Magnetic moments of decuplet baryons

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Using the momentum projection technique of Peierls and Yoccoz in the non-scaling *color dielectric model* (CDM), the decuplet baryons magnetic moments are calculated and compared with the available experimental and theoretical results. The calculated values of the magnetic moments of Δ^{++} and Ω^- in CDM are close to the observed values.

Keywords: Decuplet baryon; magnetic moment

Usando la técnica de proyección del momento de Peierls y Yoccoz en el modelo dieléctrico de color sin escala (CDM), calculamos los momentos magnéticos del decuplete de bariones y comparamos los resultados teóricos y experimentales disponibles. Los valores calculados de los momentos magnéticos de Δ^{++} y Ω^- en CDM están cerca de los valores observados.

Descriptores: Decuplete de bariones; momento magnético

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1. Introduction

Baryon magnetic moments provide very significant information concerning the structure of the hadrons. The magnetic moments of the baryon octet have already been precisely measured. More recently the magnetic moments of Δ^{++} and Ω^- have been measured. The Δ^{++} magnetic moment [1] is measured precisely through pion bremsstrahlung analysis. The E756 Collaboration measured the Ω^- magnetic moment [2], where the Ω^- was produced by a polarised neutral transfer reaction. The theoretical predictions of the magnetic moments of decuplet baryons have also been given by many models [3–7]. The simple additive quark models give the ratio $\mu_{\Delta^{++}}/\mu_p = 2$ or more, whereas the analysis of the experimental data gives the ratio 1.62 ± 0.18. In this work we have calculated the decuplet baryon magnetic moments in the color dielectric model (CDM) and compare with the available experimental and theoretical results. Our calculation shows that, for certain sets of parameter, the CDM result is comparable with the lattice result. The CDM also predicts correctly the ratio μ_{Ω^-}/μ_p.

The CDM [8–15] has the features of confining absolutely the quarks and gluons simultaneously. The absolute confinement is generated dynamically through the vanishing of the dielectric (scalar) field in the non-parturbative vacuum. So this model does not need any artificial boundary like the bag model for confinement. Thus, this model has the advantage of extending to many nucleon systems and hence to nuclear matter calculation. The CDM has been extensively used to study the baryonic properties, both in the free state [8–14], as well as in the medium [15]. In ref [16] Bae and McGovern have calculated the octet and Ω magnetic moments in the CDM by taking into account the pion and kaon contributons perturbatively and obtained good agreement with the experimental values.

The paper is organised as follows. In Sec. 2 we discuss briefly about the CDM and momentum projection technique. Section 3 contains the calculation for decuplet baryon magnetic moments. Numerical calculations and a brief conclusion is given in the Sec. 4.

2. CDM and the momentum projection

The Lagrangian density for the non-scaling CDM is given as [8–10]

$$\mathcal{L} = \sum_f \left\{ \bar{\psi}_f(x) \left[\gamma^\mu i\partial_\mu - \left(m_{su} + \frac{m_u}{\chi} \right) - g_s \gamma^\mu \frac{\lambda^a}{2} A^a_\mu(x) \right] \psi_f(x) \right\} - \frac{1}{4}\kappa(\chi) F^a_{\mu\nu}(x) F^{a,\mu\nu}(x) + \frac{1}{2}\sigma_v{}^2 (\partial_\mu \chi)^2 - U(\chi), \quad (1)$$

where ψ, χ and A_μ are the quark, scalar and the gluon fields. m_u is the $u(d)$ quark mass and m_{su} is the mass parameter which is non-zero for non-strange baryons and zero for nucleon and delta. The effective strange quark mass is given by $m_{su} + m_u/\chi(0)$. The sum is over all flavor and $\kappa(\chi) = \chi^4$ is the dielectric functional through which the scalar field couples to the gluon field. The strong coupling constant $\alpha_s = g_s^2/4\pi$ and σ_v is the parameter related to the dielectric field mass (glueball mass). The scalar fields interact among themselves non-linearly through the scalar potential,

$$U(\chi) = B\left[\alpha\chi^2 - 2(\alpha - 2)\chi^3 + (\alpha - 3)\chi^4\right], \quad (2)$$

where $U(\chi)$ has two minima, one at $\chi = 0$, the absolute minimum and the other at $\chi = 1$, the local minimum. The energy density difference between these two minima is the bag constant $\mathbf{B}$, the parameter α determines the height of the potential and its variation does not change the result very much [8]. So we choose it to be 36 here. The glueball mass is given as $m_{\rm GB} = \partial^2 U(\chi)/\partial\chi^2|_{\chi=0}$. The behaviour of the dielectric field is such that it confines the quark and the dielectric field simultaneously in the region where $\chi > 0$. As in this model the dielectric field takes care of the long range order effect of the QCD vacuum, the gluon contribution is treated parturbative. Also we consider only one gluon exchange to avoid double counting, which is already included through the scalar field χ.

