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Artículo

# Autoaclareo y manejo de la densidad en rodales coetáneos de *Pinus patula* Schiede *ex* Schlechtdl. & Cham.

# Self-thinning and density management in even-aged *Pinus patula* Schiede *ex* Schlechtdl. & Cham. stands

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**Resumen:**

La comunidad de Ixtlán de Juárez, Oaxaca, posee un potencial maderable importante debido a la productividad alta de su bosque. Una de las especies con mayor importancia es *Pinus patula* debido a la distribución y valor comercial alto; además, es de crecimiento rápido. Por lo tanto, es necesario determinar los límites de la densidad máxima posible que los rodales pueden sustentar para dirigir acciones encaminadas al control de la competencia y redistribución del espacio de crecimiento. En este estudio se estimó la línea de densidad máxima (límite superior del autoaclareo) con el modelo de Reineke mediante dos enfoques de estimación de los parámetros: 1) mínimos cuadrados ordinarios (MCO) y 2) regresión de frontera estocástica (RFE) con los modelos de tipo seminormal (*half-normal*) y normal truncado (*truncated-normal*). Para el ajuste del modelo se utilizaron datos de 64 parcelas permanentes de investigación silvícola de 400 m$^2$ de rodales puros y coetáneos de *P. patula*. La estimación del límite superior del autoaclareo con el modelo de RFE en la forma seminormal fue eficiente y permitió conocer el índice de densidad máxima de los rodales estudiados. La línea de densidad máxima representó el insumo primordial para construir una guía de densidad; herramienta indispensable para la definición de regímenes de aclareo y optimización del espacio de crecimiento.

**Palabras clave:** Competencia, guía de densidad, índice de densidad del rodal, régimen de aclareos, regresión de frontera estocástica, Reineke.

**Abstract:**

The community of *Ixtlán de Juárez, Oaxaca* has a significant timber potential due to the high productivity of the forest. One of the most important species is *Pinus patula*, because of the abundant distribution, high commercial value and rapid growth. Therefore, it is necessary to determine the limits of the possible maximum density that the stands can sustain to lead actions to control competition and growth space. In this study, the maximum density line (upper limit of self-thinning) was estimated under the Reineke model through two approaches: 1) ordinary least squares (OLS) and 2) stochastic frontier regression (SFR), the last with half-normal and truncated-normal models. A total of 64 permanent sampling plots of 400 m$^2$ in even-aged stands of *P. patula* were used. The estimate of the upper bound of the self-thinning with SFR approach with half-normal form was more efficient and let to know the maximum density index of even-aged stands. The upper bound of self-thinning line is the primary input for the construction of a stand density management diagram, which is essential tool for the definition of regimes of thinning and growth space optimization.

**Key words***:* Competition, density management diagram, stand density index, thinning regime, stochastic frontier regression model, Reineke.

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# Introduction

The law of self-thinning or the law of -3/2 describes the relationship between the growth and death of the trees. This quantitative relationship derived from ecological studies concerning intraspecific competition (Yoda *et al.,* 1963). In forest studies, the self-thinning line has been expressed in a logarithmic scale with a theoretical slope of -1.605 (Reineke, 1933); the value of the intercept varies with the species, but only within narrow logarithmic limits (White, 1985). Consequently, this rule has been considered one of the most important principles in the ecology of plant populations (Drew and Flewelling, 1977; Long and Smith, 1983; Jack and Long, 1996).

The debate about the classical methods of estimation of the self-thinning line has focused on the fact that the slope should not vary (Zeide, 1987; Weller, 1987; Lonsdale, 1990), and it is emphasized that the data used to estimate the functional relationship size-maximum density should be carefully selected (Zhang *et al.,* 2005).

Reineke (1933) developed a stand density index (SDI) when establishing the density-size functional relationship in stands with maximum densities. The SDI allows to compare densities of stands regardless of the age and site quality. It is obtained by means of a potential equation, and allows to determine the number of trees that would have a quadratic mean diameter of reference for a regular stand of a certain species (Daniel *et al.,* 1979; Chauchard *et al.*, 1999).

