

# Production of the MSSM higgs bosons at next generation linear $e^+e^-$ colliders

A. Gutiérrez-Rodríguez, M. A. Hernández-Ruiz  
*Facultad de Física, Universidad Autónoma de Zacatecas  
 Apdo. Post. C-580, 98060 Zacatecas, Zac. México.*

O. A. Sampayo  
*Departamento de Física, Universidad Nacional del Mar del Plata  
 Funes 3350, (7600) Mar del Plata, Argentina.*

Recibido el 29 de octubre de 2001; aceptado el 28 de febrero de 2002

We study the production of the Higgs bosons predicted in the Minimal Supersymmetric extension of the Standard Model ( $h^0, H^0, A^0, H^\pm$ ), with the reactions  $e^+e^- \rightarrow b\bar{b}h^0(H^0, A^0)$ , and  $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$ , using the helicity formalism. We evaluate the cross section of  $h^0, H^0, A^0$  and  $H^\pm$  in the limit when  $\tan\beta$  is large. The numerical computation is done considering two stages of a possible Next Linear  $e^+e^-$  Collider: the first with  $\sqrt{s} = 500 \text{ GeV}$  and design luminosity  $50 \text{ fb}^{-1}$ , and the second with  $\sqrt{s} = 1 \text{ TeV}$  and luminosity  $100\text{-}200 \text{ fb}^{-1}$ .

**Keywords:** Non-standard model Higgs bosons; supersymmetric Models.

Estudiamos la producción de los bosones de Higgs predichos en la extensión supersimétrica mínima del modelo estándar ( $h^0, H^0, A^0, H^\pm$ ), con las reacciones  $e^+e^- \rightarrow b\bar{b}h^0(H^0, A^0)$  y  $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$ , usando el formalismo de helicidad. Se evalúa la sección transversal de  $h^0, H^0, A^0$  y  $H^\pm$  en el límite cuando  $\tan\beta$  es grande. El cálculo numérico se hace considerando dos etapas de un posible próximo colisionador lineal  $e^+e^-$ : la primera con  $\sqrt{s} = 500 \text{ GeV}$  y luminosidad designada de  $50 \text{ fb}^{-1}$ , y la segunda con  $\sqrt{s} = 1 \text{ TeV}$  y luminosidad de  $100\text{-}200 \text{ fb}^{-1}$ .

**Descriptores:** Bosones de Higgs del modelo no estándar; modelos supersimétricos.

PACS: 14.80.Cp; 12.60.Jv

## 1. Introduction

Higgs bosons [1] play an important role in the Standard Model (SM) [2]; they are responsible for generating the masses of all the elementary particles (leptons, quarks, and gauge bosons). However, the Higgs-boson sector is the least tested one in the SM. If Higgs bosons are responsible for breaking the symmetry from  $SU(2)_L \times U(1)_Y$  to  $U(1)_{EM}$ , it is natural to expect that other Higgs bosons are also involved in breaking other symmetries. One of the more attractive extensions of the SM is Supersymmetry (SUSY) [3], mainly because of its capacity to solve the naturalness and hierarchy problems while maintaining the Higgs bosons elementary.

The theoretical framework of this paper is the Minimal Supersymmetric extension of the Standard Model (MSSM), which doubles the spectrum of particles of the SM, and the new free parameters obey simple relations. The scalar sector of the MSSM [4] requires two Higgs doublets, thus the remaining scalar spectrum contains the following physical states: two CP-even Higgs scalars ( $h^0$  and  $H^0$ ) with  $m_{h^0} \leq m_{H^0}$ , one CP-odd Higgs scalar ( $A^0$ ) and a charged Higgs pair ( $H^\pm$ ), whose detection would be a clear signal of new physics. The Higgs sector is specified at tree-level by fixing two parameters, which can be chosen as the mass of the pseudoscalar  $m_{A^0}$  and the ratio of vacuum expectation values of the two doublets  $\tan\beta = v_2/v_1$ , then the masses  $m_{h^0}$ ,  $m_{H^0}$  and  $m_{H^\pm}$  and the mixing angle of the neutral Higgs sector  $\alpha$  can be fixed. However, since radiative corrections produce substantial effects on the predictions of the

model [5], it is necessary to specify also the squark masses, which are assumed to be degenerated. In this paper, we focus on the phenomenology of the neutral CP-even and CP-odd scalars ( $h^0, H^0, A^0$ ) and charged ( $H^\pm$ ) scalars.

The search for these scalars has begun at LEP, and the current low energy bound on their masses are  $m_{h^0}, m_{A^0} > 90 \text{ GeV}$  and  $m_{H^\pm} > 120 \text{ GeV}$  for  $\tan\beta > 1$  [6]. At  $e^+e^-$  colliders the signals for Higgs bosons are relatively clean and the opportunities for discovery and detailed study will be excellent. The most important processes for the production and detection of the neutral and charged Higgs bosons  $h^0, H^0, A^0$  and  $H^\pm$ , are:

$$\begin{aligned} e^+e^- &\rightarrow Z^* \rightarrow h^0, \quad H^0 + Z^0, \\ e^+e^- &\rightarrow Z^* \rightarrow h^0, \quad H^0 + A^0, \\ e^+e^- &\rightarrow \nu\bar{\nu} + W^{+*}W^{-*} \rightarrow \nu\bar{\nu} + h^0, \quad H^0 \end{aligned}$$

(the later is conventionally referred to as  $WW$  fusions), and  $e^+e^- \rightarrow H^+H^-$  [7]; precise cross-section formulas appear in Ref. 8. The main decay modes of the neutral Higgs particles are in general  $b\bar{b}$  decays ( $\sim 90\%$ ) and  $\tau^+\tau^-$  decays ( $\sim 10\%$ ) which are easy to detect experimentally at  $e^+e^-$  colliders [9–11]. Charged Higgs particles decay predominantly into  $\tau\nu_\tau$  and  $t\bar{b}$  pairs.

In previous studies, the two-body processes of the neutral and charged Higgs bosons

$$e^+e^- \rightarrow h^0(H^0) + Z^0, \quad e^+e^- \rightarrow h^0(H^0) + A^0$$

and  $e^+e^- \rightarrow H^+H^-$  have been evaluated [8] extensively. However, the inclusion of three-body processes with heavy fermions

$$f, \quad e^+e^- \rightarrow (f\bar{f}h^0, f\bar{f}H^0, f\bar{f}A^0), \quad [12, 13],$$

and

$$e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \quad \tau^+\nu_\tau H^-, \quad [14, 15],$$

at future  $e^+e^-$  colliders energies [16–18] is necessary in order to know their impact on the two-body mode processes and also to search for new relations that could have a cleaner sign of the Higgs bosons production.

