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ALP-PHOTON INTERACTION IN THE STELLAR ENVIRONMENT

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ABSTRACT

The spin zero, very light bosons, like the pseudoscalar axion, are collectively grouped into the term axion-like particle (ALP). These elusive particles qualify as candidates for dark matter. ALPs also show their presence in higher dimensional theories, as K.K. particles in Kaluza Klein theory, moduli in string theory and chameleons in cosmology. They can interact with the photon via dimension-five coupling. In this text we provide an alternative method for investigating the signatures of the dark matter candidate particles (i.e., ALP) by estimating the expressions of the probabilities of conversion between different degrees of freedom of interacting photons into ALP and into each other in presence of a stellar magnetized background. We have also tried to explain the contribution of pseudoscalar ALP-photon mixing to the luminosity function of stars like red giants, supergiants, white dwarfs, etc.

RESUMEN

Los bosones ligeros de espín cero, como el axión seudoescalar, se agrupan con el término “partículas similares al axión (ALP)”. Estas partículas son candidatos para explicar la materia oscura. Los ALP muestran su presencia en teorías de muchas dimensiones, como partículas K.K. en la teoría de Kaluza-Klein, como módulos en la teoría de cuerdas o como camaleones en cosmología. Pueden interactuar con los fotones mediante un acoplamiento de dimensión 5. Presentamos un método alternativo para investigar las señales de los ALP como candidatos a materia oscura, estimando las expresiones para la probabilidad de conversión entre varios grados de libertad de un fotón en interacción con un ALP o de fotones entre sí, en presencia de un fondo interestelar magnetizado. Intentamos explicar la contribución de la mezcla seudoescalar ALP-fotón a la función de luminosidad de las gigantes rojas, supergigantes, o enanas blancas.

Key Words: galaxies: photometry — methods: numerical — white dwarfs

1. INTRODUCTION

For quite some time now the evidences in favour of the existence of dark matter (DM) have been mounting from astrophysical and cosmological observations (e.g. flatness of galaxy rotation curves or the light spectrum from bullet clusters etc.). According to the current understanding, nearly twenty two percent of the matter content of the universe is in the form of DM, that needs proper identification. There are many candidate particles to explain it, like neutrino, millicharge particles, dark photons, to name a few. However, compared to them axions are more appealing because they were the ones initially postulated to cure the strong CP and the $U_A(1)$ problem of quantum chromodynamics (QCD). Owing to their origin and connection to theories of unification, as well as particle physics (Conlon & David 2013-Sikivie 2021), confirmation of their existence has remained a sought after activity in laboratories, based as well on astrophysics experiments.

Owing to their anomaly-related origin, the structure of the axion (ϕ') and the photon (γ interaction Lagrangian) when expressed in terms of the coupling constant $g_{\phi'\gamma\gamma}$ and the field strength tensor $F^{\mu\nu}$, turns out to be of the form,

$$L_{int} = \frac{1}{4} g_{\phi'\gamma\gamma} \phi' F_{\mu\nu} \tilde{F}^{\mu\nu}.$$

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For L_{int} to remain invariant under charge C conjugation, parity P and time T reversal symmetry transformations, the axion field turns out to be CP violating ($\not{C}\not{P}$) and PT preserving. As a result, in a magnetized vacuum (i.e. vacuum with an external magnetic field B), or in an unmagnetized medium, only those photons having their polarization plane parallel ($\gamma_{\|}$) to B interact with ϕ' and the rest, i.e., photons having a polarization plane orthogonal ($\gamma_{\perp}$) to B remain free. Therefore, in a magnetized vacuum or in an unmagnetized medium, only the $\gamma_{\|}$ and ϕ' undergo conversion into each other and back. Once converted, the axions residing in the medium stream out almost freely due to their extremely small cross section $\sigma \propto g^2_{\phi' e}$ (i.e., due to smallness of the $e\phi'$ coupling constant $g_{\phi' e} < 10^{-13}$ GeV^{-1} and the $\gamma\ \phi'$ coupling constant $g_{\phi'\gamma\gamma} \sim 10^{-11}$ GeV^{-1}). Thus they turn out to be ideal candidates as astrophysical cooling agents next to neutrinos ($\bar{\nu}_e$) or other agents mentioned already. Therefore, studies of axion-aided cooling physics for multiple astrophysical sources were initiated to vindicate their existence.

