Information swapping scheme in cavity QED

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We present a method to swap information between atomic states by manipulating the interaction time between a quantum cavity field and two two-level atoms. We show that quantum information carried by one atom can be written onto a 'blank' state of the second atom, and the information contained in atom one is completely erased.

Keywords: Quantum information swapping; teleportation; 2-level atoms interacting with light.

Presentamos un método para intercambiar información entre estados atómicos mediante la manipulación del tiempo de interacción entre dos átomos de dos niveles y un campo electromagnético cuántizado. Mostramos que la información contenida en un átomo puede ser escrita en un estado "en blanco de un segundo átomo, borrando por completo la información del primero.

Descriptores: Intercambio de información cuántica; teleportación; interacción de 2 átomos con campos cuantizados.

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1. Introduction

The main idea in quantum information swapping is to pass the information from an individually addressable system to a similar system [1], it can be carried out by cascading three quantum controlled-NOT gates [2] and could serve as an auxiliary tool in the quantum measurement problem [3]. Also, swapping is useful to implement a universal quantum logic gates [4–6]. There have been proposals to implement it in atoms [7], nuclear spins [8] and there are studies on how intrinsic decoherence may degrade the purity and fidelity of the quantum swap gate in quantum-dot systems [9].

Here we will show how to swap (or to write the information an atom carries to a blank state of another atom) information between two atoms interacting with a quantized field. The main difference between this process (swapping) and teleportation, where the purpose is also to pass (teleport) information from a system in a given (unknown) state to another system, is that in the swapping mechanism both systems interact during the process, while in teleportation the systems are never in contact (there is no direct interaction between them), so that it is needed a third system to carry the information from the system to be teleported to the receiver and a classical channel in order to know how to manipulate the system that receives the information (that is, how to make the Bell state measurement [10–12]) to finally produce the teleportation.

Recently there was a proposal for entanglement and quantum information processing for two-atoms passing an empty cavity [13], that makes the entanglement an efficient process because of the fact that cavity losses are of no importance in this case (the cavity is in a vacuum state). Following Zheng *et al.*'s proposal [13] for an empty cavity, Osnaghi *et al.* [14] have realized an experiment where they control the collision of two Rydbeg atoms in a process assisted by a non-resonant cavity and have demonstrated that the atoms get entangled while they cross the cavity. They also show that the cavity makes the entanglement process 10^4 times more efficient than the one of free space atomic collisions with the same impact parameter (as both process are quite similar) [14].

2. A field interacting with two atoms

The interaction Hamiltonian between two identical two-level atoms and a single-mode cavity field in the interaction picture is

$$\hat{H}_I = g\sum_{j=1,2}(e^{-i\delta t}\hat{a}^\dagger\hat{\sigma}_j^- + e^{i\delta t}\hat{a}\hat{\sigma}_j^+), \tag{1}$$

where g is the atom-cavity interaction strength, δ is the detuning between the atomic transition frequency and the cavity frequency, $\hat{\sigma}_j^+ = |e_j><g_j|$ ($\hat{\sigma}_j^- = |g_j><e_j|$) is the raising (lowering) atomic operator for atom j ($j=1,2$) and $\hat{a}^\dagger$ ($\hat{a}$) is the creation (annihilation) operator of the cavity field mode. In the dispersive regime, $\delta \gg g$, one can write an effective Hamiltonian [13]

$$\hat{H}=\lambda\left[\sum_{j=1,2}(|e_j\rangle\langle e_j|\hat{a}\hat{a}^\dagger - |g_j\rangle\langle g_j|\hat{a}^\dagger\hat{a}) + \hat{\sigma}_1^+\hat{\sigma}_2^- + \hat{\sigma}_1^-\hat{\sigma}_2^+\right], \tag{2}$$

where λ is the effective interaction constant [13].

The solution to the Schrödinger equation (we have set $\hbar = 1$)

$$i\frac{\partial|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle \tag{3}$$

for the Hamiltonian (2) subject to the initial condition

$$|\psi(0)\rangle = (\psi_1(0)|e_1\rangle|e_2\rangle + \psi_2(0)|g_1\rangle|g_2\rangle + \psi_3(0)|e_1\rangle|g_2\rangle + \psi_4(0)|g_1\rangle|e_2\rangle) \times |\psi_F(0)\rangle, \tag{4}$$

where $|\psi_F(0)\rangle$ is the initial state of the field, may be written as

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} C_n|n\rangle \times (\psi_1(0)e^{-2i\lambda t(n+1)}|e_1\rangle|e_2\rangle + \psi_2(0)e^{2i\lambda tn}|g_1\rangle|g_2\rangle + \frac{\psi_3(0)[1+e^{-2i\lambda t}] + \psi_4(0)[e^{-2i\lambda t}-1]}{2}|e_1\rangle|g_2\rangle + \frac{\psi_3(0)[e^{-2i\lambda t}-1] + \psi_4(0)[e^{-2i\lambda t}+1]}{2}|g_1\rangle|e_2\rangle), \tag{5}$$

with

$$C_n = \langle n|\psi_F(0)\rangle \tag{6}$$

and $|n\rangle$ a number state.

