A STUDY OF THE LONG TERM BEHAVIOR OF THE SX PHE STAR KZ HYA[1]

J. H. Peña[2,3,4], D. S. Piña[2,4], A. Rentería[2,4], C. Villarreal[2,4], A. A. Soni[2,4], J. Guillen[2,4], and J. Calderón[2,4]

ABSTRACT

From the newly determined times of maximum light of the SX Phe star KZ Hya and others from the literature, as well as from $uvby - \beta$ photoelectric photometry, we determined the nature of this star and its physical parameters.

RESUMEN

A partir de los recién determinados tiempos de máximo en fotometría de la estrella SX Phe KZ Hya y una recopilación de tiempos de máximo de la literatura, así como datos con fotométria $uvby - \beta$, hemos determinado la naturaleza de esta estrella y sus parámetros físicos.

Key Words: binaries: close — stars: variables: delta Scuti — techniques: photometric

1. INTRODUCTION

Although KZ Hya (= HD 94033 = CD -24 9357) has been extensively studied and its nature well-established, the last thorough study of this star was that of Boonyarak et al. (2011). It is a well-known fact that this type of star needs continuous monitoring in order to deduce its nature correctly. In view of that, we present recent observations that, as will be seen, confirm the proposed behavior.

There has been some controversy with respect to the SX Phe, a high-amplitude Delta Scuti (HADS) star KZ Hya. Yang et al. (1985) suggested that there is a small amplitude light time effect and this was confirmed by Fu et al. (2008) with a much larger amplitude and longer period. Therefore, as has been said, it is understood that the light time effect with a short period and a small amplitude was occasionally covering the most changeable part of the light time effect of longer period and larger amplitude. Hence it can be seen that the observation time span should be long enough to confirm the light time effect. In a later study, the conclusions of Boonyarak et al. (2011) differed drastically from this interpretation. With one fundamental mode and its harmonics, they

explained the observed light curves well without invoking the contribution of any companions. Hence, they confirmed that KZ Hya is a single-mode SX Phe star.

With a longer time span we expect to be able to dissipate any doubts on this matter.

2. OBSERVATIONS

Although some of the times of maximum light of this star have been reported elsewhere (Peña et al, 2015), we present new times of maxima and the detailed procedure followed to acquire the data. The observations were done at both the Observatorio Astronómico Nacional of San Pedro Mártir (SPM) and Tonantzintla (TNT), in México. Table 1 presents the log of observations as well as the new times of maximum light. Column 1 presents the date of the observation; Column 2 specifies the observer or group of observers. These correspond to the Advanced Observational Astronomy (AOA) course in 2015, and Escuela de Astronomía Observacional para Estudiantes de LatinoAmerica (ESAOBELA, LatinAmerican School of Observational Astronomy) in 2016 and 2017; the subsequent columns list the number of points, the time span in days and the number of maxima in each night; the time of maximum in HJD minus 2400000.0. The final columns list the technical details, telescope, filter and camera. The last column indicates in which observatory the data were acquired.

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[2]Instituto de Astronomía, Universidad Nacional Autónoma de México, Cd. México.

[3]Observatorio Astronómico Nacional, Tonantzintla, México.

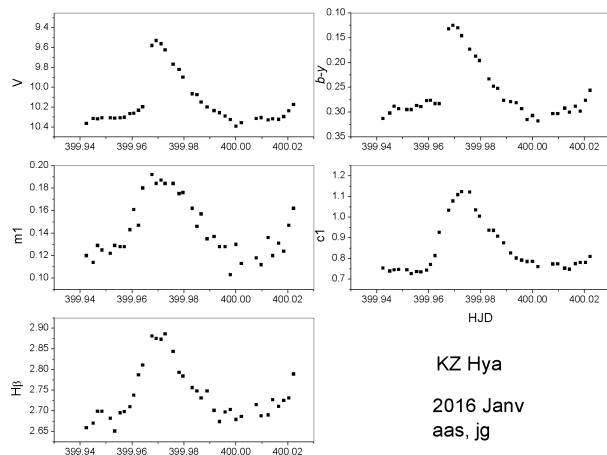[4]Facultad de Ciencias, Universidad Nacional Autónoma de México, México.

Fig. 1. Light curves of KZ Hya obtained in $uvby - \beta$ absolute photometry.

2.1. *The Data Acquisition and Reduction at Tonantzintla*

At the TNT Observatory two 10-inch Meade telescopes were utilized (denoted by m1 and m2). These telescopes were equipped with CCD cameras and the observations were done using V and G filters. During all the observational nights this procedure was followed. The integration time was 1 min. There were around 11,000 counts, enough to secure high precision. The reduction work was done with AstroImageJ (Collins, 2012). This software is relatively easy to use and has the advantage that it is free and works satisfactorily on the most common computing platforms. With the CCD photometry two reference stars were utilized, whenever possible in a differential photometry mode. The results were obtained from the difference $V_{\text{variable}} - V_{\text{reference}}$ and the scatter calculated from the difference $V_{\text{reference1}} - V_{\text{reference2}}$. Light curves were also obtained. The new times of maximum light are listed in Table 1.

2.2. *Data Acquisition and Reduction at SPM*

As was stated in Peña et al. (2016) reporting on BO Lyn, the observational pattern, as well as the reduction procedure, have been employed at the SPM Observatory since 1986 and hence, have been described many times. A detailed description of the methodology can be found in Peña et al. (2007). Over the three nights of observation the following procedure was used: each measurement consisted of at least five ten-second integrations of each star and one ten-second integration of the sky for the *uvby* filters and the narrow and wide filters that define Hβ. What must be emphasized here are the trans-

formation coefficients for the observed season (Table 7) and the season errors which were evaluated using the ninety-one observed standard stars. These uncertainties were calculated through the differences in magnitude and colors for (V, $b - y$, m_1, c_1 and Hβ), which are (0.054, 0.012, 0.019, 0.025, 0.012), for a total of 94 points in *uvby* and 68 points in Hβ, which provide a numerical evaluation of our uncertainties. Emphasis must be made on the large range of the standard stars in magnitude and color indexes: V:(5.62, 8.00); $(b-y)$:(-0.09, 0.88); m_1:(-0.09, 0.67); c_1:(-0.02, 1.32) and Hβ:(2.50, 2.90).

Table 8 lists the photometric values of the observed star. In this table Column 1 reports the time of the observation in HJD, Columns 2 to 5 the Strömgren values V, $(b - y)$, m_1 and c_1, respectively; Column 6, Hβ, whereas Columns 7 to 9 the unreddened indexes $[m_1]$, $[c_1]$ and $[u - b]$ derived from the observations. When analyzing observations obtained over such a long time base, correction to the Barycentric Julian Ephemeris Date (IAU SOFA) should be done; however, we have calculated both the barycentic and heliocentric dates and the difference is smaller than the errors. Therefore we have presented our results only in HJD. The photometry is presented in Figure 1.

3. PERIOD DETERMINATION

To determine the period behavior of KZ Hya the following methods were employed: (1) differences of consecutive times of maximum light were evaluated to determine a coarse period; (2) time series analyses of two different data sets were utilized with time series data; (3) O-C differences were calculated utilizing a compiled collection of times of maximum light; and finally (4) phase dispersion minimization of the ASAS Catalogue of Variable stars in the Kepler field served to estimate a period.

The previously determined ephemerides equations, as well as the newly determined ones are listed in Table 2.