Theoretically, the maximum size-density reference line should be represented by the upper limit of the selected points or data. One of the traditional statistical methods for estimating the self-thinning line is by ordinary least squares (OLS), in which the number of trees per hectare (Na) is a direct function of the a quadratic mean diameter (Dq). However, this function describes a central trend line of the observed data. With this methodology, the value of the intercept must be varied to adequately estimate the upper limit of self-thinning, so it is subjective, that is, the values of the lines are calculated proportionally for the stand density indexes (Weller, 1987; Vargas, 1999; Santiago *et al*., 2013).

Thomson *et al.* (1996) discussed statistical methods to study aspects of population ecology, such as density and competition, and suggested alternative functional methods for correct estimation, such as frontier production functions of econometric theory proposed by Aigner *et al.* (1977), Meeusen and Van den Broeck (1977) and Färe *et al.* (1994).

Bi *et al.* (2000) and Bi (2001; 2004) developed a stochastic frontier production function to calculate the upper limit of self-thinning line in both pure and monospecific stands of pine, and concluded that it is possible to use all data without subjective selection when obtaining an effective estimation of the upper limit of self-thinning. Santiago *et al.* (2013) referred to the stochastic frontier regression method as an alternative to efficiently estimate the upper limit of self-thinning, which, in addition, has advantages because the number of useful data for the construction of density management diagram is extended, by eliminating the subjectivity that implies only sampling stands with evident maximum density.

In this context, the objective of the present study was to estimate the upper limit of self-thinning with the Reineke model (1933), through stochastic frontier regression and ordinary least squares for pure and even-aged stands of *Pinus patula* Schiede ex Schlechtdl. & Cham., and with this, have the main element to build a density management diagram in forests of *Ixtlán de Juárez*, *Oaxaca*.

# Materials and Methods

## Study area

The study was carried out in the communal forest of *Ixtlán de Juárez*, *Oaxaca*, located in the *Sierra Norte* region, between the coordinates 17°18'16'' - 17°34'00'' N and 96°31'38'' - 96°20'00 '' W, with an altitudinal range of 2 350 to 2 960 m. The predominant climate types in the area are temperate subhumid and temperate subhumid with summer rain, and the vegetation corresponds to pine, pine-oak and oak forests (Rzedowski, 2006). *Pinus patula* has the greatest distribution at the study area.

## Forest mensuration

The data used in this research were taken from 64 permanent forestry research plots (Table 1). The plots were established during 2015 in even-aged pure stands of *Pinus patula* and were re-measured in 2016 and 2017; they were delimited of square form of 400 $m^2$, divided in 4 quadrants of 10 × 10 m with 5 control points (the center and the vertices). Diameter at breast height (Dn) was measured with approximation to the millimeter with Haglöf™ brand caliper, and the total height with an electronic Haglöf 15-102-1011™ clinometer of all living trees present within the site. With this information, the state variables of the stand were calculated for the fit of the models, in this case, the number of trees (Na, trees $ha^{-1}$), basal area (AB, $m^2$ $ha^{-1}$) and quadratic mean diameter was determined (Dq, cm), calculated as $Dq = \sqrt{\frac{40000}{\pi} \times \frac{AB}{Na}}$ to fit Reineke's model.

**Table 1**. Descriptive statistics in the permanent plots that were used.

| Variable | Mean | Standard deviation | Minimum | Maximum |
|---|---|---|---|---|
| Na | 1567.78 | 1030.07 | 300.00 | 6050.00 |
| AB | 23.75 | 12.04 | 4.77 | 69.67 |
| Dq | 15.39 | 7.06 | 7.11 | 43.07 |

## Ordinary least squares model

The method of ordinary least squares estimation (OLS) (MCO, for its acronym in Spanish) provides a function that fits the point cloud and leaves observed values, both above and below it (Álvarez, 1998). That is, a central tendency line.

Obtaining the parameters to estimate density by OLS requires fitting a curve that shows the number of trees per hectare for the quadratic mean diameter. This curve was represented with Reineke's functional relationship $Na = f(Dq)$ (Equation 1) (Reineke, 1933; Santiago *et al*., 2013):

$$Na = \beta_0 Dq^{\beta_1} \qquad (1)$$

When natural logarithms were applied to Equation 1, the linear form of Reineke's model was obtained:

$$\ln(Na) = \beta_0 + \beta_1 \ln(Dq) + \varepsilon \qquad (2)$$

$$\varepsilon \sim iid\ N\ (0, \sigma_\varepsilon^2)$$

Where:

*Na* = Number of trees (trees $ha^{-1}$)

*Dq* = Quadratic mean diameter (cm)

*ln* = Natural logarithm

$\beta_i$ = Parameters to be estimated

$\varepsilon$ = Term of error in the model

## Stochastic frontier regression models

A frontier production function is an empirical formulation of the production function concept, in other words, a frontier model provides the maximum hypothetically achievable production from observed data (Álvarez, 1998).

