Felipe J. Almuna Salgado

THE ROLE OF CONTEXT AND CONTEXT FAMILIARITY ON MATHEMATICS PROBLEMS

RESUMEN

La literatura en educación matemática promueve el uso de problemas matemáticos en diferentes contextos, y de ahí que en diferentes programas internacionales de estudios escolares de la asignatura de matemática han incorporado dicha recomendación. Un número de argumentos teóricos avalan el uso de contexto en problemas de matemáticos, sin embargo, la influencia del contexto y en especial el rol de la familiaridad del contexto en el rendimiento estudiantil es una problemática aún no entendida completamente. Después de una revisión de literatura se argumenta, en este artículo, que alrededor de noventa años de investigación del impacto del contexto de un problema matemático en el rendimiento estudiantil, nada concreto puede aún ser afirmado sobre esta relación; lo anterior, se debe a escasa evidencia en esta relación. Dado que el término contexto posee múltiples significados asociados, el artículo clarifica primeramente este término y lo diferencia de otros. Luego, argumentos teóricos y de investigación empírica son revisados en relación al rol del contexto y la familiaridad del contexto de un problema matemático en el rendimiento estudiantil.

PALABRAS CLAVE:

- *Contexto de un problema matemático*
- *Familiaridad del contexto*
- *Rendimiento estudiantil*

ABSTRACT

The mathematics education literature advocates the use of mathematics problems embedded in different contexts and therefore different mathematics curricula reflect this recommendation. A number of theoretical arguments support this, but the influence of context, and specifically the role of context familiarity, on students' performance is an issue that is not yet fully understood. After a literature review, it is argued in this paper that ninety - odd years of research on problem context and students' performance suggest that nothing firm can be said about this relationship, because evidence about this relationship is undeniably sparse. Given that context takes on a number of meanings in the literature, this paper starts by clarifying and differentiating this term from others. Then, theoretical arguments and empirical research are reviewed in relation to the role of context and context familiarity on students' performance.

KEY WORDS:

- *Mathematical problem context*
- *Context familiarity*
- *Students' performance*

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RESUMO

A literatura sobre a educação de matemática defende o uso de problemas matemáticos incorporados em diferentes contextos e, portanto, vários currículos de matemática refletem esta recomendação. Uma série de argumentos teóricos suportam a afirmação anterior, mas a influência do contexto e, especificamente, o papel da familiaridade com o contexto sobre o desempenho dos alunos é uma questão que ainda não está totalmente compreendido. Depois de uma revisão da literatura, argumenta-se neste artigo que noventa e tantos anos de pesquisa sobre o contexto dos problemas e do desempenho dos estudantes sugerem que não podemos concluir nada decisivo sobre essa relação, porque a evidência sobre essa relação é inegavelmente escassa. Atendendo ao fato que contexto tem vários significados na literatura, este artigo começa por esclarecer e diferenciar este termo de outros. Em seguida, argumentos teóricos e pesquisas empíricas são analisados em relação ao papel do contexto e da familiaridade com o contexto sobre o desempenho dos alunos.

PALAVRAS CHAVE:

- *Contexto de problemas matemáticos*
- *Familiaridade com o contexto*
- *Desempenho dos alunos*

RÉSUMÉ

De récentes recherches en didactique des mathématiques semblent indiquer qu'il serait bénéfique d'inciter les élèves à travailler sur des problèmes mathématiques lorsque ceux-ci sont intégrés dans des contextes authentiques ; une recommandation conséquemment souvent miroitée dans les réformes curriculaires. Nombreux sont les arguments théoriques qui soutiennent ce point de vue, cependant l'influence du contexte, et plus précisément le rôle du degré de « familiarité » (des élèves) avec ledit contexte sur la performance reste floue et incertaine, un problème qui est adressé dans cet article. Je suggère ici de retracer les quelques dernières quatre - vingt dix années de recherche à ce sujet et proposer une définition plus approfondie des termes jusqu'alors employés dans ce domaine (en particulier celle de « contexte ») pour ensuite conduire une étude théorique et empirique afin d'étudier plus finement le rôle que joue le contexte d'un problème mathématique et le degré de familiarité dudit contexte perçu par les élèves sur leur rendement.

MOTS CLÉS:

- *Contexte d'un problème mathématique*
- *Degré de familiarité*
- *Rendement des élèves*

1. Introduction

The existing research in mathematics education recommends measuring how well students are able to apply their knowledge and mathematical skills and use them to solve mathematical problems embedded in meaningful contexts for students

(Blum, Galbraith & Niss, 2007). Thus, the incorporation of context in problems have been highly recommended by current reform documents and mathematics curricula around the globe (see for example, NCTM, 2011 and OECD, 2013) which started to develop new forms of connectedness of the instructional mathematical content by focussing on problem solving, applications and modelling on school mathematics.

The latter was not only because of the potential for "motivating students and for the meaningful development of new mathematics concepts and skills" (Depaepe, De Corte, & Verschaffel, 2010, p.138), but also to develop in students the capability to apply and communicate efficiently the mathematics they know in different real - world and everyday contexts (Blum, Galbraith & Niss, 2007; Boaler, 1993; Depaepe et al., 2010; OECD, 2013; Wedege, 1999). This is a major educational aim that continues being highlighted globally through curriculum documents (Galbraith, 2012).

Despite the frames for recommendations, the fact that the influence of context on students' performance in mathematical problems is a matter that cannot be disregarded in school mathematics is confirmed by ample research. For instance, De Lange (2007) examines the use of the real - world as a context for problems in international studies. After reviewing concerns expressed from researchers, this author concludes that:

> The influence of contexts should be studied much more systematically than is presently the case, and we researchers should refrain from strong statements that we have proven to be of disputable quality until we have firmer evidence (De Lange, 2007, p. 1120).

Additionally, factors affecting problems set in context, such as context familiarity of a problem, have been investigated as earlier as 1920s (e.g., Washburne & Osborne, 1926a, 1926b). In general, evidence is sparse. This may be because knowledge of the findings of individual studies (rather than the body of evidence) highlights that there is a lack of a firm body of convincing empirical evidence for the effects (in any direction) of the context of a problem on students' performance (Stacey, 2015).

In this vein, this paper represents an attempt to review and outline the existing literature related to the influence of problem context and problem context familiarity on students' performance. The purpose of this paper is therefore:

- to examine problem context definitions
- to examine arguments for embedding mathematical problems in contexts, and
- to identify and describe the influence and implications of students' context familiarity of a problem on students' performance.