Sources like giants, supergiants, red giants, horizontal branch stars or white-dwarfs (WD) have shown a statistically significant amount of extra energy loss, which is difficult to explain using standard physics arguments. Their existence has been inferred from various phenomena, e.g., from the excess spin-down rate of the WDs, from their luminosity function (Gianotti et al. 2016) ($L_\gamma = C_\gamma L_{sun} T_7^{3.5}$) or from the ratio of the number of stars in the horizontal branch (HB) and the red giant branch (RGB) existing in globular clusters (Gianotti et al. 2016-Cho & Lee 2005). Labelling the ratio of these two numbers as R, it was realised that its observed value is smaller than the expected range, that is supposed to lie between 1.44 to 1.5. So these observations initiated a need to look for some extra cooling mechanisms operating in the stellar interior, so that extra channels of energy transport open up that could, in principle, compensate the mismatch between various theoretical and observational estimates.

Independently of this cooling anomaly, it was noted in earlier studies (Chaubey et al. 2024-Ganguly et al. 2009) that the standard picture of single channel $\gamma_{\|} \to \phi'$ oscillation undergoes a paradigm shift once the parity violating correction to the photon self-energy tensor (PSET) is incorporated in the effective axion-photon Lagrangian. With the incorporation of the parity violating piece, all the existing degrees of freedom of the system (i.e. photon's $\|$, $\perp$ and longitudinally (L) polarized states represented by $\gamma_{\|}, \gamma_{\perp}$ and γ_L - and axion ϕ') would get coupled with each other and hence they would be oscillating into each other. Thus, these oscillations would initiate a new ($\gamma_{\perp} - \phi'$) channel of energy loss. The goal of this study is to explore the role of this extra energy loss channel to explain the required anomalous cooling of these stars.

2. EFFECTIVE LAGRANGIAN WITH MAGNETIZED MEDIUM EFFECTS

In the interior of a compact star, pseudoscalar axions are produced abundantly due to Compton, Bremsstrahlung and Primakoff processes. The presence of a non-zero electron fraction ($Y_e > 5$), a magnetic field B and a core temperature of the order of 10^{7o} K, make an ideal physical situation for the production of axion-like particles. The extent of this magnetized medium introduces a parity violating part to the PSET that was estimated in Ganguly et al. (1999).

The pseudoscalar ALP (ϕ')-photon(γ) mixing dynamics can now be studied by employing the effective Lagrangian provided below:

$$L_{eff,\phi'} = \frac{1}{2}\phi'[k^2 - m^2_{\phi'}]\phi' - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} + \frac{1}{2}A_\mu \Pi^{\mu\nu}(k,\mu,T,eB)A_\nu - \frac{1}{4}g_{\phi'\gamma\gamma}\phi'\tilde{\bar{F}}^{\mu\nu}f_{\mu\nu}, \quad (1)$$

where the term defined as $A_\mu \Pi^{\mu\nu}(k,\mu,T,eB)A_\nu$ corresponds to the photon self energy correction to the effective Lagrangian of the pseudoscalar ALP-photon system, $g_{\phi'\gamma\gamma}$ is the coupling constant of pseudoscalar ALP-photon mixing. The other terms have their usual meanings as found in the literature.

3. EQUATIONS OF MOTION

The equations of motion of an ALP-photon interacting system are obtained by using the standard variational principle. They can be expressed in terms of the form factors ($\gamma_{\|}, \gamma_{\perp}, \gamma_L$) of the gauge potential A_μ in the orthonormal basis vectors $\hat{b}^{(1)\nu} = k_\mu \bar{F}^{\mu\nu}$, $\hat{I}^\nu = (b^{2\nu} - \frac{\tilde{u}b^{(2)}}{\tilde{u}^2}\tilde{u}^\nu)$, $\hat{\tilde{u}}^\nu = (g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2})u_\mu$, $b^{(2)\nu} = k_\mu \tilde{\bar{F}}^{\mu\nu}$ constructed out of the available four vectors and tensors of the system. In matrix form, they can be written as:

$$\left[(\omega^2 + \partial_z^2)\mathrm{I} - \mathrm{M}\right] \begin{bmatrix} \gamma_{\|}(\omega,z) \\ \gamma_{\perp}(\omega,z) \\ \phi'(\omega,z) \\ \gamma_L(\omega,z) \end{bmatrix} = 0, \quad (2)$$

where I is an identity matrix and matrix M is the 4×4 mixing matrix. The symbols $\|$ and $\perp$ correspond to plane parallel and perpendicular to the direction of the magnetic field.

