^{\pm}$. The most evident contribution is the one of the $J^{\pi} = 2^{-}$ multipole.

The previous results, for the decay of ^{76}Ge, show a tendency which is not restricted to this case. We have found the same features in the other cases. The details of the calculations and the discussion of the results are presented in [10].

4. Conclusions

We may summarize in the following:

a) The matrix elements governing the mass sector of the $0\nu\beta\beta$ decay do not depend much on g_{pp}, rather, they are dominated by few multipole contributions, which are not so sensitive to the parametrization of the two-body interaction in the particle-particle channel. This feature may be of some importance in dealing with the experimental possibilities of measuring single beta decay transitions of these few multipolarities;

b) The matrix elements of the $2\nu\beta\beta$ and $0\nu\beta\beta$ decays are not related, therefore the adjustment of the nuclear parameters by the fit to the observed half-life of the $2\nu\beta\beta$ does not necessarily guarantees that the theoretical uncertainties in the calculation of the $0\nu\beta\beta$ are reduced.

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