2. Problem context

Context is a term that takes a number of meanings in the mathematics education literature. For example, Bishop (1993) discusses a range of ways in which the term context is used, to describe different aspects of the learning environment. He suggests various layers of contextual influence that impinge on the student. Layers suggested by Bishop (1993) are, namely: the socio - political context in which learning is situated, the physical context of a mathematical activity and the socio and cultural context of the classroom. Although it is acknowledged that mathematical problems are embedded within a social context, and the influence of social contexts and how students' individual perception of a problem context on students' solutions cannot be denied, this paper is primarily concerned with the context in which a mathematical problem is embedded. For more information on these aspects see Busse (2011), Niss, Bruder, Planas, Turner, and Villa - Ochoa (2016), and Civil and Planas (2004).

Greatorex (2014) points out that problem context is a term that is particularly difficult to define. As a matter of fact, in the literature can be found several names and meanings for problem context. Terms such as: cover history[1] (Chapman, 2006; Fuchs, Fuchs, Hamlett & Appleton, 2002; Lewis & Mayer, 1987; Silver, 1981), thematic content[2] (Pollard & Evans, 1987; Ross, McCormick, & Krisak, 1986), content effects (Chipman, Marshall & Scott, 1991), situation[3] (OECD, 2013) and setting[4] are used as alternatives names for the term problem context on the research literature.

Given the plethora of names for problem context, Clarke and Helme (1996) use the term figurative context to define the context of a mathematical problem. They define figurative context as "the scenario where the task [problem] is encountered" (Clarke and Helme, 1996, p. 4) to clarify and distinguish it from the others terms stated above. Busse and Kaiser (2003) further refine the notion

1 This term was usually used in mathematics word - problems and problem solving.

2 This denomination is frequently employed in reasoning studies related to problem context.

3 This name is occasionally put to use when mathematical problems are embedded in real-world contexts.

4 *Situation* and *setting* are also used frequently when referring to context. For instance, within PISA mathematics, situation is used "alternatively as context" (Stacey, 2015, p. 74). However, according to the author of this paper, they relate to a different matter, particularly in the constructivism research and theories of situated learning. In this vein, *situations* are characterised by "social, physical, historical, and temporal aspects" (Roth, 1996, p. 491) under which students operate. On the other hand, the term *setting* is used specifically to refer the physical real-world sites in which human activities take place (Lave, 1988).

of figurative context by distinguishing between objective figurative context and subjective figurative context. According to these authors, the objective figurative context refers to "the description of the scenario given in the task [problem]" (Busse & Kaiser, 2003, p. 4) contrasting with the subjective figurative context associated to the "individual interpretation of the objective figurative context" (Busse & Kaiser, 2003, p. 4). According to these authors, the objective figurative context is "often implicitly meant by researchers when referring to the context" (Busse & Kaiser, 2003, p. 4).

However, it is considered by the author of this paper that the definition of objective figurative context, close to what researchers may intuitively call *context*, is limited. This is because, the objective figurative context, in the way it is stated, seems to draw only attention on what is described in a problem's statement rather than on extra information that might be packed within the context in which a mathematical problem, which students may also need to decode sensibly when mathematising a problem.

The above stance relates to an issue of continuous debate within the Mathematics Education community; this issue connects to the value of the real - world when doing Mathematics. To some, Mathematics is a universal practice that emphasises (factual) content knowledge and procedural skills; from the latter, this position, context is used evidently as a mean to put it in practice. This author, however, focuses on the mathematical relevance of contexts to put in practice mathematical thinking for solving mathematical problems. That is to say, the attention is on the application and communication of mathematics in a variety of contexts in order to perceive the links and transfer between mathematical concepts and procedures, and the real - world. The latter position is taken, for example, by the OECD on its PISA frameworks for mathematical literacy.

To make sense of the above contention, consider the following example. A problem can be related to the estimation of the number of fans attending to a sold out rock concert taking place at a given rectangular field (see Table i below). The rock concert context of the problem is required to find the estimation of the number of people that can be accommodated per square metre. In this problem, context provides a chance to identify assumptions and constraints to use a mathematical model and validate the answer in relation to the context in which the problem is embedded. Of course, the above is a very precise example that highlights that what is described in a scenario of a given problem cannot always be regarded exclusively as problem context; there is information that surrounds an objective figurative context, which may be also used in the problem.

Therefore, and for the purpose of this paper, an operational definition bounding what problem context means is required. Thus, (problem) context has been defined previously as follows:

> Context is the information that is contained and, at the same time surrounds the statement of a mathematical problem that needs to be mathematised. The containing and surrounding information might be necessary or unnecessary for the mathematisation of the problem, but is independent from the problem's syntax and stimulus (Almuna Salgado, 2016, p. 109).

In the above definition, problem's *syntax* refers to the problem's grammar structure whereas stimulus refers to the actual material about the problem that is presented to the student. While syntax encompasses words, stimulus can involve pictures, graphs, diagrams and formulas, or even to its physical and visual layout, and multimedia material. To clarify and exemplify the intended definition of context, consider the example provided in Table i below:

TABLE I
Problem context example

For a rock concert, a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert?
A 2 000
B 5 000
C 20 000
D 50 000
E 100 000
Source OECD (2006, p. 94)

In the example above, the context is related to a rock concert to be held in a rectangular field of size 100 m by 50 m with all the fans standing. Context involves aspects such as dimensions of the rectangular field, facilities for the crowd (e.g., inside or outside the rectangular field, emergency exists, etc.), and more general aspects of the concert including the purchasing of the tickets, and venue details (e.g., in a stadium). However, not all of them are necessary to mathematise this problem. In fact, only the lengths of the field (which are provided to students) and the density of the crowd in a rock concert are needed.

The estimation of a static crowd, in theory, is straightforward (i.e., area of the field multiplied by density of the crowd). The density rule for static crowd estimation[5] is that in a loose crowd the density is about 1 person/m^2, in a

[5] The American journalist, Herbert Jacobs, originally introduced this rule in 1967 when estimating the size of the crowd of the Berkeley riots. The court where students gathered to protest the Vietnam War was marked into grid squares, then "a simple way to estimate the crowd was to count the number of squares and estimate how many students were in each square on average" (Watson & Yip, 2011, p. 105).

solid crowd has about 2 persons/m^2 and very dense crowds have about 4 persons/m^2 (Watson & Yip, 2011). The problem requires students to make and relate their own estimation of the amount of area that a person would take up in such a type of concert in order to solve this problem. The clues *field was full, completely sold out* and *fans standing* are there to guide students in their estimation. The fact that this is a multiple - choice question further helps them. In the above example the words and the gramatical structure give the syntax, whereas the physical and visual layout (i.e., the set of words that is presented to the students) provide the stimulus.