From the econometric works of Aigner *et al.* (1977) as well as Meeusen and Van den Broeck (1977), the stochastic frontier method emerged. This approach consists in postulating an efficient production function, to which two disturbances are added (Sanhueza, 2003): the technical inefficiency and the possible sources of random variation of the data (Irizo and Ruiz, 2011).

Santiago *et al.* (2013) point out that frontier regression models estimate the extreme values of a data set, instead of the mean or the quantiles of a function. In the stochastic method, the frontier itself is a random variable in such a way that each observation has its frontier function that deviates from the general function.

The advantage of this approach is that it considers that the frontier may be a consequence of external factors not measured. In the stochastic frontier model the error components are: 1) one associated with the measurement of the individual observations and 2) one that is assumed to account for the technical inefficiency in the data (Kumbhakar and Lovell, 2000).

According to Aigner *et al.* (1977), the stochastic frontier regression model (RFE, for its acronym in Spanish) has the following general formulation:

$$y_i = f(x_i;\ \beta) + \varepsilon_i$$

The structure of the error is defined by:

$$\varepsilon_i = v_i + u_i \qquad i = 1, \dots \dots \dots N$$

Where:

$y_i$ = Term of production (output)

$x_i$ = $k \times l$ vector of input amounts

$\beta$ = Vector of unknown parameters

$v_i$ = Symmetrical disturbance independently distributed from $u_i$

$u_i$ = Asymmetrical term that takes the technical inefficiency of the observations; an assumption is made of an independent distribution of $v_i$ and of the regressors

The Reineke model that was used to estimate the self-thinning line with the RFE formulation assumes that no observation could be above the upper limit of the self-thinning, and that those below this line would indicate technical inefficiency, that is, the difference in stand density at a given time and the maximum achievable density (Santiago *et al.,* 2013). This means that all residuals must be negative or equal to zero. The Reineke model with non-positive residuals has the following structure (Santiago *et al*., 2013):

$$\ln(Na) = \beta_0 + \beta_1 \times \ln(Dq) - u + v \qquad (3)$$

$$v \sim iid\ N\ (0, \sigma_V^2)$$

$$u \sim\ iid\ \ N^+\ (0, \sigma_u^2) \quad \text{or} \quad iid\ N^+(\mu, \sigma_u^2)$$

Where:

*Na* = Number of trees

*Dq* = Quadratic mean diameter

*ln* = Natural logarithm

$\beta_i$ = Parameters to be estimated

*u* and *v* = Terms of error in the model

According to Santiago *et al.* (2013) and Quiñonez *et al.* (2018), the distribution of the asymmetric term for the error ($u_i$) can be adapted to the half-normal, exponential, gamma and normal-truncated statistical distributions. In this work, the half-normal and truncated-normal distribution were used.

## Statistical analysis

The SAS / ETS 9.3™ (SAS, 2011) statistical package was used to fit the models. The OLS model was fitted by the REG procedure, and the stochastic frontier models were fitted with the QLIM procedure and the Conjugate-Gradient optimization method to ensure the convergence of both models.

The selection of the best options was made based on the statistical indicators: Akaike information criterion (AIC), Schwarz criterion (ShcC), and the significance of the parameters; in addition, the graphic behavior of the self-thinning lines was verified by overlapping them on the observed data.

## Construction and use of the density management diagram

A density management diagram is a graphic model that allows to program thinning regimes. In this work a density management diagram was built based on the Reineke model (1933) with a stochastic frontier formulation.