The mean-field (MF) solutions to the CDM are localized and do not correspond to the momentum eigenstates [8, 9, 14]. The localized states contain spurious center-of-mass energy and center-of-mass fluctuational motion. The center-of-mass energy adds to the total energy of the soliton. So we use Peierls-Yoccoz momentum projection technique [18] to project onto good momentum eigenstate and the finite momentum eigen state is used for the calculation of baryon magnetic moments. This technique has been used in MIT bag model [19], Friedberg-Lee soliton model [20] and also in CDM [11, 13].

In order to construct states of good momentum we need a quantum description of the dielectric field, which is related to MF solution. The simplest wave function which maintains many features of the MFA is a "coherent state." The coherent state is defined as

$$|\sigma\rangle = \exp\left[\int d^3k\,\sqrt{\frac{\omega_k}{2}}\,f_k a_k^\dagger\right]|0\rangle. \tag{3}$$

We choose the expectation value of the dielectric field to coincide with the MF value *i.e.*

$$\begin{aligned}\langle\chi\rangle &= \frac{\langle\sigma|\hat{\chi}|\sigma\rangle}{\langle\sigma|\sigma\rangle}\\ &= \frac{1}{(2\pi)^{3/2}}\int d^3k\,\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} f_k\\ &= \chi(\mathbf{r}).\end{aligned} \tag{4}$$

Using PY projection a baryon state with finite momentum $\mathbf{P}$ is given as

$$|\mathbf{P}\rangle = \int d^3\,\mathbf{X}\mathrm{e}^{i\mathbf{P}\cdot\mathbf{X}}|\mathbf{X}\rangle, \tag{5}$$

where $|\mathbf{X}\rangle$ is a localized baryon state centered at $\mathbf{X}$ and it has the form

$$|\mathbf{X}\rangle = \exp\left[\int d^3k\,\sqrt{\frac{\omega_k}{2}}\,f_{\mathbf{k}}(\mathbf{X})a_{\mathbf{k}}^\dagger\right] \times b_1^\dagger(\mathbf{X})b_2^\dagger(\mathbf{X})b_3^\dagger(\mathbf{X})|0\rangle. \tag{6}$$

The creation operator for scalar field (a_k^+) and the quark fields (b_i^+) are centered on $\mathbf{X}$. The exponential part in Eq. (6) is the coherent state for the dielectric field. Then the expectation value of an operator $\widehat{O}$ in finite momentum state is

$$\langle O\rangle = \frac{\langle\mathbf{P}|\widehat{O}|\mathbf{P}\rangle}{\langle\mathbf{P}|\mathbf{P}\rangle}, \tag{7}$$

provided O is translational invariant.

3. Magnetic moment

The baryon electromagnetic current in terms of the quark current is given as

$$\begin{aligned}J_\mu(x) &= \sum_i Q_i\bar{\psi}_i\gamma_\mu\psi_i\\ &= \sum_i e\left(\frac{\lambda_3}{2}+\frac{\lambda_8}{\sqrt{3}}\right)_i\bar{\psi}_i\gamma_\mu\psi_i.\end{aligned} \tag{8}$$

By using the Eq. (7), the magnetic moment is given as

$$\frac{\left\langle\frac{\mathbf{P}}{2}\middle|J_i(0)\middle|-\frac{\mathbf{P}}{2}\right\rangle}{\left\langle\frac{\mathbf{P}}{2}\middle|\frac{\mathbf{P}}{2}\right\rangle} = i(\mathbf{P}\times\mu)+\mathcal{O}(\mathbf{P}^2). \tag{9}$$

where μ is the magnetic moment operator. Here we work in the Breit frame (brick wall) so that the magnetic moment is free of longitudinal current. Expanding the L.H.S. of Eq. (9), keeping only the first order in $\mathbf{P}$ and comparing with the R.H.S., the z component of the magnetic moment will be

$$\mu_z = \frac{\int d^3Z\left\langle\frac{1}{2}\mathbf{Z}\middle|\frac{1}{2}\int d^3r\,\left[\mathbf{r}\times\mathbf{J}(\mathbf{r})\right]_z\middle|\frac{1}{2}\mathbf{Z}\right\rangle}{\int d^3Z\left\langle-\frac{1}{2}\mathbf{Z}\middle|\frac{1}{2}\mathbf{Z}\right\rangle}. \tag{10}$$