The mixing matrix M in terms of the newly defined variables $\Pi_L = \omega_p^2\left(1-\frac{\omega_p^2}{\omega^2}\right)$, $G = g_{\phi'\gamma\gamma}\omega B\sin(\pi/4)$, $F = \omega_B\omega_p^2\cos(\pi/4)/\omega$ and $L = g_{\phi'\gamma\gamma}\omega_p B\sin(\pi/4)$ can be cast in the following form:

$$\mathrm{M} = \begin{bmatrix} \omega_p^2 & -iF & 0 & 0 \\ iF & \omega_p^2 & iG & 0 \\ 0 & -iG & m_{\phi'}^2 & -iL \\ 0 & 0 & iL & \Pi_L \end{bmatrix}. \qquad (3)$$

Here ω_p is the plasma frequency of the medium. In order to get the dynamics of the available degrees of freedom of the system, the mixing matrix M has been diagonalized by a unitary transformation $\mathbf{U}^\dagger\mathrm{M}\mathbf{U} = M_D$ (see Chaubey et al. 2024). The matrix M_D is the diagonalized matrix that contains four eigenvalues λ_1, λ_2, λ_3, and λ_4 of the mixing matrix M. Matrix $\mathbf{U}$, given by:

$$\mathbf{U} = \begin{bmatrix} \hat{u}_1 & \hat{u}_2 & \hat{u}_3 & \hat{u}_4 \\ \hat{v}_1 & \hat{v}_2 & \hat{v}_3 & \hat{v}_4 \\ \hat{w}_1 & \hat{w}_2 & \hat{w}_3 & \hat{w}_4 \\ \hat{x}_1 & \hat{x}_2 & \hat{x}_3 & \hat{x}_4 \end{bmatrix}, \qquad (4)$$

is the unitary matrix constructed from the eigenvectors of matrix M. The elements (for $i = 1, 2, 3, 4$) are as follows:

$$\hat{u}_i = \big[(\omega_P^2-\lambda_i)(m_{\phi'}^2-\lambda_i)(\Pi_L-\lambda_i) - (\omega_P^2-\lambda_i)(L)^2 - (\Pi_L-\lambda_i)(G)^2\big]\times N_i, \qquad (5)$$

$$\hat{v}_i = \big[(\omega_P^2-\lambda_i)[(m_{\phi'}^2-\lambda_i)(\Pi_L-\lambda_i) - (L)^2]\big]\times N_i, \qquad (6)$$

$$\hat{w}_i = \big[(\Pi_L-\lambda_i)[(\omega_P^2-\lambda_i)(\omega_P^2-\lambda_i) - (F)^2]\big]\times N_i, \qquad (7)$$

$$\hat{x}_i = \big[(\omega_P^2-\lambda_1)(\omega_P^2-\lambda_i)(m_{\phi'}^2-\lambda_i) - (\omega_P^2-\lambda_i)(G)^2 - (m_{\phi'}^2-\lambda_i)(F)^2\big]\times N_i. \qquad (8)$$

Here N_i's are the normalization constants, which can be obtained by the expression $N_i = \frac{1}{\sqrt{u_i^2+v_i^2+w_i^2+x_i^2}}$.

The mixing pattern that leads to the mixing matrix M obtained in eq. (3) from the effective Lagrangian given in eq. (1) can be understood in the following way. Due to the presence of PSET in L_{eff}, the two transverse degrees of freedom of the photon (i.e., $\gamma_\|$ and $\gamma_\perp$) would mix with each other. Next, due to the presence of the tree level interaction Lagrangian L_{int}(in an external magnetic field) the $\perp$ component of photons and the one due to the unmagnetized medium effect: the longitudinal component of photon mix with ϕ'. Thus, all four degrees of freedom of the ALP-photon system mix with each other. These mixings are represented by: $\gamma_\| \to \phi'$, $\gamma_\perp \to \phi'$, $\gamma_L \to \phi'$, $\gamma_\perp \to \gamma_L$, $\gamma_\| \to \gamma_\perp$ and $\gamma_\perp \to \gamma_L$. The evaluation of probabilities of these mixings is provided in the following section.