In the present paper we study the production of SUSY Higgs bosons at  $e^+e^-$  colliders. We are interested in finding regions that could allow the detection of the SUSY Higgs bosons for the set parameter space ( $m_{A^0} - \tan \beta$ ). We shall discuss the neutral and charged Higgs bosons production  $b\bar{b}h^0(H^0, A^0)$ , and  $\tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$  in the energy range of a future  $e^+e^-$  collider [16–18] for large values of the parameter  $\tan \beta$ , where one expects to have a high production. Since the coupling  $h^0 b\bar{b}$  is proportional to  $\sin \alpha / \cos \beta$ , the cross-section will receive a large enhancement factor when  $\tan \beta$  is large. A similar situation occurs for  $H^0$ , whose coupling with  $b\bar{b}$  is proportional to  $\cos \alpha / \cos \beta$ . The couplings of  $A^0$  with  $b\bar{b}$  and of  $H^\pm$  with  $\tau^-\bar{\nu}_\tau, \tau^+\nu_\tau$  are directly proportional to  $\tan \beta$ , thus the amplitudes will always grow with  $\tan \beta$ . We consider the complete set of Feynman diagrams at tree-level and use the helicity formalism [19–25] for the evaluation of the amplitudes. Succinctly, our aim in this work is to analyze how much the results of the Bjorken mechanism [Fig. 1, (1.4)] would be enhanced by the contribution from the diagrams depicted in Figs. 1.1–1.3, 1.5 and 1.6 in which the SUSY Higgs bosons are radiated by a  $b(\bar{b})$  quark. For the case of the charged Higgs bosons the two-body mode

[Figs. 3.1 and 3.4] would be enhanced by the contribution from the diagrams depicted in Figs. 3.2, 3.3, and 3.5, in which the charged Higgs boson is radiated by a  $\tau^-\bar{\nu}_\tau (\tau^+\nu_\tau)$  lepton.

Recently, it has been shown that for large values of  $\tan \beta$  the detection of SUSY Higgs bosons is possible at FNAL and LHC [26]. In the papers cited in Ref. 26, the authors calculated the corresponding three-body diagrams for hadron collisions. They pointed out the importance of a large bottom Yukawa coupling at hadron colliders and showed that the Tevatron collider may be a good place for detecting SUSY Higgs bosons. In the case of the hadron colliders, the three-body diagrams come from gluon fusion and this fact makes the contribution from these diagrams more important, due to the gluon abundance inside the hadrons. The advantage for the case of  $e^+e^-$  colliders is that the signals of the processes are cleaner.

This paper is organized as follows. We present in Sec. 2 the relevant details of the calculations. In Sec. 3 we evaluate the cross section for the processes  $e^+e^- \rightarrow b\bar{b}h^0(H^0, A^0)$  and  $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$  at future  $e^+e^-$  colliders. Finally, Sec. 4 contains our conclusions.

## 2. Helicity Amplitude for Higgs Bosons Production

When the number of Feynman diagrams is increased, the calculation of the amplitude is a rather unpleasant task. Some algebraic forms [27] can be used in it to avoid manual calculation, but sometimes the lengthy printed output from the computer is overwhelming, and one can hardly find the required results from it. The CALKUL collaboration [28] suggested the Helicity Amplitude Method (HAM) which can simplify the calculation remarkably and hence make the manual calculation realistic.

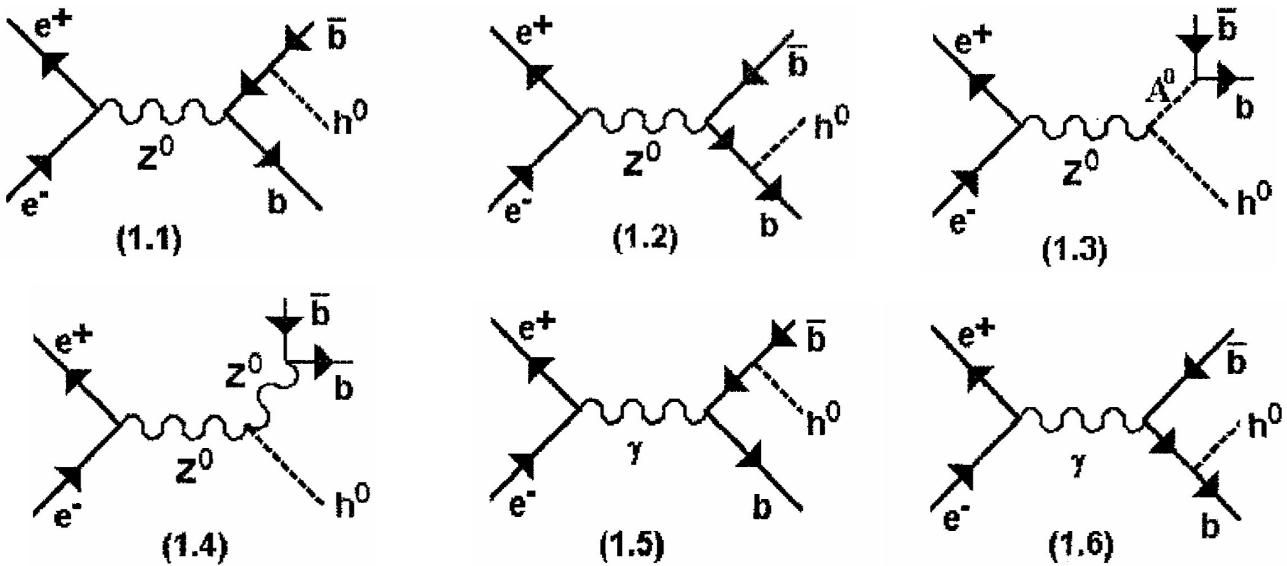
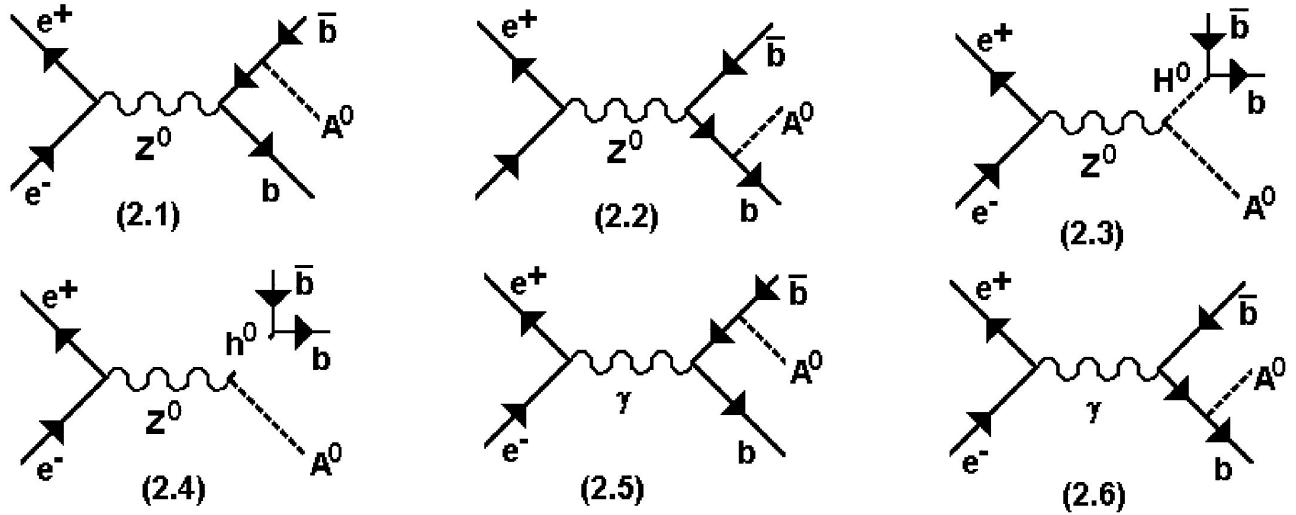
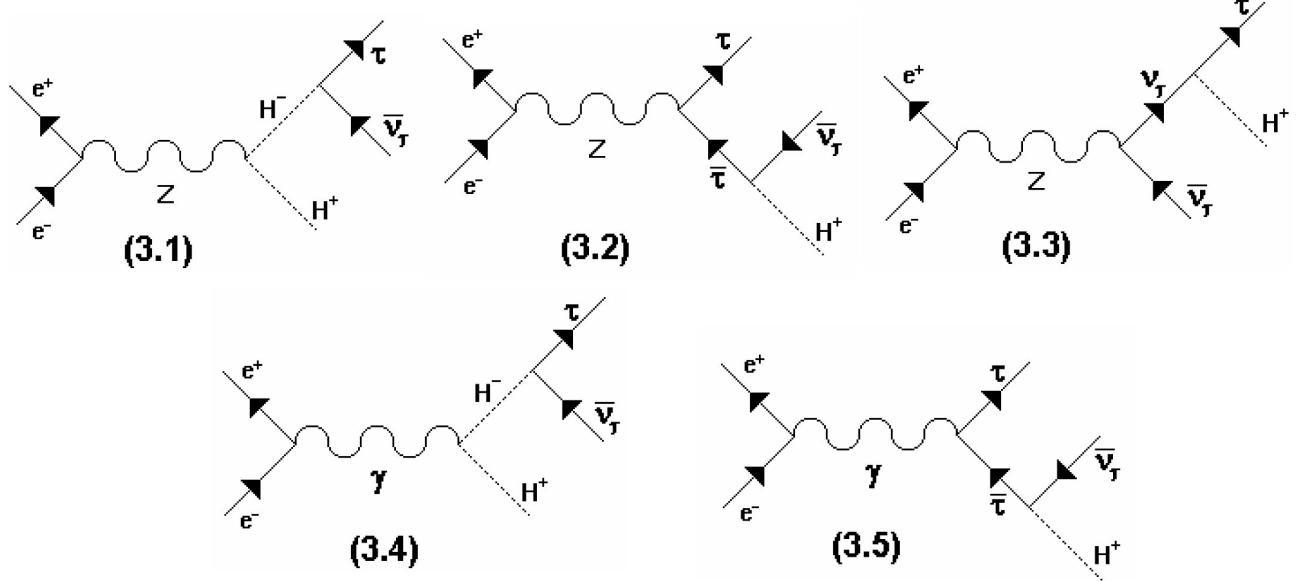


