



Sampling estimators for the inventory of *Pinus chiapensis* (Martínez) Andresen plantations

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Abstract:

The assessment of commercial forest plantations (PFC) requires accurate quantitative tools (sampling estimators) to know the timber inventory, to generate reliable information for planning actions and make the best decisions on sustainable forest management of forest resources. In this context the aim of this study was to compare four estimators to propose an efficient sampling strategy that allows calculation of timber stocks of *Pinus chiapensis* growing in a commercial forest plantation (PFC), in *Tlatlauquitepec, Puebla*. To quantify the inventory of *Pinus chiapensis* plantations, we used the mensuration information (*D* and *H*) of 44 permanent sampling plots, established in 2014 in an 87 ha^{-1} surface area and re-measured in 2015. Two classical estimators based on designs: Simple Random Sampling (MSA) and Stratified Sampling (ME), and two based on models: Reason Estimators (ERaz) and Regression Estimators (EReg), the latter employing a population mean that is estimated under ME. The results show that the four estimators are statistically different and that the EReg is the most accurate to estimate the stock in volume when using the basimetric area as an auxiliary variable. EReg allowed to update the inventory of a total of $4\ 806 \text{ m}^3$ for the first year of measurement to $6\ 496 \text{ m}^3$ in the second year. In ERaz (V_2/V_1) under ME a Reason (R) of volume two (2015) is obtained between volume one (2014) which suggests a percentage increase of 35 % of the volume for 2015.

Key words: Sampling estimators, forest inventory, simple random sampling, permanent plots, *Pinus chiapensis* (Martínez) Andresen, sampling sites.

Introduction

The assessment of commercial forest plantations requires precise quantitative tools (sampling estimators) to know timber stocks, generate reliable information to plan actions and make the best decisions in sustainable forest management. In an infinite population of trees it is impractical to measure all existing trees, so it is essential to carry out the inventory through a sample, which allows obtaining the parameters of interest of a population with a specific reliability and reduces costs to a minimum (Van Laar and Akça, 2007; Roldán *et al.*, 2013; Tamarit, 2013). Sampling consists of selecting a certain number of inventory sites (independent) that can be random or systematic. The stratification reduces the variation in the stratum and, therefore, it is required that said strata be heterogeneous among them, but homogeneous within each stratum (Bell, 1998).

A forest inventory consists of the systematic collection of data on the forest resources of a given area. These data constitute the starting point of a sustainable forest management and with them the evaluation of the current state of the resources is made. In Mexico, much of the research focused on obtaining inventories of forest resources implements a sampling based on designs, such as simple random sampling (MSA) and stratified sampling (ME) estimators; and in estimators based on models, such as ratio estimators (ERaz) and regression (EReg). Based on the type of data taken in the field (sites of fixed or variable dimensions) different levels of accuracy can be obtained (Schreuder *et al.*, 1992, Schreuder *et al.*, 2004).

The MSA assumes that the sampling n units are selected from the N units of the population at random, so that the combination of the n units is equally likely to be the selected samples of the population (Cochran, 1977; Kish, 1995; Thompson, 2002). For the design of the MSA and from the parsimony of its estimators, several authors such as Schumacher and Chapman (1942); Sukhatme (1956); Freese (1962); Scheaffer *et al.* (1987); Särndal *et al.* (1992); Kish (1995); Torres and Magaña (2001); Thompson (2002); Köhl *et al.* (2006); Schreuder *et al.* (2006); Roldán *et al.* (2013) have used this methodology to quantify forest resources.

According to Tamarit (2013), when using the ME, variances that are smaller than the estimated average are obtained and, in addition, the variances of the main or interest variable are weighted, which represents an advantage compared to the MSA. The term "stratify" means that, within the population of interest, there are differences such that it is possible to distinguish subpopulations of it. These populations are called "strata" (Schumacher and Chapman, 1942; Särndal *et al.*, 1992; Kish, 1995; Johnson, 2000; Thompson, 2002; Roldán *et al.*, 2013).

On the other hand, the ERaz is a recommendable method to estimate the mean when the variable y_i , which is generally the volume, correlates linearly with the variable x_i that passes through the origin, increases the accuracy of the estimators in terms of greater precision and reliability than conventional estimation methods (MSA and ME). These estimators are biased by definition, but with small variances. While the EReg is accurate when the relation between the variable y_i and the variable x_i does not necessarily pass through the origin, but with a linear relationship. This sampling will be efficient if the auxiliary variable is easy to measure (Valdez *et al.*, 2006; Roldán *et al.*, 2013; Tamarit, 2013; Muñoz *et al.*, 2014; Ortiz *et al.*, 2015).