3.1. *Differences of Consecutive Times of Maximum Light, DCTM*

In view of the disagreement about the proposed equations and since there is no description on how these proposed ephemerides were determined, we developed our own method to reproduce previous findings.

To determine the period starting from scratch, as a first guess we considered the period determined through the differences of two or three times of maxima that were observed in the same night. Since

TABLE 1

LOG OF OBSERVING SEASONS AND NEW TIMES OF MAXIMA OF KZ HYA

Date yr/mo/day	Observers	Npoints	Time span (day)	Nmax	Tmax 2400000+	Telescope	Filters	Camera	Observatory
15/03/0607	AOA15	155	0.16	1	57088.7971	m1	V	ST-1001	TNT
					57088.8565				
15/03/0708	AOA15	194	0.12	1	57089.7493	m1	V	ST-1001	TNT
15/03/3031	DSP	150	0.13	2	57112.7789	m1	V	ST-1001	TNT
					57112.8385				
16/01/1112	AAS, JG	41	0.08	1	57399.9693	0.84 m	uvbyβ		SPM
16/01/2425	ESAOBELA16	113	0.07	1	57412.8265	m2	V	ST-8300	TNT
16/03/1112	PS, KL	82	0.10	2	57459.8400	m2	V	ST-8300	TNT
					57459.8989				
16/03/1213	PS, KL	52	0.06	1	57460.7921	m2	V	ST-8300	TNT
16/03/2223	DSP	154	0.13	2	57470.7894	m1	V	ST-8300	TNT
					57470.8491				
16/03/2324	DSP	194	0.17	3	57471.7419	m1	V	ST-8300	TNT
					57471.8013				
					57471.8611				
17/01/1617	ESAOBELA17	75	0.07	1	57770.9547	m2	G	ST-8300	TNT
17/01/1819	ESAOBELA17	202	0.16	3	57772.8602	m1	V	ST-1001	TNT
					57772.9198				
					57772.9791				

Notes: DSP, D.S. Piña; AAS, A.A. Soni; JG, J. Guillén; PS, P. Santillán; KL, K. Lozano; AOA15: S. Arellano, J. Diaz, X. Moreno, J. Ramirez, F. Ruíz, C. Téllez, K. Vargas, V. Vázquez; ESAOBELA16: A. Rodríguez, V. Valera, A. Escobar, M. Agudelo, A. Osorto, J. Aguilar, R. Arango, C. Rojas, J. Gómez, J. Osorio, M. Chacón; ESAOBELA17: V. Ramírez, M. Rodríguez, S. Vargas, C. Castellón, R. Salgado, J. Mata, R. Santa Cruz, L. Gonzáles, K. Chipana, R. Rodríguez, A. García, D. de la Fuente.

TABLE 2

KZ HYA EPHEMERIS EQUATIONS

Method	T_0	P	β	$(O-C)mean$	std dev	Residuals
1 DCTM	2442516.1593	0.059399998		0.003253	0.018285	0.1633
2 Period04 ($uvby - \beta$ data)	2442516.18330	0.060742815		0.000734	0.018338	0.1565
2 Period04 (CCD data)	2442516.15850	0.059509655		0.003253	0.018285	0.0607
3 Doncel et al. (2005)	2442516.15850	0.05911157		-0.001801	0.017032	0.2200
3 Chulee Kim et al. (2007)	2442516.18525	0.059510388		0.003557	0.016428	0.2141
3 Fu et al. (2008)	2442516.1833	0.059510413		0.002379	0.016162	0.2329
3 PDDM (Tmax, linear)	2457772.8602	0.059510384		0.006624	0.016137	0.1881
3 PDDM (Tmax, quadratic)	2457772.8602	0.059510367	6.47×10^{-13}	0.000590	0.017290	0.0352
4 Kepler	2442516.1833	0.059511385		0.007421	0.015331	0.1095

they are separated by only one cycle they are, by definition, one period apart. The sample of periods determined in this fashion is constituted of 102 determined differences. The mean value was 0.0594 ± 0.0006 (d). The uncertainty is merely the standard deviation of the mean.

With this period as seed and utilizing as T_0 the first time of maximum of Przybylski & Bessel (1979), we calculated new epochs and through a linear regression of epoch vs. HJD calculated values of T_0 and P. The values we determined in this fashion (2442516.1593 d, 0.059399998 d) are in agreement

TABLE 3

OUTPUT OF PERIOD04.[*]

Nr.	Frequency	Amplitude	Phase
$F1$	16.4628524	0.303093807	0.448028
$F2$	$3.61296764 \times 10^{-5}$	0.236208293	0.926073
$F3$	32.2438773	0.124308647	0.068152

[*]With the V magnitude of McNamara and Brudge (1985) and the present paper's $uvby - \beta$ data.

with those assumed period values reported in the literature. Hence, in view of the concordant results we have verified period values reported in the literature from our basic methodology.

The results of this method are presented in the first row of Table 2.

3.2. *Time Series Analyses*

As a second method to determine the period we used a time series method amply utilized by the δ Scuti star community: Period04 (Lenz & Breger, 2005). Two data sets were analyzed with this code: (1) The combined data in $uvby - \beta$ of McNamara & Brudge (1985) with the present paper's data, and (2) the CCD data obtained at the Observatory of Tonantzintla.

In the first case, we were fortunate enough in that KZ Hya had been observed in 1978 by McNamara & Brudge (1985) so a time series analysis was possible using their data along with the observations reported in the present paper carried out in 2015. Start time is HJD 2443601.6864 and end time HJD 2457400.0222 with a total of 85 data points. The time span covered is then 13798.3358 d or 38 years which in cycles is 232295, an enormous gap during which no observations were done on this system. Combining both seasons we realized that there is an appreciable magnitude discrepancy in the mean value as well as in the amplitude. This might be because the variation is mounted on a large sinusoidal variation as some results seem to indicate that the variation is going through a possible large envelope.

The analysis of these data gave the results listed in Table 3 with a zero point of 10.1313, residuals of 0.08033 and 20 iterations. The analysis of Period04 is presented in Figure 2. At the bottom is the window function. Going upwards, the second is the periodogram of the original data. The next, the first set of residuals and finally, at the top, the residuals of the second set of residuals. The frequencies obtained from this analysis are presented in Table 3.
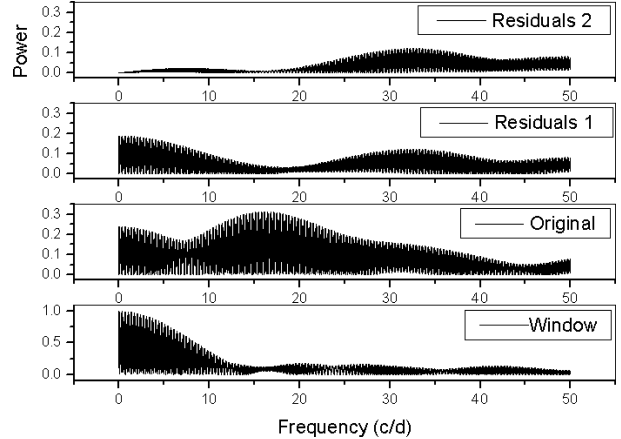

Fig. 2. The analysis of Period04 is presented. At the bottom is the window function. Going upwards, the second panel shows the periodogram of the original data. The next, the first set of residuals and finally, at the top, the residuals of the second set of residuals.

TABLE 4

OUTPUT OF PERIOD04.[*]

Nr.	Frequency	Amplitude	Phase
$F1$	16.8040003	0.324464978	0.690475
$F2 = 2F1$	33.6079961	0.13813718	0.853153
$F3 = 3F1$	50.4120215	0.0837371322	0.495932

[*]With in the V magnitude of the present paper's CCD data.