4. PROBABILITIES OF CONVERSIONS

The amplitude of the evolution of a state corresponding to one degree of freedom [say $|\gamma_\perp(\omega,0)\rangle$] of an ALP-photon system into another (say $|\phi'(\omega,z)\rangle$) with respect to the photon path length z is obtained by $\langle\gamma_\perp(\omega,0)|\phi'(\omega,z)\rangle$. The probability of this evolution is estimated from the square of the modulus of the obtained amplitude, that is given by:

$$P_{\gamma_\perp\to\phi'} = |\langle\gamma_\perp(\omega,0)|\phi'(\omega,z)\rangle|^2. \qquad (9)$$

Since due to the photon self-energy correction introduced in the effective Lagrangian, the pseudoscalar ALP-photon system has four physical degrees of freedom (i.e., $\gamma_\perp$, $\gamma_\|$, γ_L and ϕ') that can produce C $\binom{4}{2=6}$ number of combinations of such evolution amplitudes, and hence the corresponding oscillation probabilities represented by: $P_{\gamma_\perp\to\phi'}$ for oscillation between perpendicularly polarized photon to ALP, $P_{\gamma_\|\to\phi'}$ for oscillation between parallel polarized photon to ALP, $P_{\gamma_L\to\phi'}$ for oscillation between longitudinally polarized photon to ALP, $P_{\gamma_\|\to\perp}$ for oscillation between parallel polarized photon to perpendicularly polarized photon, $P_{\gamma_\perp\to\gamma_L}$ for oscillation between perpendicularly polarized photon to longitudinally polarized photon and lastly $P_{\gamma_\|\to\gamma_L}$ for oscillation between parallel polarized photon to longitudinally polarized photon - are generated.

Using the exact solutions of the equations of motion for an ALP-photon system, the full expressions of the probabilities of conversion can be written as follows.

Defining the variables $A = |\hat{v}_1|\hat{w}_1|$, $B = |\hat{v}_2|\hat{w}_2|$, $C = |\hat{v}_3|\hat{w}_3|$ and $D = |\hat{v}_4|\hat{w}_4|$ from eqns. (5)-(8), the probability of conversion of the perpendicular polarization state of the photon $\gamma_\perp$ into the pseudoscalar axion ϕ' turns out to be:

$$\begin{aligned} P_{\gamma_\perp\to\phi'} = {} & (A^2+B^2+C^2+D^2) + 2AB\cos((\Omega_\|-\Omega_\perp)z) \\ & + 2BC\cos((\Omega_\perp-\Omega_{\phi'})z) + 2CD\cos((\Omega_{\phi'}-\Omega_L)z) \\ & + 2AC\cos((\Omega_\|-\Omega_{\phi'})z) + 2BD\cos((\Omega_\perp-\Omega_L)z) \\ & + 2AD\cos((\Omega_\|-\Omega_L)z), \end{aligned} \qquad (10)$$

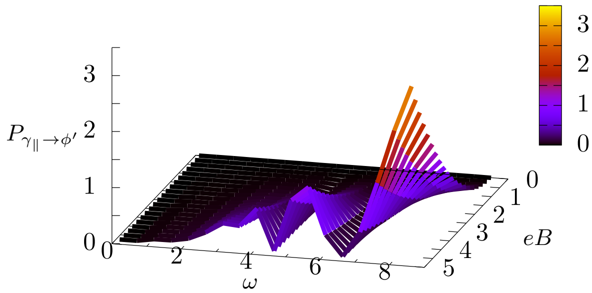

Fig. 1. Plot of oscillation probability $P_{\gamma_{\parallel}\to\phi'}$ versus photon energy ω (in units of 10^{-5} GeV) and magnetic field eB (in units of 10^{11} Gauss). The probability $P_{\gamma_{\parallel}\to\phi'}$ has been scaled by the factor 10^{+7}. Plasma frequency ω_p is taken to be $\approx 10^{-10}$ GeV, mass of axion $m_{\phi'} \approx 10^{-11}$ GeV and coupling constant $g_{\phi'\gamma\gamma} \approx 10^{-11}$ GeV^{-1}. The colour figure can be viewed online.

where the variables $\Omega_{\parallel}$, $\Omega_{\perp}$, Ω_L and $\Omega_{\phi'}$ in terms of photon energy ω are defined as:

$$\Omega_{\parallel} = \left(\omega - \frac{\lambda_1}{2\omega}\right),\ \Omega_{\perp} = \left(\omega - \frac{\lambda_2}{2\omega}\right),$$
$$\Omega_{\phi'} = \left(\omega - \frac{\lambda_3}{2\omega}\right) \text{ and } \Omega_L = \left(\omega - \frac{\lambda_4}{2\omega}\right). \quad (11)$$

It is to be noted that $P_{\gamma_{\perp}\to\phi'}$ is the only probability (out of six) that survives in absence of any background medium. The other probabilities would vanish.