The objective of this study was to compare four estimators to propose an efficient sampling strategy that allows to calculate the timber stocks of *Pinus chiapensis* (Martínez) Andresen in a commercial forest plantation (PFC), in the *Tlatlauquitepec* municipality, *Puebla*. The hypothesis was that the four estimators are not statistically different.

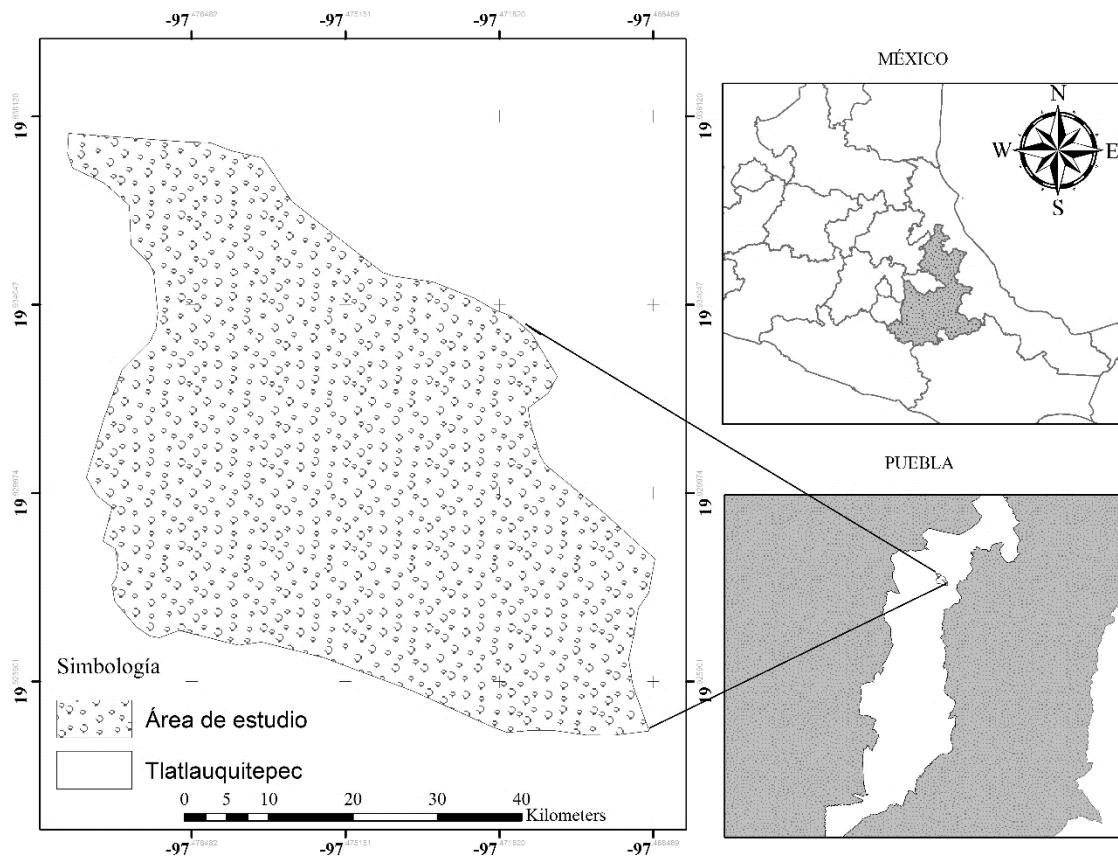
The results will support the administrators when making better decisions in the planning of the plantation, besides having a tool that allows them to keep updated information of the inventory at an accessible cost.

Materials and Methods

Study area

The mensuration information comes from 44 permanent sampling plots of 400 m² (20 m × 20 m) systematically distributed in ages of 3 to 8 years in 87 ha⁻¹ planted with *Pinus chiapensis* in the *Tlatlauquitepec* municipality of *Puebla* State. The seedlings were obtained from seeds of selected trees of the region and were established in lands that were previously used for fruit and coffee production. The plantation was established at a density of 1 100 trees ha⁻¹, at real frame with a spacing of 4 m between rows and 2.25 m between plants. These plantations are located at 19°36'24" N and 97°14'42" W and average altitude of 1 900 m (Figure 1). The climate of the place is semi-warm with rain all year round, with an average annual temperature of 17 °C and an average annual rainfall of 2 350 mm (Inegi, 2009).





