Matthews' theorem in effective Yang-Mills theories

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We study the quantization of effective Yang-Mills theories within the path integral formalism. In particular, the equivalence of the Hamiltonian and Lagrangian path integral quantization (Matthews' theorem) is probed for an effective Yang-Mills Lagrangian without matter fields, which includes all the invariant terms up to dimension six. This theorem is probed from point of views of both the gauge and BRST symmetries. The importance of the BRST symmetry in probing this theorem is stressed. We found that the functional integration on the generalized momenta are of Gaussian type and that they do not contribute to physical quantities as a consequence of the symmetries of the effective Lagrangian, which leads to a Lorentz and BRST invariant Lagrangian path integral.

Keywords: Effective lagrangians; constraints

Se estudia la cuantización de teorías efectivas de Yang-Mills en el contexto del formalismo de integral de trayectoria. En particular, se prueba la equivalencia de la cuantización por las integrales de trayectoria hamiltoniana y lagrangiana (teorema de Matthews) para un lagrangiano efectivo de Yang-Mills sin campos de materia, el cual incluye todos los términos invariantes de hasta dimensión seis. Este teorema es probado desde los puntos de vista de la simetría de norma y de la simetría BRST. Se enfatiza la importancia de la simetría BRST en la prueba de este teorema. Se encuentra que las integrales funcionales en los momentos generalizados son de tipo gaussiano y que no contribuyen a cantidades físicas como consecuencia de las simetrías del lagrangiano efectivo, lo cual conduce a una integral de trayectoria Lagrangiana invariante de Lorentz y BRST.

Descriptores: Lagrangianos efectivos; constricciones

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1. Introduction

Effective field theories is a well-motivated framework to parametrize in a model-independent manner the virtual effects of heavy particles lying beyond a given low-energy theory [1], which has been successfully used both in hadron physics [2] and in the electroweak theory [3]. An effective Lagrangian is nonrenormalizable under the Dyson prescription of power counting since it contain all terms of dimension greater than four, constructed out only with the fields of the dimension-four theory, which respect the symmetries of this theory. This scheme has extensively been used to make predictions within perturbation theory not only at tree level, but also at one-loop level [4]. Due to the presence of terms of arbitrary dimension in the Lagrangian, it is important to investigate their quantum properties, mainly those aspects related with the constraints associated with the gauge freedom. In particular, it is important to know what is the correct structure of the Feynman rules and if they arise from a generating functional possessing the Lorentz and BRST [5] symmetries, like in renormalizable Yang-Mills theories.

Canonical quantization leads to a generating functional defined on the phase space coordinates [Hamiltonian path integral (HPI)]. Though it is the fundamental quantity, it has the inconvenient that the symmetries involved are not manifest. Hence it is very important to have our deposition a generating functional defined on the configuration space coordinates [Lagrangian path integral (LPI)], because in most of cases is manifestly covariant, does not depends on the generalized momenta, and directly implies the Feynman rules. The technical procedure of deriving the LPI from the HPI, when it is possible, is known in the literature as Matthews' theorem [6]. The proof of this theorem may be impossible for a HPI depending arbitrarily on the generalized momenta, but this is not the case for renormalizable Yang-Mills theories in virtue that the involved functional integrals are of Gaussian type. This is another key aspect in effective Yang-Mills theories that must be treated with care because the corresponding HPI could depends arbitrarily on the momenta. Even in the case of Gaussian integrals, their solutions could give contributions with nontrivial physical consequences if the coefficients of the quadratical part of the momenta depend on the involved fields, since then the contributions would modify the action of the theory and thus the corresponding Feynman rules. Moreover, this class of terms could lead to a quantum action without explicit Lorentz and BRST invariance.

Gauge theories possesses interesting properties at classical level because of the presence of first-class constraints [7], that have nontrivial implications at the quantum domain. In particular, to have a well-behaved unitary S-matrix, it is necessary to introduce new unphysical degree of freedom, the Faddeev-Popov ghosts fields. Therefore, it is important to study the implications of these type of constraints in quantizing effective Yang-Mills theories. We will extend on this later on, for the moment, let us argue that the structure, as well as the number, of first-class constraints in the effective theory must be the same as those appearing in the conventional theory, since the effective Lagrangian is, by construction, gauge invariant and does not involve more degrees of freedom that those already present in the dimension-four theory. On the other hand, as we will see below, in general the consistence requirements to be satisfied by the gauge-fixing procedure will depend on the specific structure of the effective Lagrangian. Though interesting, it is an intricate problem to put the gauge theory in the Hamiltonian form to quantize it, mainly because it is not possible to introduce covariant gauge-fixing functions, but only canonical ones, such the Coulomb or axial gauges, which in turn implies the use of the Faddeev-Popov trick to recover manifest Lorentz covariance in the LPI [8]. One would expect that the situation becomes more complicated for an effective theory, although, as we will see below, the new aspects in probing Matthews' theorem arise not from the constraints of the effective theory, but from the dependence of the effective Hamiltonian on the generalized momenta.

Due to the complications that arise from the first-class constraints, it is convenient to lift the degeneration of the gauge invariant effective Lagrangian not on the phase space, but on the configuration one because one can introduce a covariant gauge-fixing procedure. To carry out this it is necessary to extend the configuration space by introducing new degrees of freedom, the Faddeev-Popov anticommuting fields, and auxiliary bosonic scalar fields. In this framework, the theory is characterized by a larger Lagrangian, which contains the gauge invariant part and two new terms (gauge-fixing and Faddeev-Popov) whose structure is dictated by the so-called BRST symmetry [5]. Unlike the gauge-invariant Lagrangian, the BRST-invariant one represents a system subject to second-class constraints only [7] and no gauge-fixing procedure is necessary in the phase space because the corresponding Hamiltonian is unique, *i.e.*, these systems are not degenerate. One of the main goals of this work is to show that the proof of Matthews' theorem is very transparent from the point of view of this symmetry. In particular, it makes evident that any possible modification to the BRST-invariant action only arises from the functional integrals on the generalized momenta and not from the nature of the constraints of the theory.

