A simple method for changing the state of polarization from elliptical into circular

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Changes of polarization occur as a consequence of the interaction of light and the various optical elements through which it passes. A circularly polarized light beam may change its state to slightly elliptically polarized for many reasons. To correct this is not always easy but we show a very simple method for correcting circular polarization that has changed slightly into elliptic polarization. In this paper we propose to restore the circular state of polarization of an elliptically polarized light beam back to circular by means of a glass plate properly oriented while polarization is being measured. The basic idea is to modulate the transmittances of the electric field in both the major and minor axes of the ellipse of polarization. It is done by means of glass plates at non-normal incidence. Experimental results are consistent with theory.

Keywords: Interference; polarization; polarizers; interferometers; interferometry.

Al pasar la luz a través de diferentes elementos ópticos, ocurren cambios de polarización. Un haz circularmente polarizado puede cambiar ligeramente a elípticamente polarizado por muchas razones. Corregir esto puede ser complicado. Aquí mostramos un método sencillo para recuperar la polarización circular del haz de luz cuando ha sufrido pequeños cambios de polarización. El método consiste en atravesar una placa de vidrio convenientemente orientada mientras medimos la polarización del haz. La idea básica es modular las transmitancias del campo eléctrico en los dos ejes de la elipse de polarización. Esto se logra con una placa de vidrio a incidencia no normal. Los resultados experimentales son consistentes con la teoría.

Descriptores: Interferencia, polarización; polarizadores; interferómetros; interferometría.

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1. Introduction

When designing an experiment, it is important to preserve the state of polarization, but sometimes it is not possible to avoid complications. The interaction with optical elements changes the state of polarization to a certain degree. Although most photodetectors are almost insensitive to polarization, a change in the state of polarization could be important in interferometry and other fields. The methods we show here are based on the fact that glass windows modify the state of polarization of a light beam passing through them, as demonstrated by Holmes in his study on rotary compensators [1].

A problem indirectly related to polarization effects similar to those described here is considered by Schechner [2], who considered a problem of image recovery where two white light images overlap. This problem arises when a polychromatic scene is imaged through a glass plate. The main scene, illuminated by white light, is transmitted, but a secondary overlapping scene is reflected by the glass plate. In order to eliminate the spurious image or to separate them, the glass plate reflectance is calculated for s and p polarizations. Finally, in a digital iterative procedure, the magnitude of the cross correlation between the two images is calculated and used to eliminate the spurious image. So the cross correlation between them is zero. Considering these aspects allowed the authors to construct better images.

To control the state of polarization of light, many light phenomena can be used. For example, it is possible to introduce a variable phase delay by means of a Soleil compensator or a modified Babinet compensator, the Faraday Effect modulator [3], the Kerr [4] and Pockels cells [5], wave plate retarders, etc. Zhuang [6], shows a method for changing any state of polarization of light from one arbitrary state to another. He uses three liquid crystal cells to vary the state of polarization along three lines on the Poincaré sphere by modulating the birefringence of the liquid crystal cells.

In this work, we use the change of the state of polarization of a light beam that has passed through a glass plate [1]. For a circularly polarized light beam that has slightly changed its state to elliptically polarized, we propose to restore the circular state of polarization by means of a glass plate properly oriented while polarization is being measured. It can be done under certain considerations by utilizing the difference in the transmittances for s and p polarization components of light modifying the major and minor axes of the ellipse until it becomes a circle. The analysis is made for monochromatic, spatially and temporally coherent light.

We must take into account the interference between the main transmitted beam and those transmitted after multiple reflections. This phenomenon is used for measuring angles by fringe counting [7], in which a light beam is directed onto an oblique glass plate that partially reflects the light at both the first and second surface by using a Murty lateral shearing interferometer. The glass plate is slowly rotated, producing interference oscillations in the intensity of the reflected light.

By measuring these changes in intensity, it is possible to calculate the change in angle of incidence.

Other work related to polarization compensation was reported by Azzam [8, 9]. He proposes using a thin layer on a glass substrate in order to control the ratio of s and p transmittances. By selecting of an appropriate material for the thin layer, depending on the substrate, he made a beam-splitter that does not change the state of polarization in either the reflected or the transmitted beam. Nevertheless, the ratio of transmittance to reflectance is 50-50% for only one angle of incidence. He also designed a beam splitter that introduces a retardation of half a wave in the reflected beam. In both cases, the thin layer could be thought of as a Transmittance Modulator (TM), except that it is immersed in different media.