Área de estudio = Study area; Simbología = Symbology

Figure 1. Location of the study area in *Tlatlauquitepec* municipality, *Puebla* State.

Mensuration data

The establishment and measurement of the 44 plots (n) was carried out in 2014 with ages of 3, 5, 6 and 7 years, and for the year 2015 the first re-evaluation of the same plots was obtained, with ages of 4, 6, 7 and 8 years. Of all the trees in each plot the following variables were measured: number of trees (*NA*), normal diameter (*D* in cm) with bark at the height of 1.30 m above the soil surface with a *Haglöf*® caliper; and for the total height (*H*), 10 dominant and co-dominant trees were measured with a Vertex Laser *Haglöf*®. To estimate the total height of each tree from the 2014 and 2015 inventory, a two-parameter Chapman-Richards model proposed and discussed by Fierros *et al.* (2017), for the same database described was used.

The mathematical structure of the model is the following:

$$H = 17.8020 \times e^{\left(\frac{0.0694}{D}\right)}$$

Where:

D = Normal diameter (cm)

H = Total height (m)

The H function had a fitted coefficient of determination (R_{adj}^2) equal to 88.50 % and 2.328 as root mean square error (RMSE).

The mensuration data (D and H) from the 2014 and 2015 inventories served as a basis to estimate the basimetric area (AB_1 , AB_2) for such years as well as the volume from the barked total stem at the tree level (V_1 , V_2). After it, the AB_1 , AB_2 , V_1 y V_2 values of each tree were summed to find the numbers per site. AB was calculated with the following equation:

$$AB = \left(\frac{\pi}{40000} \times D^2 \right)$$

Where:

AB = Basimetric area (m^2)

D = Normal diameter (cm)

π = Constant with a value of 3.1416

V was estimated with the equation of local volume of the Schumacher and Hall (1933) type formulated by Martínez (2016) for *P. chiapensis*, which has the following structure:

$$V = 0.000065 \times D^{1.630512} \times H^{1.15635}$$

Where:

V = Volume of the total barked stem (m^3)

D = Normal diameter (cm)

H = Total height (m)

The V function had a fitted coefficient of determination (R_{adj}^2) equal to 99.66 % and 0.0058 as root mean square error (RMSE).

Sampling estimators

To determine the timber stocks of v_1 and v_2 in the *P. chiapensis* plantations, we two classical estimators based on designs were used: Simple Random Sampling (MSA) and Stratified Sampling (ME), and with two based on models: Ratio estimators (ERaz) and Regression Estimators (EReg); the latter uses a population mean ($\mu_x = AB$ and E) that is estimated under ME, which is assumed to be the true population value. For the statistical analysis the statistical program R version 3.4.3 was used. (<https://cran.r-project.org/bin/windows/base/>). Table 1 shows the mathematical structure of the equations that describe the MSA, ME, ERaz and EReg estimators.

Table 1. Sampling estimators used to quantify the V inventory (m^3).

Estimators	Parameter	Equation
	Mean	$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ 1
MSA	Sampling variance	$S_y^2 = \sum_{i=1}^n y_i^2 - \left(\frac{\bar{y}_i - \bar{y}}{n} \right)^2 / n - 1$ 2
	Mean variance	$S_{\bar{y}}^2 = \frac{S^2}{n} \left(\frac{N-n}{N} \right)$ 3
	Sampling mean per h stratum	$N = \sum_{h=1}^L N_h ; n = \sum_{h=1}^L n_h$ 4
ME	Sampling mean in the h stratum	$\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} \bar{y}_{h,i}$ 5
	Sampling variance in the h stratum	$S_h^2 = \frac{\sum_{i=1}^{n_h} (y_{hi} - \bar{y}_h)^2}{n_h - 1}$ 6
	Mean variance in the h stratum	$S_{\bar{y}_h}^2 = \frac{S_h^2}{n_h} \left(\frac{N_h - n_h}{N_h} \right)$ 7
	Ratio	$R = \frac{\mu_y}{\mu_x}$ 8
ERaz	Ratio estimator	$\hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{\sum_{i=1}^n y_i}{n} / \frac{\sum_{i=1}^n x_i}{n}$ 9
	Population mean of the ratio	$\bar{y}_{\hat{R}} = \hat{R} \times \mu_x$ 10
	Mean variance of the ratio	$S_{\hat{R}}^2 = \frac{\sum_{i=1}^n y_i^2 + \hat{R}^2 \sum_{i=1}^n x_i^2 - 2\hat{R} \sum_{i=1}^n x_i y_i}{n-1} \times \left(1 - \frac{n}{N} \right)$ 11
	Mean	$\bar{y}_{REG} = \bar{y} + \hat{\beta}(\mu_x - \bar{x})$ 12
EReg	Slope	$\hat{\beta} = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$ 13
	Mean variance	$S_{\bar{y}_{REG}}^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2 - \hat{\beta}^2 \sum_{i=1}^n (x_i - \bar{x})^2}{n-2} \times \left(1 - \frac{n}{N} \right)$ 14