The expression to estimate the SDI for any stand according to the number of trees ha$^{-1}$ and its quadratic mean diameter (Dq), in relation to the quadratic reference mean diameter (Dqr) is:

$$SDI = Na \times \left(\frac{Dqr}{Dq}\right)^{\beta_1} \qquad (4)$$

And, to obtain the number of trees of a given SDI:

$$Na = SDI \times \left(\frac{Dq}{Dqr}\right)^{\beta_1} \qquad (5)$$

Where:

*Na* = Number of trees

*Dq* = Quadratic mean diameter

*Dqr* = Quadratic reference mean diameter

$\beta_i$ = Parameters to be estimated

The definition of the limits of the growth zones in the density management diagram was based on the Langsaeter theory (Daniel *et al.,* 1979), which establishes four growth zones on which it is possible to manipulate the density to promote the growth of the remaining stand: (I) area of underutilization of the site, (II) transition zone, (III) constant growth zone, and (IV) self-thinning zone (maximum competence). In the construction of the density management diagram (Figure 2) a quadratic reference mean diameter of 15 cm was used according to the average of the observed data.

The density management diagram presents the quadratic mean diameter (Dq) and the number of trees per hectare (Na) in the main axes, which makes easier its use and interpretation because it is particularly useful for the characterization of site occupation, and not dependent of the age and site quality (Curtis, 1982; Jack and Long, 1996).

In the thinnings simulation, to calculate the timber possibility, the total height of the trees for each diameter value was estimated with the allometric height-diameter model for *P. patula* proposed by López *et al.* (2017):

$$h = 46.06167[1 - exp(-0.023647\ d_n)]^{0.936185}$$

The volume of the total stem with bark (V) was obtained by means of the equation designed by Rodríguez (2017):

$$V = 0.000074 \times D^{1.610374} \times H^{1.213333}$$

The timber cutting was defined as: removal (m$^3$ ha$^{-1}$) × area (ha) of the stand. In all the treatments, a relative growth space (ER) with a three bobbin distribution was idealized:

$$ER = \sqrt{\frac{\left(\frac{10000}{Na}\right) \times 2}{\sqrt{3}}}$$

# Results and Discussion

## Estimation of the self-thinning line

The self-thinning lines estimated for the Reineke model were three, one by MCO and two by stochastic frontier regression (RFE) (half-normal and truncated-normal). The p-values to evaluate the significance of the parameters were lower than the value of ɑ = 5 %; therefore, the parameters are reliable and accurate, and the standard errors associated with the parameters are small (Table 2).

**Table 2**. Values of the parameter estimators and goodness-of-fit statistics for the Reineke model under MCO and RFE.

| Fit method | Parameters | Estimation | Standard error | T value | Pr>\|t\| |
|---|---|---|---|---|---|
| MCO | $\beta_0$ | 10.03534 | 0.18418 | 54.49 | <0.0001 |
| | $\beta_1$ | -1.07452 | 0.06878 | -15.62 | <0.0001 |
| | $\sigma_\varepsilon^2$ | 0.13601 | | | |
| | *AIC* | -357.110 | | | |
| | *SchC* | -355.065 | | | |
| RFE | $\beta_0$ | 11.591549 | 0.235565 | 49.21 | <0.0001 |
| *Half-normal* | $\beta_1$ | -1.398203 | 0.086837 | -16.1 | <0.0001 |
| | $\sigma_v^2$ | 0.107746 | 0.021197 | 5.08 | <0.0001 |
| | $\sigma_u^2$ | 1.001068 | 0.138544 | 7.23 | <0.0001 |
| | *AIC* | 251.27060 | | | |
| | *SchC* | 264.04243 | | | |
| RFE | $\beta_0$ | 11.119823 | 0.370065 | 30.05 | <0.0001 |
| *Truncated-normal* | $\beta_1$ | -1.270809 | 0.073507 | -17.29 | <0.0001 |
| | $\sigma_v^2$ | 0.298518 | 0.147271 | 2.03 | 0.0427 |

| Fit method | Parameters | Estimation | Standard error | T value | Pr>\|t\| |
|---|---|---|---|---|---|
| | $\sigma_u^2$ | 0.235683 | 0.212173 | 1.11 | 0.2667 |
| | $\mu$ | 0.543446 | 0.168244 | 3.23 | 0.0012 |
| | *AIC* | 168.08746 | | | |
| | *SchC* | 184.05224 | | | |

MCO = Ordinary least squares; RFE= Stochastic frontier regression; $\beta_i$ = Estimated parameters; $\sigma_\varepsilon^2$, $\sigma_v^2$, and $\sigma_u^2$ parameters of the variance for the terms of error of the MCO and RFE models; *AIC*= Akaike´s information criterion; *SchC* = Schwarz criterion.