In order to show the advantages of probing the Matthews' theorem within the BRST-symmetry framework, we will probe it also in the context of the gauge symmetry. Previous studies of this theorem have already been presented for a scalar effective theory in [9] and more recently for a gauge theory, using the gauge-invariant point of view [10]. To our knowledge the proof from the point of view of BRST symmetry has never been studied before. We will restrict our study to a $\mathrm{SU}(N)$ invariant Lagrangian containing all the terms up to dimension six, since it is sufficient to discuss the main difficulties encountered in quantizing this type of theories. The study of the most general case[a] will be presented elsewhere [11].

Our paper is organized as follows. The proof of Matthews' theorem from the point of view of the gauge symmetry is presented in Sec. 2, following the conventional scheme used in renormalizable theories. We will discuss with certain detail the structure of the first-class constraints arising from the effective theory, as well as the properties on the consistency requirements of the gauge-fixing procedure. The same theorem is probed from the BRST symmetry point of view in Sec. 3. Finally, the conclusions are presented in Sec. 4.

2. Gauge invariant effective Lagrangian and Matthews' theorem

2.1. The effective Lagrangian

It is convenient to start with a brief discussion of the renormalizable theory, which allows us to present our notation and conventions. A dimension-four Yang-Mills theory without matter fields is characterized by the following Lagrangian:

$$\mathcal{L}_4 = -\frac{1}{2}\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}], \tag{1}$$

where $F_{\mu\nu} = t^a F^a_{\mu\nu}$, with $F^a_{\mu\nu}$ and t^a being the strength tensor and the generators associated with the $\mathrm{SU}(N)$ group, respectively. The equations of motion are given by

$$\mathcal{D}^{ab}_\mu F^{\mu\nu}_b = 0, \tag{2}$$

where $\mathcal{D}^{ab}_\mu = \delta^{ab}\partial_\mu - gf^{abc}A^c_\mu$ is the covariant derivative in the adjoint representation of the group, f^{abc} are the respective structure constants, and g is the coupling constant.

The general structure of the effective Lagrangian is given by

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_4 + \sum_{n=5}\epsilon_n\mathcal{L}_n, \tag{3}$$

where $\mathcal{L}_n$ are Lorentz and $\mathrm{SU}(N)$ invariant structures of dimension greater than four which are constructed with the fields of the dimension-four theory. Here $\epsilon_n = \alpha_n/\Lambda^{n-4}$, where Λ is the new physics scale and the α_n are unknown parameters which depend on the details of the underlying physics, typically $\leq O(1)$ in a weakly coupled fundamental theory. To have a predictive theory, it is fundamental that $\epsilon_n \ll 1$, since then we would have S-matrix elements depending on a finite number of unknown parameters. In practice, it is assumed that the effective Lagrangian technique is valid only to describe physical processes at energies $E \ll \Lambda$, so the ϵ_n parameters are small in this sense and they decrease when the dimension n of the invariant structures $\mathcal{L}_n$ is increased. The building blocks necessary to construct the effective Lagrangian are, in this theory, the gauge and Lorentz covariant objects $F^a_{\mu\nu}$ and their covariant derivatives, *i.e.*,

$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}(F^a_{\mu\nu}, \mathcal{D}^{ab}_\mu F^{\mu\nu}_b, \ldots)$. This means that effective Lagrangians contain an arbitrarily high number of time derivatives, *i.e.*, they are higher-order Lagrangians, which have quit unsatisfactory properties [12]. For instance, the solutions of the equations of motion are not uniquely determined by the initial values of the coordinates and their first time derivatives. In this respect, it has been show in [13] that an effective higher-order Lagrangian can be reduced always to a first-order one by using a certain type of transformations, without affecting the S-matrix elements. These transformations are valid only at first order in the ϵ_n parameters and it is an essential consistency requirement as any effective Lagrangian is assumed to describe the effects of well-behaved new physics just in that approximation. Since any invariant higher-order term has a structure that resembles the covariant form of the equations of motion, the procedure is indeed equivalent to use the equations of motion arising from the dimension-four theory (at first order in the ϵ_n parameters). Consequently, after performing the required transformation, the equations of motion can be used to eliminate any higher-order structure and, in this way, to transfer its coefficient to other first-order invariant terms, already present in the effective Lagrangian. In the following, we will make use of this procedure.

We now proceed to construct the effective Lagrangian up to dimension six. The effective Lagrangian would contain only invariant terms of even dimension, so there are no dimension-five terms. There is only one first-order invariant dimension-six term,[b] given by

$$\mathcal{L}_6 = -\frac{4i}{3!}\mathrm{Tr}[F^\mu{}_\nu F^\nu{}_\lambda F^\lambda{}_\mu] = \frac{f^{abc}}{3!}F^{a\mu}{}_\nu F^{b\nu}{}_\lambda F^{c\lambda}{}_\mu. \quad (4)$$

In the electroweak theory, the analogous of this term has interesting phenomenological implications, since it induce anomalous electromagnetic and weak effects on the charged $W^\pm$ gauge boson, which have extensively been studied in the literature.

On the other hand, using the covariant derivative, it is possible to construct the following two second-order structures given by

$$\mathcal{L}^1_6 = \mathcal{D}^{ab}_\mu F^{\mu\nu}_b \mathcal{D}^\lambda_{ac} F^c_{\lambda\nu}, \quad (5)$$

$$\mathcal{L}^2_6 = F^{\mu\nu}_a \mathcal{D}^{ab}_\mu \mathcal{D}^\lambda_{bc} F^c_{\lambda\nu}. \quad (6)$$

However, these structures are not independent since they are related through a surface term. The remaining one can be eliminated using the equations of motion given in Eq. (2). So, we will probe the Matthews' theorem for an effective Lagrangian which includes all first-order invariant terms of up to dimension-six, namely