The other data set analyzed with Period04 was that of the CCD data from TNT observatory. It was homogeneously reduced to the same reference stars. As we can see in Table 1, the first observation of the star was done in March, 2015 and the last in January, 2017, separated by 685 days. The whole sample of CCD data is constituted of ten nights. The analysis gave the results listed in Table 4 with a zero point of -0.0102. If we consider $50.412021 (= 3F1)$ the residuals diminish to 0.03736.

3.3. *O-C Differences*

Before calculating the coefficients of the ephemeris equation, we studied the existing literature related to KZ Hya. Several authors have conducted studies of the O-C behavior of these particular objects, and in this preliminary stage we took the existing equations and reproduced the diagrams with our updated list of times of maxima taken from literature plus the data that we observed. These reported ephemerides are listed
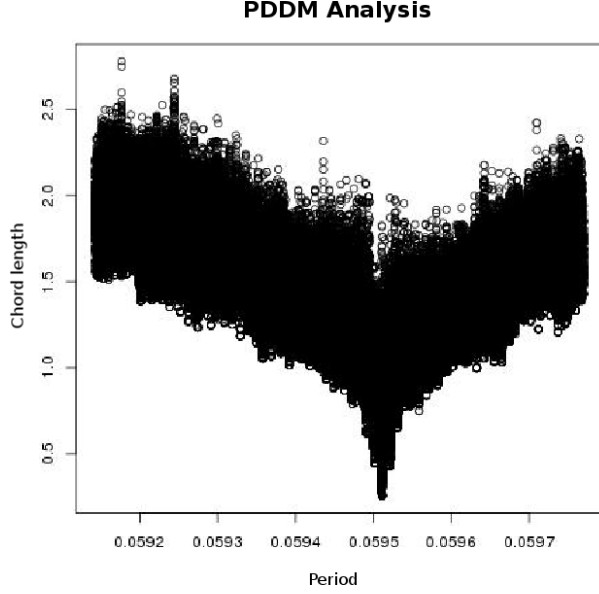
PDDM Analysis

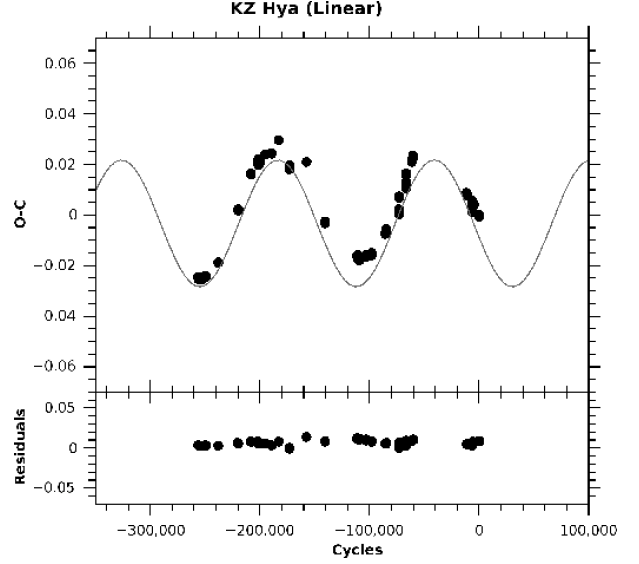Fig. 3. Period longitude, PDDM.

KZ Hya (Linear)

Fig. 4. Period determination through an O-C differences minimization with a sinusoidal fit assuming a linear behavior in the ephemerides equation and the resiudals, PDDM.

in Table 2 and shown schematically in Figure 7 in which the newly observed ephemerides are also presented.

Table 2 contains a compilation of the equation ephemerides reported in the literature, namely, those of Doncel et al. (2005), Chulee Kim et al. (2007) and Fu et al. (2008) as well as the results of the analysis of the present paper. The goodness of each was evaluated by calculating the O-C mean standard deviation value which is presented in the fifth column of Table 2 utilizing all the available times of maximum data, a more extended set than that the pioneer researchers had in the past. Columns two, three and four list the equation ephemerides elements. Finally, the goodness of each proposed period was evaluated in Period04 through the reported residuals provided in the code. These are listed in last column of Table 2.

Since the study of Fu et al. (2008) (with a time basis of 11670 days, almost 32 years), more observations have been realized, some of them carried out recently, and they are presented in this paper in Table 1 (the time basis has been extended to 14955 days or nearly 9 years more, almost one third more than what Fu et al. (2008) used for their calculations). We tested the old proposed ephemerides equations with the complete set of times of maximum light which includes 221 times of maximum light.

3.4. *Period Determination through O-C Differences Minimization (PDDM)*

We also employed a method based on the idea of searching the minimization of the chord length linking all the points in the O-C diagram for different values of changing periods, looking for the best period which corresponds to the minimum chord length (See the case of BO Lyn, Peña et al. 2016).

A set of 221 times of maxima was considered to perform this analysis. Taking this into consideration, we determined an interval span in which the period is located at 0.0594 ± 0.0003 days. Maintaining a period precision of a billionth and taking the interval span period into consideration, 626541 periods were used to perform this method. The T_0 used to calculate the O-C diagrams is 2457772.8602. Then the best period is the one with the smallest chord length and it is shown in Figure 3. The resulting linear ephemerides equation is

$$T_{max} = 2457772.8602 + 0.059510384 \times E \,.$$

Figure 4 shows the respective O-C diagram of the period found. As a part of the analysis, we also performed a search for a possible sinusoidal behavior in the O-C diagram. This kind of behavior is related to the light-travel time effect (LTTE) due to the presence of a second body orbiting the main body.

As can be seen in Figure 4, the diagram appears to have a clear sinusoidal behavior. Then, we ad-
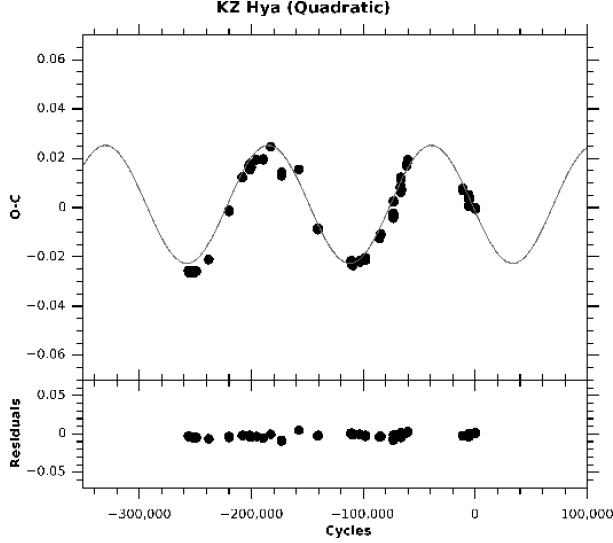
Fig. 5. Period determination through an O-C differences minimization with a sinusoidal fit over a quadratic behavior in the ephemerides equation and the residuals, PDDM.