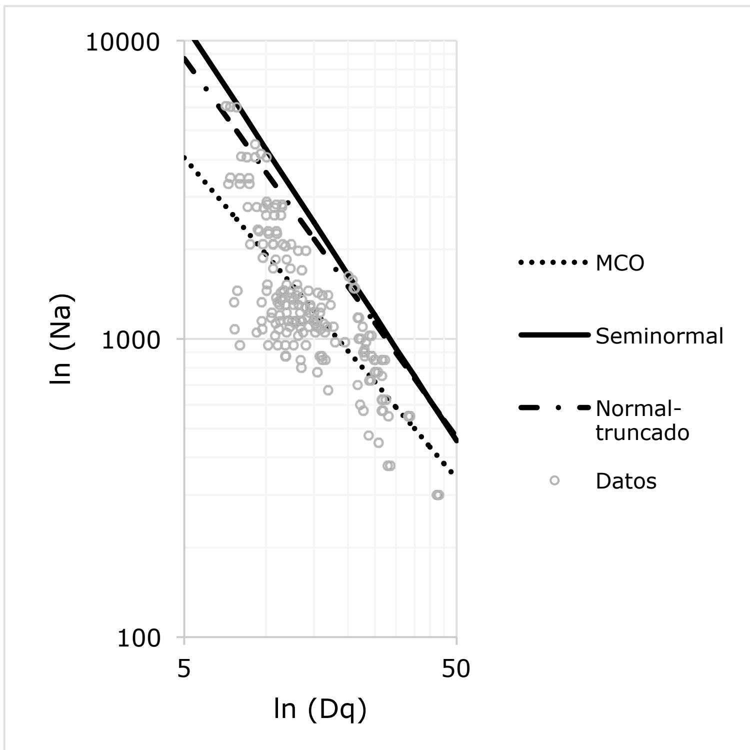
The linear structure of the Reineke model was used in the fit because this logarithmic transformation allowed to control the heterogeneity of variances of the residuals (Gezan *et al*., 2007, Santiago *et al*., 2013).

It can be observed that the Akaike´s information criterion (AIC) and Schwarz (SchC) are lower for the normal-truncated model. However, the half-normal model presented lower standard errors for error variances (Table 2); in addition, the graphical behavior had better relation with the observed data (Figure 1), so this model was the most adequate to represent the line of the upper limit of self-thinning.

Santiago *et al.* (2013) mention that overlapping the lines of self-thinning to the observed data is important for the selection of the model, because it must be verified that it is able to correctly describe the upper limit marked by the data and the entire range of natural variation that exists. That is, that there are no data that go beyond the frontier.

The lines of the upper limit of self-thinning obtained by MCO and RFE can be differentiated graphically because the estimated parameters have different values. As indicated by several authors, among them Jack and Long (1996), Drew and Flewelling (1977), Zeide (1987), Gezan *et al*. (2007) and Corvalán (2015), each species must be evaluated by its own parameters of intercept and slope when the Reineke model is used.

When comparing the RFE models with the OLS regression model, a noticeable difference in the graphic behavior is observed for the estimation of the upper limit of self-thinning, coinciding with the studies of Santiago *et al.* (2013), who mention that RFE estimators provide a direct estimation and without subjective selection of density in pure and even-aged stands; whereas the OLS-based method requires a data set at maximum density, and the model describes a central trend line (Figure 1).




MCO = Ordinary least squares; Seminormal (*Normal-truncado*) = Truncated normal; *Datos* = Data

**Figure 1**. Self-thinning lines obtained by MCO and RFE for the Reineke model.

By using all the data from the sampling plots, good fits were obtained for the RFE models. On the other hand, authors such as Zhang *et al.* (2005) indicate that when making the estimation of the self-thinning line with other methods, some inappropriate estimation of the slope is made; thus, they corroborated different methods where it was found that the deterministic frontier regression better estimates the slope and intercept for the construction of the self-thinning line.