Although it is acknowledged that mathematical problems are embedded within a social context, and the influence of social context on students solutions cannot be denied, this paper is primarily concerned with the context in which a mathematical problem is embedded.

2.1. *What are problems set in context? A general view*

A clear meaning of mathematical problems and ideas (not entirely within the mathematical world e.g., the idea of addition) in context is needed, because of their close relationship to the literature to be reviewed. In this manner, Galbraith (1987) establishes that mathematical problems embedded in contexts are often called applications. Generally, applications require a translation of the problem into a suitable representation to produce the problem comprehension, interpretation, and a mental representation of the problem. Then they require a formulation of a mathematical model, which is linked with that representation, and the successful choice and use of relevant mathematics involved in solving the problem. Taking the above into account, four different kinds of applications are distinguished in the literature, namely: (a) Word Problems, (b) Standard Applications problems, (c) Realistic Mathematics Education (RME) problems and (d) Modelling problems. These kinds of applications sometimes blur because of the degree of variation of context considerations to be incorporated and then required in the solution process of the application problems, but in the examination of their nature, there are fine distinctions that reveal essential differences among them. As Stillman and Galbraith (1998) explain correspondingly:

> Various intermediate stages exist between completely structured word problems and open modelling problems where the structuring must be supplied entirely by the modeller. One such stage involves contexts where the aim of the problem is well defined, where the problem is couched in everyday language, but where some additional mathematical information must

> be inferred on account of the real world setting in which the problem is presented. This is a level between textbook word problems and modelling problems contextualised fully within real - life settings (p.158).

Word problems are often presented as applications of mathematics. They usually involved a question for finding a solution with a context added. They are just dressing up to purely mathematical problems in words trying to link them to a context real or imagined (Blum, Galbraith, Henn & Niss, 2007). In word problems, context can act merely as a camouflage, because the intention of the writer or teacher is to practise, through word problems, mathematical concepts and ideas. Hence, the solving process consists only of the direct use of mathematics; hence word problems are a fixed procedure (a mathematical recipe approach) of translating mathematics and words. Example (1) in Table ii below is a pattern of a word problem.

TABLE II
Example of a word problem and a standard application problem

Type of applications	*Example*
Word problems	(1) Jim has 16 marbles and wins 10 more. How many does he have now? Source Van den Heuvel-Panhuizen (2005, p. 5).
Standard Applications	(2) A return train ticket to Sydney is $145.00. From next 1st of March, the price will be increased by 5%. What will be the new price of a return train ticket?

Example (1), from Table ii, is a problem involving marbles; context is not relevant to practise addition. Besides, as a matter of fact, if the word marble was blocked out, this problem still can be solved. Additionally, the context can be exchanged for another context without altering the demands of problem. To some extent different to word problems, standard applications problems are embedded in either real - world contexts. Nevertheless, standard application problems tends to "focus on the direction: mathematics → reality" (Blum, Galbraith & Niss, 2007, p. 10) and therefore they generally emphasises the mathematical concepts involved. In simple words, "with applications we are standing inside Mathematics looking out: Where can I use this particular piece of mathematical knowledge" (Blum, Galbraith & Niss, 2007, p. 10). They are characterised by the fact that the "appropriate [mathematical] model is immediately at hand" (Blum, Galbraith

& Niss, 2007, p. 12). The latter suggests that in standard applications, students need to be taught specifically about how a mathematical concept applies before practising it. Example (2) in Table ii above, provides an illustration of this kind of problems. The mathematical concept of percentages is embedded in a real life situation (the purchasing of travel tickets); the mathematical model to solve the problem is immediately at hand (145 x 1.05 = 152.5, therefore the new price is $152.50), extra information is not needed because it is widely assumed by the teacher that students know how to use a particular model in a range of contexts (i. e., the mathematical model has been taught to students for its relevance to everyday life). Hence, the context plays a secondary role, that is to say, the context is treated routinely because students have been taught how to use a particular piece of mathematics which fits into a predetermined model or technique.

Features of word problems and standard application problems referred previously indicated that context is a mere add-on to these categories of mathematical problems; this is because, context provides a conservative condition to put in practice mathematical knowledge within acknowledged mathematical models by students. This is not the case for Realistic Mathematics Education problems (RME) or modelling problems, which will be now reviewed. Within these sorts of application problems, context plays a central role in the solving process, although there is a degree of variation in which solving these problems requires different engagement with the context. This variation determines the differences between RME and modelling problems. In RME problems, contexts are required for students to develop understandings of mathematical concepts through 'educational modelling'. To accomplish the mathematical conceptual understandings in students, contexts must be rich in terms of mathematical organisation because in RME, contexts need to be mathematised (De Lange, 1999; Treffers, 1987; Van den Heuvel-Panhuizen, 1999) by students. Before proceeding, it is worth considering what is understood by mathematisation. Briefly and conventionally, the verb mathematising or the noun thereof mathematisation denotes organising reality using mathematics ideas and concepts (De Lange, 1999). Mathematisation is also referred in PISA documents as a mathematical competency which is related to the process of "transforming or interpreting a problem, a mathematical object or information in relation to the situation [context] presented into a mathematical form" (Turner, 2011, p. 4).