We begin with a brief review of the theory behind the proposed method for changing polarization. Then we will describe two methods for preserving a circular polarization state.

2. Theory

For the polarization of a TE (Transverse Electric), a monochromatic and uniform wave can be represented in a right hand coordinate system x, y, z, as in Fig. 1. Here, the electric wave $\mathbf{E}(z,t)$ moves along the z axis, normal to the page and pointing at the reader. The electric wave $\mathbf{E}(z,t)$ rotates in the x, y surface describing an ellipse whose major and minor axes are in the x' and y' directions respectively. The orientation of the ellipse in the x, y plane is defined by α, the angle between the major axis of the ellipse and the positive direction of the x axis.

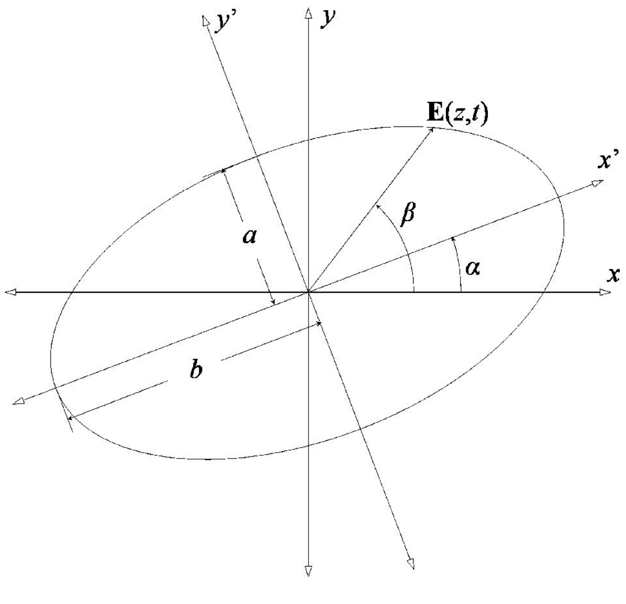

FIGURE 1. Electric wave elliptically polarized.

Mathematically, the electric wave $\mathbf{E}(z,t)$ can be represented as

$$\mathbf{E}(z,t) = [E_x \cos(\omega t - kz - \delta_x)]\hat{\mathbf{x}} + [E_y \cos(\omega t - kz - \delta_y)]\hat{\mathbf{y}}, \quad (1)$$

where the electric wave vector is the superposition of two orthogonal wave vectors, the first with amplitude E_x, oscillating in the direction of the unit vector $\hat{\mathbf{x}}$, and a second with amplitude E_y, oscillating in the direction of the unit vector $\hat{\mathbf{y}}$. Both waves have the same angular frequency ω, the same wave number $k = 2\pi/\lambda$ (where λ is the wave length), but can have different initial phases δ_x and δ_y. If we use a Jones representation for polarized light, we can write from Eq. (1)

$$\mathbf{E} = \begin{bmatrix} E_x\, e^{-j\delta_x} \\ E_y\, e^{-j\delta_y} \end{bmatrix}. \quad (2)$$

This representation allows us to see that any elliptical polarization can be represented either by differences between the initial phases δ_x and δ_y, the amplitudes E_x and E_y, or a combination of the two.

Under certain circumstances, an elliptical polarization can be transformed into a circular one by means of a transmittance modulator (TM) given as

$$\mathbf{T'} = \begin{bmatrix} t_{x'} & 0 \\ 0 & t_{y'} \end{bmatrix}, \quad (3)$$

where $t_{x'}$ is the transmittance in amplitude for the component of an electric wave oscillating in the x' direction, and $t_{y'}$ is the transmittance for the y' component.