Where:

\bar{y} = Sampling mean of the main variable V ($m^3 \text{ ha}^{-1}$) observed in the i^{th} sampling site and extrapolated per ha^{-1}

\bar{x} = Sampling mean of the auxiliary variable

N_h = Sampling frame in the h^{th} h stratum

L = Total number of strata in the population

n_h = Total number of sampling units in total number of included in the sample

n_{hi} = Observed value of the main variable V ($m^3 \text{ ha}^{-1}$) in the i^{th} sampling unit in the stratum h^{th}

μ_x = Population mean of the auxiliary variable

n = Sample unit

N = Population size

$\hat{\beta}$ = Parameters of the model estimated from the data of the sample through ordinary least squares (MCO, acronym in Spanish)

t = Parameter that represents the Student distribution at 95 % of confidence with $n-1$ degrees of freedom (gl, acronym in Spanish)

The upper limit and the lower limit (L.S. and L.I.) for the mean (\bar{y}) of the volume ($m^3 \text{ ha}^{-1}$) were calculated, with their respective accuracy (P in %) and the total inventory ($m^3 \text{ ha}^{-1}$), corresponding to the sampling estimators (MSA, ME, ERaz and EReg) used in this study. L.S. and L.I. were determined by the following expression:

$$L.S \text{ and } L.I = \bar{y} \pm t_{n,gl} \sqrt{S_{\bar{y}}^2} \quad (\text{Equation 15})$$

Where:

$L.S$ = Upper limit of V expressed ($m^3 ha^{-1}$)

$L.I.$ = Lower limit of V expressed ($m^3 ha^{-1}$)

$S_{\bar{y}}^2$ = Variance of the V mean ($m^3 ha^{-1}$)

\bar{y} = Sampling mean of V ($m^3 ha^{-1}$)

$t_{n,gl}$ = Level of confidence at 95% ($1 - \alpha = 0.95$)

(gl) = Degrees of freedom which are equal to the number of sampling sites ($n = 44$ sites)

The accuracy is described as:

$$P = t_{a,gl} \sqrt{S_{\bar{y}}^2 / \bar{y}} \times 100 \quad (\text{Equation 16})$$

Where:

P = Accuracy of the mean (%)

$S_{\bar{y}}^2$ = Variance of the V mean ($m^3 ha^{-1}$)

\bar{y} = Sampling of the V mean ($m^3 ha^{-1}$)

Finally, the total inventory was obtained with the following expression:

$$\hat{y} = N \times \bar{y} \quad (\text{Equation 17})$$

Where:

\hat{y} = Total inventory in volume (m^3)

N = Population size

\bar{y} = V sampling mean ($m^3 ha^{-1}$)

Under the ME estimator, the age of the plantation (E) was used as an auxiliary variable, from which four age strata (3, 5, 6 and 7) were obtained, for the 2014 inventory, and four strata (4, 6, 7 and 8) for the 2015 inventory, which are known variables without error (Roldán *et al.*, 2013). In each stratum, the sample of inventory sites was systematically selected. According to Cochran (1977) and Thompson (2002), stratification reduces the variance of the mean in the stratum from an inventory design that groups and weights the variances.

ERaz and EReg use auxiliary information that provides estimates of greater reliability and accuracy than simple estimation methods (MSA and ME). This is the best method to estimate the population mean of the main Y_i (V_1 and V_2), with greater accuracy, when the auxiliary variable X_i (AB and E) has a high correlation with the main variable Y_i . In this context, to estimate V_1 the auxiliary variable that was used were AB_1 and E_1 , while for V_2 it was AB_2 , E_2 and V_1 , auxiliary parameters estimated under ME that assume as true known value at the population (N) level (μ_x) without sampling error.