FIGURE 1. Feynman Diagrams at tree-level for  $e^+e^- \rightarrow b\bar{b}h^0$ . For  $e^+e^- \rightarrow b\bar{b}H^0$  one only has to make the change  $\sin \alpha / \cos \beta \rightarrow \cos \alpha / \cos \beta$ .

FIGURE 2. Feynman Diagrams at tree-level for  $e^+e^- \rightarrow b\bar{b}A^0$ .FIGURE 3. Feynman Diagrams at tree-level for  $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$ .

In this section we describe in brief the evaluation of the amplitudes at tree-level, for  $e^+e^- \rightarrow b\bar{b}h^0(H^0, A^0)$  and  $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$  using the HAM [19–25]. This method is a powerful technique for computing helicity amplitudes for multiparticle processes involving massless spin-1/2 and spin-1 particles. Generalization of this method that incorporates massive spin-1/2 and spin-1 particles is given in Ref. 25. This algebra is easy to program and is more efficient than computing the Dirac algebra.

A Higgs boson  $h^0, H^0, A^0$ , and  $H^\pm$  can be produced in scattering  $e^+e^-$  via the following processes:

$$e^+e^- \rightarrow b\bar{b}h^0, \quad (1)$$

$$e^+e^- \rightarrow b\bar{b}H^0, \quad (2)$$

$$e^+e^- \rightarrow b\bar{b}A^0, \quad (3)$$

$$e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-. \quad (4)$$

The diagrams of Feynman, which contribute at tree-level to the different reaction mechanisms, are depicted in Figs. 1 - 3.

However those diagrams with exchange of Higgs bosons instead of gauge bosons (photon or  $Z^0$ ) have been neglected because of the smallness of the Higgs fermion coupling. Using the Feynman rules given by the Minimal Supersymmetric Standard Model (MSSM), as summarized in Ref. 8, we can write the amplitudes for these reactions. For the evaluation of the amplitudes we have used the spinor-helicity technique of Xu, Zhang and Chang [20] (denoted henceforth by XZC), which is a modification of the technique developed by the CALKUL collaboration [28]. Following XZC, we introduce

a very useful notation for the calculation of the processes (1)-(4). The complete formulas of the processes (1)-(3) are given in Ref. 13, while those for the case of process (4) are given in Ref. 14. We are going to use the same notation and the formulas given in Ref. 13 and 14 and do not reproduce them here.

After writing down the Feynman diagrams corresponding to a given amplitude  $\mathcal{M}$  one usually proceeds to derive an analytic expression for the cross section  $\sum |\mathcal{M}|^2$ , with an appropriate spin and/or color sum or average. The result, which is usually a function of Minkowsky products of the particle four-momenta, is then evaluated numerically at phase-space points in the region of interest.

It is clear that any method which evaluates  $\mathcal{M}$  instead of  $\sum |\mathcal{M}|^2$  will eventually become superior to the standard approach. Indeed, several authors have stressed this point and proposed alternatives.

We want to argue that in the evaluation of the amplitude, it is useful to employ not vector products like  $p_1 \cdot p_2$  but rather spinor products like  $\bar{u}(p_1)u(p_2)$ .

After the evaluation of the amplitudes of the corresponding diagrams, we obtain the cross-sections of the analyzed processes for each point of the phase space using Eqs. (12)-(17), (23)-(28) and (11)-(15) [13, 14] by a computer program, which makes use of the subroutine RAMBO (Random Momenta Beautifully Organized) [29]. The advantages of this procedure in comparison to the traditional “trace technique” are discussed in Refs. 19 to 25.

