Emission of electron density waves by neutrinos in a dense medium

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We study the production of electron density waves by neutrinos propagating through a plasma. We treat this process in field theoretically as a Cherenkov emission of phonons (quanta of electron density waves) by neutrinos. The energy transferred to a supernova shock wave by phonon emission from neutrinos is a factor 2×10^2 times the corresponding energy deposited by plasmon (longitudinal photon) emission.

Keywords: Neutrino; supernova; phonon

Nosotros estudiamos la producción de densidad de ondas de electrón mediante la propagación de neutrinos a traves de un plasma. Tratamos este proceso teórico como emisión de fonones Cherenkov (el cuanto de densidad de electrones) por neutrinos. La energía transferida a una supernova por medio de ondas de choque mediante emisión de fonones de neutrinos es un factor de 2×10^2 veces la correspondiente energía depositada por la emision de un plasmón (fotones longitudinales).

Descriptores: Neutrino; supernova; fonón

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1. Introduction

High energy particles propagating in a dense plasma can loose their kinetic energy by perturbing the particle distribution of the plasma. The fluctuation in the number densities of protons and neutrons is small as they are heavy but for electrons the fluctuations are significant. Neutrinos propagating through a plasma will scatter electrons by weak interactions. The primary energy transfer process is inelastic scattering of neutrinos by nucleons ($\nu_e n \rightarrow e^- p$) which was first studied by Bethe and Wilson [1]. The Bethe-Wilson (BW) process was introduced as a mechanism for transfering energy to the shock wave of supernova which would enable it to overcome the gravity of the supernova core [2]. The energy delivered by the BW process is however not enough to revive the stalled shock wave [2].

Bingham *et al.* [3] have introduced a semiclassical mechanism analogous to the electromagnetic pondermotive force where by a inhomogeneous flux of neutrinos passing through plasma transfers energy collectively to the medium. The energy transfer by the neutrino pondermotive force is sufficient for the shock wave explosion of supernovae. A field theoretic derivations of this concept is not yet been achieved [4].

In this paper we study another collective process where neutrinos can transfer energy to plasma through which they propagate. We study the generation of electron density waves by the passage of neutrinos. In field theoretic terms we study the emission of *phonons* (the quanta of electron density waves) by neutrinos by the weak interaction process. Such a single vertex emission of a quanta which is possible only in a medium is called a Cherenkov process [5–10]. This is kinematically allowed when the emitted "particles" have a space-like dispersion relation ($k^2 > \omega^2$) in the medium.

We show that for a range of frequencies the refractive index of the electron density waves $n(\omega) > 1$ and therefore the Cherenkov emission of these waves is kinematically allowed. We compute the rate of emission of phonons and the energy transfer by phonon emission from neutrinos. It has been shown earlier [11] that neutrinos in a plasma acquire an effective charge. Due to this effective charge the neutrino emits a longitudinal photon by the Cherenkov process, which in turn deposits its energy to the plasma. The energy deposited by the phonon emission process in a supernova shock wave is larger than the corresponding plasmon process [7–9] by a factor $\dot{E}_{\rm phonon}/\dot{E}_{\rm plasmon} \sim 2 \times 10^2$. We see therefore that phonon emission is the dominant collective mode that is generated by the passage of neutrinos through a plasma.

The paper is organised in the following manner. In Sec. 2 we derive the dispersion relation of the electron density waves in a plasma. The Cherenkov emission of phonon is considered in Sec. 3. We calculate the amount of energy deposited by the Cherenkov emission of phonon, in the stalled supernova medium and compare this with the BW mechanism in Sec. 4. Our results are briefly discussed in conclusions.

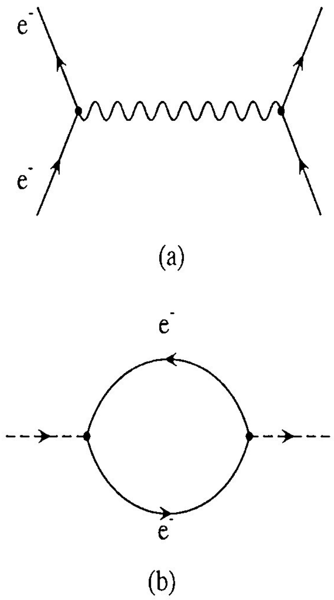

FIGURE 1. a) Feynman diagram for $e^- + e^- \leftrightarrow e^- + e^-$ and b) phonon polarisation in the medium.