To make sense of the mathematisation process, in RME problems, it is important to acknowledge that two processes articulate it, namely: (i) horizontal and (ii) vertical mathematisation. The first, horizontal, is the process of going from the context to the mathematical world. It occurs when students use their informal strategies to describe and solve the problems. Horizontal mathematisation demands activities such as: identifying the specific mathematics in a general context, schematising, formulating and visualising the problem, discovering relations and regularities, recognising similarities in different problems (De

Lange, 1999; Treffers, 1987). In contrast to horizontal mathematisation, vertical mathematisation arises within the mathematical world with the development of mathematical tools in order to solve a situation that requires to be mathematised. In this process, the students' informal strategies to solve the problems influenced students to solve them using mathematical language and tools (Treffers, 1987). As De Lange (1999) highlights, the process of vertical mathematisation can be recognised by the following activities: representing a relation in a formula, proving regularities, refining, combining, adjusting, and integrating mathematical models, and generalising. It is important to acknowledge that for mathematisation purposes reality is not conceived necessarily as a synonymous of the real - world. Instead, reality denotes that "the context of the problems is imaginable for students" (Van den Heuvel - Panhuizen, 2005). This implies that non real - world contexts can be suitable contexts for mathematical tasks [problems] as long as they are "real in the students' minds and they can experience them as real for themselves" (p. 2); this is because of the 'educational modelling' approach rather than for a practical purpose.

From the students' point of view, students' experience of reality consent a sense of problem's meaningfulness to them which assists students to learn, organise, and apply mathematics flexibly (Van den Heuvel - Panhuizen, 2005). This flexibility should not be understood superficially, instead it reflects the fact that mathematical problems can be solved in different ways rather than conducting a fixed procedure, in this manner the later then offers opportunities to students to develop high order reasoning through the mathematisation process (De Lange, 1999; Treffers, 1987).

Finally, it should be recognised that the related use of context in RME problems is dependent on how a real - world context can be inspiration of the learning of a mathematical concept or "for a mathematical theory or an application of it, or both" (Stacey, 2015, p.74). As De Lange (1999) acknowledges, in problems in context the mathematisation process varies according the complexity of a problem's demands.

To a certain extent different from RME problems, modelling problems tend to focus on the direction: real - world → mathematics rather than reality → mathematics as realistic mathematics problems do. Therefore, modelling problems generally highlight an interaction process between context and mathematics. In the formulation stage of these kind of problems, the students face a question situated in a real - word context, and then by trimming away gradually aspects of the real - word context a mathematical model must be formulated, solved, and interpreted (modelling process). Then, the proposed solution must be evaluated mathematically and in terms of the real - world context in which the problem is presented. Modelling (i.e., applied modelling) and RME problems (i.e., educational modelling) are reasonably analogous; they involve an entire process consisting of structuring, also working mathematically, interpreting, and validating (Blum, 2002). This can be explained by the fact that the aspiration of modelling problems are to "develop skills appropriate to obtaining a mathematically productive outcome for a problem with genuine real - world connections" (Galbraith, Stillman & Brown, 2006, p. 237). However, RME problems provide an alternative scenario for students to "learn mathematical concepts and structures that are relevant for the problem situation" (Van den Heuvel - Panhuizen, 2003, p. 13), where mathematical models are seen as vehicles to support progressive mathematisation (Treffers, 1987). The essence of modelling problems, which make them particularly unique in the spectrum of applications, is that, in simple words, [with modelling problems] "we are standing outside mathematics looking in: Where can I find some mathematics to help me with this problem?" (Blum, Galbraith & Niss, 2007, p. 10). Figure 2 below illustrates an example of a modelling problem. As described above, modelling problems highlights the process of students working through the problem in which interaction with the problem context, techniques as well as meta - knowledge are just as important as the result. Although in standard applications problems, a translation into a suitable mathematical representation of the problem statement is required (i. e., students have learned how to do this in context), modelling problems require much more; a real - world context needs to be trimmed away by the solver to recognise and employ mathematical relations and models in order to solve the problem in mathematical terms. Then, mathematical results need to be interpreted and validated with explicit reference to the context in order to produce a solution that addresses the problem in terms of the problem context. The latter is crucial to modelling problems.

Along this vein, in modelling problems students do not know either data or the mathematical model already. It has not been taught because the problem is not common or important enough in real life to teach all students. However, in modelling problems[6] the context, which is derived from the real - world, plays an important role because the either information (data) or mathematical model to solve the task is usually found in the problem context. To conclude, in modelling problems the context plays an important role because the information (data) to solve the problem is usually found in the problem context (Almuna Salgado, 2010). This reference to the context involves a purposeful interpretation of contexts in order to produce a relevant mathematical representation of the underlying problem and therefore a solution that addresses the problem, as exemplified in the problem presented in Table i.

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[6] The characteristics of modelling problems offered in the above paragraphs are a very simplified interpretation of them. For a detailed insight into modelling, see for example Stillman (2002). She offers, among other insights, a comprehensive literature review of how modelling has been understood since its inception in different educational systems.

While the importance of using mathematical problems in context seems to be well acknowledged, the "degree to which the context of a task [problem] affects students' performance is widely underestimated" (Boaler, 1993, p. 13). Although Boaler made this statement more than twenty years ago, it has not lost its relevance; many issues remain to be resolved about the effects of problem context on students' performance. These issues will be discussed along next sub-sections.

2.2. *Arguments for embedding mathematical problems in contexts*

The emphasis of the curriculum documents on problems in context can be furthered by a set of theoretical arguments for which context should be used in mathematics. These are:

- *The formative argument*
  The emphasis is put on the application of mathematics in context as a means for developing general competencies, attitudes, and skills orientated towards fostering creative and problem solving abilities as well as "open - mindedness, self - reliance, and confidence in their [students'] own powers" (Blum & Niss, 1991, p. 42).
- *The critical competence argument*
  This argument highlights the importance of preparing mathematically literate students to enable them to "see and judge independently, to recognise, understand, analyse, and assess representative examples of the uses of mathematics, including solutions to socially significant problems" (Blum & Niss, 1991, p. 43).
- *The utility argument*
  Problems in context may enhance the transfer of mathematics to other contexts. They may increase the chance of students applying mathematics that they had learned at school in other areas in later studies, everyday contexts or future employments. Mathematics is seen under this argument as a service subject or as a subject of instrumental interest (Helme, 1994). This argument relies on the assumption that the ability to use mathematics in context "does not result automatically from the mastering of pure mathematics but requires some degree of preparation and training" (Blum & Niss, 1991, p. 43).
- *The picture of mathematics argument*
  This argument stresses the importance of providing students with a rich and comprehensive picture of mathematics in all its facets, "as a science, as a field of activity in society and culture" (Blum & Niss, 1991, p, 43). That is to say, mathematics in context reflects the nature of mathematics as a human activity (Van den Heuvel - Panhuizen, 2005).