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \epsilon\frac{f_{abc}}{3!}F^{a\mu}{}_\nu F^{b\nu}{}_\lambda F^{c\lambda}{}_\mu, \quad (7)$$

where $\epsilon = \alpha_6/\Lambda^2$. Taking into account that this is a first-order Lagrangian, the corresponding equations of motion can be calculated in the usual form. We obtain

$$\mathcal{D}^{ab}_\mu \hat{F}^{\mu\nu}_b = 0, \quad (8)$$

where we have found convenient to define the antisymmetric tensor $\hat{F}^{\mu\nu}_a$ as

$$\hat{F}^{\mu\nu}_a = F^{\mu\nu}_a + \epsilon f_{abc}F^{b\mu}{}_\lambda F^{c\lambda\nu}. \quad (9)$$

2.2. The effective Hamiltonian

In this section we will study the structure of the constraints arising from the effective Lagrangian of Eq. (7). To put the theory in the Hamiltonian form it is necessary to introduce the generalized momenta given by

$$\pi^\mu_a = \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \dot{A}^a_\mu} = F^{\mu 0}_a + \epsilon f_{abc}F^{b\mu}{}_\lambda F^{c\lambda 0}. \quad (10)$$

Due to the antisymmetry of both the strength tensor and the structure constants, the above expression leads to the primary constraints[c]

$$\Phi^{(1)}_a = \pi^0_a \approx 0, \quad (11)$$

which means that the $\dot{A}^a_0$ velocities can not be expressed in terms of coordinates and momenta. Notice that these constraints do not depend on the ϵ parameter, so the effective term does not modifies the structure of the primary constraints arising from the dimension-four theory. This result is true not only for this particular theory, but also in the general case, since all invariant terms are constructed with the strength tensor only, which does not depends on the $\dot{A}^a_0$ velocities and thus $\partial\mathcal{L}_{\text{eff}}/\partial\dot{A}^a_0 = 0$ always. On the other hand, the momenta associated with the spatial components of the fields are given by

$$\pi^a_i = \left(\delta^{ac}\delta_{ij} + \epsilon f^{abc}F^b_{ij}\right)F^c_{0j}. \quad (12)$$

In the following, we will use the letters $i, j, k, \ldots$ to denote spatial indices. It is difficult to solve these equations for the $\dot{A}^a_i$ velocities for an arbitrary parameter ϵ, but working at first order in it, we obtain

$$\dot{A}^a_i = \left(\delta^{ac}\delta_{ij} + \epsilon f^{abc}F^b_{ij}\right)\pi^c_j + \partial_i A^a_0 - g f^{abc}A^b_0 A^c_i. \quad (13)$$

This approximation is equivalent to make the substitution $F^a_{0i} \to \pi^a_i$ anywhere, valid in the general case because a structure of arbitrary dimension would be made of combinations of the strength tensor only. We will use this result later when we demand consistency conditions on the constraints.

In order to classify all constraints of the theory, we introduce the primary Hamiltonian, defined as [7]

$$H^{(1)}_{\text{eff}} = \int d^3x \mathcal{H}^{(1)}_{\text{eff}} = \int d^3x \left(\mathcal{H}_{\text{eff}} + \lambda^a \Phi^{(1)}_a\right), \quad (14)$$

where $\mathcal{H}_{\text{eff}}$ is the canonical Hamiltonian, constructed out with the expressible velocities given by Eq. (13), and λ^a are arbitrary Lagrange multipliers. The canonical Hamiltonian $\mathcal{H}_{\text{eff}}$ can be conveniently expressed as

$$\mathcal{H}_{\text{eff}} = \mathcal{H} + \hat{\mathcal{H}}, \quad (15)$$

being $\mathcal{H}$ and $\hat{\mathcal{H}}$ the Hamiltonians arising from the dimension-four theory and from the dimension-six term, respectively, expressed by

$$\mathcal{H} = \frac{1}{2}\pi_i^a\pi_i^a - A_0^a\mathcal{D}_i^{ab}\pi_i^b + \frac{1}{4}F_{ij}^aF_{ij}^a, \tag{16}$$

$$\hat{\mathcal{H}} = \epsilon f_{abc}(\pi_i^a\pi_j^bF_{ij}^c + \frac{1}{3!}F_{ij}^aF_{jk}^bF_{ki}^c). \tag{17}$$

A basic consistency requirement is the preservation of the constraints in time. Since the constraints do not depend explicitly on time, this requirement means that

$$\begin{aligned}\dot{\Phi}_a^{(1)} &= \left\{\Phi_a^{(1)}, H_{\text{eff}}^{(1)}\right\} \\ &= \int d^3y\,\left\{\Phi_a^{(1)}(\vec{x}\,), \mathcal{H}(\vec{y}\,) + \hat{\mathcal{H}}(\vec{y}\,) + \lambda^b(\vec{y}\,)\Phi_b^{(1)}(\vec{y}\,)\right\} \\ &= \int d^3y\,\left\{\Phi_a^{(1)}(\vec{x}\,), \mathcal{H}(\vec{y}\,)\right\} \approx 0,\end{aligned} \tag{18}$$

where $\{\cdot\}$ denotes Poisson brackets (PB). The last expression comes from the fact that neither $\Phi_a^{(1)}$ nor $\hat{\mathcal{H}}$ depend on the A_0^a fields, as it is clear from Eqs. (11) and (17). The consistency condition in Eq. (18) does not determines the Lagrange multipliers but leads to secondary contraints given by

$$\Phi_a^{(2)} = \mathcal{D}_i^{ab}\pi_i^b \approx 0, \tag{19}$$

whose structure is completely determined by the dimension-four theory, as in the primary constraints case. From comments presented after Eq. (13), we can conclude that this result is valid in the general case. By noting the structure of the secondary constraints in Eq. (19), we can rewrite the dimension four canonical Hamiltonian in the suggestive form

$$\mathcal{H} = \frac{1}{2}\pi_i^a\pi_i^a - A_0^a\Phi_a^{(2)} + \frac{1}{4}F_{ij}^aF_{ij}^a, \tag{20}$$

which reflect the role played for the A_0^a fields as Lagrange multipliers. Taking into account that the λ^a multipliers are indeed the $\dot{A}_0^a$ velocities [7], it is clear that both A_0^a and $\dot{A}_0^a$ fields play the role of Lagrange multipliers.