TABLE 5

EQUATION PARAMETERS FOR THE SINUSOIDAL FIT OF THE O-C RESIDUALS.[*]

	Value	Error
Z	-3.338×10^{-3}	2.416×10^{-4}
Ω	7.005×10^{-6}	2.431×10^{-8}
A	2.505×10^{-2}	3.116×10^{-4}
Φ	5.354×10^{-1}	2.614×10^{-3}
RSS residuals	2.695×10^{-3}	

[*]For the linear fit.

justed a sinusoidal function to the O-C diagram performing a fit using the Levenberg-Marquardt algorithm. The fitting equation for the O-C diagram is

$$O - C = Z + A\sin(2 \times \pi \times (\Omega \times E + \Phi)),$$

where Z is the Y interception; A is the amplitude in days; ω, the frequency in $1/E$, and ϕ, the phase. The continuous line in Figure 4 shows the sinusoidal fit over the O-C diagram.

The sinusoidal function parameters are listed in Table 6. According to this, the period of the sinusoidal behavior is 8495.42 days or 23.28 years.

Looking at the O-C diagram and the sinusoidal adjustment of Figure 4, it is possible to see an ascending branch in the O-C diagram, a behavior that can be caused by a secular variation in the intrinsic
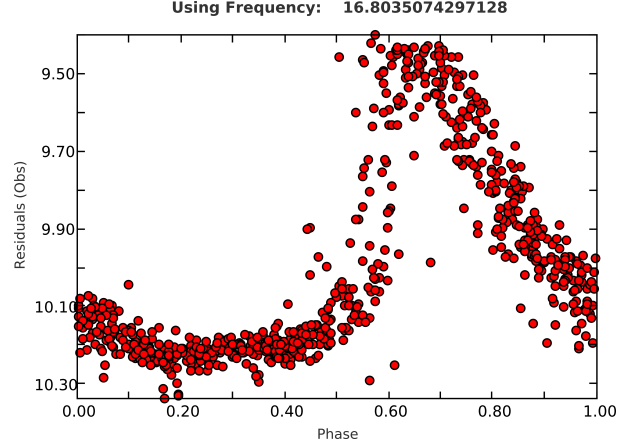

Fig. 6. Phase-magnitude diagram of the Kepler's field data set. The frequency 16.80357430 c/d was utilized.

TABLE 6

EQUATION PARAMETERS FOR THE SINUSOIDAL FIT OF THE O-C RESIDUALS.[*]

	Value	Error
Z	3.629×10^{-3}	2.554×10^{-4}
Ω	6.921×10^{-6}	2.641×10^{-8}
A	2.429×10^{-2}	2.821×10^{-4}
Φ	5.288×10^{-1}	2.487×10^{-3}
β	6.473×10^{-13}	7.983×10^{-14}
dP	7.056×10^{-8}	$1.06 \ \times 10^{-8}$
RSS residuals	2.370×10^{-3}	

[*]For the quadratic adjustment.

period of the pulsating star, i.e., KZ Hya. Following the same PDDM method, a sinusoidal adjustment was performed this time including a quadratic term and a correction term for the period. These components are the β in the quadratic ephemerides equation of this system and a dP. The fit is shown in Figure 5 and the sinusoidal function parameters are listed in Table 6. According to this new model, the period of the second body that is observed by the LTTE is 8659.83 days or 23.73 years and the resulting ephemerides equation is

$$
\begin{aligned}
T_{max} &= 2457772.8602 + 0.059510382 \times E + (1) \\
&+ (1/2) \times 6.47 \times 10^{-13} \times E^2
\end{aligned}
$$

The goodness of the model is determined by the RSS residuals listed in Table 5 and Table 6 showing that the model with the secular variation (quadratic) is the one that fits the O-C behavior better.

3.5. Phase Dispersion Minimization of Kepler's Data

Another independent source of data was considered: the Kepler field. This is a set with little data on KZ Hya although it has been densely observed. In their package http://www.astrouw.edu.pl/asas/?page=aasc, a visualization of both the data set and a window to test different periods directly is provided. Starting from scratch, we took as seed the root of all previously determined periods up to four decimal figures. Looking at the phase diagrams we were refining, the periods were modified considering the phase dispersion as criterion of goodness. Finally, a value of 0.059511385 d gave the best phase diagram (Figure 6). The result is included in Table 2 along with the other methods and data sets.

3.6. Period Determination Conclusions

Up to now, KZ Hya had been observed extensively and analyzed carefully. In the past there were extensive studies which have been previously mentioned (see Boonyarak et al. (2011) and bibliography therein).

Since then, more information was gathered but no period analysis was done. In the present paper, different approaches were utilized to determine the stability of the pulsation.

In the first approach, differences of consecutive times of maximum light were evaluated to determine a coarse period; the second method utilized time series analyses of two different data sets. The first set is that of the V filter of the photometry of McNamara & Brudge (1985) and that of the $uvby - \beta$ photoelectric of the present paper. In this case the time span covered is 13798.3358 d or 38 years, which in cycles is 231864. The amazing result is that the $uvby - \beta$ photoelectric photometry both in magnitude and color indexes fit remarkably well the phase diagram despite the large time gap in between. The other set is that of the present paper's CCD photometry.

With respect to the third method, utilizing the set of 222 times of maximum light, including those recently observed (15256 days or 256372 cycles apart), there are three studies in the literature, Chulee et al. (2007), Fu et al. (2008) and Boonyarak et al. (2011), whose residuals show a very distinct sinusoid behavior or residuals that could be interpreted as a LTTE of a binary nature. This result is derived with the PDDM method constructed in the present paper.

The remaining methods like those of Doncel et al. (2005) and those of Period04 show residuals without such a pattern. The fourth method utilized the data collected by Kepler for which we adjusted the period by an eye-ball phase dispersion minimization. The summary of the behavior of all the proposed period calculations (O-C) vs Epoch is presented in Table 2.

Basing our analysis of the most simple results we must consider the overall behavior presented in Table 2 and Figure 7. To begin with, we preferred those ephemerides equations with O-C sinusoidal residuals because they are supported by physical arguments. With this idea in mind, we discarded the period proposed from the O-C differences of the consecutive times of maxima (0.059399998 d) because this analysis only pursued the corroboration of the already published results. Given the relatively few pairs of data, the resulting accuracy is not sufficient, but served its purpose to coarsely verify the correctness of the values of the proposed periods in the literature. Now, analyzing the different O-C residuals of the different ephemerides equations proposed, we have to discard those of Doncel et al. (2005), because it seems that the times of maximum light they published are incorrect, and hence cause an erroneous ephemerides equation. In fact, Fu et al. (2008) states that "the observations of Doncel et al. (2005) covered 12 maximum phases of KZ Hya. However, this work had a significant error in the computation of Julian Dates which took UT = 0h as the reference time of the Julian Date calculation, instead of UT=12h". Two more proposed ephemerides should not be taken into consideration: the two obtained by the time series analysis by Fourier transforms in Period04. It is encouraging that with such a long time of separation between the considered two $uvby$ data sets: McNamara & Brudge (1985) and the present paper's data, this analysis gave an amazing phase diagram. However, the period (0.060742815 d), corresponding to the best frequency in Period04, differs significantly from the O-C results of the times of maxima and is not even close to the period obtained by the O-C differences.

Such a large discrepancy is not found when the CCD data is analyzed by Period04. This set, however, has the disadvantage of having a very short time span and hence the result cannot be as accurate as that obtained from the O-C analysis that utilizes all the times of maximum light in a larger time span. The same conclusion can be drawn from Kepler's data and methodology.