- *Promoting mathematical learning argument*
  This argument insists that mathematics in context is well suited to assist students in "acquiring, learning, and keeping mathematical concepts, notions, methods, and results, by providing motivation for and relevance of mathematical studies" (Blum & Niss, 1991, p, 44); contributing to train students who can think mathematically within and outside of mathematics.
- *The use of mathematics in real - worlds contexts argument*
  The use of contexts may assist in overcoming the common perception of mathematics as a "remote body of knowledge" (Boaler, 1993, p. 13) with no connection to the real - world. Mathematical problems in real - world contexts may allow students to understand the connection between mathematics and the real - world (Felton, 2010, p. 61) highlighting that mathematics has a relevant meaning in the real - world. Moreover, when assessing mathematics embedded in real - world contexts it allows students to "discover whether students have been well prepared mathematically for future challenges in life and work" (Stacey & Turner, 2015, p. 7).
- *The halo - effect argument*
  Last but not least, Pierce and Stacey (2006) show that some teachers use contexts that appeal to students (for example a problem about a dog) to improve students' attitude towards learning mathematics by associating the subject with pleasant things. This association of mathematics with pleasurable parts of students' lives is what Pierce and Stacey (2006) call the *halo - effect.*

3. The role of context on students' performance

Research studies in the field of cognition (Fiddick, Cosmides & Tooby, 2000; Marsh, Tood, & Gigerenzer, 2004) account in general that the contexts in which problems are embedded influence the strategies that "individuals choose to solve problems and the success of those strategies" (Leighton & Gokiert, 2005, p. 2). Cognitive experiments started to take place in the early 1970s. These experiments aimed to study the role of context in reasoning. They were stimulated mainly by the work of Piaget's theory of formal operations[7]. British psychologists Peter Wason and Philip Johnson - Laird devise an experiment on deductive reasoning

[7] According to Inhelder and Piaget (1958) at the age of 11 years old approximately, children gain the ability to think in an abstract manner, the ability to combine and classify pieces of information in a more sophisticated way, and the capacity for higher-order reasoning. Therefore, problem solvers should be guided by problem's logic, content and structure rather than problem's context.

which is known today as the four - card problem (Johnson - Laird & Wason, 1970). The original experiment -and its later variations- show that people, when solving a problem in context, usually rely on some problem's contextual features (e.g., context familiarity) rather than abstracting from the content as suggested early by Piaget's theory of formal operations (Johnson - Laird & Wason, 1970).

At the broad - spectrum, the experiment conducted by the British researchers aimed to test people's deductive reasoning by applying the logic conditional rule (i.e., if… then) when following an introduced rule. In the original version of the experiment -presented in Table iii below- a problem involving whether or not cards which contained vowel / consonant letters printed on one side have odd / even numbers on the other side.

TABLE III

Example of the four - card problem

Here are four cards. You know that each has a number in one side and a letter on the other. The uppermost face of each card is like this:

A	D	4	7

The cards are supposed to be printed according the following rule: If a card has a vowel on one side, it has an even number on the other side. Which among the cards do you have to turn over to be sure that all four cards satisfy the rule?

Source Johnson-Laird & Wason (1970, p. 134)

Participants were presented with four cards, showing respectively A, D, 4, 7. It is known that every card has a letter on one side and a number on the other. Participants were then given the rule: *If a card has a vowel on one side, then it has an even number on the other side, and were told: Your task is to say which of the cards you need to turn over to find out whether the rule is true or false.* Out of 128 undergraduate university students, only five chose the two right cards (Johnson - Laird & Wason, 1970).

Due to the low frequency of correct answers, a later study examines the effects of adopting a more realistic appearance of the card problem. In this manner, Johnson - Laird, Legrenzi and Legrenzi (1972) decide to employ a different context, a postal context. At the time of the experiment, there were two rates for mailing envelopes -first and second class rates[8] - in England and Ireland. In this manner, researchers used the following rule in the experiment:

[8] At that time, if a person mails an envelope sealed it requires a first - class stamp, but if a person decides to mail the envelope unsealed then a second - class stamp was needed.

If a letter is sealed, it has a 50 Lire stamp on it. Then participants were asked to imagine themselves as post - office workers sorting letters; then five envelopes were presented to them and they were instructed to: *select those envelopes that you definitely need to turn over to find out whether or not they violate the rule.* This cognitive problem embedded in a realistic context resulted easier than the symbolic one, 22 out of 24 undergraduate university students at one university in London turned over the correct envelopes (Johnson - Laird et al., 1972). These authors then infer that the better rate of response in this problem compared to the four - card problem can be attributed to the postal context. They refer to the improvement in correct responses as the *thematic - materials effects.*

Some American researches in the 1980s questioned the reliability of this effect as no facilitation of correct response was observed in their replication experiments using context (Manktelow & Evans, 1979). One hypothesis for no such a replication of results could be the fact that participants were inexperienced with postal regulations. Hence, Griggs and Cox (1982) create a closer context version (see Table iv below) to their participants (one - hundred and forty undergraduate university students at one American university). Within this context, what needs to be checked is both the type of drink of the person who is under 19 years old and the age of the person who drinks beer.

TABLE IV
Example of a familiar context to the four - card problem

You are in charge of a party that is attended by people ranging in age. The party is being held in a state where the following law is enforced: *If you are under 21 you cannot drink alcohol.*

Under 19	Over 19	Drinking soda	Drinking beer

Of these four, who do you need to check in order to make sure that the law is not broken?

Source Griggs and Cox (1982).

The results on this experiment show that "74% of the participants made a correct selection of the drinking problem while no one did for the abstract problem" (Griggs & Cox, 1982, p. 415). The data also provides evidence that "participants' extra - experimental experience has a significant impact on the performance on the Wason selection task [problem]" (Griggs & Cox, 1982, p. 501).

The results of the cognitive experiments presented do not show that human reasoning is not logical, but that the traditional logic is not a proper normative under certain conditions. Highlights on these well - designed and tested cognitive experiments show that context can aid or obstruct a solver getting the correct response. These experiments have the same logic structure, but they appear to be solved using different approaches. The original four - card experiment is a problem in pure logic; whereas the envelope and drinking versions of the original experiment may not be problems in pure logic. In the variations, the success of given the right answer has been shown to depend whether or not the context of the problem is familiarly meaningful to the solver. It seems that embedding the original problem in sufficient familiarity with the context, prevented participants from making the logical errors occurred in the four - card problem. For example, the drinking rule in Florida -the state from which the participants of the drinking - age problem took place- was well debated in 1980. At that year, this state in its general legislation raised the age of drinking from 18 to 19 years of age (for more information see The Florida Legislature Service Bureau, 1980). Hence, it can be inferred that undergraduate students had specific familiarity of the context to reason adequately about it.