When an elliptically polarized light beam passes through a transmittance modulator, the output electric wave as a function of x', y' is

$$\mathbf{E}_o = \begin{bmatrix} E_{ox'}\, e^{-j\delta_{ox'}} \\ E_{oy'}\, e^{-j\delta_{oy'}} \end{bmatrix} = \begin{bmatrix} t_{x'} & 0 \\ 0 & t_{y'} \end{bmatrix} \cdot \begin{bmatrix} E_{ix'}\, e^{-j\delta_{ix'}} \\ E_{iy'}\, e^{-j\delta_{iy'}} \end{bmatrix} = \begin{bmatrix} t_{x'} & 0 \\ 0 & t_{y'} \end{bmatrix} \cdot \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \cdot \begin{bmatrix} E_{ix}\, e^{-j\delta_{ix}} \\ E_{iy}\, e^{-j\delta_{iy}} \end{bmatrix}, \quad (4)$$

where $\delta_{ox'} = \delta_{ix'}$, $\delta_{oy'} = \delta_{iy'}$ and, since we suppose that **T'** does not have any phase term, $\delta_{ox'} - \delta_{oy'}$ must be equal to 90° in order to have circular polarization.

Equation (4) shows that the electric field as a function of x', y', z' is amplitude modulated as indicated by the Jones matrix **T'**. It is clear from Fig. 1 that, after a rotation is applied and expressed in the coordinate system x', y', z', to have circular polarization we require that $E_{ox'} = E_{oy'}$. So, with this method, the amplitudes $E_{ix'}$ and $E_{iy'}$ are both reduced in magnitude.

3. A First Solution

A first step toward making a TM like the one represented in Eq. (3) is to consider a single interface. From the Fresnel equations we have that, for non-normal incidence, the transmittance is different for the p and s planes of polarization:

$$t_p = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}, \quad (5)$$

and

$$t_s = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}, \quad (6)$$

where, n_i is the refractive index of the medium of the incident beam, n_t is the refractive index of the medium after the interface and θ_i and θ_t are the angles of incidence and refraction given by Snell's law. Figure 2 is a plot of t_s, t_p and its ratio versus the angle of incidence θ_i for an air-glass interface, say

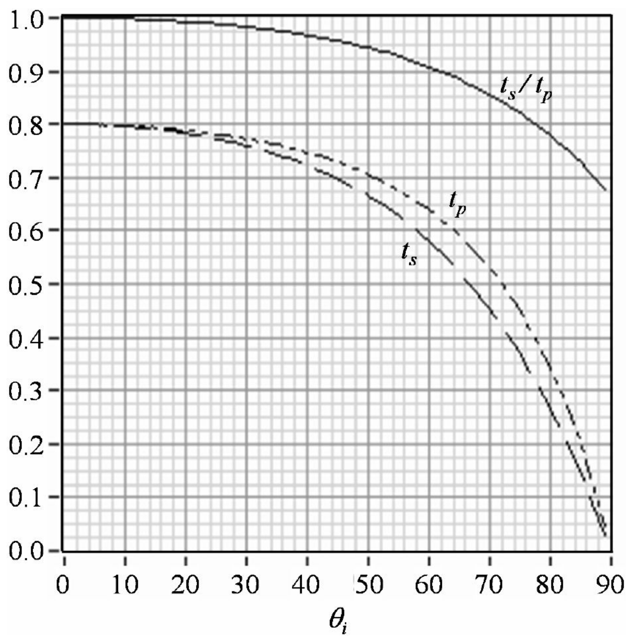

FIGURE 2. Amplitude transmittance coefficients for an air-glass interface ($n_i = n_a = 1$ and $n_t = n = 1.5$).

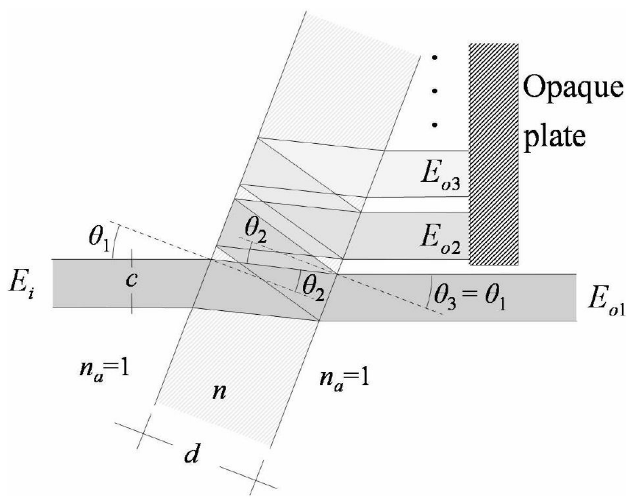

FIGURE 3. Transmission through a glass plate ($n_a = 1$ and $n = 1.5$).

the first surface in Fig. 3. The refractive index of air is $n_i = n_a = 1$, the refractive index for the glass is $n_t = n = 1.5, \theta_i = \theta_1$ and $\theta_t = \theta_2$.