ERaz offers the advantage that when using the AB as an auxiliary variable there is an R (ratio) that describes the amount of V in $m^3 \text{ ha}^{-1}$ standing for each $m^2 \text{ ha}^{-1}$ of AB . While with E (age) a measure of expected annualized growth is obtained in $m^3 \text{ ha}^{-1}$. It is also interesting to evaluate V_2 as a function of V_1 , an R that involves the main variable measured at time two (V_2) and its same variable measured at time one (V_1); in this way, a reading is obtained on the percentage increase of V_2 according to the initial volume (V_1).

The comparison of the four estimators was based on a numerical analysis: lower mean of the inventory ($m^3 \text{ ha}^{-1}$), greater accuracy (P, %) and lower amplitude (A, $m^3 \text{ ha}^{-1}$) of the confidence intervals. This allows to propose a precise and practical sampling strategy (sampling estimators), which during the survey of field sites is efficient in terms of time and cost.

Results and Discussion

Correlations of the variables

Through an analysis of Pearson's Correlation Matrix, the associated correlations between V , AB and E , which is a statistical measure to evaluate if two quantitative variables have a linear relationship, were studied. The information from this analysis made it possible to observe the trends of the data, which suggests that V and AB had a higher correlation (99.52 and 99.58 %, for the measurement of 2014 and 2015, respectively); secondly, because of the correlation (98.5 %) between the V_2 of 2015 and the V_1 2014, while the E showed the lowest correlation (64.37 and 68 % for 2014 and 2015, respectively). Roldán *et al.* (2013) mention that the previous trends suggest that AB is a potential variable that can be used as an auxiliary variable in the estimation of the inventory, either through ERaz or EReg.

Results of the samplings made

The values of the parameters of the mean of the volume (m^3) of each evaluated sampling estimator and the comparative in terms of gain in precision and amplitude of the confidence intervals are shown in Table 2. The estimators showed a greater variability in terms of P (statistical precision), with values that ranged from 1.55 to 21.75 % for 2014 and from 1.35 to 20.30 % for 2015. EReg (V/AB) under ME was comparatively better, followed by ERaz (V/AB) under ME, both showed the best accuracies, higher values of gain in precision and amplitudes of small confidence intervals.

Table 2. Comparison of the four studied estimators.

Estimator	Mean ($m^3 ha^{-1}$)	R	L.S.	L.I.	P (%)	Inventory (m^3)	G (%)	A ($m^3 ha^{-1}$)
MSA*	56.28	-	68.52	44.04	21.75	4896.68	-	24.48
ME*	55.14	-	60.99	49.29	10.62	4796.94	11.13	11.7
ERaz V/AB ME*	55.35	5.10	56.83	53.87	2.68	4815.39	19.07	2.96
ERaz V/E ME*	56.03	9.58	65.38	46.68	16.70	4874.29	5.05	18.7
EReg V/AB ME*	55.24	-	56.10	54.38	1.55	4806.38	20.2	1.72
EReg V/E ME*	55.84	-	63.22	48.46	13.21	4859.58	8.54	14.76
MSA**	75.83	-	91.22	60.44	20.30	6597.17	-	30.78
ME**	74.52	-	81.38	67.66	9.21	6483.57	11.09	13.72
ERaz V/AB ME**	74.77	5.35	76.50	73.04	2.31	6505.29	17.99	3.46
ERaz V/E ME**	75.69	11.05	87.52	63.86	15.62	6585.62	4.68	23.66
ERaz V ₂ /V ₁ ME**	74.28	1.35	76.53	72.03	3.03	6462.66	17.27	4.5
EReg V/AB ME**	74.67	-	75.68	73.66	1.35	6496.01	18.95	2.02
EReg V/E ME**	75.44	-	84.16	66.70	11.58	6562.94	8.73	17.46
EReg V ₂ /V ₁ ME**	74.40	1.25	76.31	72.49	2.57	6472.60	17.73	3.82

* = Results of the 2014 inventory; ** = Results of the 2015 inventory; R = Ratio; L. S. = Upper limit; L. I. = Lower limit; P. = Accuracy (%); G = Gain (%) in regard to MSA; A. = Amplitude between L. S. and L. I. ($m^3 ha^{-1}$).