We use the Breit-Wigner propagators for the  $Z^0$ ,  $h^0$ ,  $A^0$  and  $H^\pm$  bosons. For the SM parameters we adopted the following:  $m_b = 4.25 \text{ GeV}$ ,  $m_t = 175 \text{ GeV}$ ,  $m_\tau = 1.78 \text{ GeV}$ ,  $m_\nu = 0$ ,  $m_{Z^0} = 91.2 \text{ GeV}$ ,  $\Gamma_{Z^0} = 2.4974 \text{ GeV}$ ,  $\sin^2 \theta_W = 0.232$ , which are taken as inputs. The widths of  $h^0$ ,  $H^0$ ,  $A^0$  and  $H^\pm$  are calculated from the formulas given in Ref. [8]. In the next sections we present the numerical computation of the processes  $e^+e^- \rightarrow b\bar{b}h$ ,  $h = h^0, H^0, A^0$  and  $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$ .

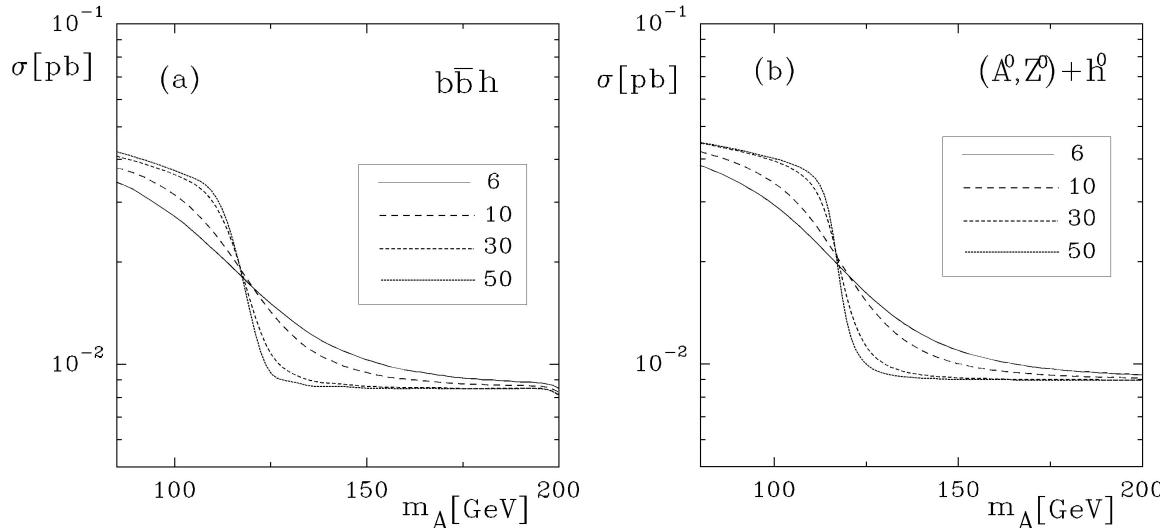


FIGURE 4. Total Higgs production cross sections  $e^+e^- \rightarrow b\bar{b}h^0$  and  $e^+e^- \rightarrow (A^0, Z^0) + h^0$  with  $\sqrt{s} = 500$  and 4 different values of  $\tan\beta$ , 6, 10, 30, 50. We have taken  $m_t = 175 \text{ GeV}$  and  $M_t = 500 \text{ GeV}$  and neglected squark mixing.

### 3. Production of the MSSM Higgs Bosons at Next Generation Positron-Electron Colliders

In this paper, we evaluate the total cross section of neutral and charged MSSM Higgs bosons at next generation  $e^+e^-$  colliders, including three-body mode diagrams [Figs. 1.1-1.3, 1.5, and 1.6; Figs. 2.1-2.3, 2.5 and 2.6; Figs. 3.2, 3.3, and 3.5] besides the dominant mode diagram [Fig. 1.4; Fig. 2.4; Figs. 3.1, and 3.4] and consider two stages of a possible Next Linear  $e^+e^-$  Collider: the first with  $\sqrt{s} = 500 \text{ GeV}$  and design luminosity  $50 \text{ fb}^{-1}$ , and the second with  $\sqrt{s} = 1 \text{ TeV}$  and luminosity  $100\text{-}200 \text{ fb}^{-1}$ . We consider the complete set of Feynman diagrams (Figs. 1-3) at tree-level and utilize the helicity formalism for the evaluation of their amplitudes. In the next subsections, we present our results for the case of the different Higgs bosons.

#### 3.1. Cross section of $h^0$

In order to illustrate our results on the production of the  $h^0$  Higgs boson, we present graphs of the cross section as functions of  $A^0$  Higgs boson mass  $m_{A^0}$ , assuming  $m_t = 175 \text{ GeV}$ ,  $M_t = 500 \text{ GeV}$  and  $\tan\beta > 1$  for NLC. Our results are displayed in Fig. 4, for the process at three-body  $e^+e^- \rightarrow b\bar{b}h^0$  and for the  $e^+e^- \rightarrow (A^0, Z^0) + h^0$  dominant mode.

The total cross section at colliders energies of  $500 \text{ GeV}$  and 4 different values of the fundamental supersymmetry parameter  $\tan\beta$ , 6, 10, 30, 50 is of the order of  $0.1 \text{ pb}$ . We note from this figure that the effect of the reaction  $b\bar{b}h^0$  is slightly more important than  $(A^0, Z^0) + h^0$ , for the interval  $80 \text{ GeV} \leq m_{A^0} \leq 110 \text{ GeV}$  and  $\tan\beta = 30, 50$ . Nevertheless, there are substantial portions in which the discovery of the  $h^0$  is not possible using either  $b\bar{b}h^0$  or  $(A^0, Z^0) + h^0$ .

For the case of  $\sqrt{s} = 1 \text{ TeV}$ , the results of the total cross section of the  $h^0$  are shown in Fig. 5. It is clear from this figure that the contribution of the process  $e^+e^- \rightarrow b\bar{b}h^0$

becomes dominant in the interval  $80 \text{ GeV} \leq m_{A^0} \leq 120 \text{ GeV}$ . However, they could provide important information on the Higgs bosons detection. For instance, we present in the next section tables that illustrate the number of events for the process  $e^+e^- \rightarrow b\bar{b}h^0$ , with  $m_{A^0} = 100 \text{ GeV}$ ,  $\sqrt{s} = (500, 1000) \text{ GeV}$  and  $\tan\beta = 10, 30$ .