2. Phonon dispersion relation

The Lagrangian for the elastic electron-electron scattering is given by

$$\mathcal{L}_{\rm int} = \frac{e^2}{K^2}(\bar{u}_e \gamma_\mu u_e)(\bar{u}_e \gamma^\mu u_e), \tag{1}$$

where the Feynman diagram for electron-electron elastic scattering is given in Fig. 1a. Now considering free electron scattering from the background electrons (electron plasma) then, we have to average the background electrons which will give

$$\mathcal{L}_{\rm int} = \frac{e^2}{K^2}\bar{u}_e \gamma_\mu u_e \langle \bar{u}_e \gamma^\mu u_e \rangle, \tag{2}$$

where $\langle \bar{u}_e \gamma^\mu u_e \rangle = n^\mu$ denotes the averaging over the background electrons and $n^\mu = (n_e, \mathbf{j})$. The quantity n_e is the mean electron density in the medium and $\mathbf{j}$ is the current density. Fluctuation in n^μ is given by $n^\mu + \delta n^\mu$, where δn^μ is the fluctuation over the mean n_μ. We are interested in the part of the Lagrangian, which corresponds to the coupling of the δn^μ with the electron. From the above equation, we obtain for the fluctuation part

$$\delta\mathcal{L}_{\rm int} = \frac{e^2 n_e}{K^2 m_e}\bar{u}_e \gamma_\mu u_e \left(\frac{\delta n^\mu m_e}{n_e}\right), \tag{3}$$

where $\delta n^\mu m_e / n_e$ defines the phonon field ϕ^μ. From Eq. (3) the effective coupling of the field ϕ with the electrons, is given by

$$g = \frac{e^2 n_e}{K^2 m_e} = \frac{4\pi\alpha n_e}{K^2 m_e}. \tag{4}$$

The phonon polarisation in the medium is given by the Feynman diagram Fig. 1b. Evaluation of this diagram gives,

$$i\Pi_{00}(K) = -g^2 \int \frac{d^4 p}{(2\pi)^4} {\rm Tr}[\gamma_0 S_F(p) \gamma_0 S_F(p+K)], \tag{5}$$

where $S_F(p)$ is the electron propagator at finite temperature and density. Using the real time formalism of the finite-temperature field theory [12,13], the electron propagator in the medium is given by

$$S_F(p) = (\not{p} + m)\left[\frac{1}{p^2 - m^2} + 2\pi i \delta(p^2 - m^2) f_{\rm F}(p \cdot u)\right], \tag{6}$$

where u_μ is the four-velocity of the center of mass of the medium and $f_{\rm F}$ denotes the Fermi distribution function

$$f_{\rm F}(x) = \frac{\theta(x)}{{\rm e}^{\beta(x-\mu)} + 1} + \frac{\theta(-x)}{{\rm e}^{-\beta(x-\mu)} + 1}. \tag{7}$$

Here, θ is the unit step function, β is the inverse temperature and μ denotes the chemical potential of the electron. After carrying out the trace and computing the integral over $d^3 p$ in Eq. (5) becomes,

$$\Pi_{00}(K) = -\frac{g^2}{2\pi^2 k}\int dp_0\, f_{\rm F}(p_0)\left[\left(p_0^2 + p_0\omega + \frac{K^2}{4}\right)\ln\left(\frac{K^2 + 2p_0\omega + 2\sigma k}{K^2 + 2p_0\omega - 2\sigma k}\right) + \left(p_0^2 - p_0\omega + \frac{K^2}{4}\right)\ln\left(\frac{K^2 - 2p_0\omega + 2\sigma k}{K^2 - 2p_0\omega - 2\sigma k}\right) - 2\sigma k\right], \tag{8}$$

where $p_\mu = (p_0, \mathbf{p})$, $k_\mu = K = (\omega, \mathbf{k})$, $|\mathbf{k}| = k = n\omega$, with n denoting the refractive index for the phonon and $\sigma = (p_0^2 - m_e^2)^{1/2}$. In the limit $p_0 \gg m_e$ and ω, the lograthmic terms in Eq. (8) will be simplified to

$$\ln\left(\frac{K^2 + 2p_0\omega + 2\sigma k}{K^2 + 2p_0\omega - 2\sigma k}\right) \simeq \ln\left(\frac{\omega + k}{\omega - k}\right) - \frac{k}{p_0} + \frac{\omega k}{2p_0^2} + \ldots, \tag{9}$$

and

$$\ln\left(\frac{K^2 - 2p_0\omega + 2\sigma k}{K^2 - 2p_0\omega - 2\sigma k}\right) \simeq -\ln\left(\frac{\omega + k}{\omega - k}\right) - \frac{k}{p_0} - \frac{\omega k}{2p_0^2} + \ldots \tag{10}$$