With further simplification this will give

$$\mu_z^B = \frac{\int d^3Z\left\langle-\frac{Z}{2},B\middle|\frac{1}{3}\int\sum_i Q_i\sigma_{zi}\left[r^2\mathcal{G}_i(r_+,r_-)-\left(\frac{\vec{r}\cdot\mathbf{Z}}{2}\right)\mathcal{F}_i(r_+,r_-)\right]d^3r\middle|\frac{Z}{2},B\right\rangle}{\int d^3Z\left\langle-\frac{Z}{2},B\middle|\frac{Z}{2},B\right\rangle}, \tag{11}$$

where

$$\mathcal{G}_i(r_+,r_-) = \frac{g_i(\mathbf{r}_+)f_i(\mathbf{r}_-)}{|\mathbf{r}_-|}+\frac{g_i(\mathbf{r}_-)f_i(\mathbf{r}_+)}{|\mathbf{r}_+|}, \tag{12}$$

TABLE I. The decuplet baryon magnetic moments for the three parameter sets are shown here. The NRQM and CBM results are from the Ref, 5. For CBM we have considered only the result for $R = 1$ fm. The lattice result is from Ref. 7. The experimental value of $\mu_{\Delta^{++}} = 4.52 \pm 0.50$ and $\mu_{\Omega^-} = -1.94 \pm 0.17$. All the magnetic moments are in units of nuclear magneton.

	$\mu_{\Delta^{++}}$	$\mu_{\Delta^{+}}$	$\mu_{\Delta^{0}}$	$\mu_{\Delta^{-}}$	$\mu_{\Sigma^{*+}}$	$\mu_{\Sigma^{*0}}$	$\mu_{\Sigma^{*-}}$	$\mu_{\Xi^{*0}}$	$\mu_{\Xi^{*-}}$	$\mu_{\Omega^{-}}$
A	4.63	2.32	0.0	−2.32	2.54	0.23	−2.08	0.46	−1.84	−1.61
B	4.54	2.27	0.0	−2.27	2.47	0.22	−2.04	0.43	−1.82	−1.60
C	3.44	1.72	0.0	−1.72	1.84	0.13	−1.59	0.25	−1.45	−1.32
NRQM	5.58	2.79	0.0	−2.79	3.11	0.32	−2.47	0.64	−2.15	1.83
CBM	3.78	1.65	−0.49	−2.63	2.78	−0.11	−2.52	0.60	−2.26	−1.57
Skyrme	4.53	2.09	−0.36	−2.80	2.55	−0.02	−2.60	0.40	−2.31	−1.98
Lattice	4.91	2.46	0.0	−2.46	2.55	0.27	−2.02	0.46	−1.68	−1.40

and

$$\mathcal{F}_i(r_+, r_-) = \frac{g_i(\mathbf{r}_+) f_i(\mathbf{r}_-)}{|\mathbf{r}_-|} - \frac{g_i(\mathbf{r}_-) f_i(\mathbf{r})}{|\mathbf{r}_+|}; \qquad (13)$$

with

$$\mathbf{r}_+ = r + \frac{Z}{2}, \qquad (14)$$

and

$$\mathbf{r}_- = r - \frac{Z}{2} \qquad (15)$$

For $Z = 0$, the Eq. (11) reduces to the MF result. Using the spin-flavor wave functions of the SU(3) baryon octet and decuplet the magnetic moments can be calculated. We use the Variation Before Projection (VBP) method [9] to calculate the baryon properties. In this method we use the mean field solutions of the quark and dielectric fields to calculate the static properties. But in principle the energy of the system should be minimized with respect to these solutions. However a better approach is the Variation After Projection (VAP) method $[11, 13]$, where variation of the quark and the dielectric field solutions are needed to minimize the energy. But in VAP method we need to solve a set of equations which are complicated and time consuming. Apart from that extending this procedure to SU(3) baryonic sector is too complicated. So we choose VBP method to treat the momentum projection in our calculation. Earlier he wave used VBP method to calculate the baryon masses and charge radii in color dielectric model [9]. Of course it is important to take the mesonic correction into account to have a complete calculation and we have left this for a future work.