### 3.2. Cross section of $H^0$

To illustrate our results regarding the detection of the heavy Higgs bosons  $H^0$ , we give the total cross section for both processes  $e^+e^- \rightarrow b\bar{b}H^0$  and  $e^+e^- \rightarrow (A^0, Z^0) + H^0$  in

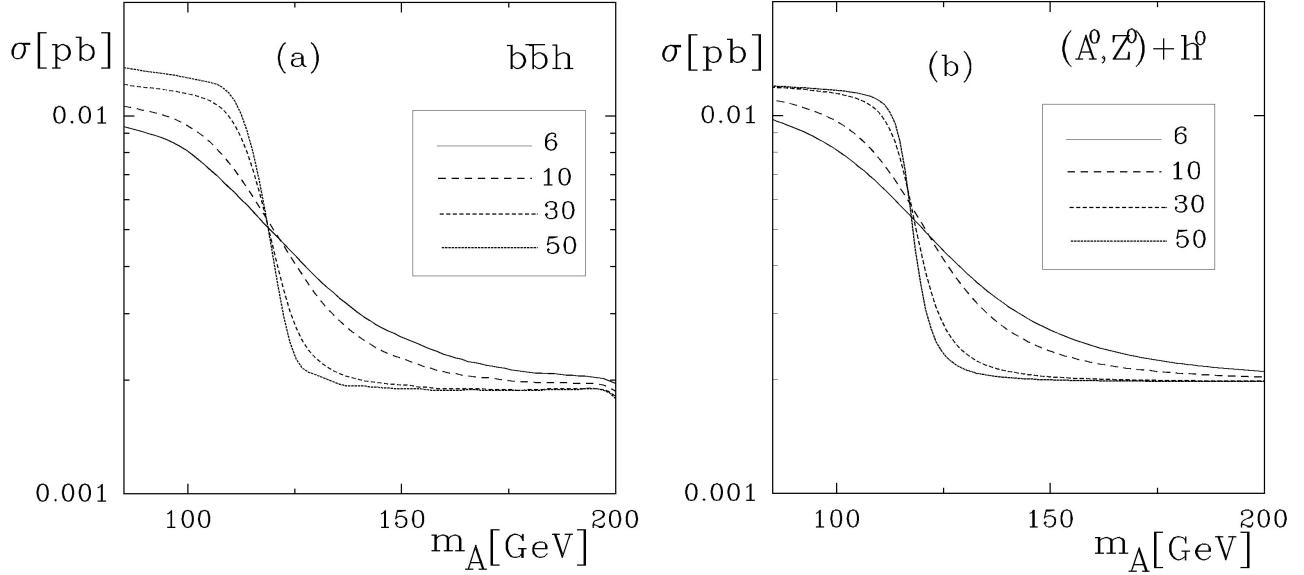


FIGURE 5. Total Higgs production cross sections for  $\sqrt{s} = 1 \text{ TeV}$  and 4 different values of  $\tan\beta$ , 6, 10, 30, 50. We have taken  $m_t = 175 \text{ GeV}$ ,  $M_{\tilde{t}} = 500 \text{ GeV}$  and neglected squark mixing. We display contours for  $e^+e^- \rightarrow b\bar{b}h^0$  and  $e^+e^- \rightarrow (A^0, Z^0) + h^0$ , in the parameters space  $(\sigma_T - m_{A^0})$ .

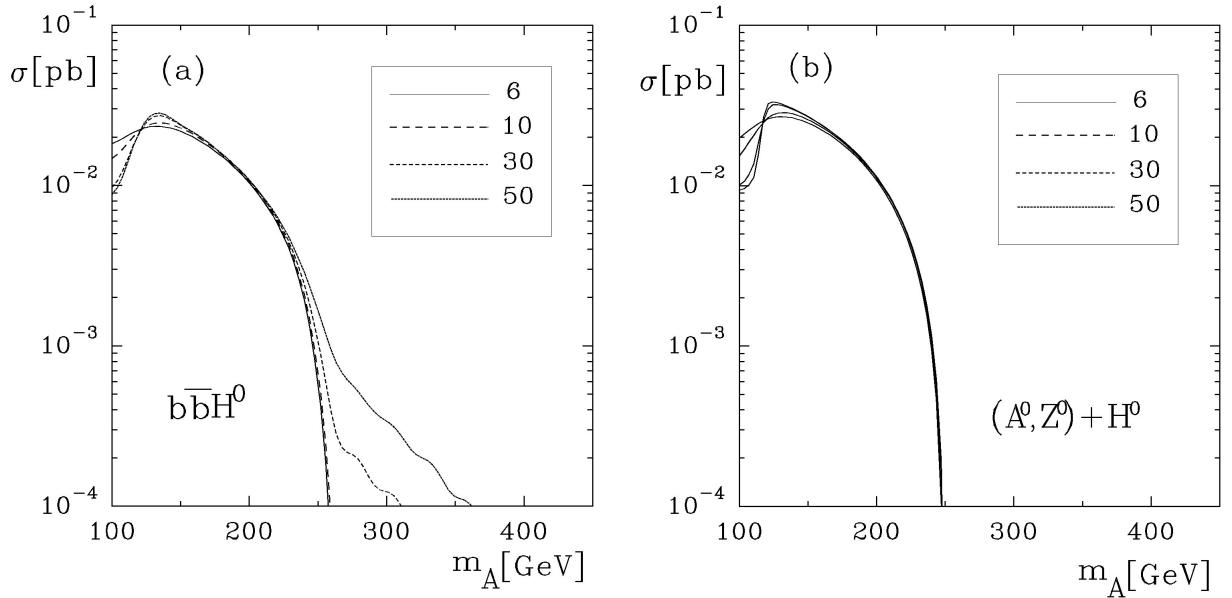


FIGURE 6. Same as in Fig. 4, but for  $e^+e^- \rightarrow b\bar{b}H^0$  and  $e^+e^- \rightarrow (A^0, Z^0) + H^0$ .

Fig. 6 for  $\sqrt{s} = 500 \text{ GeV}$ .

The total cross section for this case is of the order of  $0.01 \text{ pb}$ . In this figure, we observed that the effect of incorporating  $b\bar{b}H^0$  in the detection of the Higgs boson  $H^0$  is more important than the case of the two-body mode  $(A^0, Z^0) + H^0$ , because  $b\bar{b}H^0$  is dominant throughout the interval of  $100 \text{ GeV} \leq m_{A^0} \leq 400 \text{ GeV}$ .

For the case of  $\sqrt{s} = 1 \text{ TeV}$ , the results are shown in Fig. 7. In this case, the three-body mode  $b\bar{b}H^0$  is slightly more important than  $(A^0, Z^0) + H^0$  for  $100 \text{ GeV} \leq m_{A^0} \leq 450 \text{ GeV}$ . In the next section we present tables with the number of events for  $e^+e^- \rightarrow b\bar{b}H^0$ .