In conclusion, there are three analyses with congruent results: Chulee et al. (2007), Fu et al. (2008)
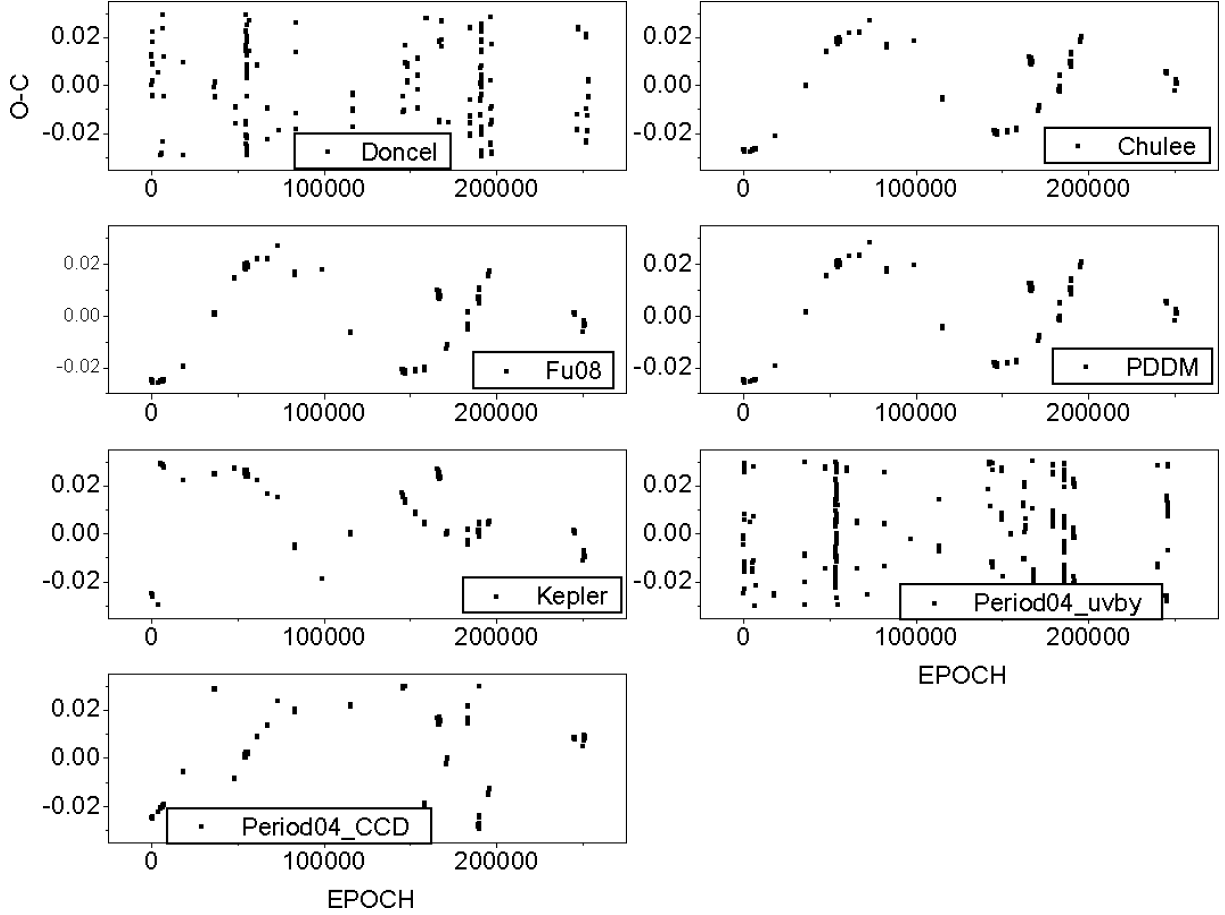
Fig. 7. Behavior of all O-C values with the different proposed ephemerides.

TABLE 7

TRANSFORMATION COEFFICIENTS
OBTAINED FOR THE OBSERVED SEASON

Season	B	D	F	J	H	I	L
Jan 2016	0.031	1.008	1.031	-0.004	1.015	0.159	-1.362
σ		0.028	0.003	0.015	0.017	0.005	0.004	0.060

and PDDM (present paper). The three methods analyzed the extended data sets up to the current date and gave the same behavior of the residuals. They are also numerically very close: 0.059510388 d for Chulee et al. (2007); 0.059510413 for Fu et al. (2008) and 0.059510396 d for PDDM. Their average value gives $0.059510399 \pm 1.3 \times 10^{-8}$ and the corresponding differences with mean value of -1.1×10^{-8}, 1.4×10^{-8} and -3×10^{-9}, almost negligible but not equal. We feel that with a longer time basis this puzzle will be solved.

4. PHYSICAL PARAMETERS

4.1. *Data Acquisition and Reduction at SPM*

The observations were done in January, 2016 along with some other variable stars and clusters. The procedure to determine the physical parameters has been reported elsewhere (Peña et al., 2016). If the photometric system is well-defined and calibrated, it provides an efficient way to investigate physical conditions such as effective temperature and surface gravity via a direct comparison of the unreddened indexes with the theoretical models. These calibrations have already been described and used in previous analyses, (Peña & Peniche; 1994; Peña & Sareyan, 2006).

A comparison of theoretical models, such as those of Lester, Gray & Kurucz (1986), (hereinafter LGK86) with intermediate or wide band photometry measured for each star, allows the determination of reddening. LGK86 calculated grids for stellar atmospheres for G, F, A, B and O stars for different values of [Fe/H] in a temperature range from 5500

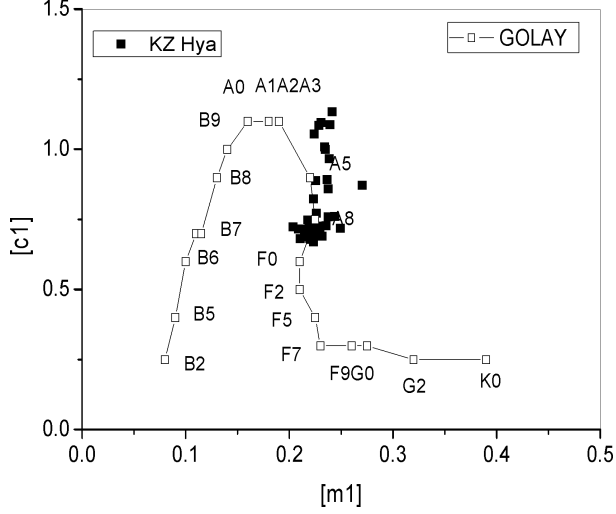Fig. 8. Location of KZ Hya in the $[m_1]$ vs. $[c_1]$ diagram of Golay (1974).

Fig. 9. Location of the unreddened points of KZ Hya (dots) in the LGK86 grids.

up to 50 000 K. The surface gravities vary approximately from the Main Sequence values to the limit of the radiation pressure in 0.5 intervals in $\log g$. A comparison between the photometric unreddened indexes $(b - y)_0$ and c_0 obtained for the star with the models allows us to determine the effective temperature T_e and surface gravity $\log g$.

These calibrations might either those proposed by Balona & Shobbrook (1984) and Shobbrook (1984) for early type stars, or those proposed by Nissen (1988) for A and F type stars. Therefore, it is necessary to first determine the range of variation in spectral class of KZ Hya. This will be accomplished by means of the unreddened [m1], [c1] and [u-b] color indexes.

According to Simbad, KZ Hya has a spectral type of B9 III/IV, and therefore, the first method should be adequate. However, if the $uvby - \beta$ photoelectric photometry obtained in the present work is utilized to determine the spectral type, for example in the $[m_1]$ vs. $[c_1]$ diagram of Golay (1974), Figure 8, it is immediately seen that KZ Hya is not such an early A type star as was reported, but a later type, and hence Nissen's (1988) procedure is applicable.