The secondary constraints must also satisfy consistency conditions, similar to the primary ones. As the PB between the primary and secondary constraints are trivially zero, those conditions are given by

$$\begin{aligned}\dot{\Phi}_a^{(2)} &= \int d^3y\{\Phi_a^{(2)}(\vec{x}\,), \mathcal{H}(\vec{y}\,) + \hat{\mathcal{H}}(\vec{y}\,)\} \\ &= \int d^3y\{\Phi_a^{(2)}(\vec{x}\,), \hat{\mathcal{H}}(\vec{y}\,)\} \approx 0,\end{aligned} \tag{21}$$

where the last expression arises after using the well-known result of the dimension-four theory. The remaining PB can be calculated as follows. Using the following relations:

$$\left\{\mathcal{D}_i^{ab}\pi_i^b(\vec{x}\,), \pi_j^c(\vec{y}\,)\right\} = -gf_{abc}\pi_j^b\delta^3(\vec{x}-\vec{y}\,), \tag{22}$$

$$\left\{\pi_i^a(\vec{x}\,), F_{jk}^b(\vec{y}\,)\right\} = (\delta_{ij}\mathcal{D}_k^{ab} - \delta_{ik}\mathcal{D}_j^{ab})\delta^3(\vec{x}-\vec{y}\,), \tag{23}$$

and the Jacobi identity satisfied by the structure constants of the group, we arrive at

$$\begin{aligned}\{\Phi_a^{(2)}, \hat{H}\} = {}&\epsilon f_{bcd}\left[gf_{aed}F_{ij}^e - \left(\mathcal{D}_i^{ae}\mathcal{D}_j^{ed} - \mathcal{D}_j^{ae}\mathcal{D}_i^{ed}\right)\right]\pi_i^b\pi_j^c \\ &+ \frac{\epsilon}{3!}f_{bcd}\left[\left(\mathcal{D}_j^{ae}\mathcal{D}_i^{ed} - \mathcal{D}_i^{ae}\mathcal{D}_j^{ed}\right)F_{jk}^bF_{ki}^c + \left(\mathcal{D}_k^{ae}\mathcal{D}_j^{eb} - \mathcal{D}_j^{ae}\mathcal{D}_k^{eb}\right)F_{ij}^dF_{ik}^c + \left(\mathcal{D}_i^{ae}\mathcal{D}_k^{ec} - \mathcal{D}_k^{ae}\mathcal{D}_i^{ec}\right)F_{ij}^dF_{jk}^b\right].\end{aligned} \tag{24}$$

This expression vanishes after using again the Jacobi identity together with the identity $\mathcal{D}_i^{ae}\mathcal{D}_j^{ed} - \mathcal{D}_j^{ae}\mathcal{D}_i^{ed} = -gf_{ade}F_{ij}^e$. This show that there are no more constraints. It can be shown that the same result is obtained in the general case [11]. So, the primary and secondary constraints of the effective theory are the same to those appearing in the dimension-four theory, as it was anticipated in the introduction. As it is well-known, these contraints are first-class ones, since all their PB vanish, implying that the Lagrange multipliers remain undetermined. The primary Hamiltonian describes a degenerate system in the sense that given a state at an initial time, it evolves following many histories. These histories must be recognized as physically equivalent because they are consequence of arbitrary Lagrange multipliers in the Hamiltonian. At a later time, the corresponding physically equivalent states on the histories form an orbit. The states on the orbit are related one to another through a gauge transformation, the generators being the first-class constraints. It is clear that only one set of coordinates, corresponding to a representative point of the orbit, is necessary to specify the state of the system at a given time. In order to specify a representative set of variables it is necessary to introduce supplementary conditions, known in the literature as gauge fixing-conditions, which lift the degeneration of the system. For this aim, the number of gauge-fixing functions must be equal to the number of the first-class constraints. These functions can not be arbitrary at all, they must have nonvanishing Poisson brackets with the first-class constraints, which implies that first-class constraints and gauge-fixing functions together form a set of second-class constraints. Since we can lift the degeneration in many different ways, it is clear that there exist many physically equivalent classical theories, each one of them determined once a specific gauge fixing procedure has been chosen. According to canonical quantization, there is a quantum theory corresponding to each classical Hamiltonian and since all of them are physically equivalent, one may believe that the quantum versions of these classical theories must also be physically equivalent. It is, therefore, reasonable to quantize only one of the physically equivalent classical theories.

We will define a specific classical theory using the Coulomb gauge to lift the degeneration. This supplementary condition is defined by

$$\chi_a^{(1)} = \partial_i A_i^a \approx 0. \tag{25}$$

We demand that these constraints be also preserved in time:

$$\begin{aligned}\dot{\chi}_a^{(1)} &= \int d^3y \left\{\chi_a^{(1)}(\vec{x}\,), \mathcal{H}(\vec{y}\,) + \hat{\mathcal{H}}(\vec{y}\,) + \lambda^b(\vec{y}\,)\Phi_b^{(1)}(\vec{y}\,)\right\} \\ &= \int d^3y \left[\{\chi_a^{(1)}(\vec{x}\,), \mathcal{H}(\vec{y}\,)\} + \{\chi_a^{(1)}(\vec{x}\,), \hat{\mathcal{H}}(\vec{y}\,)\}\right] \\ &\approx 0. \end{aligned}\tag{26}$$

A direct calculation shows that the second PB in the last expression vanishes. However, as it was pointed out in the introduction, in the general case this PB would give a nonvanishing result on the constraint surface and thus the consistency requirements on the gauge-fixing procedure could be affected for the higher-dimension terms [11]. The first PB in this expression is a well-known result of the dimension-four theory, given by

$$\chi_a^{(2)} = \partial_i \pi_i^a + \mathcal{D}_i^{ab}\partial_i A_0^b \approx 0, \tag{27}$$

which constitutes a new constraint. It is not difficult to convince ourselves that consistency conditions imposed on these constraints does not leads to new constraints, but to the determination of the Lagrange multipliers. The first-class constraints [Eqs. (11) and (19)] together with the supplementary conditions [Eqs. (25) and (26)] represent indeed a set of second-class constraints, because the matrix formed with all PB among the constraints is nonsingular for a configuration of small fields.[d] The determinant of this matrix is given by

$$\mathrm{Det}\|\{\Phi_a, \chi_b\}\| = \det{}^4\left[\partial_i \mathcal{D}_i^{ab}\delta^3(\vec{x} - \vec{y}\,)\right] \neq 0, \tag{28}$$

for small fields, which is sufficient for perturbation theory.