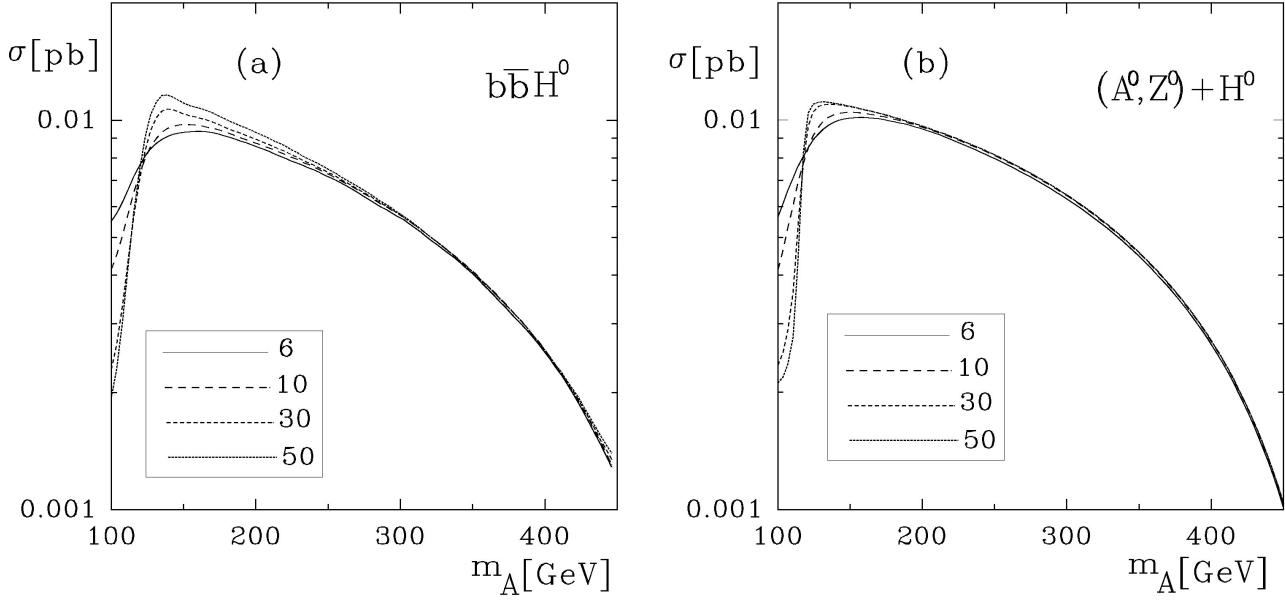


FIGURE 7. Same as in Fig. 5, but for  $e^+e^- \rightarrow b\bar{b}H^0$  and  $e^+e^- \rightarrow (A^0, Z^0) + H^0$ .

### 3.3. Cross section of $A^0$

For the pseudoscalar  $A^0$ , it is interesting to consider the production mode in  $b\bar{b}A^0$ , since it can have large a cross-section due to the fact that the coupling of  $A^0$  with  $b\bar{b}$  is directly proportional to  $\tan \beta$ ; thus will always grow with it. In Fig. 8, we present the total cross section for the process of our interest  $b\bar{b}A^0$  at  $\sqrt{s} = 500$  GeV and  $\tan \beta = 6, 10, 30, 50$ .

We note that for  $\tan \beta = 6, 10, 30, 50$ , both the  $b\bar{b}A^0$  mode and the  $(h^0, H^0) + A^0$  mode have cross sections of the order of 0.1 pb, and for  $m_{A^0} > 175$  GeV the process  $b\bar{b}A^0$

cover a major region in the parameters space  $(m_{A^0}, \tan \beta)$ .

On the other hand, if we focus the detection of the  $A^0$  at  $\sqrt{s} = 1$  TeV, the panorama for its detection is more extensive. Figure 9 shows the contours lines in the plane  $(\sigma_T - m_{A^0})$ , to the production cross section of  $b\bar{b}A^0$ . The contours for this cross section correspond to  $\tan \beta = 6, 10, 30, 50$ . The total cross section is of the order of 0.01 pb and the three-body mode  $e^+e^- \rightarrow b\bar{b}A^0$  is more important than the two-body mode  $e^+e^- \rightarrow (h^0, H^0) + A^0$  in all the parameters space.

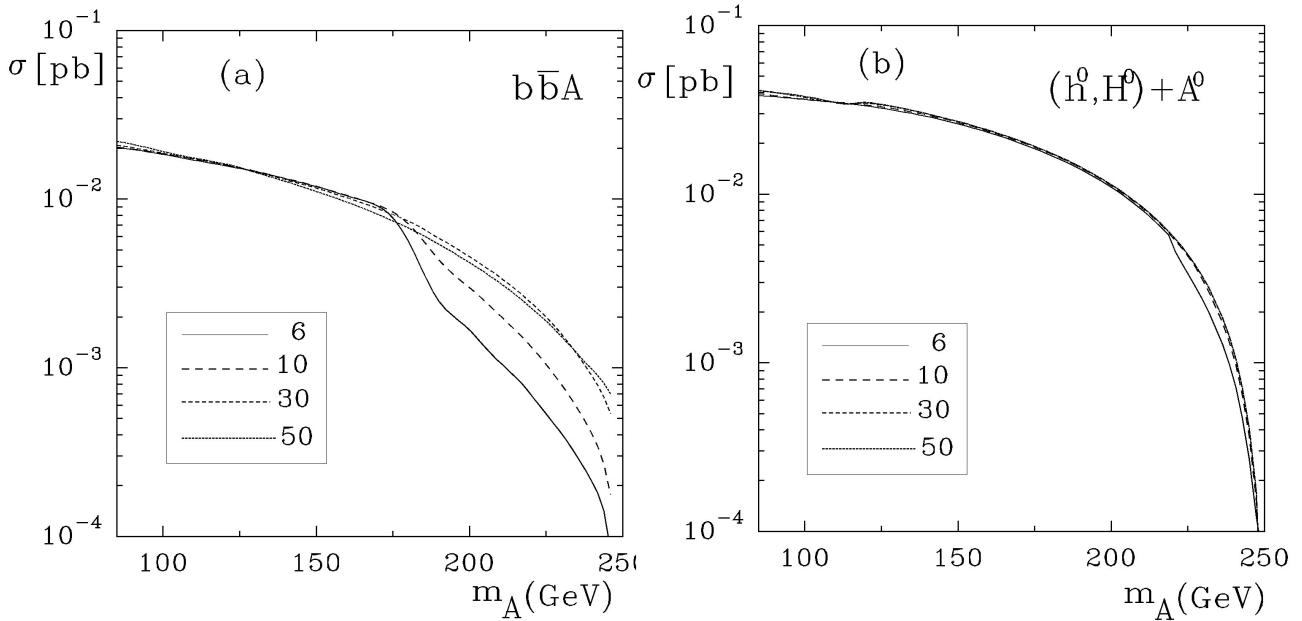
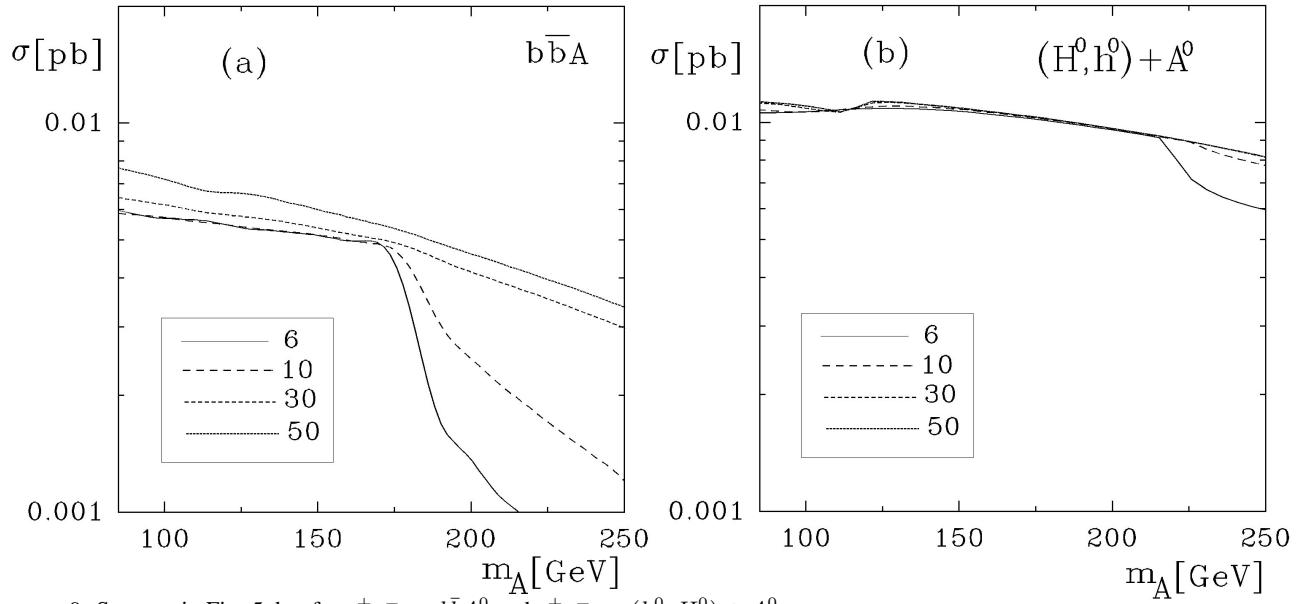