Keeping only the leading order terms in p_0 in Eq. (8) we obtain

$$\Pi_{00} = -\frac{2g^2}{\pi^2}\int dp_0\, p_0 f_{\rm F}(p_0)\left[\frac{\omega}{2k}\ln\left(\frac{\omega + k}{\omega - k}\right) - 1\right]. \tag{11}$$

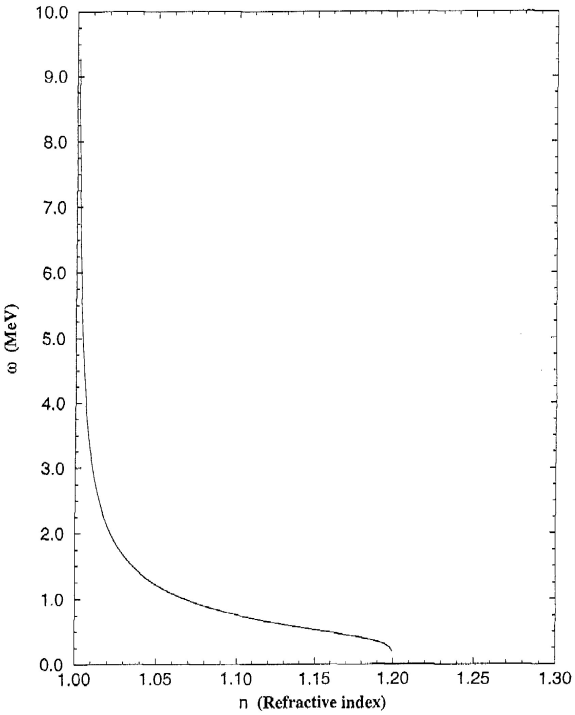

FIGURE 2. Dispersion curve for the phonon.

The Fermi momentum is given by $p_{\rm Fe} = (3\pi^2 n_e)^{1/3}$. For matter density $\rho \simeq 10^8$ gm/cm^3 the electron Fermi momentum is approximately 1,6 MeV. So the electron chemical potential $\mu \simeq p_{\rm Fe} = 1{,}6$ MeV. Thus we consider $p_0 \gg \mu$ here. Also in the stalled shock wave medium there is no positron. With the above approximation we obtain,

$$\mathcal{R}\mathrm{e}\, \Pi_{00} = -\frac{g^2T^2}{6}\left(\frac{\omega}{2k}\ln\left|\frac{\omega+k}{\omega-k}\right| - 1\right), \qquad (12)$$

where the plasma has the temperature T of order MeV. The dispersion relation satisfied by the phonon is

$$w^2 - k^2 - \mathcal{R}\mathrm{e}\, \Pi_{00} = 0. \qquad (13)$$

Putting the value of g from Eq. (4) in Eq. (13) we obtain the following dispersion relation:

$$(n^2-1)^3 = \left(\frac{4\pi\alpha n_e}{m_e}\right)^2 \frac{T^2}{6\omega^6}\left(\frac{1}{2n}\ln\left|\frac{1+n}{1-n}\right| - 1\right). \qquad (14)$$

Figure 2, shows that for the refractive index n in the range $1 < n < 1{,}2$ the phonon frequency ω is positive. On the other hand for $n > 1{,}2$ the phonon frequency is no longer positive and phonon propagation is Landau damped. We are interested only in the range of refractive index $n > 1$ for which phonon frequency is positive, because this is the range in which Cherenkov emission of phonon is kinematically allowed.