2.3. Matthews' theorem

We now proceed to probe Matthews' theorem. The fundamental HPI for a system subject to first-class constraints only is [15]

$$Z[J] = \int \mathcal{D}A_\mu^a \mathcal{D}\pi_\mu^a \mathrm{Det}\left[\{\Phi_a(\vec{x}\,), \chi_b(\vec{y}\,)\}\delta(x_0 - y_0)\right]\delta\big(\Phi_a^{(1)}\big)\delta\big(\Phi_a^{(2)}\big)\delta\big(\chi_a^{(1)}\big)\delta\big(\chi_a^{(2)}\big) \times \exp\left[i\int d^4x \left(\pi_\mu^a \dot{A}_a^\mu - \mathcal{H}_{\rm eff} + J\cdot A\right)\right], \tag{29}$$

where $J \cdot A = J_\mu^a A_a^\mu$, with J representing the sources associated with the gauge fields. The determinant appearing in this expression can be directly calculated using the Eqs. (11), (19), (25), and (27). The result is

$$\mathrm{Det}\left[\{\Phi_a(\vec{x}\,), \chi_b(\vec{y}\,)\}\delta(x_0 - y_0)\right] = \mathrm{Det}^2\left[\partial_i \mathcal{D}_i^{ab}\delta^4(x-y)\right]. \tag{30}$$

For subsequent development, it is convenient to rewrite the effective Hamiltonian as follows:

$$\mathcal{H}_{\rm eff} = \frac{1}{2}K_{abij}\pi_i^a\pi_j^b + \frac{1}{4}F_{ij}^a F_{ij}^a - A_0^a\Phi_a^{(2)} + \epsilon\frac{f_{abc}}{3!}F_{ij}^a F_{jk}^b F_{ki}^c, \tag{31}$$

where $K_{abij} = \delta_{ab}\delta_{ij} + \epsilon f_{abc}F_{ij}^c$. Before carrying out the momenta integrations, the following remarks are in order. The structure of the effective Hamiltonian leads to functional integrals of the Gaussian type, though the coefficients of the quadratic parts depend on the gauge fields, which may contribute to the action of the theory. Besides, these terms are not covariant. In the general case, more complicate Hamiltonians with arbitrary dependence on the generalized momenta would appear and use of dimensional regularization would be necessary in order to eliminate non-covariant terms [9]–[11].

Since the structure of the constraints are the same as in the renormalizable theory, we follow the standard procedure to remove the delta functions. We present only some comments on the relevant steps. The integration on the generalized momenta π_0^a is immediate due to the simple structure of the primary constraints. Next, we integrate on the A_0^a fields to remove the delta function on the constraints $\chi^{(2)}$. For this, we use the following relation:

$$\delta(\partial_i\pi_i^a + \mathcal{D}_i^{ab}\partial_i A_0^b) = \frac{\delta(A_0^b - \hat{A}_0^b)}{\mathrm{Det}[\mathcal{D}_i^{ab}\partial_i\delta^4(x-y)]}, \tag{32}$$

where $\hat{A}_0^b$ is the solution of the differential equation

$$\partial_i\pi_i^a + \mathcal{D}_i^{ab}\partial_i A_0^b = 0. \tag{33}$$

The resulting integral is modified by using the exponential representation for the delta function corresponding to the secondary constraint, as follows:

$$\delta(\mathcal{D}_i^{ab}\pi_i^b) = \int \mathcal{D}V_a \exp\left[-i\int d^4x\, V_a \mathcal{D}_i^{ab}\pi_i^b\right], \tag{34}$$

where V_a are auxiliary scalar fields, which allow us to reincorporate into the measure of integration the A_0^a fields by means of the change of variables $A_0^a = V^a + \hat{A}_0^a$. After these considerations, we obtain

$$Z[J]=\int \mathcal{D}A_\mu^a\mathcal{D}\pi_i^a\mathrm{Det}\big[\partial_i\mathcal{D}_i^{ab}\delta^4(x-y)\big]\delta(\partial_iA_i^a)$$
$$\times\exp\left\{i\int\left[-\frac{1}{2}K_{abij}\pi_i^a\pi_j^b+\pi_i^a\big(\dot{A}_a^i+\mathcal{D}_i^{ab}A_0^b\big)-\frac{1}{4}F_{ij}^aF_{ij}^a-\epsilon\frac{f_{abc}}{3!}F_{ij}^aF_{jk}^bF_{ki}^c+J\cdot A\right]\right\}. \quad (35)$$

The determinant and the delta function appearing in this expression can be treated following the standard procedure by using the Faddeev-Popov trick to express them in covariant form. In particular, one can use their four-dimensional covariant version, which is equivalent to make the following change in the generating functional:

$$\mathrm{Det}\big[\partial_i\mathcal{D}_i^{ab}\delta^4(x-y)\big]\delta(\partial_iA_i^a)\to\mathrm{Det}\big[\partial_\mu\mathcal{D}_\mu^{ab}\delta^4(x-y)\big]\delta(\partial_\mu A_\mu^a-B^a), \quad (36)$$

where we have introduced the real functions on the space-time $B^a(x)$, which do not alter the previous results. As a field-independent term multiplying the generating functional does not contributes to physical quantities, we can introduce the following constant term into the generating functional:

$$\int\mathcal{D}B_a\exp\left\{-\frac{i}{2\xi}\int d^4xB_aB_a\right\}, \quad (37)$$

where ξ is a positive real parameter. We can then solve for the delta function. On the other hand, the determinant can be expressed as a Gaussian functional integration on anticommuting c^a and $\bar{c}^{\,a}$ fields. So we finally obtain

$$Z[J]=\int\mathcal{D}A_\mu^a\mathcal{D}\bar{c}^{\,a}\mathcal{D}c^a\mathcal{D}\pi_i^a$$
$$\times\exp\left\{i\int\left[-\frac{1}{2}K_{abij}\pi_i^a\pi_j^b+\pi_i^a(\dot{A}_a^i+\mathcal{D}_i^{ab}A_0^b)-\frac{1}{4}F_{ij}^aF_{ij}^a-\epsilon\frac{f_{abc}}{3!}F_{ij}^aF_{jk}^bF_{ki}^c-\frac{1}{2\xi}(\partial_\mu A_a^\mu)^2-\bar{c}^{\,a}\partial^\mu\mathcal{D}_\mu^{ab}c^b+J\cdot A\right]\right\}. \quad (38)$$