Bailes and Brooks (2004) indicate that the consequence of using auxiliary variables (AB) with a greater correlation with the main variable (V) leads to obtain more efficient estimators; in addition, it offers the advantage that it is easy, fast and cheap to measure in the field, which leads to optimize the investment in time and cost during the execution of inventories. The answer to the above is explained in the statistics obtained in the EReg (V/AB) when using the AB estimated under ME, with AB with the highest correlation with V, a P of 1.55 % for 2014 and 1.35 % for 2015, and smaller intervals amplitude (A, $m^3 ha^{-1}$) equal to 1.72 for 2014 and 2.02 for 2015, followed by ERaz (V/AB) under ME. MSA showed the lowest accuracy (P) with 21.75 % for 2014 and 20.30 % for 2015.

However, ME produces results lightly conservative in terms of total inventories compared to EReg (V/AB) under ME (optimistic inventory); undoubtedly, it is feasible to ascribe to stratification, and when using estimators of ME, smaller variances to the estimated mean result; in addition, when the variances of the interest variable are weighed, a lightly conservative inventory is achieved, but such a variation is minimal. For future projects, it would be attractive to continue exploring in EReg (V/AB) a real estimated population value under ME in order to get more precise estimators, a conservative inventory, more accuracy a confidence intervals of greater amplitude.

It is also important to note that the value of the $V AB^{-1}$ ratio from that obtained with the ERaz (V/AB) under ME provides punctual information on the average volume (5.10 and $5.35\text{ m}^3\text{ ha}^{-1}$ for 2014 and 2015, respectively) in existing foot for each m^2 of AB . This information becomes relevant when fast inventories are made by the relascopy technique, whose aim is to estimate the AB inventory in sites of varying dimensions.

On the other hand, $V E^{-1}$ ratio suggests that the average annual increase in plantations of *P. chiapensis* is $9.58\text{ m}^3\text{ ha}^{-1}\text{ year}^{-1}$ for 2014 and $11.05\text{ m}^3\text{ ha}^{-1}\text{ year}^{-1}$ for 2015. Despite the low correlation of volume with age, it is possible to use it as an auxiliary variable to quantify total inventories. When this happens, ERaz (V/E) works as a simplified growth and yield system, in which, based on the weighted age (5.8 years), which is calculated upon the age classes and the size of each stratum, the inventory can be estimated on an annualized basis (Roldán *et al.*, 2013).

ERaz (V_2/V_1) which involves volume two (V_2) of 2015 as the main variable, and volume one (V_1) of 2014 as auxiliary variable, suggests that for every m^3 that was in 2014 (initial volume), the volume grew by approximately 35 %. However, the EReg (V_2/V_1) assumes a more conservative value of 25 % percentage increase.

When comparing the traditional estimators based on design (ME vs. MSA), it is observed that ME presented a substantial gain in precision (11.13 %) and confidence intervals ($11.7\text{ m}^3\text{ ha}^{-1}$) when stratifying by age, in addition it offers the advantage of that generates total inventories more conservative with respect to the MSA. Several authors concluded that when stratifying by age, the accuracy

increases as the variance decreases, which allows obtaining values of the most reliable and precise parameters (Achard *et al.*, 1998; Roldán *et al.*, 2013; Tamarit, 2013). One of the strategies to optimize an inventory is to resort to stratification with some variable strongly related to the variable of interest, in this case it was the volume with the age of the plantations (Lencinas and Mohr, 2007).

Of the four estimators studied and under the evaluation mentioned criteria, EReg (V/AB) presented the best sample estimators. The inventory in estimated volume with EReg is 4 806.38 m³ for 2014 and 6 496.01 m³ for 2015 and, when age is used as auxiliary variable, the inventory is 4 859.5 for 2014 and 6 562.94 m³ for 2015.

Conclusions

When there are differences in accuracy, confidence intervals, means of the variable of interest and total inventory, it is concluded that the four estimators are statistically different, being EReg (V/AB) under ME more accurate to calculate the timber stocks of *Pinus chiapensis* in a Commercial Forest Plantation. EReg offers accurate estimators in forest inventory estimation when the auxiliary variable (basimetric area) has the highest correlation with the variable of interest.

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Conflict of interest

The authors declare no conflict of interest.

Contribution by author

Reynol Fierros Mateo: supervision of the data analysis and writing of the original manuscript; Héctor Manuel De Los Santos Posadas: conceptualization and design of the study; Aurelio Manuel Fierros González: review of the manuscript; Francisco Cruz Cobos: development of the methodology; Luis Martínez Ángel: review and follow-up of results, writing and review of the final manuscript; Efraín Velasco Bautista: development of the methodology, review of the manuscript; Gerónimo Quiñonez Barraza: review of the manuscript.