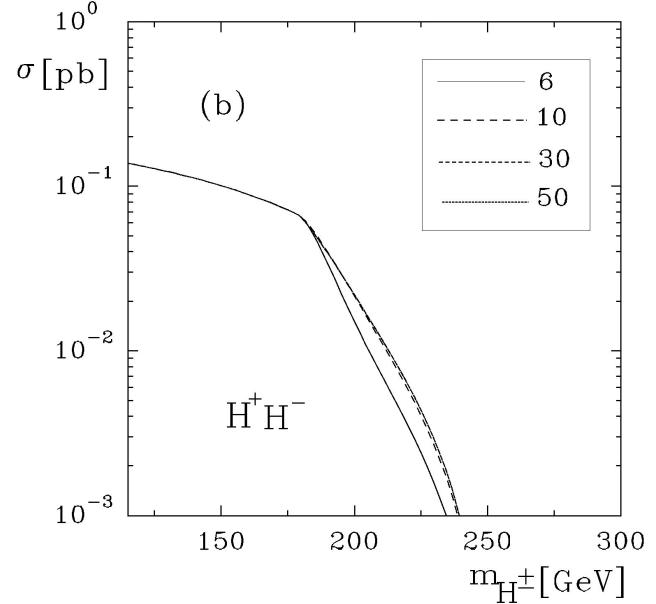
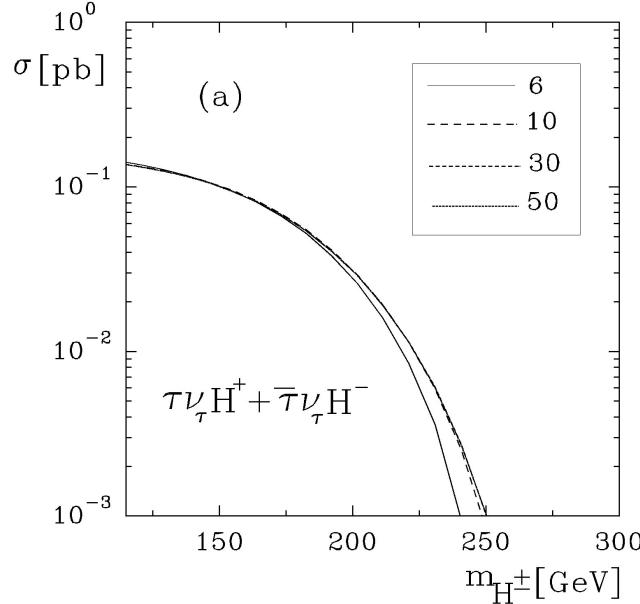
FIGURE 8. Same as in Fig. 4, but for  $e^+e^- \rightarrow b\bar{b}A^0$  and  $e^+e^- \rightarrow (h^0, H^0) + A^0$ .

FIGURE 9. Same as in Fig. 5, but for  $e^+e^- \rightarrow b\bar{b}A^0$  and  $e^+e^- \rightarrow (h^0, H^0) + A^0$ .

### 3.4. Cross section of $H^\pm$

Our results for the  $H^+H^-$  scalars are displayed in Figs. 10, 11 for the processes at three-body  $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$  and for the  $e^+e^- \rightarrow H^+H^-$  dominant mode.

The total cross section for this reaction with  $\sqrt{s} = 500$  GeV and 4 different values of  $\tan\beta$ , 6, 10, 30, 50 are shown in Fig. 10. The cross section is of the order of  $\sigma_T \approx 0.1$  pb for  $m_{H^\pm} \approx 100$  GeV and  $\tan\beta$  large, while for  $m_{H^\pm} \approx 200$  GeV is of  $\sigma_T \approx 0.01$  pb. We can see from this figure that the effect of the reactions  $\tau^-\bar{\nu}_\tau H^+$  and  $\tau^+\nu_\tau H^-$  is slightly more important than  $H^+H^-$  for most of the ( $m_{A^0} - \tan\beta$ ) parameter space regions. It is precisely

FIGURE 10. Same as in Fig. 4, but for  $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$  and  $e^+e^- \rightarrow H^+H^-$ .

in this curve where the contribution of the processes at three-body is notable. Nevertheless, there are substantial portions of parameter space in which the discovery of the  $H^\pm$  is not possible using either  $H^+H^-$  or  $\tau^-\bar{\nu}_\tau H^+$  and  $\tau^+\nu_\tau H^-$ .

For the case of  $\sqrt{s} = 1$  TeV, the results are shown in Fig. 11. In both cases ( $\tau^-\bar{\nu}_\tau H^+$ ,  $\tau^+\nu_\tau H^-$  and  $H^+H^-$ ) the curves with  $\tan\beta$ , 6, 10, 30, 50 given  $\sigma_T \approx 0.1$  pb for  $m_{H^\pm} \approx 175$  GeV, while for  $m_{H^\pm} \approx 350$  GeV the cross section drops until  $0.001$  pb $^{-1}$  for  $\tan\beta = 6$ . These cross sections are small, however, the contribution of the processes at three-body is slightly more important than the process at two-body. The most important conclusion from this figure is that detection of the charged Higgs bosons will be possible at future  $e^+e^-$  colliders.