The results of the above experiments do not suggest necessarily that students need to have familiarity with the context present in a mathematics problem. On the contrary, it can be suggested that the relevant question raised by the findings of these experiments is in line with the contemporary issue of embedding mathematics problems in context, that is to say: how context and context factors such as context familiarity, of a problem may influence students' performance? Some insights to this question have been made previously, although the next section offers more understandings on role of context familiarity on students' performance.

4. The role of context familiarity on students' performance

The effects of context familiarity on students' performance had been researched and reported as early as 1920s. In general, evidence is sparse and findings are inconclusive. This is because knowledge of the findings of individual studies highlights that there is a lack of a firm body of convincing empirical evidence for the effects (in any direction) of familiarity of the context of a problem on students' performance. Thus, this section represents an attempt to scrutinise a possible effect of context familiarity in the students' performance.

Whether performance in solving problems is affected by the familiarity of the context has been studied by many. One early classic study on familiarity was carried out by Washburne and Osborne (1926a,1926b) who report a two years research on the difficulties that students from Year 3 to Year 7 have when solving arithmetic problems. The study involved 23 American schools. The number of students tested in all the study varied from "three hundred to more than a thousand" (Washburne & Osborne, 1926a, p. 219). One of the difficulties studied was the effect of the unfamiliarity of the problem context, or with the materials with which the problem deals causing failure to solve the problem correctly. This aspect was studied in two schools by giving students across Year 3 and Year 7 ten arithmetic problems. The problems consisted in a pair of five problems with the same mathematical difficulty; one problem dealt with a less familiar situation or with less familiar materials than the other. The way in which the familiarity of the problems was determined was not stated. Presumably, the researchers classified familiar vs. unfamiliar contexts from their point of view.

Results indicated that in average, 80.5% of students in schools answered correctly problems embedded in more familiar context to students and 67.5% answered correctly the problems embedded in unfamiliar contexts. These authors conclude that unfamiliarity with materials and contexts is a small factor in causing difficulty with problem - solving, but unfamiliarity is not a large element as may be supposed (Washburne & Osborne, 1926a, 1926b).

At the beginning of research into the influence of familiarity of context on performance, there was conflicting evidence on this relationship. For example, Brownell and Stretch (1931) research whether the success in performance of Year 5 students in America (n = 256) in solving the arithmetic of problems was conditioned by either the familiarity or lack of familiarity of the four contexts in which the problems were embedded. It should be noted that the original problem was in a context of boys scouts from which students needed to decode the expression to compute (i.e., $3 \cdot 34 - 91$) and the three remained presented students the arithmetic expression to compute but it was embedded in different contexts (i.e., soldiers cavalry, refining oil plant, and Hindu village). The variation in familiarity of the contexts presented to students was determined from the researchers' point of view. Students solved all of the four versions of the problem. Results indicated that significant increase in difficulty was observable as context familiarity decreased. Eighty percent of the students were unaffected by the changes in familiarity. This can be explained by the fact that students could have recognised the problems presented to them as similar, as they had to solve all the four versions, or by the fact that students were given the arithmetic expression to compute in the unfamiliar contexts. In any case, the conclusion of these researchers was that problems were not made unduly difficult for children by unfamiliar contexts.

Although, there were few studies between the 1930s and the 1960s investigating the effects of familiarity on performance (see for example, Post, 1958; Sutherland, 1942), one of them highlights in the merit of its conclusions. Lyda and Franzén (1945) in their study involving approximately two thousands

Year 7 to Year 11 American students provide an interesting connection for the triad context familiarity, performance and students' age. These researchers find students' age as a major factor conditioning students' performance. Their findings suggested that as students developed in age -from Year 7 to Year 11, their performance in problems set in familiar / unfamiliar contexts "gradually diminishes for the obvious reason that the pupils [students] have the ability to see similarities between the situation [contexts] of the problem, those of other problems, and those they had in real life" (Lyda & Franzén, 1945, p. 295). These authors do not discuss the exact nature of this effect, although this can be explained by the fact that in their research they used arithmetic and algebraic problems in which procedural knowledge can be applied in different contexts from remembering methods and recognised when they needed to be applied.

The introduction of large - scale assessments in America and the new approaches to data analysis and interpretation have opened new potentials to revisit the study of context and its impact on performance from different perspectives. Hembree (1992), for example, conducts a meta - analysis of forty - four studies, involving Year 4 to undergraduate American students, in which the problem context differed in terms of (i) abstract (using symbolic or intangible subjects and objects) vs. concrete (involving a real situation and objects) contexts, (ii) factual (simply describing) vs. hypothetical (not only describing but using if - then statements to contemplate possible changes) contexts, (iii) familiar vs. unfamiliar contexts, and (iv) imaginative (using fantasy or unusual circumstances) vs. personalised (using the solver's own interests and characteristics to write the problem) while the corresponding mathematical structure remained constant.

The meta - analysis results show that better performance was statistically significant and most strongly associated with familiar contexts, whereas mean effects with borderline significance was associated in (i) and (ii) categories, and no context effects were found in category (iv) (Hembree, 1992).

However, in this meta - analysis from the forty - four studies analysed, only four of them (n=1608) corresponded to studies of standard mathematical problems embedded in familiar vs. unfamiliar contexts with students of Year 5, 6 and 12. However, theoretical considerations of familiarity were omitted in his meta - analysis which may suggest that changes in problem contexts (familiar vs. unfamiliar) could result in statistical significance difference of performance under certain statistical conditions.

The treatment of familiarity and its impact on statistical results is an issue that Chipman, Marshall, and Scott (1991) address in their research. In a careful design study, these authors analyse the way in which the context of problems might affect solving performance in undergraduate students (n=256) at one American university. Sixty-four algebra problems were embedded in four different contexts, namely: masculine, feminine, neutral familiarity and neutral unfamiliar. The researchers test two hypotheses. One was that students' performance might be affected by contexts typed as appropriate for the opposite sex. No statistical support was found for this hypothesis.