So far, none of the manipulations made in the HPI were related directly with the effective theory, but only with the renormalizable one. Hence, any possible change introduced by the effective theory to the LPI would arise from the Gaussian integrals on the π_i^a momenta in Eq.(38), as a consequence of the dependence on the gauge fields of the coefficients K_{abij}. Solving these integrals, we obtain

$$Z[J]=\int\mathcal{D}A_\mu^a\mathcal{D}\bar{c}^{\,a}\mathcal{D}c^a\mathrm{Det}^{-\frac{1}{2}}\left(\frac{K_{abij}}{2\pi}\right)\exp\left\{i\int\left[\mathcal{L}_{\mathrm{eff}}-\frac{1}{2\xi}(\partial_\mu A_a^\mu)^2-\bar{c}^{\,a}\partial^\mu\mathcal{D}_\mu^{ab}c^b+J\cdot A\right]\right\}, \quad (39)$$

where Eq. (12) was used to determine the “stationary” point of the Gaussians. The determinant appearing in this expression is the sole effect arising from the dimension-six term, which can not be removed from the integral since it depends on the gauge fields. This term can be added explicitly to the action of the theory by using the well-known formula $\mathrm{Det}(A)=\exp\{\mathrm{Tr}(\ln(A)\}$, valid for any nonsingular A matrix. After calculating the continuous trace and ignoring a constant factor, we arrive at

$$Z[J]=\int\mathcal{D}A_\mu^a\exp\left[i\int d^4x\left\{\mathcal{L}_{\mathrm{eff}}-\frac{1}{2\xi}(\partial_\mu A_a^\mu)^2-\bar{c}^{\,a}\partial^\mu\mathcal{D}_\mu^{ab}c^b-\frac{1}{2}\delta^4(0)\mathrm{Tr}\big[\ln\big(\delta_{ab}\delta_{ij}-\epsilon f_{abc}F_{ij}^c\big)\big]+J\cdot A\right\}\right], \quad (40)$$

where here Tr indicates the trace on the discrete indices and the divergent term $\delta^4(0)=\delta^4(x-x)$ comes from the space-time trace. The logarithm in this expression is determined by its Taylor series: $\ln(\delta_{ab}-U_{ab})=U_{ab}+U_{ac}U_{cb}+\ldots$ At first order in the ϵ parameter, its contribution vanishes due to the antisymmetry of the term $f_{abc}F_{ij}^c$:

$$\mathrm{Tr}\big[\ln\big(\delta_{ab}\delta_{ij}-\epsilon f_{abc}F_{ij}^c\big)\big]\simeq\epsilon\mathrm{Tr}\big(f_{abc}F_{ij}^c\big)=0. \quad (41)$$

Thus, the non-covariant terms arising from the dimension-six term disappear from the LPI. Then, Matthews' theorem says, for this theory, that the correct Feynman rules are the naive ones, *i.e.*, those obtained directly from the gauge invariant effective Lagrangian together with the usual gauge-fixing and Faddeev-Popov terms.

To conclude this section, we would like to mention that the same result is obtained if the dimensional regularization scheme is used, since in this case the divergent term vanishes: $\delta^4(0)=0$. Though in our case it is unnecessary to recurs to this regularization scheme to remove the non-covariant terms, in the general case, it plays a fundamental role, not only in eliminating non-covariant terms arising from Gaussian integrals, as in the present work, but, more importantly, in dealing with a HPI depending arbitrarily on the generalized momenta [9–11].

3. BRST invariant effective Lagrangian and Matthews' theorem

In this section, we will probe the Matthews' theorem for the effective Lagrangian given in Eq. (7), focusing from the BRST symmetry point of view. The corresponding effective Lagrangian would be defined in a configuration space extended by the ghosts (c^a) and anti-ghosts ($\bar{c}^{\,a}$) fields, as well as the auxiliary B^a-fields, which allow to lift the degeneration of the gauge invariant Lagrangian in a covariant and quite general way.

3.1. The BRST invariant effective Lagrangian

Under the BRST symmetry [5], the gauge fields (and also the matter ones) are transformed according the infinitesimal form of the gauge symmetry. The ghost fields are related to the gauge group parameters (α^a) through $c^a = \eta\alpha^a$, where η is an anticommuting constant. These fields are subject to the requirement $c^{a\dagger} = c^a$ and $\bar{c}^{\,a\dagger} = -\bar{c}^{\,a}$, which guarantees a Hermitian action. The corresponding BRST transformations are given by

$$\delta_{\rm BRST}A^a_\mu = \eta s A^a_\mu = \eta\mathcal{D}^{ab}_\mu c^b, \tag{42}$$

$$\delta_{\rm BRST}c^a = \eta s c^a = \eta\left(-\frac{g}{2}f_{abc}c^b c^c\right), \tag{43}$$

$$\delta_{\rm BRST}\bar{c}^{\,a} = \eta s\bar{c}^{\,a} = \eta B^a, \tag{44}$$

$$\delta_{\rm BRST}B^a = \eta s B^a = 0, \tag{45}$$

where s is the BRST operator. These transformations are nilpotent in the sense that $s^2 = 0$, leading to the existence of an unitary S-matrix [16].