TABLE I. Total production of Higgs bosons  $h^0, H^0, A^0, H^\pm$  of the MSSM for  $\sqrt{s} = 500 \text{ GeV}$  and  $m_{A^0} = 100 \text{ GeV}$ .

Total Production of Higgs Bosons	$\mathcal{L}=50 \text{ fb}^{-1}$	
	$\tan \beta = 10$	$\tan \beta = 30$
$h^0$	1600	1800
$H^0$	700	470
$A^0$	900	935
$H^+ H^-$	7000	6500

### 3.5. Total Production of the Higgs Bosons

In Tables I and II we present our results for the total production of  $h^0, H^0, A^0, H^\pm$  Higgs bosons, taking different values of the center-of-mass energy  $\sqrt{s}$ , fundamental supersymmetry parameter  $\tan \beta$ , luminosity  $\mathcal{L}$ , and the mass of the pseudoscalar  $m_{A^0}$ . We take the following values as representative of the supersymmetric parameters  $m_{A^0} = 100 \text{ GeV}$  and  $\tan \beta = 10, 30$ . From these results we observed a strong dependence of the supersymmetric parameter  $\tan \beta$ , in particular for  $\tan \beta$  large, as well as of the center-of-mass energy  $\sqrt{s}$ , in the production of the different Higgs bosons ( $h^0, H^0, A^0, H^\pm$ ). These results make apparent the great importance of investigating the possibility of de-

tecting the Higgs bosons with the reactions at three-body  $e^+ e^- \rightarrow b\bar{b} h^0 (H^0, A^0)$  and  $e^+ e^- \rightarrow \tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$ , at next generation linear  $e^+ e^-$  colliders.

## 4. Conclusions

In this paper, we have calculated the production of the neutral and charged Higgs bosons in association with  $b$ -quarks and with  $\tau \nu_\tau$  leptons via the processes  $e^+ e^- \rightarrow b\bar{b} h$ ,  $h = h^0, H^0, A^0$  and  $e^+ e^- \rightarrow \tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$  using the helicity formalism. We find that these processes could help to detect the possible neutral and charged Higgs bosons at energies of a possible Next Linear  $e^+ e^-$  Collider when  $\tan \beta$  is large.

In summary, we conclude that the possibilities of detecting or excluding the neutral and charged Higgs bosons ( $h^0, H^0, A^0, H^\pm$ ) of the Minimal Supersymmetric Standard Model in the processes  $e^+ e^- \rightarrow b\bar{b} h$ ,  $h = h^0, H^0, A^0$  and  $e^+ e^- \rightarrow \tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$  are important and in some cases are compared favorably with the dominant mode  $e^+ e^- \rightarrow (A^0, Z^0) + h$ ,  $h = h^0, H^0, A^0$  and  $e^+ e^- \rightarrow H^+ H^-$  with  $\tan \beta$  large. The detection of the Higgs boson will require the use of a future high energy machine like the Next Linear  $e^+ e^-$  Collider.

TABLE II. Total production of Higgs bosons  $h^0, H^0, A^0, H^\pm$  of the MSSM for  $\sqrt{s} = 1 \text{ TeV}$  and  $m_{A^0} = 100 \text{ GeV}$ .

Total Production of Higgs Bosons	$\mathcal{L}=100 \text{ fb}^{-1}$		$\mathcal{L}=200 \text{ fb}^{-1}$	
	$\tan \beta = 10$	$\tan \beta = 30$	$\tan \beta = 10$	$\tan \beta = 30$
$h^0$	960	1150	1920	2300
$H^0$	370	220	740	440
$A^0$	560	615	1120	1230
$H^+ H^-$	4700	4900	9400	9800

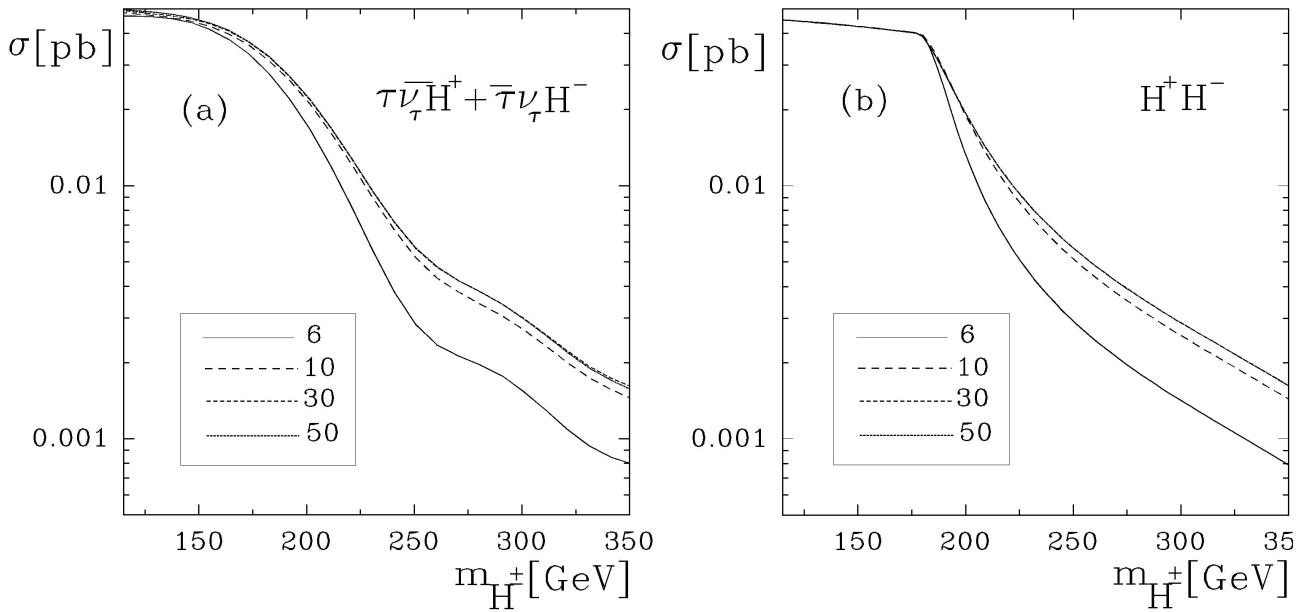


FIGURE 11. Same as in Fig. 5, but for  $e^+ e^- \rightarrow \tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$  and  $e^+ e^- \rightarrow H^+ H^-$ .

## Acknowledgments

This work was supported in part by *Consejo Nacional de Ciencia y Tecnología* (CONACyT), *Sistema Nacional de Investigadores* (SNI) (México) and Programa de Mejoramiento al Profesorado (PROMEP). A.G.R. would like to thank the

organizers of the Summer School in Particle Physics and Sixth School on non-Accelerator Astroparticle Physics 2001, Trieste Italy for their hospitality. O. A. S. would like to thank CONICET (Argentina). The authors would also like to thank Anna Maria D'Amore for revising the manuscript.

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