3. Cherenkov radiation of phonon

For neutrino-electron scattering the Lagrangian is given by

$$\mathcal{L} = \frac{G_{\rm F}}{\sqrt{2}}\bar{u}_{\nu_e}\gamma_\mu(1-\gamma_5)u_{\nu_e}\bar{u}_e\gamma^\mu(1-\gamma_5)u_e, \qquad (15)$$

where $G_{\rm F} = 1{,}166 \times 10^{-5}/\mathrm{GeV}^2$ is the Fermi coupling constant. For the background electrons we take the average $\langle \bar{u}_e\gamma_\mu u_e\rangle$ (assuming the electrons to be unpolarised). Then perturbing the electron density as done in Eq. (3), we obtain the effective neutrino-phonon interaction Lagrangian as

$$\mathcal{L}_{\nu\phi} = \frac{iG_{\rm F}n_e}{\sqrt{2}m_e}\bar{u}_{\nu_e}\gamma_0(1-\gamma_5)u_{\nu_e}\phi, \qquad (16)$$

where ϕ is the field defined in the previous section. As we are interested only in the fluctuation part of the electron density *i.e.* δn_e which is the zeroth component of δn_e^μ, the field ϕ is the zeroth component of ϕ^μ. Equation (16) gives the effective neutrino-phonon coupling as

$$g_{\nu\phi} = \frac{G_{\rm F}n_e}{\sqrt{2}m_e}. \qquad (17)$$

Let us consider the ϕ emission process from neutrino in the medium

$$\nu_e(p_1) \rightarrow \nu_e(p_2) + \phi(K), \qquad (18)$$

where $p_1 = (E_1, \mathbf{p_1})$, $p_2 = (E_2, \mathbf{p_2})$ and K are the four-momenta of incoming neutrino, outgoing neutrino and outgoing phonon respectively. The matrix element for the process in Eq. (18) is

$$i\mathcal{M} = ig_{\nu\phi}\epsilon_\mu\bar{u}_{\nu_e}(p_2)\gamma_\mu(1-\gamma_5)u_{\nu_e}(p_1), \qquad (19)$$

and after averaging over the initial neutrino spin this gives

$$|\mathcal{M}|^2 = 4g_{\nu\phi}^2\sum\epsilon_\mu\epsilon_\nu^*\big(p_{2\mu}p_{1\nu} + p_{1\mu}p_{2\nu} - p_1p_2g_{\mu\nu}\big). \qquad (20)$$

Here ϵ_μ is the polarisation vector for the field ϕ. As we consider only the longitudinal mode, the polarisation sum will also be on the longitudinal mode only. The polarisation sum for longitudinal mode in the medium is given by

$$\sum\epsilon_\mu^L(k)\epsilon_\nu^{L*}(k) = \frac{1}{n^2\omega}(u_\mu k_\nu + k_\mu u_\nu) - \frac{1}{n^2\omega^2}k_\mu k_\nu, \qquad (21)$$

with $u_\mu = (1, 0)$ the center of mass velocity of the medium. Using Eq. (21) in Eq. (20), $|\mathcal{M}|^2$ is given by

$$|\mathcal{M}|^2 = \frac{4g_{\nu\phi}^2}{n^2}E_1\Big[2E_1 + 2E_1n^2 - \omega + n^2\omega + n\cos\theta\big(1 - 4E_1 - n^2\omega\big)\Big]. \qquad (22)$$

The total energy emitted from a single process is

$$\dot{S} = \frac{1}{2E_1}\int \frac{d^3p_2}{2E_2(2\pi)^3}\frac{d^3k}{2\omega(2\pi)^3} \times\omega(2\pi)^4\delta^4(p_1-p_2-K)|\mathcal{M}|^2. \quad (23)$$

We use the identity

$$\int \frac{d^3p_2}{2E_2} = \int d^4p_2\Theta(E_2)\delta(p_2^2-m_\nu^2), \quad (24)$$

where $\Theta(E_2)$ is the step function and m_ν is the neutrino mass. Putting Eq. (24) in Eq. (23) and integrating over d^4p_2 we obtain the rate of energy radiated per neutrino as

$$\dot{S} = \frac{1}{16\pi^2E_1}\int \frac{d^3k}{2|\mathbf{p_1}|k} \times\delta\left(\frac{2E_1\omega-\omega^2+k^2}{2|\mathbf{p_1}|k}-\cos\theta\right)|\mathcal{M}|^2. \quad (25)$$