For example, in order to change the ellipticity, $e = t_p/t_s$, from any arbitrary value, say $e = 0.89$, to $e = 1$, we must insert the air-glass interface in the beam with an angle of incidence $\theta_i = 64°$. This gives us $t_{x'} = t_s = 0.535$ and $t_{y'} = t_p = 0.601$, that is, $t_s/t_p = 0.89$.

Now we proceed to a solution in the simple case in which we use a thick plane-parallel glass plate to alter the polarization of a laser beam. The glass plate must be thick enough so that the main transmitted laser beam, E_{o1} in Fig. 3, can be isolated from the multiple reflection beams. The condition for this separation is

$$\frac{c}{\cos\theta_1} \leq 2d \tan\theta_2, \quad (7)$$

where, from Fig. 3, c is the beam diameter, θ_1 is the angle of incidence, θ_2 is the angle of refraction, and d is the glass thickness. The total transmittance for the main beam is the product of both transmittances calculated at the air-glass (t) and glass-air (t') interfaces. Both transmittances must be calculated for p and s by means of Eqs. (5) and (6).

4. Thin Plane-Parallel Glass Plate in a Wide Light Beam

Assume that a flat, infinitely wide, coherent wavefront strikes an oblique plane parallel glass plate as shown in Fig. 4. The glass plate, with thickness d and refractive index n, is immersed in air $n_a = 1$ and is inclined at an angle θ_1 in the y', z' plane, Fig. 1. If the glass plate is perfectly plane and parallel, then the angle of refraction θ_2 in the interface air-glass is also the angle of incidence in the interface glass-air, and consequently $\theta_3 = \theta_1$. Then, several rays from the incident wavefront contribute, after multiple internal reflections, to the light ray transmitted at point P,

$$\widetilde{E}_t = \sum_{m=0}^{\infty} E_{im}\, t\, r'^{\,2m}\, t'\, e^{-jm\delta}, \quad (8)$$

where E_{im} is the amplitude of the incident electric wave at a given point indexed by m (m goes from 0 to infinity), t and t' are the amplitude transmittance coefficients for the air-glass and glass-air interfaces respectively, r' is the internal reflectance coefficient, and $\delta = \delta_d + \delta_F$, is the relative phase delay δ_d, due to the optical path difference (OPD), in addition to the phase change δ_F, which occurs for internal reflection under certain conditions that will be shown later. We must remember that Eq. (8) must be solved for both p and s polarizations. The result is a measure of the interference between the directly transmitted ray and those transmitted after multiple reflections.

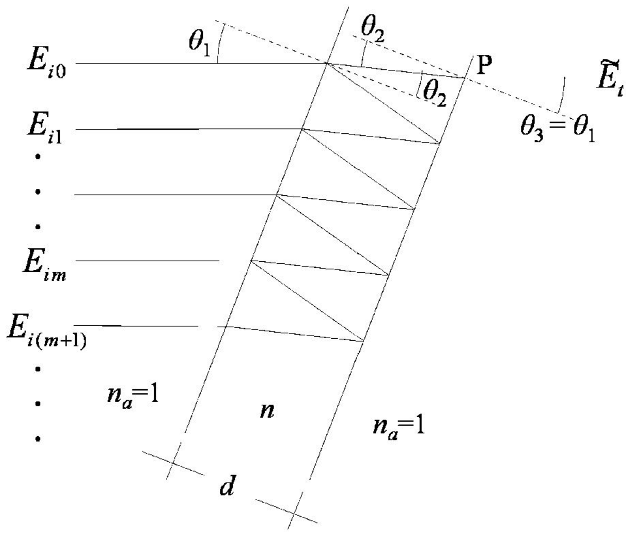

FIGURE 4. Several points from the incident wavefront contribute to the output wavefront at point P.

We must use Eqs. (5) and (6) to calculate t and t' in order to solve Eq. (8). This must be done for both interfaces and both polarizations s and p. To calculate r', we need the Fresnel amplitude reflection coefficients

$$r'_p = \frac{\cos\theta_2 - n\cos\theta_1}{n\cos\theta_1 + \cos\theta_2}, \tag{9}$$

and

$$r'_s = \frac{n\cos\theta_2 - \cos\theta_1}{n\cos\theta_2 + \cos\theta_1}, \tag{10}$$

both expressed only for internal reflection.