The oscillation between the parallel polarization state of photon into the pseudoscalar ALP is possible due to the presence of PSET introduced due to magnetized medium effects in the effective Lagrangian. We have numerically evaluated and show in Figure 1 the dependence of it on the magnetic field strength eB and photon frequency ω in the $1-10$KeV range.

Defining a new set of variables, $E = |\hat{u}_1|\hat{w}_1|$, $F = |\hat{u}_2|\hat{w}_2|$, $G = |\hat{u}_3|\hat{w}_3|$ and $H = |\hat{u}_4|\hat{w}_4|$ the probability of finding the ALP ϕ' evolved from the parallel polarization state of photon $\gamma_{\parallel}$ after travelling a distance z turns out to be:

$$\begin{aligned} P_{\gamma_{\parallel}\to\phi'} = & (E^2+F^2+G^2+H^2)+2EF\cos((\Omega_{\parallel}-\Omega_{\perp})z) \\ & +2FG\cos((\Omega_{\perp}-\Omega_{\phi'})z)+2GH\cos((\Omega_{\phi'}-\Omega_L)z) \\ & +2EG\cos((\Omega_{\parallel}-\Omega_{\phi'})z)+2FH\cos((\Omega_{\perp}-\Omega_L)z) \\ & +2EH\cos((\Omega_{\parallel}-\Omega_L)z). \end{aligned} \quad (12)$$

The unique feature of the presence of a medium is the activation of the longitudinal component of the photon. This interacts directly with the pseudoscalar ALP at the tree level of the effective Lagrangian. Defining the variables $I = |\hat{x}_1|\hat{w}_1|$, $J = |\hat{x}_2|\hat{w}_2|$, $K = |\hat{x}_3|\hat{w}_3|$ and $L = |\hat{x}_4|\hat{w}_4|$, the probability of conversion of the longitudinal polarization state of photon into an axion or pseudoscalar ALP can be written in terms of the variables defined above as:

$$\begin{aligned} P_{\gamma_L\to\phi'} = & (I^2+J^2+K^2+L^2)+2IJ\cos((\Omega_{\parallel}-\Omega_{\perp})z) \\ & +2JK\cos((\Omega_{\perp}-\Omega_{\phi'})z)+2KL\cos((\Omega_{\phi'}-\Omega_L)z) \\ & +2IK\cos((\Omega_{\parallel}-\Omega_{\phi'})z)+2JL\cos((\Omega_{\perp}-\Omega_L)z) \\ & +2IL\cos((\Omega_{\parallel}-\Omega_L)z). \end{aligned} \quad (13)$$

It needs to be emphasised here that this probability of conversion is absent in the case of scalar ALP-photon interaction, and when the mixing background of ALP(scalar or pseudoscalar)-photon is a vacuum.

The other three oscillations happening between the photon (which is initially in a particular polarization state) and the photon of a different polarization state after travelling a path of length z, denoted by $\gamma_{\parallel} \to \gamma_{\perp}$, $\gamma_{\parallel} \to \gamma_L$ and $\gamma_{\perp} \to \gamma_L$ - imply a transformation in the plane of the photon's polarization state. The expressions of their oscillation probabilities turn out to be:

$$\begin{aligned} P_{\gamma_{\parallel}\to\gamma_{\perp}} = & (M^2+N^2+O^2+P^2)+2MN\cos((\Omega_{\parallel}-\Omega_{\perp})z) \\ & +2NO\cos((\Omega_{\perp}-\Omega_{\phi'})z)+2OP\cos((\Omega_{\phi'}-\Omega_L)z) \\ & +2MO\cos((\Omega_{\parallel}-\Omega_{\phi'})z)+2NP\cos((\Omega_{\perp}-\Omega_L)z) \\ & +2MP\cos((\Omega_{\parallel}-\Omega_L)z), \end{aligned} \quad (14)$$