The BRST invariant action associated with the effective gauge theory in consideration can be written as follows:

$$S^{\rm eff}_{\rm BRST} = S^{\rm eff}_{\rm GI} + s\Psi, \tag{46}$$

where $S^{\rm eff}_{\rm GI}$ is the gauge invariant action for the effective theory, which was studied in the previous section, and the so called gauge-fermion functional Ψ has the form

$$\Psi = \int d^4x\left(f^a + \frac{\xi}{2}B^a\right)\bar{c}^{\,a}, \tag{47}$$

where f^a are the covariant gauge-fixing functions and ξ is a real positive number. Taking into account the BRST transformations given by Eqs.(42-45), the action $s\Psi$ takes the form

$$s\Psi = \int d^4x\left[B^a f^a + \frac{\xi}{2}B^a B^a + (sf^a)\bar{c}^{\,a}\right], \tag{48}$$

which define the usual gauge-fixing and Faddeev-Popov-ghost terms, given by

$$\mathcal{L}_{\rm GF} = B^a f^a + \frac{\xi}{2}B^a B^a, \tag{49}$$

$$\mathcal{L}_{\rm FPG} = (sf^a)\bar{c}^{\,a}. \tag{50}$$

In a dimension-four theory, the gauge-fixing functions f^a are restricted to satisfy, besides Lorentz covariance, the Dyson prescription of renormalizability. In our case, we are restricted to use only a covariant gauge, though for our purpose it is sufficient to use the simplest gauge, namely, the Lorenz one, given by

$$f^a = \partial_\mu A^{a\mu}. \tag{51}$$

In this gauge, the BRST invariant effective Lagrangian can be written as

$$\mathcal{L}^{\rm BRST}_{\rm eff} = \mathcal{L}_{\rm eff} + B^a\partial_\mu A^{a\mu} + \frac{\xi}{2}B^a B^a - \bar{c}^{\,a}\partial_\mu\mathcal{D}^\mu_{ab}c^b, \tag{52}$$

where $\mathcal{L}_{\rm eff}$ is the gauge invariant effective Lagrangian given by Eq. (7). The corresponding equations of motion can be written as

$$\mathcal{D}^{ab}_\mu\hat{F}^{b\nu\mu} + \partial^\nu B^a = g f_{abc}\bar{c}^{\,b}\partial^\nu c^c, \tag{53}$$

$$\xi B^a = -\partial^\mu A^a_\mu, \tag{54}$$

$$\partial^\mu\mathcal{D}^{ab}_\mu\bar{c}^{\,b} = 0, \tag{55}$$

$$\mathcal{D}^{ab}_\mu\partial^\mu\bar{c}^{\,b} = 0, \tag{56}$$

where the tensor $\hat{F}^a_{\mu\nu}$ was already presented in Eq. (9).

3.2. The effective Hamiltonian

The structure of the configuration space in which the BRST symmetry is defined implies the existence of a corresponding phase space extended by the $c^a, \bar{c}^{\,a}, B^a$ fields and their generalized momenta. The momenta associated with this new phase space are given by

$$\pi^a_0 = \frac{\partial\mathcal{L}^{\rm BRST}_{\rm eff}}{\partial\dot{A}^a_0} = B^a, \tag{57}$$

$$\pi^a_B = \frac{\partial L^{\rm BRST}_{\rm eff}}{\partial\dot{B}} = 0, \tag{58}$$

$$\pi^a_i = \frac{\partial\mathcal{L}^{\rm BRST}_{\rm eff}}{\partial\dot{A}^a_i} = (\delta^{ac} + \epsilon f^{abc}F^b_{ij})F^c_{0j}, \tag{59}$$

$$\pi^a_c = \frac{\partial\mathcal{L}^{\rm BRST}_{\rm eff}}{\partial\dot{c}^a} = \dot{\bar{c}}^{\,a}, \tag{60}$$

$$\pi^a_{\bar{c}} = \frac{\partial\mathcal{L}^{\rm BRST}_{\rm eff}}{\partial\dot{\bar{c}}^{\,a}} = \mathcal{D}^{ab}_0 c^b. \tag{61}$$

The Eqs. (59-61) can be solved for the corresponding velocities (at first order in ϵ) as follows:

$$\dot{A}^a_i = \big(\delta^{ac}\delta_{ij} + \epsilon f^{abc}F^b_{ij}\big)\pi^c_j + \partial_i A^a_0 - g f^{abc}A^b_0 A^c_i, \tag{62}$$

$$\dot{c}^a = \pi^a_{\bar{c}} + g f_{abc}c^b A^c_0, \tag{63}$$

$$\dot{\bar{c}}^a = \pi^a_c, \tag{64}$$

whereas the remaining ones lead to the following primary constraints:

$$\Phi^a_1 = \pi^a_0 - B^a \approx 0, \tag{65}$$

$$\Phi^a_2 = \pi^a_B \approx 0. \tag{66}$$

We immediately see that these constraints, besides their algebraic simplicity, are of the second-class type. In fact, after a simple calculation one obtains

$$\mathrm{Det}\big\|\big\{\Phi^a_m(\vec{x}\,), \Phi^b_n(\vec{y}\,)\big\}\big\| = \delta^{ab}\delta^3(\vec{x} - \vec{y}\,), \tag{67}$$

where $m, n = 1, 2$.

The primary effective Hamiltonian can be written as

$$\mathcal{H}^{(1)}_{\rm eff} = \mathcal{H}_{\rm BRST} + \lambda^a_m \Phi^a_m, \tag{68}$$

where

$$\mathcal{H}_{\rm BRST} = \mathcal{H}_{\rm eff} - B^a \partial_i A^a_i - \frac{\xi}{2} B^a B^a + \pi^a_{\bar{c}} \pi^a_c + \partial_i \bar{c}^{\ a} \mathcal{D}^{ab}_i c^b + g f_{abc} \pi^a_c c^b A^c_0. \tag{69}$$

Here $\mathcal{H}_{\rm eff}$ is the effective Hamiltonian given by Eqs. (15)–(17). From Eq. (67), we can see that the consistency conditions on the constraints determine the Lagrange multipliers.