Müller *et al.* (2013) estimated the self-thinning for *Nothofagus oblique* (Mirb.) Oerst, in forests of the Biobío region, Chile, where they varied the intercept and the slope was assumed to be constant for the upper limit of self-thinning line. Pretzsch and Biber (2005) argued that between species there is a significant difference in slope change, as corroborated for *Fagus sylvatica* L., *Picea abies* (L.) Karst and *Pinus sylvestris* L.

Therefore, in studies of competition and density management, values should be estimated in the intercept and slope of the particular Reineke model for each species and by climatic and soil conditions. For several species of conifers, Reineke (1933) determined the equation $\ln(N) = \beta - 1.605 \times \ln(Dq)$ where β is a constant that varies with the species; while Rodríguez *et al.* (2009) obtained the model for *Pinus montezumae* Lamb.: $N = 43645.9 \times Dq^{-1.0151}$ and Hernández *et al.* (2013) for *Pinus teocote* Schiede ex Schltdl. & Cham: $N = 105550.708 \times Dq^{-1.534711}$.

In plantations of several species, Harper (1977) found that the coefficients of the slopes for the density of stands ranged from -1.74 to -1.82, so there is no direct rule on the law of self-thinning when considering the theoretical slope of - 1.605, but this parameter must be separately assessed for each species or degree of association of species, density levels and specific management characteristics.

The main discussions around the slope of Reineke (1933) focus on whether the slope should be invariant (Zeide, 1987). In the present study it is demonstrated that both, the value of the slope and the value of the intercept, are particular, given the physiographic characteristics of the region, as well as the habits and the rate of growth of *Pinus patula*.

Some authors consider that stands with densities below the average should not be used to estimate the self-thinning line (Westoby, 1984; Osawa and Allen, 1993). However, with the RFE-based method for the relationships between all possible densities and the size of the trees, it is feasible to estimate a frontier function that limits the values of the parameters and thereby determine the maximum possible density.

## Density management diagram

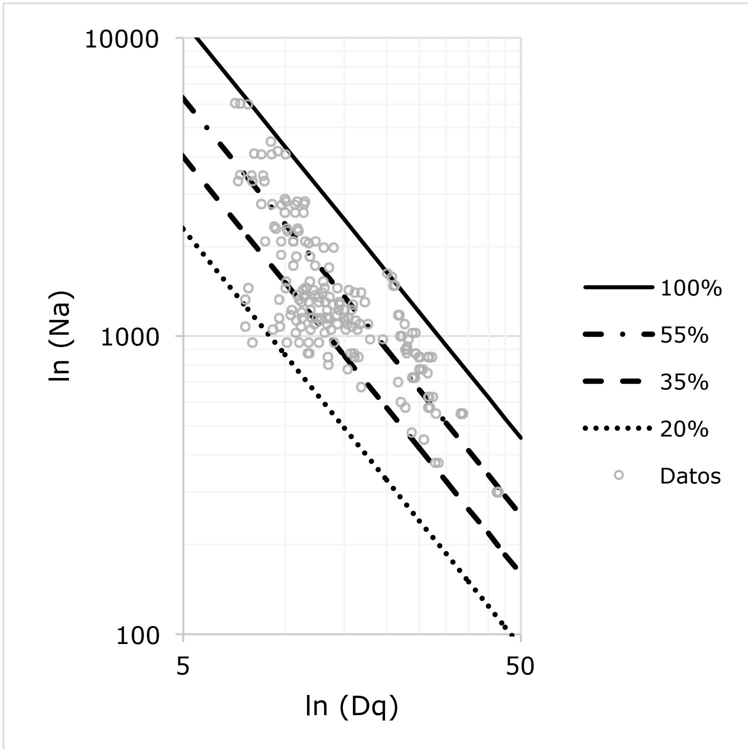
The estimation of the growth zones was derived from the percentage that has been used for other conifers in different regions of Mexico and the world; the maximum density line or upper limit of self-thinning line was fixed at 100 % of the SDI, while the lower limit by 55 %. The lower limit of the zone of constant growth was established at 35 % and at 20 % the upper limit of the zone of free growth (Drew and Flewelling, 1977; Vacchiano *et al.,* 2008). Several authors have used similar percentages such as Long and Shaw (2005) and Santiago *et al.* (2013). Growth zones are presented in tabular form in Table 3.