$$\begin{aligned} P_{\gamma_{\parallel}\to\gamma_L} = & (Q^2+R^2+S^2+T^2)+2QR\cos((\Omega_{\parallel}-\Omega_{\perp})z) \\ & +2RS\cos((\Omega_{\perp}-\Omega_{\phi'})z)+2ST\cos((\Omega_{\phi'}-\Omega_L)z) \\ & +2QS\cos((\Omega_{\parallel}-\Omega_{\phi'})z)+2RT\cos((\Omega_{\perp}-\Omega_L)z) \\ & +2QT\cos((\Omega_{\parallel}-\Omega_L)z), \end{aligned} \quad (15)$$

$$\begin{aligned} P_{\gamma_{\perp}\to\gamma_L} = & (U^2+V^2+W^2+X^2)+2UV\cos((\Omega_{\parallel}-\Omega_{\perp})z) \\ & +2VW\cos((\Omega_{\perp}-\Omega_{\phi'})z)+2WX\cos((\Omega_{\phi'}-\Omega_L)z) \\ & +2UW\cos((\Omega_{\parallel}-\Omega_{\phi'})z)+2VX\cos((\Omega_{\perp}-\Omega_L)z) \\ & +2UX\cos((\Omega_{\parallel}-\Omega_L)z), \end{aligned} \quad (16)$$

where we have defined the variables (in terms of variables present in eqns. 5-8) as follows $M = |\hat{u}_1|\hat{v}_1|$, $N = |\hat{u}_2|\hat{v}_2|$, $O = |\hat{u}_3|\hat{v}_3|$ and $P = |\hat{u}_4|\hat{v}_4|$, $Q = |\hat{u}_1|\hat{x}_1|$, $R = |\hat{u}_2|\hat{x}_2|$, $S = |\hat{u}_3|\hat{x}_3|$ and $T = |\hat{u}_4|\hat{x}_4|$, $U = |\hat{v}_1|\hat{x}_1|$, $V = |\hat{v}_2|\hat{x}_2|$, $W = |\hat{v}_3|\hat{x}_3|$ and $X =$

$|\hat{v}_4|\hat{x}_4|$. The nonzero magnitudes of the last three probabilities verify the prominent contributions of magnetized medium effects on the ALP-photon oscillation.

5. ASTROPHYSICAL APPLICATIONS AND CONCLUSION

To conclude, in this work we noted that the hot dense core of WDs is capable of providing a suitable environment for the photon axion conversion due to the presence of a non-zero fraction of charged fermions and He ions. Axions produced in this environment, would stream out from the core of the WDs carrying a non-trivial amount of energy from the WD interior.

These streaming axion-flux from the core of WDs would contribute not only to the cooling of the star but in turn may also affect the same in two ways. First, by a reduction of mass, hence increasing the extent of the radius (since they are connected by the mass radius relationship). Hence the star period is likely to slow down. G117-B15A, a WD, has been noted to spin down at a rather fast rate, $\dot{P} \approx (12.0 \pm 3.5) \times 10^{-15}$sec. (sec.)$^{-1}$ (c.f. Gianotti et al. 2016 and references therein). If the same happens due to axionic energy loss, it would leave open possibilities of explaining other phenomena (like the electromagnetic torque decay) due to axion physics.

Second. A change in the effective surface temperature of the star, because the total luminosity ($L_{tot} = L_{photon} + L_{neutrinos}$) would undergo a modification through axion luminosity (L_{axion}). As a result, the characteristic cooling time ($\tau_{chr} \propto L_{tot}^{-1}$) of the star would be modified. Therefore one may expect to see changes in the luminosity distribution curve obtained from the expressions provided in Isern et al. (2018).

In this work we have presented some of the possible ways that the magnetized media present in the environment of any compact star can render some observable effects, prominently due to axion mediated interactions, those predicted in the past. We have numerically shown that the presence of magnetized media causes oscillations between the axion and the parallel component of the photon, which are absent when the effect of a magnetized medium is not considered. The amplitude of the probability of this oscillation increases with an increase in eB and ω (see Figure 1).

We expect that, with the availability of more observational data, the significance level of some of the past predictions can be improved.

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