It should be emphasized the fact that these second-class constraints arise directly from the gauge-fixing procedure, *i.e.*, it is the structure of the $s\Psi$ action which determine their nature and structure, which have nothing to do with the nonrenormalizable terms.

3.3. Matthews' theorem

The fundamental HPI for a system subject to second-class constrainst only is given by [17]

$$Z[J] = \int \mathcal{D}A^a_\mu \mathcal{D}\bar{c}^{\ a} \mathcal{D}c^a \mathcal{D}B^a \mathcal{D}\pi^a_\mu \mathcal{D}\pi^a_{\bar{c}} \mathcal{D}\pi^a_c \mathcal{D}\pi^a_B$$
$$\times {\rm Det}^{\frac{1}{2}} \left[\left\{ \Phi^a_m(\vec{x}\,), \Phi^b_n(\vec{y}\,) \right\} \delta(x_0 - y_0) \right] \delta(\Phi^a_1) \delta(\Phi^a_2) \exp \left\{ \int d^4x \left[\pi^a_\mu \dot{A}^\mu_a + \pi^a_c \dot{c}^a + \pi^a_{\bar{c}} \dot{\bar{c}}^a + \pi^a_B \dot{B}^a - \mathcal{H}_{\rm BRST} + J\phi \right] \right\}, \tag{70}$$

Since the determinant appearing in this expression does not depends on the fields, it can be neglected in the HPI. On the other hand, the integrations on the π^a_B momenta are trivial, while those on the π^a_0 momenta restore the covariant form of the gauge-fixing functions. Besides, the integrals on the ghost and anti-ghost fields momenta are of Gaussian type and can be solved immediately. Since the coefficients of the quadratical parts do not depend on the fields and using Eqs. (63) and 64) for the "stationary point" of the Gaussians, we arrive at

$$Z[J] = \int \mathcal{D}A^a_\mu \mathcal{D}\bar{c}^{\ a} \mathcal{D}c^a \mathcal{D}B^a \mathcal{D}\pi^a_i \exp \left\{ i \int \left[-\frac{1}{2} K_{abij} \pi^a_i \pi^b_j + \pi^a_i (\dot{A}^i_a + \mathcal{D}^{ab}_i A^b_0) \right. \right.$$
$$\left. \left. -\frac{1}{4} F^a_{ij} F^a_{ij} - \epsilon \frac{f_{abc}}{3!} F^a_{ij} F^b_{jk} F^c_{ki} - \bar{c}^{\ a} \partial^\mu \mathcal{D}^{ab}_\mu c^b + B^a \partial^\mu A^a_\mu + \frac{\xi}{2} B^a B^a + J \cdot A \right] \right\}. \tag{71}$$

Noting that the integrations on the auxiliary B^a-fields are also of Gaussian type and taking into account that the coefficients of the quadratics parts do not depend on the fields, we obtain

$$Z[J] = \int \mathcal{D}A^a_\mu \mathcal{D}\bar{c}^{\ a} \mathcal{D}c^a \mathcal{D}\pi^a_i \exp \left\{ i \int \left[-\frac{1}{2} K_{abij} \pi^a_i \pi^b_j + \pi^a_i (\dot{A}^i_a + \mathcal{D}^{ab}_i A^b_0) \right. \right.$$
$$\left. \left. -\frac{1}{4} F^a_{ij} F^a_{ij} - \epsilon \frac{f_{abc}}{3!} F^a_{ij} F^b_{jk} F^c_{ki} - \bar{c}^{\ a} \partial^\mu \mathcal{D}^{ab}_\mu c^b - \frac{1}{2\xi} (\partial_\mu A^\mu_a)^2 + J \cdot A \right] \right\}, \tag{72}$$

where, as before, constant factors arising from field-independent determinants have been neglected. This expression coincides with those given in Eq.(38) of Sec. 2 which contain the non-covariant contributions arising from the dimension-six term. Hence, the conclusions concerning Matthews' theorem are the same of Sec. 2. It should be stressed that BRST-symmetry greatly simplifies its probe, making evident that any non-covariant contribution to the LPI only can arises from the structure of the HPI on the generalized momenta associated with the gauge fields and not from the constraints of the theory.

4. Conclusions

In this paper we have presented a study of Matthews' theorem for an effective Yang-Mills theory without matter fields, whose Lagrangian includes all invariant terms up to dimension six. This theorem was probed from both the gauge and BRST symmetries point of views. The nature and structure of the constraints arising from the effective Lagrangian were studied with certain detail. It was shown that the presence of nonrenormalizable invariant terms can not modify, neither in their structure nor in their number, the constraints arising from the dimension-four theory. It was found that any possible source of non-covariant effects to the LPI can arises only from the specific dependence of the HPI on the generalized momenta. It was stressed that these facts are more transparent from the BRST-symmetry point of view. In the special effective Lagrangian considered in this work, it was found that the HPI has a dependence of the Gaussian type on the generalized momenta and can explicitly be solved. Their non-covariant implications on the LPI can be eliminated using the dimensional regularization scheme, though in our case it was unnecessary, since this term vanish at first order in the unknown ϵ parameter as a consequence of the symmetries of the effective Lagrangian. This regularization scheme would play a fundamental role in eliminating non-covariant structures arising from a HPI depending arbitrarily on the generalized momenta. For the effective Lagrangian studied here, Matthews' theorem says that the correct Feynman rules are those obtained directly from the BRST-invariant effective Lagragian. This conclusion would be valid in the general case if the effective theory is regularized using the dimensional scheme.

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[a]. In the following, always we refer to the general case, it must be understood that the effective Lagrangian in consideration is gauge invariant and contain all invariant structures of arbitrary dimension, including matter fields.

[b]. It is possible to construct another independent term by substituting in $\mathcal{L}_6$ one of the strength tensor by its dual: $\widetilde{F}^a_{\mu\nu} = (1/2)\epsilon_{\mu\nu\lambda\rho}F^{a\lambda\rho}$, but this class of structures will be not considered here, for simplicity.

[c]. Through the paper, we will write weak equations using the symbol $\approx$.

[d]. For large values of the fields, the Gribov phenomenon arises and no gauge-fixing is possible [14].

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