The application of the above mentioned numerical unreddening package gives the results listed in Table 9 for KZ Hya. This table lists, ordered in decreasing Hβ, in the first column, the HJD; subsequent columns present the reddening, the unreddened indexes, the unreddened and the absolute magnitudes. Mean values were calculated for $E(b - y)$ for two cases: (i) the whole data sample and (ii) in phase limits between 0.3 and 0.8,
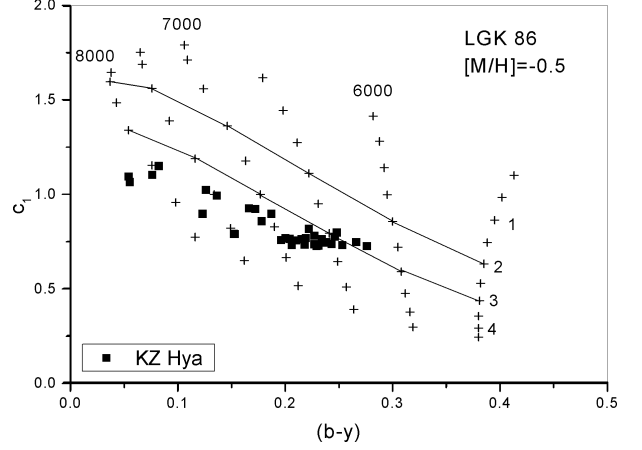
which is customary for pulsating stars to avoid the maximum. This gave values for the whole cycle of 0.075 ± 0.027; 9.04 ± 0.80 and 687 ± 256 for $E(b - y)$, DM and distance (in pc), respectively, whereas for the mentioned phase limits we obtained, 0.073 ± 0.030; 9.02 ± 0.93 and 696 ± 297 respectively. The uncertainty is merely the standard deviation. In the case of the reddening, most of the values in the spectral type in the F stage of KZ Hya produced negative values which is un-physical. In those cases we forced the reddening to be zero. If the negative values are included, the mean $E(b - y)$ is 0.009 ± 0.038.

The same analysis for determining the physical parameters was performed by Fu et al. (2008) by means of $uvby - \beta$ photoelectric photometry, utilizing Crawford's (1979) calibrations. They derived an $E(b - y)$ of 0.04 ± 0.01, $T_e(k)$ of 7300 ± 530 K, and $\log g$ (dex) of 4.00 ± 0.18.

In order to locate our unreddened points in the theoretical grids of LGK86 a metallicity has to be assumed. LGK86 calculated outputs for several metallicities. The metallicity of KZ Hya can be determined from the $uvby - \beta$ photometry when the star passes through the F type stage (Nissen, 1988). Particularly, in the case of KZ Hya for which we determined a mean metallicity of $[\mathrm{Fe/H}] = -0.275 \pm 0.106$ there are two models which were applicable, either $[\mathrm{Fe/H}] = 0.0$ or -0.5. We tested both since our determined mean metallicity lies in-between.

To diminish the noise and to see the variation of the star in phase, mean values of the unreddened colors were calculated in phase bins of 0.1 starting at 0.05 as the initial value. As can be seen in Figure 9, for the case of $[\mathrm{Fe/H}] = -0.5$ the star varies between effective temperature 6800 K and 8100 K;

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TABLE 8

$uvby - \beta$ PHOTOELECTRIC PHOTOMETRY OF KZ HYA

HJD	V	$(b-y)$	m_1	c_1	β	$[m1]$	$[c1]$	$[u-b]$
-2457000								
399.8477	11.875	0.130	0.195	0.928	2.706	0.237	0.902	1.375
399.8506	11.865	0.160	0.154	0.952	2.754	0.205	0.920	1.330
399.8546	11.897	0.149	0.186	0.872	2.743	0.234	0.842	1.310
399.8567	11.894	0.162	0.180	0.865	2.791	0.232	0.833	1.296
399.8591	11.901	0.172	0.169	0.881	2.774	0.224	0.847	1.295
399.8617	11.937	0.160	0.167	0.893	2.804	0.218	0.861	1.297
399.8638	11.944	0.166	0.177	0.881	2.751	0.230	0.848	1.308
399.8662	11.974	0.164	0.157	0.882	2.683	0.209	0.849	1.268
399.8706	11.980	0.193	0.139	0.877	2.730	0.201	0.838	1.240
399.8728	11.974	0.210	0.123	0.877	2.695	0.190	0.835	1.215
399.8747	11.999	0.195	0.157	0.833	2.682	0.219	0.794	1.233
399.8768	12.012	0.192	0.167	0.824	2.664	0.228	0.786	1.242
399.8790	12.015	0.206	0.135	0.856	2.731	0.201	0.815	1.217
399.8809	12.056	0.155	0.222	0.802	2.691	0.272	0.771	1.314
399.8846	12.076	0.157	0.215	0.784	2.675	0.265	0.753	1.283
399.8864	12.064	0.174	0.190	0.812	2.713	0.246	0.777	1.269
399.8895	12.057	0.188	0.178	0.791	2.698	0.238	0.753	1.230
399.8926	12.059	0.196	0.155	0.830	2.694	0.218	0.791	1.226
399.8946	12.061	0.181	0.171	0.823	2.715	0.229	0.787	1.245
399.8966	11.964	0.253	0.121	0.717	2.739	0.202	0.666	1.070
399.9008	11.956	0.247	0.118	0.739	2.730	0.197	0.690	1.084
399.9027	11.962	0.232	0.115	0.791	2.708	0.189	0.745	1.123
399.9049	11.950	0.224	0.139	0.751	2.719	0.211	0.706	1.128
399.9073	11.939	0.205	0.148	0.795	2.715	0.214	0.754	1.181
399.9093	11.907	0.221	0.124	0.792	2.717	0.195	0.748	1.137
399.9114	11.955	0.173	0.162	0.869	2.777	0.217	0.834	1.269
399.9157	11.916	0.167	0.159	0.878	2.731	0.212	0.845	1.269
399.9176	11.918	0.161	0.173	0.907	2.811	0.225	0.875	1.324
399.9193	11.878	0.168	0.151	0.913	2.792	0.205	0.879	1.289
399.9208	11.911	0.135	0.190	0.912	2.743	0.233	0.885	1.351
399.9224	11.857	0.163	0.163	0.891	2.757	0.215	0.858	1.289
399.9240	11.840	0.153	0.160	0.950	2.742	0.209	0.919	1.337
399.9257	11.828	0.151	0.163	0.925	2.777	0.211	0.895	1.317
399.9276	11.833	0.142	0.176	0.911	2.770	0.221	0.883	1.325
401.9013	11.842	0.155	0.171	0.918	2.764	0.221	0.887	1.328
401.9033	11.838	0.159	0.164	0.923	2.796	0.215	0.891	1.321
401.9052	11.845	0.180	0.146	0.939	2.799	0.204	0.903	1.310
401.9070	11.863	0.167	0.166	0.901	2.773	0.219	0.868	1.306
401.9086	11.879	0.157	0.177	0.913	2.789	0.227	0.882	1.336
401.9103	11.888	0.169	0.151	0.934	2.779	0.205	0.900	1.310
401.9135	11.909	0.152	0.195	0.878	2.730	0.244	0.848	1.335
401.9152	11.897	0.171	0.173	0.895	2.794	0.228	0.861	1.316
401.9168	11.914	0.187	0.144	0.909	2.774	0.204	0.872	1.279
401.9187	11.927	0.173	0.173	0.895	2.794	0.228	0.860	1.317
401.9207	11.921	0.189	0.164	0.866	2.783	0.224	0.828	1.277