The OPD between the ray $E_{i(m+1)}$ and the ray E_{im} is given by

$$\delta_d = 2\, k\, n\, d\, \cos\theta_2. \tag{11}$$

The phase change due to internal reflection can be calculated with the help of the Fresnel equations [10]. From Fig. 4, we can see that the ray $E_{i(m+1)}$ suffers two more internal reflections than E_{im}, so that the total phase change for the p polarization is

$$\delta_{Fp} = \begin{cases} 2\pi & 0 < \theta_2 < \theta'_p \\ 0 & \theta'_p < \theta_2 < \theta_c \end{cases}, \tag{12}$$

where θ'_p is the internal polarizing angle [10] and θ_c occurs when $\theta_1 = \pi/2$. But a phase change of 2π is equivalent to zero due to coherence, and no phase change occurs for the s component. We can then set $\delta_F = 0$ in all cases, independently of polarization.

Finally, assuming that the incident electric field is uniform, that is, $E_{im} = E_{i(m+1)} = E_i$ for any m, from a geometric progression we can represent Eq. (8) as

$$\widetilde{E}_t = \frac{E_i\, t\, t'}{1 - r'^{\,2}\, e^{-j\delta}}, \tag{13}$$

or

$$\widetilde{E}_t = E_i\, t_t\, e^{-j\delta_t}, \tag{14}$$

where

$$t_t = \frac{t\, t'}{\sqrt{1 - 2\, r'^{\,2}\, \cos\delta_d + r'^4}}, \tag{15}$$

and

$$\delta_t = \tan^{-1} \frac{r'^{\,2} \sin\delta_d}{1 - r'^{\,2}\, \cos\delta_d}, \tag{16}$$

here δ_d is defined in Eq. (11). As expected, the total transmittance t_t changes with the angle of incidence θ_1. In addition, the $\cos\delta_d$ term in the denominator produces an oscillation in amplitude as θ_1 increases. Figure 5a shows the theoretical total transmittances t_{ts} and t_{tp} for a plate $n = 1.5239$, $d = 147\mu$ and $\lambda = 632.8$nm. These data were chosen equal to those from a real plate used later to experimentally demonstrate the method. From Eq. (11), we can see that the frequency of the oscillation depends on the thickness d, the wave number k, and the refraction index n, and it is modulated by $\cos\theta_2$. The maximum frequency occurs at approximately $\theta_1 = 48°$ and the minimum frequency when θ_1 is nearly 0° or 90°, in good agreement with Ref. 7 for a similar plate. Figure 5b shows a detail of this oscillation. Notice that there are many angles for which the plate is completely transparent, and the transmittance for both the s and p components

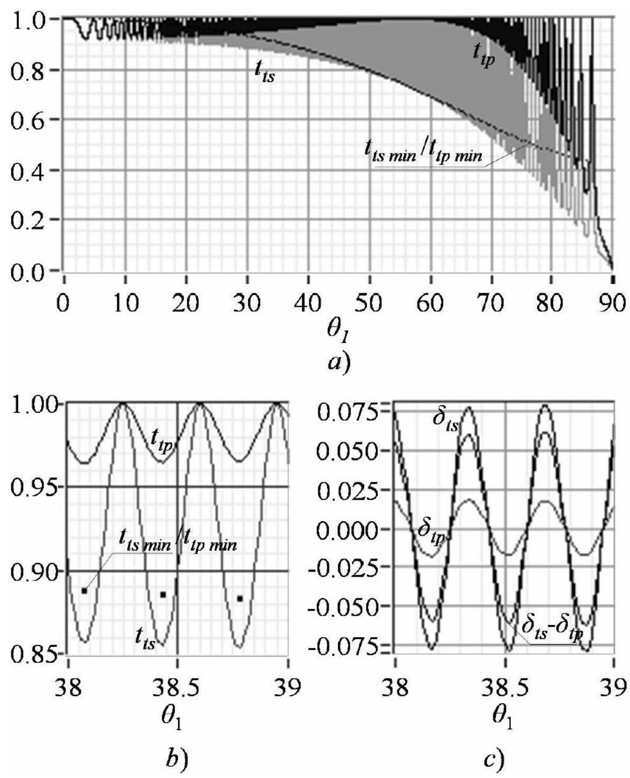

FIGURE 5. a) Theoretical total transmittances vs. angle of incidence (in deg) for a plate n=1.5239, d=147μ and λ=632.8nm. b) Detail and c) total phase change introduced by the plate (in radians) vs. angle of incidence (in deg).