**Table 3**. Stand density calculated with the Reineke model under the RFE approach in its half-normal form to delimit the competition zones in the density management diagram.

| Dq (cm) | Tree density by SDI class | | | |
|---|---|---|---|---|
| | **100 %** | **55 %** | **35 %** | **20 %** |
| 1 | 108 180 | 59 499 | 37 863 | 21 636 |
| 5 | 11 398 | 6 269 | 3 989 | 2 280 |
| 10 | 4 325 | 2 379 | 1 514 | 865 |
| 15 | 2 453 | 1 349 | 859 | 491 |
| 20 | 1 641 | 902 | 574 | 328 |
| 25 | 1 201 | 661 | 420 | 240 |
| 30 | 931 | 512 | 326 | 186 |
| 35 | 750 | 413 | 263 | 150 |
| 40 | 623 | 342 | 218 | 125 |
| 45 | 528 | 290 | 185 | 106 |
| 50 | 456 | 251 | 159 | 91 |

*Dq* = Quadratic mean diameter (cm); *SDI* = Stand density index.

The density management diagram allows to interpret the different density conditions of the stands under management (Figure 2).




*Datos* = Data

**Figure 2**. Density management diagram for even-aged stands of *Pinus patula* Schiede *ex* Schlechtdl. & Cham. of *Ixtlán de Juárez*, *Oaxaca*.

## Simulation of thinning regimes

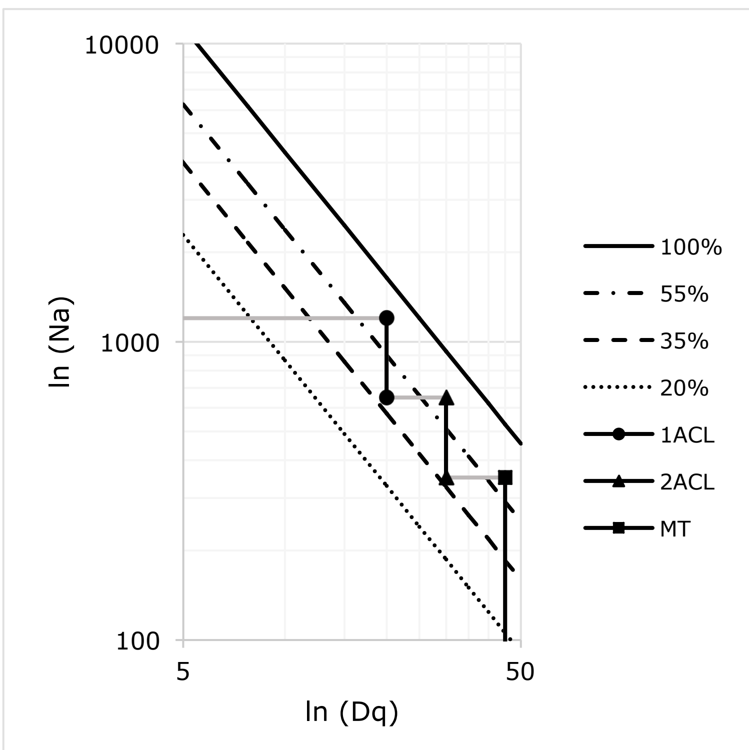
Vincent *et al.* (2000) concluded that a stand responds well to intense thinnings in plots with an average initial spacing of 2.5 × 2.5 m (1 111 trees ha$^{-1}$), by applying 48 % cutting intensity. Kanninen *et al.* (2004) recommended, in the first thinning, to eliminate between 40 and 60 % of the young trees in bad general conditions.

When simulating thinning regimes, it is considered that a sufficient quadratic mean diameter should be satisfied to carry or manage the stand in the maximum growth zone, *i. e*., from 35 to 55 % of the SDI.

With the use of density diagram, multiple suggestions for density management can be made, based on the production objectives.

Figure 3 exemplifies a regime of thinnings and final harvest, in which a stand with 5 ha of surface area is assumed, where the initial density is 1 200 trees ha$^{-1}$ (NA ha$^{-1}$) with a quadratic mean diameter (Dq) of 5 cm, in the first thinning (1ACL) it is decided to reduce the density to 650 trees ha$^{-1}$ when the Dq = 20 cm with a cutting intensity (IC %) of 45.8 %, ($IC\ \% = \left[\frac{(Na_1 - Na_2)}{Na_1}\right] \times 100$ ).