The other hypothesis was that student's performance might be affected negatively by unfamiliar contexts. Students of both sexes were more likely to "omit problems of neutral but unfamiliar content and less likely to solve such problems correctly" (Chipman et al., 1991, p. 910). This hypothesis was supported statistically, but small in magnitude. Hence, context familiarity assisted in the performance of both genders. As the authors report, the result was obtained in an experiment ($n_1 = 128$) in which the problems' context familiarity was controlled, hence they can attribute this result to context.

Along with these hypotheses and results, these researchers carried out a preliminary rating study on two variables under study (i) sex stereotype of the context and (ii) personal familiarity of the context. This was done primarily in order to guide the construction of the sixty - four problems. Nevertheless, when analysing the results of the rating study on familiarity, researchers realise that students' judgements on context familiarity seemed to measure familiarity of the underlying problem structure (i.e., problem familiarity) rather than context familiarity. Therefore, in factoring context - familiarity out the problem, Chipman et al. (1991) find that problem familiarity might strengthen the problem difficulty and hence, students' performance.

The exact nature of this effect is not discussed explicitly by these authors, but it can be inferred from their work that a familiar problem structure might induce well establish solving routines, which can account for producing correct solutions; consequently, these two different types of familiarity / unfamiliarity need to be distinguished at all times.

The literature that relates real - world problems and the students' performance also support the positive effects of problems set in familiar contexts. Empirical studies such as those by Cooper and Dunne (1998) and Carraher, Carraher and Schliemann (1985, 1987) that show that students' socioeconomic background can influence students' activation of the real - world knowledge, and hence the use of the context familiarity, when they solve mathematical problems set in a more realistic or a real - world context. It seems that activation of real - world knowledge in such socioeconomic disadvantage students (e.g., street sellers) dealing with familiar contexts in a direct mathematical experience appears to be as supportive of effective problem solving. These students have presented to exhibit more advance mathematical reasoning as well as better performance.

A seminal couple of studies in this area are: Mathematics in the streets and in the schools (Carraher et al., 1985) and Written and oral mathematics (Carraher et al., 1987). In Carraher at al. (1985) study, young Brazilian street sellers (n = 5, aged 9 to 15 years old) performance on mathematical problem presented in real - life contexts was greater to that on school - type mathematical problems and on context - free computational problems involving the identical numbers and operations. From this study, Carraher at al. (1985) infer that students might benefit from contexts designed to activate real - world knowledge.

Based on the findings and the hypothesis above, Carraher et al. (1987) conduct a follow - up study with 16 Brazilian Year 3 students. Three sets of arithmetic problems were given to students, but embedded in three different contexts, namely: (i) in a simulated store situation in which students played either the role of the store owner or the customer, (ii) embedded in a standard application word exercise, and (iii) in symbolic computation exercises. In that way, Carraher, et al. (1987) find that Brazilian students showed significant differences in performance when they solved simulated store contexts (outside school contexts, usually presented in verbal form), than problems inside school contexts presented in written form, and symbolic computation exercises.

Results confirmed that students performed better in solving store problems than in solving symbolic computation exercises; the average difference in facility being about 20% between store problems and symbolic computation ones. Finally, differences in the way students approached the altered problems versions were also detected by the researches because in the stimulated store contexts students had to deal with money (a concrete real - world construct), which changed the arithmetic demand of the problems. In that case, Carraher, et al. (1987) suggest that embedding problems in contextualised real - world contexts can be meaningful for students due to the activation of real - world knowledge facilitates problem's accessibility, hence it can lead them to a greater performance.

Some studies tried to replicate the above finding; however, it was found that students did not normally performed better on mathematical problems embedded in real - world contexts, which conflicted with the findings reviewed in the above paragraphs. For instance, Baranes, Perry, and Stigler (1989) intend to replicate Carraher et al. (1987)'s findings with Year 3 American students. Baranes et al. (1989) find that no contextual effects were found in either performance or strategy use for success with the American sample (n = 18); that is to say, the students did not generally activate their real - world knwoledge and representation of it in the solution of the problems. Participating students in this research activated their real - world knowledge in some specific cases. It took place when numbers used in the word problems presented to them made it possible to induce students to stimulate knowledge of "a culturally constituted system of quantification, such as money" (Baranes et al., 1989, p. 316).

McNeil, Uttal, Jarvin and Sternberg (2009) acknowledge that although the results of the study above differed from the findings of Carraher et al. (1985, 1987), the results do correspond with other research (Carpenter, Lindquist, Matthews & Silver, 1983; Verschaffel, De Corte & Greer, 2000; Verschaffel, De Corte & Lasure, 1994; Yoshida, Verschaffel & De Corte, 1997) in terms of students' activation of the real - world knowledge. The above body of replication studies highlight overall that students show difficulties in activating their real - world knowledge and it has been found that students do not normally performed better on mathematical problems embedded in real - world contexts. Nonetheless, this can be explained probably by the fact that students in those studies did not work as street sellers and almost certainly had more consistent schooling than the Brazilian students that Carraher and colleagues' studies had. However, some empirical research evidence points out that familiarity of the context may be associated with either negative or neutral impact on students' performance. For instance, Helme (1994) investigates the impact of context familiarity on the responses of nine adult women students (full - time - return - to - study program at a vocational education and training provider) to eighteen mathematical problems in six different content areas. Findings reveal that students did not perform better on more familiar problems to them, on average than problems without context or problems set in unfamiliar contexts. Helme (1994) accounts that individual differences -such as language barriers and individual performance on specific problems- overshadowed group trends; these may be responsible for the not significant performance on more familiar contexts to students.

Along this same matter, Huang (2004) explores to what extent four everyday shopping mathematical problems set in familiar vs. unfamiliar contexts for students influenced their performance and perception of problem difficulty in forty - eight Year 4 students from two classes of a public elementary school in Taipei, Taiwan. In the study, the hypothesis of familiar contexts assist students in their performance was not supported from the data obtained. The results revealed interestingly that students did not perform better than that problems embedded in unfamiliar contexts. The difference was statistically significant. Moreover, students spent a longer time in solving problems with familiar contexts; this difference was as well statistically significant.

From this result, it might be implied that the set of familiar problems presented to students appeared more difficult to them. From the integration of the above results with the data obtained from the students' perception on problem difficulty, Huang (2004) conjectures that familiarity of the context would promote the conscious representation from a situation to its mathematical structure -as Bernardo (1994) also did-. However, the above "effect does not seem to be strong enough to influence deep - level processes, such as identification of relevant information for figuring out a correct calculation for an accurate solution" (Huang, 2004, p. 286).