4. Results

The parameters in the model are m_u, m_{su}, $B^{1/4}$, $\alpha_{\rm s}$ and $m_{\rm GB}$ and the numerical procedure is as follows. We solve the equations of motion for quark and dielectric field self-consistently with a parameter set m_u, $B^{1/4}$ and $m_{\rm GB}$. The color-magnetic energy is included perturbatively. The strong coupling constant is adjusted to fit the nucleon-delta masses, as in this case the nucleon- delta splitting comes from the gluonic contribution. After that we fit the strange baryon masses for different values of m_{su}. We calculate the baryon masses for different parameter sets to get the average fit [8]. So once the masses are calculated, the parameters in the model are fixed. Thus for magnetic moment or any other static properties calculation we need the solutions of the quark and dielectric fields as is shown in the calculation of the magnetic moments from the Eq. (11).

By using the same momentum projection technique for calculating the baryonic static properties, we have shown that the mass spectrum of the octet and decuplet baryons are in good agreement with the observed values [9]. Our previous works on this model for baryon static properties calculation also had put constraints on the parameters in the model $[8, 9, 13]$. So in this work we consider the parameters for which we obtain resonablely good fit for the mass as well as other static properties. To study the decuplet magnetic moments we have considered the following parameter sets:

Set A:

$$m_u = 70.0\,\text{MeV}, \qquad B^{1/4} = 88.0\,\text{MeV},$$
$$m_{\rm GB} = 875.0\,\text{MeV}, \qquad m_{\rm s} = 323.0\,\text{MeV}.$$

Set B:

$$m_u = 87.0\,\text{MeV}, \qquad B^{1/4} = 87.0\,\text{MeV},$$
$$m_{\rm GB} = 970.0\,\text{MeV}, \qquad m_{\rm s} = 327.0\,\text{MeV}.$$

Set C:

$$m_u = 125.0\,\text{MeV}, \qquad B^{1/4} = 125.0\,\text{MeV},$$
$$m_{\rm GB} = 2788.0\,\text{MeV}, \qquad m_{\rm s} = 358.0\,\text{MeV}.$$

The numerical results of our calculation for the above three parameter sets are shown in Table I. Also we have calculated the magnetic moments of the decuplet baryons for a wide range variation of parameters. It has been observed that, in this model the magnetic moments of the octet baryons are less than the observed values [20].

Our calculation shows that, the decuplet magnetic moments are in good agreement with the lattice calculation [7] for up(down) quark mass and $B^{1/4}$ about 100 MeV and glueball mass of about 1000 MeV as shown in the table for sets A and B. Better fitting of the baryon masses also prefer the same range of the above parameters. On the other hand it is clear from the Table I (set C) that for large quark mass, bag constant and the glueball mass, the comparison with lattice result is not good. Also the mass fitting for this parameter is bad. Similar results are obtained for very small quark mass. For the sets A and B, the magnetic moment of Δ^{++} is in better agreement with the observed value. On the other hand in CDM we obtain the ratio $\mu_{\Delta^{++}}/\mu_p = 2$, which is much larger than the observed value. This is because in this case we have $\mu_{\Delta^{++}} = 2\mu_p$ same like non-relativistic quark model (NRQM) [5]. We consider the SU(2) isospin symmetric case so the Δ^0 magnetic moment is zero. The Ω^- magnetic moment calculation for sets A and B shows that the ratio $\mu_{\Omega^-}/\mu_p = 0.70$, is in good agreement with the observed value. Also the Ω^- magnetic moment for sets A and B are close to the observed value. The comparison of CDM prediction with NRQM and cloudy bag model (CBM) [5] shows that, the NRQM predictions are much higher than CDM and the CBM prediction is below CDM . We also show the result of the Skyrme model [6]. The Σ^{*0} magnetic moment in CBM and Skyrme model is negative, whereas we find a posttive contribution which is consistent with the NRQM and lattice prediction [7]. We hope that inclusion of mesonic correction will improve the result.

5. Conclusions

We have extended the CDM to calculate the decuplet baryon magnetic moments. It shows that for certain class of parameters our result is comparable with the lattice calculation. Also the Δ^{++} and Ω^- magnetic moments are close to the observed values. It predict correctly the ratio of the Ω to proton magnetic moments.

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