TABLE 8 (CONTINUED)

HJD	V	$(b-y)$	m_1	c_1	β	$[m1]$	$[c1]$	$[u-b]$
-2457000								
401.9227	11.951	0.182	0.164	0.875	2.788	0.222	0.839	1.283
401.9245	11.964	0.185	0.153	0.893	2.806	0.212	0.856	1.280
401.9264	11.961	0.202	0.133	0.881	2.766	0.198	0.841	1.236
401.9301	11.969	0.195	0.174	0.855	2.746	0.236	0.816	1.289
401.9319	11.983	0.187	0.184	0.825	2.753	0.244	0.788	1.275
401.9337	11.992	0.199	0.169	0.829	2.802	0.233	0.789	1.255
401.9362	12.000	0.204	0.162	0.814	2.719	0.227	0.773	1.228
401.9378	12.005	0.217	0.146	0.822	2.753	0.215	0.779	1.209
401.9395	12.015	0.208	0.155	0.834	2.743	0.222	0.792	1.236
401.9411	12.039	0.195	0.159	0.827	2.728	0.221	0.788	1.231
401.9430	12.033	0.218	0.147	0.790	2.769	0.217	0.746	1.180
401.9450	12.030	0.231	0.121	0.825	2.769	0.195	0.779	1.169
401.9469	12.039	0.212	0.133	0.852	2.734	0.201	0.810	1.211
401.9509	12.033	0.231	0.130	0.837	2.672	0.204	0.791	1.199
401.9526	12.046	0.219	0.141	0.812	2.670	0.211	0.768	1.190
401.9547	12.058	0.193	0.159	0.829	2.784	0.221	0.790	1.232
401.9567	12.031	0.210	0.146	0.811	2.770	0.213	0.769	1.195
401.9586	12.025	0.203	0.151	0.830	2.773	0.216	0.789	1.221
401.9610	12.010	0.212	0.147	0.810	2.758	0.215	0.768	1.197
401.9631	12.015	0.183	0.185	0.791	2.770	0.244	0.754	1.242
401.9652	11.990	0.187	0.176	0.818	2.791	0.236	0.781	1.252
401.9677	11.959	0.213	0.137	0.845	2.779	0.205	0.802	1.213
401.9697	11.971	0.170	0.169	0.836	2.753	0.223	0.802	1.249
401.9714	11.921	0.190	0.156	0.863	2.747	0.217	0.825	1.259
401.9735	11.913	0.188	0.147	0.868	2.828	0.207	0.830	1.245
401.9772	11.891	0.185	0.148	0.885	2.779	0.207	0.848	1.262
401.9791	11.885	0.166	0.176	0.866	2.751	0.229	0.833	1.291
401.9812	11.846	0.166	0.154	0.920	2.781	0.207	0.887	1.301
401.9832	11.828	0.171	0.150	0.919	2.817	0.205	0.885	1.294
401.9852	11.814	0.179	0.132	0.948	2.810	0.189	0.912	1.291
401.9869	11.813	0.166	0.144	0.940	2.833	0.197	0.907	1.301

the surface gravity $\log g$ varies between 3.2 and 4.0. Table 9 lists these values. Column 1 shows the phase, Columns 2 and 3 list the temperature obtained from the plot for each [Fe/H] value; Column 4, the mean value and Column 5, the standard deviation of the [Fe/H] $= -0.5$ metallicity. Column 6 shows the effective temperature obtained from the theoretical relation reported by Rodriguez (1989) based on a relation of Petersen & Jorgensen (1972, hereinafter P&J72) $T_e = 6850 + 1250 \times (\beta - 2.684)/0.144$ for each value and averaged in the corresponding phase bin. The last column lists the surface gravity $\log g$ from the plot.

4.2. *Conclusions on the Physical Parameters*

New $uvby-\beta$ photoelectric photometry observations were carried out for the SX Phe star KZ Hya. From the $uvby-\beta$ photoelectric photometry we determined first its spectral type, which varies between A5V and A8V, differing from previous determinations. From Nissen's (1988) calibrations the reddening was determined as well as the unreddened indexes. These served to determine the physical characteristics of this star, $\log T_e$, in a range from 6800 K to 8100 K and $\log g$ from 3.2 to 3.6 using two methods: (1) from the location of the unreddened indexes in the LGK86 grids, and (2) through the theoretical

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TABLE 9

REDDENING AND UNREDDENED PARAMETERS OF KZ HYA

HJD	$E(b-y)$	$(b-y)_0$	m_0	c_0	$H\beta$	V_0	M_V
-2457000							
401.9869	.063	.166	.163	.927	2.833	11.54	1.76
401.9735	.074	.188	.169	.853	2.828	11.60	2.35
401.9832	.052	.171	.166	.909	2.817	11.60	1.76
399.9176	.036	.161	.184	.900	2.811	11.76	1.78
401.9852	.058	.179	.149	.936	2.810	11.57	1.40
401.9245	.055	.185	.170	.882	2.806	11.73	1.83
399.8617	.028	.160	.176	.887	2.804	11.81	1.80
401.9337	.059	.199	.187	.817	2.802	11.74	2.35
401.9052	.049	.180	.161	.929	2.799	11.63	1.31
401.9033	.024	.159	.171	.918	2.796	11.73	1.41
401.9152	.032	.171	.183	.889	2.794	11.76	1.63
401.9187	.034	.173	.183	.888	2.794	11.78	1.63
399.9193	.029	.168	.160	.907	2.792	11.75	1.44
399.8567	.017	.162	.185	.862	2.791	11.82	1.86
401.9652	.038	.187	.187	.810	2.791	11.83	2.28
401.9086	.015	.157	.182	.910	2.789	11.81	1.40
401.9227	.036	.182	.175	.868	2.788	11.80	1.73
401.9547	.039	.193	.171	.821	2.784	11.89	2.08
401.9207	.038	.189	.175	.858	2.783	11.76	1.73
401.9812	.019	.166	.160	.916	2.781	11.77	1.21
401.9103	.022	.169	.157	.930	2.779	11.80	1.06
401.9677	.057	.213	.154	.834	2.779	11.72	1.86
401.9772	.033	.185	.158	.878	2.779	11.75	1.50
399.9114	.017	.173	.167	.866	2.777	11.88	1.61
399.9257	.001	.151	.163	.925	2.777	11.82	1.11
399.8591	.015	.172	.174	.878	2.774	11.84	1.46
401.9168	.033	.187	.154	.902	2.774	11.77	1.21
401.9070	.011	.167	.169	.899	2.773	11.81	1.27
401.9586	.040	.203	.163	.822	2.773	11.85	1.91
399.9276	.000	.142	.176	.911	2.770	11.83	1.13
401.9567	.043	.210	.159	.802	2.770	11.85	2.03
401.9631	.014	.183	.189	.788	2.770	11.95	2.21
401.9430	.048	.218	.161	.780	2.769	11.83	2.21
401.9450	.065	.231	.140	.812	2.769	11.75	1.89
401.9264	.039	.202	.145	.873	2.766	11.79	1.34
401.9013	.000	.155	.171	.918	2.764	11.84	0.98
401.9610	.035	.212	.158	.803	2.758	11.86	1.86
399.9224	.000	.163	.163	.891	2.757	11.86	1.12
399.8506	.000	.160	.154	.952	2.754	11.86	0.53
401.9319	.008	.187	.186	.823	2.753	11.95	1.66
401.9378	.038	.217	.157	.814	2.753	11.84	1.68
401.9697	.000	.170	.169	.836	2.753	11.97	1.56
399.8638	.000	.166	.177	.881	2.751	11.94	1.12
401.9791	.000	.166	.176	.866	2.751	11.89	1.26