becomes unity. The minima for both transmittances have a ratio t_{tsmin}/t_{tpmin} that decreases as the angle of incidence θ_1 increases. This is plotted in Fig. 5a. In a first approximation, it is sufficient to find a proper ratio t_{tsmin}/t_{tpmin} in order to obtain an ellipticity close to unity.

Consider the presence of the phase term in the transmittance in Eq. (14). This phase term has been plotted for each polarization in Fig. 5c. From Eqs. (15) and (16) we can see that the phase δ_t is zero exactly at the minima and maxima of the transmittances t_t, which allows us to make the above mentioned first approximation. In order to improve that first approximation, a solution to the following parametric equations must be found:

$$|E_{oy'}| - |E_{ox'}| = |E_{iy'}(\alpha')t_{ty'}(\theta_1)| - |E_{ix'}(\alpha')t_{tx'}(\theta_1)| = 0$$

$$|\delta_{oy'}| - |\delta_{ox'}| = |\delta_{ix'}(\alpha') + \delta_{tx'}(\theta_1)| - |\delta_{iy'}(\alpha') + \delta_{ty'}(\theta_1)| = 90^\circ, \quad (17)$$

where $t_{ty'} = t_{tp}$, $t_{tx'} = t_{ts}$, $\delta_{ty'} = \delta_{tp}$, $\delta_{tx'} = \delta_{ts}$ can be obtained from Eqs. (15) and (16), and α' is the angle between x and x'. We calculate $E_{iy'}$, $E_{ix'}$, $\delta_{ix'}$ and $\delta_{iy'}$ with the help of Eq. (4), where α is now substituted for α'. In fact, there are many solutions to Eq. (17) that imply circular polarization. Finding these requires an iterative method as shown next.

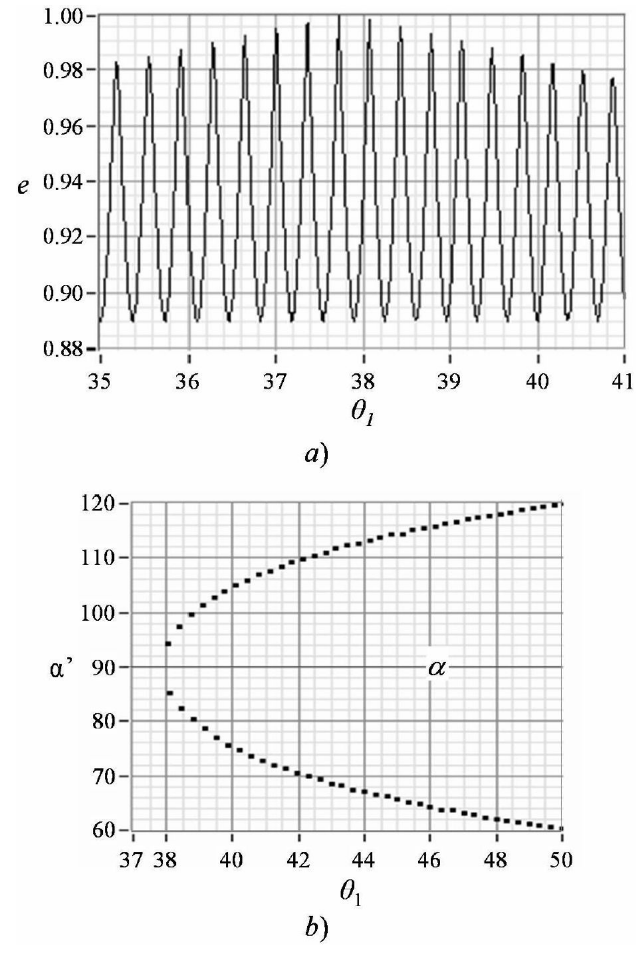

FIGURE 6. a) Ellipticity e vs. angle of incidence θ_1 with $\alpha' = \alpha$, and b) Solutions α', θ_1 where $e \cong 1$.