After this stage, the Dq continues to increase until reaching the lower limit of the self-thinning area and a second thinning (2ACL) is applied when a Dq = 30 cm is reached and 350 trees ha$^{-1}$ are left standing to take full advantage the area of constant growth. Subsequently, regeneration cutting treatment can be carried out with a clearcutting (MT) when the Dq reaches 45 cm (Table 4).




*ACL*= Thinning; *MT* = Clearcutting.

**Figure 3**. Thinning regime with final harvest to clearcutting.

**Table 4**. Calculation of the possibility in a stand of 5 ha, based on the density management diagram of Reineke.

| NA | Dq (cm) | AT (m) | ER (m) | SDI (%) | Individual volume ($m^3$) | V ($m^3$ $ha^{-1}$) | IC (%) | Silvicultural regime | Removal ($m^3$ $ha^{-1}$) | Timber cutting ($m^3$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 200 | 5 | 5.9 | 3.1 | 10.5 | 0.0085 | 10.2 | - | - | - | - |
| 1 200 | 20 | 18.5 | 3.1 | 73.1 | 0.3170 | 380.4 | 45.8 | 1ACL | 174.4 | 871.8 |
| 650 | 20 | 18.5 | 4.2 | 39.6 | 0.3170 | 206.1 | - | - | - | - |
| 650 | 30 | 24.4 | 4.2 | 69.8 | 0.8552 | 555.9 | 46.2 | 2ACL | 256.6 | 1282.9 |
| 350 | 30 | 24.4 | 5.7 | 37.6 | 0.8552 | 299.3 | - | - | - | - |
| 350 | 45 | 31.0 | 5.7 | 66.3 | 2.1926 | 767.4 | 100.0 | MT | 767.4 | 3837.0 |
| 0 | 0 | 0.0 | 0.0 | 0.0 | 0.0000 | 0.0 | - | - | - | - |

NA = Number of trees $ha^{-1}$; Dq = Quadratic mean diameter; AT = Total individual height; ER = Relative spacing; SDI = Reineke's stand density index; V = Total stem volume with bark; IC = Cutting intensity; T =Silvicultural regime; ACL = Thinning; MT = Clearcutting.

According to the model and the density management diagram, it is not feasible to perform thinnings below 5 cm of quadratic mean diameter, because the development of stand has not reached a point of visible competition, and since it is sought to obtain economic benefits from the thinnings products (Quintero and Jerez, 2013), when they are practiced at an early age, the wood extracted is of little commercial value.

Quiñonez *et al.* (2018) indicate that thinning prescriptions can be made according to the SDI with the quadratic mean diameter and the number of trees per hectare. While Santiago *et al.* (2017) mention that the use of growth models is required to generate the most important information for management decision making in time and space. In this case, for the appropriate use of the density management diagram, the growth model for *Pinus patula* generated by Santiago *et al.* (2017) for the study area should be used, and with this determine the natural mortality over time, and the age at which a certain value of Dq is accomplished.

# Conclusions

The line of the upper limit of self-thinning was estimated with a slope different from that proposed by Reineke, so it is confirmed that this depends on the species and local factors of each region. The best lines of maximum density obtained were those of the stochastic frontier regression scheme, due to the better graphic behavior. In this study, with the point estimate of the upper limit of self-thinning it was possible to build a density management diagram in *Pinus patula* stands at *Ixtlán, Oaxaca,* which will allow the simulation of thinning regimes and find the best management strategies to optimize the growth space, and therefore, the redistribution of the growth of the remaining trees.

This tool is basic for decision making when proposing cutting intensities in thinnings that prepare the forest for the final harvest. The thinning regime should be specific for each stand, depending on its quadratic mean diameter and the number of trees per hectare, as well as the type of products to which the trees are to be removed.

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## Conflict of interest

The authors declare no conflict of interest.

## Contribution by author

J. Alberto Camacho-Montoya: data collection at the field, analysis of results and writing of the manuscript; Wenceslao Santiago-García: design of the research, conduction of data collection at the field and of the analysis of results, and writing of the manuscript; Gerardo Rodríguez-Ortiz: analysis of data and writing of the manuscript; Pablo Antúnez: writing of the manuscript; Elías Santiago-García: taking field data and writing of the manuscript; Mario Ernesto Suárez-Mota: review of the manuscript.

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