TABLE 9 (CONTINUED)

HJD	$E(b-y)$	$(b-y)_0$	m_0	c_0	$H\beta$	V_0	M_V
-2457000.00							
401.9714	.010	.190	.159	.861	2.747	11.88	1.23
401.9301	.013	.195	.178	.852	2.746	11.91	1.29
399.8546	.000	.149	.186	.872	2.743	11.90	1.09
399.9208	.000	.135	.190	.912	2.743	11.91	0.73
401.9395	.022	.208	.162	.830	2.743	11.92	1.44
399.9240	.000	.153	.160	.950	2.742	11.84	0.38
399.8966	.052	.253	.137	.707	2.739	11.74	2.42
401.9469	.021	.212	.139	.848	2.734	11.95	1.09
399.8790	.014	.206	.139	.853	2.731	11.96	0.98
399.9157	.000	.167	.159	.878	2.731	11.92	0.79
399.8706	.002	.193	.140	.877	2.730	11.97	0.77
399.9008	.042	.247	.131	.731	2.730	11.77	2.01
401.9135	.000	.152	.195	.878	2.730	11.91	0.76
401.9411	.000	.195	.159	.827	2.728	12.04	1.17
399.9049	.016	.224	.144	.748	2.719	11.88	1.64
401.9362	.000	.204	.162	.814	2.719	12.00	1.07
399.9093	.015	.221	.129	.789	2.717	11.84	1.22
399.8946	.000	.181	.171	.823	2.715	12.06	0.87
399.9073	.000	.205	.148	.795	2.715	11.94	1.13
399.8864	.000	.174	.190	.812	2.713	12.06	0.91
399.9027	.020	.232	.121	.787	2.708	11.88	0.95
399.8477	.000	.130	.195	.928	2.706	11.88	−0.44
399.8895	.000	.188	.178	.791	2.698	12.06	0.60
399.8728	.000	.210	.123	.877	2.695	11.97	−0.40
399.8926	.000	.196	.155	.830	2.694	12.06	0.04
399.8809	.000	.155	.222	.802	2.691	12.06	0.20
399.8662	.000	.164	.157	.882	2.683	11.97	−0.98
399.8747	.000	.195	.157	.833	2.682	12.00	−0.49
399.8846	.000	.157	.215	.784	2.675	12.08	−0.19
401.9509	.000	.231	.130	.837	2.672	12.03	−0.89
401.9526	.000	.219	.141	.812	2.670	12.05	−0.68
399.8768	.000	.192	.167	.824	2.664	12.01	−1.02

relation (P&J72). They are similar, within the error bars, and give a good idea of the star's behavior. Furthermore, when mean values are obtained from the two closest metallicity values, the result is closer to the obtained theoretical value.

5. CONCLUSIONS

Rodriguez and Breger's (2001) in their study on δ Scuti stars, stated in § 2.2 that: "...the distribution in apparent magnitude of the δ Scuti variables known to be part of binary or multiple stellar systems. The R00 catalogue lists 86 variables (62 with CCDM identification). This represents only 14% of the total sample of known δ Scuti stars. Only five variables are fainter than $V = 10.0$. Hence, multiplicity is catalogued for 22 of all the δ Scuti known up $10{.}^{\mathrm{m}}0$. This percentage is very low because more than 50% of the stars are expected to be members of multiple systems". They later stated that "pulsating stars in eclipsing binaries are important for accurate determinations of fundamental stellar parameters and the study of tidal effects on the pulsations... During the last two decades, unusual changes in the light curves have been detected, leading to a number of different interpretations...

"Pulsation provides an additional method to detect multiplicity through a study of the light-time effects in a binary system. This method generally

TABLE 10

EFFECTIVE TEMPERATURE OF KZ HYA AS
A FUNCTION OF PHASE

Phase	T_e(0.0)	T_e(-0.5)	Mean	σT_e	P&J72	$\log g$(-0.5)
0.05	8000	6800	7400	412	7251	3.2
0.15	7900	6800	7350	319	7235	3.4
0.25	8000	6800	7400	267	7259	3.5
0.35	8000	6800	7400	387	7234	3.5
0.45	8200	7400	7800	264	7494	3.6
0.55	8400	7900	8150	284	7582	3.6
0.65	9000	8100	8550	254	7781	3.6
0.75	8200	7300	7750	396	7469	3.4
0.85	8400	7900	8150	164	7642	3.5
0.95	8200	7800	8000	340	7569	3.6

Note: Values in parenthesis specify the [Fe/H].

favors high-amplitude variables with only one or two pulsation periods (which tend to be radial). Several decades of measurements are usually required to study these (O-C) residuals in the times of maxima".

At that time they listed, in their Table 4, only six stars with determined orbital periods. Since then, with a longer time basis for those stars, and for an increased number of measured times of maximum, better defined orbital elements have been obtained. There have been numerous studies with this purpose on HADS stars. For example, Boonyarak et al. (2011) carried out a study devoted to the analysis the stability of fourteen HADS stars. Many other authors carried out analyses on a star-by-star basis. Some of the HADS stars show a behavior of the O-C residuals compatible with the light-travel time effect pointing to a binary nature: AD CMi, KZ Hya, AN Lyn, BE Lyn, SZ Lyn, BP Peg, BS Aqr, CY Aqr among others, whereas there are some that, on the contrary, vary with one period and its harmonics and do not show this effect. To this category, according to Boonyarak et al., (2011) belong GP And, AZ CMi, AE UMa, RV Ari, DY Her, DH Peg.

In the present study we found that KZ Hya has a spectral type varying between A5V and A8V, values of E(b-y) of 0.073 ± 0.030, DM of 9.02 ± 0.93. It has been proved that KZ Hya is pulsating with one stable varying period (0.059510382 d), whose O-C residuals show a sinusoidal pattern compatible with a light-travel time effect. As regards this topic, it is interesting to mention that in his excellent discussion, Templeton (2005) states that: "In all cases

except SZ Lyn, the period of the purported binarity is close to that of the duration of the (O-C) measurements, making it difficult to prove that the signal is truly sinusoidal. A sinusoidal interpretation is only reliable when multiple cycles are recorded, as in SZ Lyn. While the binary hypothesis is certainly possible in most of these cases, conclusive proof will not be available for years or even decades to come. Continued monitoring of times of maximum will be crucial, and such observations are encouraged. In the meantime, however, other possible interpretations of their behavior must also be explored".

We feel that the results presented in this paper agree with Templeton's (2005) request that "a sinusoidal interpretation is only reliable when multiple cycles are recorded" as we have found for KZ Hya.

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J. Calderón, J. Guillen, J. H. Peña, D. S. Piña, A. Rentería, A. A. Soni, and C. Villarreal: Instituto de Astronomía, Universidad Nacional Autónoma de México, Apdo. Postal 70-264, Cd. de México (jhpena@astro.unam.mx).

J. H. Peña: Observatorio Astronómico Nacional, Tonantzintla, México.

J. Calderón, J. Guillen, J. H. Peña, D. S. Piña, A. Rentería, A. A. Soni, and C. Villarreal: Facultad de Ciencias, Universidad Nacional Autónoma de México, México.