To give an experimental demonstration we use a glass plate with $n = 1.5239$ and thickness $d = 147\mu$ to change the polarization of a laser beam ($\lambda = 632.8$nm) from a measured ellipticity $e = 0.89$ and $\alpha = 90^\circ$ to $e = 1$. The first step is to calculate e vs. θ_1 maintaining $\alpha' = \alpha$, as shown in Fig. 6a. This permits us to find a first approximation for the angle of incidence at the glass plate equal to $\theta_1 = 37.7^\circ$. In order to reduce the ellipticity as much as possible and make it nearly circularly polarized we must iteratively adjust α' as well as θ_1. Following this procedure, as Fig. 6b shows, we can find multiple solutions to Eq. (17) for θ_1 between 37.7° and 90°. As we see, we can find one of these solutions when $\alpha' > \alpha$, and another when $\alpha' < \alpha$. The iterative procedure is the same since for any θ_1 greater than 37.7°, we are necessarily in the vicinity of one of these maxima. Any maximum gives circular polarization. So it is enough to iteratively change the values of θ_1 and α'. This allows us to find the nearest peak that is one of the multiple solutions.

From experimental results, perfectly plane parallel plates are not necessary, but they must be sufficiently thin and parallel as to maintain uniformity in wavefront polarization. It was sufficient to use a microscope cover glass to demonstrate this method. In Figure 7 we compare experimental and theoretical data. The experimental data were measured with a Thorlabs polarimeter, model PA410. The theoretical data are obtained from zones where the polarization reaches $e \geq 0.995$. With

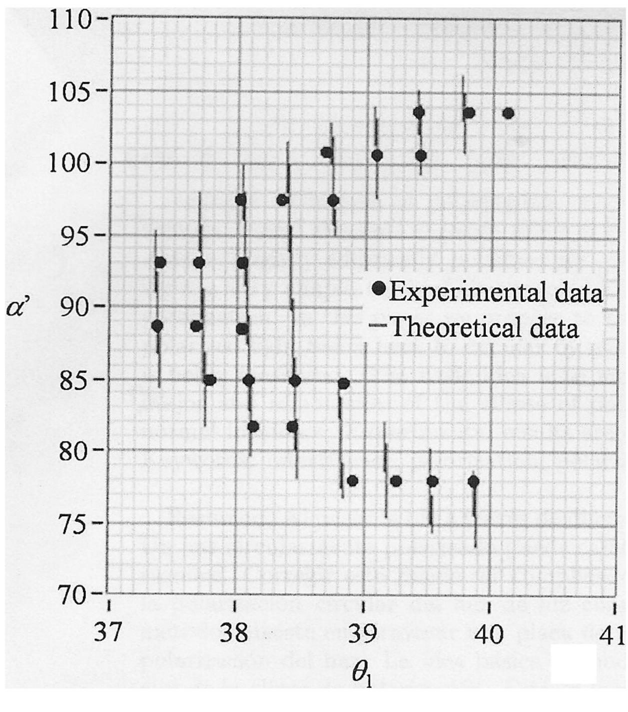

FIGURE 7. Experimental solutions (dots) vs. theoretical ones (continued) for $e \geq 0.995$, it means e=1 in the PA410 polarimeter.

this minimum ellipticity, we can find one additional solution for θ_1 less than $37.7°$. We also note that the first solutions for θ_1 greater than $37.7°$ are approximately the same, because the method's sensitivity to errors in α' is considerably less than for θ_1. As we can see, theory is in agreement with experiment.

5. Conclusions

A simple method for changing the state of polarization from elliptical to circular was applied. It is useful for restoring slight changes in circular polarization occurring due to the interaction of light and the various optical elements through which it passes. The method is simpler than other known methods, and it is based on what we call a Transmittance Modulator (TM). It consists of a homogeneous plane parallel glass plate oblique to the light beam. The TM modulates the transmittances of the electric field in both major and minor axes of the ellipse of polarization. The theoretical analysis was made for coherent light. The polarization change in transmission and reflection through the surfaces of the plate is given by the Fresnel equations. The interference was also taken into account. In order to restore the state of polarization to circular, the TM was properly oriented while the polarization was being measured. From experimental results, we show that it is possible to obtain a high degree of circular polarization even with low quality plane parallel plates such as common microscope cover glass plates.

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