3. Information swapping

3.1. Coherent state

Let us consider the initial field to be a coherent state $|\alpha\rangle$, and the atoms to be in the state

$$|\psi_A\rangle = (\beta_1|e_1\rangle + \beta_2|g_1\rangle)|e_2\rangle, \tag{7}$$

i.e., atom 2 in its excited state and atom 1 in a superposition of its excited and ground levels with unknown coefficients β_1 and β_2. Then the evolved wave function is

$$|\psi(t)\rangle = e^{-2i\lambda t}\beta_1|\alpha e^{-2i\lambda t}\rangle|e_1\rangle|e_2\rangle + \left[\beta_2\frac{e^{-2i\lambda t}-1}{2}|e_1\rangle|g_2\rangle + \beta_2\frac{1+e^{-2i\lambda t}}{2}|g_1\rangle|e_2\rangle\right]|\alpha\rangle, \tag{8}$$

that in general may be disentangled only to give back the initial state (with an interaction time $\lambda t = j\pi, \quad j = 1, 2, \ldots$). Therefore information swapping can not be realized with coherent states of arbitrary amplitude.

3.2. Field in a vacuum state

If the amplitude of the coherent field is set to zero, *i.e.*, Zheng *et al.* case [13]:

$$|\Psi(t)\rangle = \left[e^{-2i\lambda t}\beta_1|e_1\rangle|e_2\rangle + \frac{\beta_2}{2}(e^{-2i\lambda t}-1)|e_1\rangle|g_2\rangle + \frac{\beta_2}{2}(e^{-2i\lambda t}+1)|g_1\rangle|e_2\rangle\right]|0\rangle. \tag{9}$$

If we choose the interaction time to be

$$t_j = (2j+1)\pi/2\lambda, \quad j = 0, 1, 2, \cdots, \tag{10}$$

we obtain, after the passage by the cavity, the atomic state

$$|\Psi(t_j)\rangle = -|e_1\rangle(\beta_1|e_2\rangle + \beta_2|g_2\rangle), \tag{11}$$

i.e. the information has been passed from atom 1 to atom 2 or in other words, information swapping has been carried out.

Furthermore, if the atoms are initially in the entangled state

$$|\Psi(0)\rangle = \beta_1|e_1\rangle|g_2\rangle + \beta_2|g_1\rangle|e_2\rangle, \tag{12}$$

after a time t yields a wave function

$$|\Psi(t)\rangle = \left(\beta_1\frac{e^{-2i\lambda t}+1}{2} + \beta_2\frac{e^{-2i\lambda t}-1}{2}\right)|e_1\rangle|g_2\rangle + \left(\beta_2\frac{e^{-2i\lambda t}+1}{2} + \beta_1\frac{e^{-2i\lambda t}-1}{2}\right)|g_1\rangle|e_2\rangle, \tag{13}$$

and for the same interaction time as before, t_j, the state

$$|\Psi((2j+1)\pi/2\lambda)\rangle = -(\beta_2|e_1\rangle|g_2\rangle + \beta_1|g_1\rangle|e_2\rangle) \tag{14}$$

is obtained. Therefore the coefficients containing the quantum information are swapped.

4. Schrödinger cats to realize swapping of information

Schrödinger cat states or superposition of coherent states may be used to do the swapping of information. The states may be written as

$$|\psi_\alpha\rangle = \frac{1}{N_\alpha}(|\alpha\rangle + |-\alpha\rangle), \tag{15}$$

where N_α is the normalization constant. The coefficients C_n may be obtained from (6) as

$$C_n = \langle n|\psi_\alpha\rangle = \frac{e^{-\frac{|\alpha|^2}{2}}}{N_\alpha}\frac{\alpha^n}{\sqrt{n!}}[1+(-1)^n]. \tag{16}$$

By using (7) as initial atomic states and (15) as initial field, one finds as the evolved wave function

$$|\psi(t)\rangle = \frac{1}{N_\alpha}e^{-2i\lambda t}\beta_1(|\alpha e^{-2i\lambda t}\rangle + |-\alpha e^{-2i\lambda t}\rangle)|e_1\rangle|e_2\rangle + \frac{1}{N_\alpha}\left[\left[\beta_1\frac{e^{-2i\lambda t}-1}{2}|e_1\rangle|g_2\rangle + \beta_2\frac{1+e^{-2i\lambda t}}{2}|g_1\rangle|e_2\rangle\right](|\alpha\rangle + |-\alpha\rangle)\right] \tag{17}$$

Again, by using the same interaction time t_j, the state given above may be disentangled to give

$$|\Psi(t)\rangle = -|e_1\rangle(\beta_2|e_2\rangle + \beta_2|g_2\rangle)|\psi_\alpha\rangle. \qquad (18)$$

In conclusion, it has been shown a method of quantum information swapping that works in an empty cavity (making it efficient in this case) and when the field is in a photon distribution containing only even number states given by a cat state. Although this seems to complicate the problem due to how sensitive cat states are to decoherence effects due to their interaction with an environment, we believe it will be a clue to have information processing at finite temperature [15, 16] making it possible to swap information in ”hot” cavities. This becomes important because of the fact that it is experimentally difficult to reach close-to zero temperatures and that the presence of thermal noise produces errors in the experimental data [17].

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