In other study reported by Shannon (2007), she tests the same mathematical content (linear function) embedded in three different contexts, namely: supermarket trolleys, shopping baskets, and paper cups. The three problems consisted in diagrams with common objects that could be nested when stacked. Students needed to formulate the corresponding linear function describing how the height of the pile of objects would vary with the numbers of objects stacked. Despite of the similarities of the mathematical process to create a formula that represents the height of the pile of objects in every case, students were more successful when working with cups. Next, she analyses how students abstracted salient features of geometry of the contexts above into variables required to solve the problem. In her analysis, she determines that the specific geometrical structure of the cups facilitated the students' success with this variant, rather than the familiarity of the context. She also highlights, as Chipman et al. (1991) did, the issue of relative familiarity with the problem.

Almuna Salgado (2010), in other small scale study, tries to scrutinise how the performance of thirty Year 10 students on four PISA items compares with performance on variants with more familiar contexts. Results show that performance was not better when they solved problems with more familiar contexts. This might be explained by the fact that the greater familiarity of problems in this study was not empirically determined, but was only established from the researcher' opinion. It may also be explained if the new and more familiar problem were not technically as well constructed as the multiply - trialled PISA items, but as Almuna Salgado (2010) points out this may be unlikely because the PISA items were such a close model for their variants.

However, one latent issue with problems set in contexts is the potential differential effect of context familiarity on students' performance. Van den Heuvel - Panhuizen (1999), for example, acknowledges -from a theoretical point of view- that the use of mathematical problems embedded in familiar contexts are not always helpful to students and may also generate difficulties in students' performance. Some students may ignore the context, while others may focus on context aspects that are not necessary for the problem and fail to engage with the necessary mathematics required to solve the problem.

In this vein, Almuna Salgado and Stacey (2014) argue that one difficulty with familiar contexts is that they tend to elicit responses in students that may be based on integration of personal knowledge and values with mathematics in order to build an intended solution. Familiar contexts also may be borderline cases where the relatively stronger understanding of a problem plays a role when students communicate an answer; students may assume that it is not necessary to give a very detailed answer because everyone already knows the arguments.

However, it follows from above that familiar contexts may not always helpful to students and may generate difficulties in students' problem solving. Familiar contexts can hinder some students finding an answer, while others may focus on contextual aspects and fail to engage with the necessary mathematics required to solve the problem. As can be seen below, some research findings indicate that familiar contexts can distract students from a problem's mathematical structure.

For example, in an often cited small scale study, Boaler (1994) analyses the performance of 50 female students on two sets of questions intended to assess the same mathematical content (equivalence of fractions) but set in different contexts (i. e. soccer season, planting plants, cutting pieces of wood, and a fashion workshop). Results show that females underachieved in contexts with which they were probably more familiar (e. g. fashion rather than soccer). They often took excessive account of contextual information in the problems. Boaler (1994) speculates that the relative underachievement on a fashion problem was because the attractive and familiar context distracted the students from the mathematical structure.

Almuna Salgado (2010) small study supports the above. Qualitative evidence from the students' interviews on this study revealed that in more familiar contexts, some students tended to bring personal information into arguments rather than using a mathematical argument. A familiar context was in certain cases (i. e., money and robberies context) interpreted and judged as personal rather than from a mathematical point of view (Almuna Salgado & Stacey, 2014), which did not produce a very detailed answer.

The unpredictable differential effect of the context familiarity, which may be positive or negative in contextualised problems seems to be clear. The above research literature suggests that it makes sense to consider that evidence "indicate that one cannot say anything firm about the relationship context familiarity to success rate" (De Lange, 2007, p. 1119), because the results so far are variable.

5. Final comments

Although studies considered in this review are not directly comparable due to different methodologies and age and school level of participants, results of individual studies suggest that problem context can affect students' performance in variable ways. In this vein, the previous review of research and commentary on all of the aforementioned studies have raised several issues on how problem context and context familiarity might influence on students' performance, which this paper aims to examine.

From the literature, it is clear that evidence is undeniably sparse on this relationship. In general, a number of studies reviewed in this paper seem to suggest that familiarity of a context may have a larger effect than unfamiliar contexts (especially on cognitive studies); in this case, literature tends to finds that a high level of context familiarity may have a positive effect on performance. In addition, students' socioeconomic background can influence students' activation of the real - world knowledge, and hence the use of the context familiarity, when they solve mathematical problems set in a more realistic or a real - world context. Besides, it appears that familiar contexts can result in easier problems for students, but the abstraction and transfer of the corresponding mathematical structure remains difficult.

On the other hand, empirical evidence of small studies points out that either neutral or negative effect of context familiarity can be associated to the students' performance. For instance, in a more familiar context, some students may tend to bring personal information into arguments rather than using a mathematical argument. A familiar context may be in certain cases interpreted and judged as personal rather than from a mathematical point of view. Nonetheless, due to the nature of these small studies, it is difficult to make strong claims.

Ninety - odd years of floundering on research of problem context leads to infer that the relationship between context and students' performance needs more careful research with new methodologies, deeper analyses (both quantitatively and qualitatively) and experimental control of the way in which context is involved.

As an example, the line of research of the author of this paper is on the relationship of the effects of three contextual features (i.e., context familiarity, context engagement, and use of the context) on mathematical problems with the same mathematical core whilst varying contextual features. It is believed that perusing this particular line of research may generate not only answers to unsolved questions, but also it may assist to synthetise and create a body of empirical research on the relationship of problem context and the students' performance.

It also may be particularly helpful in offering another view in the way in which context is treated at the mathematics classroom by teachers and students. In this vein, it is anticipated that understanding the relationship between context and students' performance can provide deeper and finer understandings of how some context factors may influence students' performance, thereby contributing to the improvement of assessments among teachers, policy makers, and assessment writers. In addition, it is expected that this study has implications for the teaching practice of mathematics. On one hand, it is hypothesised that carefully chosen contexts can facilitate performance and promote cognitive strategies when solving problems in context. When one of the goals of mathematics is to provide a model for students to think with, problems in context provide an opportunity to do so. Hence, better information about how context affects students' performance might help to teachers to instruct students how to work more effectively with problems in context.

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Autor

Felipe J. Almuna Salgado. Universidad de Melbourne, Australia. felipe.almuna@uach.cl