The angle θ between the incoming neutrino and the emitted phonon is obtained from the delta function in Eq. (25),

$$\cos\theta = \frac{(2E_1\omega-\omega^2+k^2)}{2|\mathbf{p}_1|k} = \frac{1}{nv}\left[1+\frac{(n^2-1)\omega}{2E_1}\right], \quad (26)$$

where $v=|\mathbf{p_1}|/E_1$ is the neutrino velocity ($\simeq 1$). Neutrino mass term being small we neglect it. Since $-1\le\cos\theta\le 1$; which implies

$$-\frac{2E_1}{(n-1)} \le \omega \le \frac{2E_1}{(n+1)}. \quad (27)$$

But definitely $-2E_1/(n-1)$ can not be the lower limit for the above process as for $n>1$ this is a negative quantity and ω can not be negative. The lower limit for ω will be calculated from the dispersion relation obtained for phonon in the previous section. Figure 2 shows that, the phonon frequency is minimum for higher value of the refractive index and for $n\simeq 1{,}2, \omega=\omega_1\simeq 0{,}19$ MeV and the upper limit depends on the value of the incoming neutrino energy. Putting the value of $\cos\theta$ from the Eq. (26) in $|\mathcal{M}|^2$ and simplifying Eq. (25) we get

$$\dot{S} = \frac{g_{\nu\phi}^2}{16\pi E_1^2}\int_{\omega_1}^{\omega_2}(n^2-1)\omega \times d\omega\left[4E_1^2-4E_1\omega+\omega^2(n^2-1)\right]. \quad (28)$$

This shows that only for $n>1$ the phonon emission is possible, which is the usual Cherenkov condition. Substituting the value of (n^2-1) from the phonon dispersion relation Eq. (14) in Eq. (28) one obtains,

$$\dot{S} = \frac{G_F^2n_e^2}{16\pi m_e^2E_1^2}\left(\frac{4\pi\alpha n_e}{m_e}\right)^{2/3}\left(\frac{T^2}{6}\right)^{1/3}\int_{\omega_1}^{\omega_2}\frac{d\omega}{\omega}f(n) \times\left[4E_1^2-4E_1\omega+\left(\frac{4\pi\alpha n_e}{m_e}\right)^{2/3}\left(\frac{T^2}{6}\right)^{1/3}f(n)\right], \quad (29)$$

where we have defined

$$f(n) = \left[\frac{1}{2n}\ln\left|\frac{1+n}{1-n}\right|-1\right]^{1/3}. \quad (30)$$

The integral in Eq. (29) can not be evaluated analytically. So we have evaluated this numerically. For $\rho=10^8$ g/cm^3, $T=2$ MeV and $E_1=10$ MeV we obtain

$$\dot{S} = 3{,}23\times10^{-24}\ \mathrm{MeV}^2.$$

4. Supernova shock revival

Type-II supernovae are consequence of the collapse of the iron core of massive stars of $8M_\odot\le M\le 25M_\odot$ and lead to the formation of a neutron star or black hole. Observations of neutrino events from supernova SN1987A explosion, by Kamiokande II and IMB detectors have confirmed the fundamental aspects of the theoretical understanding of type-II supernovae. However the mechanism of causing the supernova explosion is yet to be understood satisfactorily.

The neutrino streaming up from deeper region of the supernova are supposed to deposit a small fraction of their energy in the matter between the protoneutron star and the stalled shock, which is about 100–200 Km away from the core. Recent numerical calculations in more than one dimension shows that material behind the stalled shock wave of the supernova can be heated efficiently by the neutrinos coming from the neutrinosphere and eventually expel the outer mantle causing the supernova explosion [14–17]. Matter-enhanced neutrino oscillation (MSW) effect in supernova is considered for the shock revival. The fact that, the region between the neutrino sphere and the stalled wave density is such that flavour transformation of ν_μ or ν_τ to ν_e is resonant for massive neutrinos. Since the average energy of ν_μ's and ν_τ's at the neutrino sphere is about 20 MeV whereas that of ν_e's is about 10 MeV. The oscillation of $\nu_\mu(\nu_\tau)$to ν_e would have twice as high energy as the originally emitted ones, and this extra energy would be available for heating the matter behind the shock. Also it has been proposed that spin-flavour precession of neutrinos may play an important role in the explosion of the stalled wave. In particular, it can be resonantly enhanced in the region between the neutrino sphere and the stalled matter. But it is still controversial, whether the neutrino energy is sufficient for strong enough heating to revive the stalled shock wave.

Bethe and Wilson (BW) in 1985 showed that [1], neutrinos from the hot inner core of the supernova are captured by the matter behind the shock through the process $\nu_e+n\to p+e^-$ and $\bar{\nu}_e+p\to n+e^+$ and deliver their energy. It was argued by BW that $0{,}1\,\%$ of the total energy of the neutrino and anti-neutrino capture process is sufficient to reheat the stalled shock wave and cause supernova explosion.

In a paper by one of us [9], has shown that, neutrinos propagating in the stalled medium, will emit longitudinal photons (plasmons) by Cherenkov process and deposit some energy in the stalled matter, while transverse photons emission is not possible because the transverse mode is Landau damped. Comparison of the plasmon emission process with BW process shows that the former one is a very weak process to account for the shock heating.

Here we are interested to calculate the amount of energy emitted due to Cherenkov emission of phonon, which is subsequently absorbed by the matter from the neutrinos which are propagating through this stalled medium and compare with the BW process.

The total energy deposited by Cherenkov emission of phonons by neutrinos per unit time within the stalled matter of thickness d is given by

$$\dot{E}_{\rm phonon} = \dot{S}\, dF_\nu, \quad (31)$$

where $\dot{S}$ is given in Eq. (29) and $F_\nu = 10^{52}$ erg/s/E_1 is the neutrino flux. For stalled shock density $\rho = 10^8$ gm/cm^3 [2], temperature 2 MeV and neutrino energy $E_1 = 10$ MeV we obtain $\dot{S} = 3{,}23 \times 10^{-24}$ MeV^2. This gives

$$\dot{E}_{\rm phonon} = 1{,}64 \times 10^{38}\, d_{cm}\, {\rm erg/s}. \quad (32)$$

In BW mechanism rate of energy absorbed by a gram of matter at a distance R is [1]

$$\dot{E}_{\rm BW} = 3 \times 10^{18} L_{\nu 52} \left(\frac{T_\nu^2}{R_7^2}\right) \tilde{Y}_N\ {\rm erg/g/s}, \quad (33)$$

where $L_{\nu 52}$ is the neutrino luminosity in units of 10^{52} erg/s, R_7 is the distance from the center in units of 10^7 cm, $T_\nu = 5$ MeV is the temperature of the neutrino sphere and $\tilde{Y}_N \simeq 1$ is the total mean fraction of the nucleon. Here we neglect the contribution due to electron and positron capture as they are correction to this contribution. The total energy absorbed by the stalled shock wave, which has a mass $4\pi\rho R^2 d$ (having thickness d and density ρ) is

$$\dot{E}_{\rm BW} = 3 \times 10^{18} L_{\nu 52} \left(\frac{T_\nu^2}{R_7^2}\right) \tilde{Y}_N\ {\rm erg/g/s}\ \times 4\pi\rho R^2 d. \quad (34)$$

For the neutrino luminosity $L_{\nu_e} = 10^{53}$ erg/s and the stalled shock wave density $\rho \simeq 10^8$ g/cm^3 and $R = 200$ Km, the energy absorbed by the shock wave (assuming $100\,\%$ absorption) is $\dot{E}_{\rm BW} = 9{,}4 \times 10^{43} d_{cm}$ erg/s where d_{cm} is in units of cm. Comparing phonon emission process with BW gives $\dot{E}_{\rm phonon}/\dot{E}_{\rm BW} \simeq 10^{-6}$.

Another collective mode that is emitted by neutrinos in a supernova is "plasmo" or longitudinal photon which also has $n > 1$ and is therefore produced by the Cherenkov process[7–9]. The energy deposited by the phonon emission process [Eq. (29)] in a supernova shock wave is larger than the corresponding plasmon process [7–9] by a factor

$$\frac{\dot{E}_{\rm phonon}}{\dot{E}_{\rm plasmon}} \sim 2 \times 10^2. \quad (35)$$

We see therefore that phonon emission is the dominant collective mode that is generated by the passage of neutrinos through a plasma.

5. Conclusions

It has been shown earlier [11] that neutrinos in a plasma acquire an effective charge. Due to this effective charge the neutrino emits a longitudinal photon by the Cherenkov process, which in turn deposits its energy to the plasma.

In this paper we show that there is a more direct method of energy deposition due to generation of electron density waves or "phonons." In this mechanism the energy of the neutrino is transferred to kinetic energy of the electron density waves without the mediation of photons (unlike the plasmon process). Due to this reason the phonon mechanism is the dominant energy transfer collective process by which neutrinos transfer their energy to a plasma. But still it is very small compared to BW process and therefore can not deposit suffecient energy in the stalled shock wave.

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