Barbara Garii, Fredrick Silverman

BEYOND THE CLASSROOM WALLS: HELPING TEACHERS RECOGNIZE MATHEMATICS OUTSIDE OF THE SCHOOL

RESUMEN. Los maestros de primaria con frecuencia luchan para conectar las matemáticas con las experiencias cotidianas. Esto puede ser debido a que los maestros, quienes enseñan matemáticas y los profesionales en el campo de las matemáticas constituyen dos grupos distintos, y es así que pertenecen a comunidades etnomatemáticas de práctica muy diferentes ya que usan distintos algoritmos y vocabularios para explorar conceptos matemáticos similares. Este artículo está constituido por una revisión a la literatura sobre este tema y un reporte de investigación de una iniciativa de desarrollo profesional al respecto. Participaron seis maestros en este proyecto de desarrollo profesional durante una semana, en el que se les expuso a ideas y conceptos relacionados, pero distintos de las matemáticas del salón de clases. Al término de la semana, todos los maestros articularon conexiones más amplias y más profundas entre sus prácticas de aula y las matemáticas fuera de la escuela. Este trabajo sugiere que los maestros necesitan más estímulo y apoyo para explorar los límites entre el salón de clases y los usos profesionales de las matemáticas. De esa manera, los maestros podrán ayudar a sus propios estudiantes a reconocer la importancia de las matemáticas fuera del salón de clases.

PALABRAS CLAVE: Etnomatemáticas, comunidades de prácticas, escuelas primarias de matemáticas, formación de profesores (matemáticas).

ABSTRACT. Primary school teachers often struggle to connect the mathematics they teach to meaningful real world experiences. This may be because, most of the time, classroom teachers who teach mathematics and professionals who use mathematics are members of different ethnomathematic communities of practice. Each community of practice uses different algorithms and vocabularies to explore similar mathematical ideas. A literature review of this subject and a research report of a related professional development initiative comprise this paper. Six teachers participated in a week-long professional development project that exposed them to mathematical ideas and contexts that related to, but were different from, classroom mathematics. At the end of the week, all teachers articulated wider and deeper connections between their classroom practices and mathematics outside the school. This work suggests that teachers need encouragement and support to explore the edges between classroom and professional uses of mathematics. By doing so, teachers will be better able to help their own students recognize meaning for and of mathematics outside the classroom.

KEY WORDS: Ethnomathematics, communities of practice, primary school mathematics, teacher education (mathematics).

RESUMO. Os professores dos primeiros anos do ensino fundamental têm muitas vezes dificuldades em ligar a matemática que ensinam a experiências significativas do mundo real. Isto pode acontecer porque os professores que ensinam matemática e os profissionais que usam a matemática são

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membros de diferentes comunidades etnomatemáticas de prática. Cada comunidade de prática usa diferentes algoritmos e terminologia para explorar ideias matemáticas semelhantes. Este artigo é composto por uma revisão de literatura neste assunto e um relatório de investigação de uma iniciativa de desenvolvimento profissional relacionada. Seis professores participaram num projecto de desenvolvimento profissional de uma semana que os expôs a ideias matemáticas e contextos que se relacionam com a matemática da sala de aula mas são diferentes desta. No fim da semana todos os professores articularam conexões entre as suas práticas e a matemática fora da sala de aula mais alargadas e profundas. Este trabalho sugere que os professores precisam de encorajamento e apoio para explorar as ligações entre a aula e os usos profissionais da matemática. Desse modo, os professores ficarão mais aptos a ajudar os seus próprios alunos a reconhecer significado à matemática fora da sala de aula.

PALAVRAS CHAVE: Etnomatemática, comunidades de prática, professores primários, formação de professores (matemática).

RÉSUMÉ. Les instituteurs en école primaire rencontrent fréquemment des difficultés pour faire comprendre les rapports qui existent entre les mathématiques et les expériences de la vie quotidienne. Il est peut-être possible d'expliquer ce problème en rappelant que les instituteurs (qui enseignent les mathématiques) et les professionnels des mathématiques constituent deux groupes bien distincts. Appartenant à des communautés ethnomathématiques dissemblables, leur pratique respective change car les algorithmes utilisés et le vocabulaire employé pour explorer des concepts mathématiques similaires ne sont pas les mêmes. Abordant ce sujet, cet article explore la littérature se rapportant à ce thème et présente les résultats d'un travail de recherche portant sur un projet de formation professionnelle dans le même champ d'études. 6 instituteurs ont participé à ce projet pendant une semaine. Au cours de cette période, ils se sont familiarisés avec des idées et des concepts ayant rapport avec le thème principal mais qui ne font pas partie de l'enseignement des mathématiques en classe. A la fin de la semaine, tous les instituteurs ont pu imaginer des rapports plus importants et plus profonds entre leur pratique pédagogique et les mathématiques hors de l'enceinte scolaire. Ce travail suggère donc que les instituteurs ont besoin d'être mieux stimulés et soutenus pour pouvoir repousser les limites de la salle de classe et y faire entrer les utilisations professionnelles des mathématiques. Les efforts qu'ils déploieront en classe pour montrer combien sont importantes les mathématiques en dehors de l'école n'en seront que plus efficaces.

MOTS CLÉS : Ethnomathématiques, groupes de pratique, mathématiques dans les écoles primaires, formation des professeurs (mathématiques).

1. Introduction

We face a dilemma in mathematics education in primary school: many teachers teach a mathematics they do not fully understand to students who see, recognize and use less mathematics in their lives than ever before (Hastings, 2007). For example, teachers are often unable to articulate the role of mathematics and mathematical decision making in terms of development of public and political policy, medical assessment, technology planning, and generalized problem-solving (Bajaj & Grynbaum, 2008; Bakalar, 2006; Mukhopadhuyay & Greer, 2007; Schiesel, 2005; Schoenfeld, 2007). As societies becomes more technologically reliant, the visible, day-to-day use of mathematics diminishes (Skovsmose, 2005): mathematics becomes hidden behind the tools we use to interface with society, such as internet protocols, global positioning units, automated check out machines, and the engineering specifications that underlie the design of highway interchanges. Thus the application of mathematics is less obvious and less understood (Friedman, 2005; Schiesel, 2005) by many classroom teachers thereby limiting their ability to fully support student learning sophisticated connections to real life (Greshner, 2007; Hoyles, Newman, & Noss, 2001; Koirala & Bowman, 2003).

Ethnomathematics, the study of how mathematics is used and explained in cultural groups (D'Ambrosio, 2006b), alludes to an approach we can use to begin to understand the disparity between school mathematics and mathematical practices outside of the classroom. Ethnomathematics asks how groups that share a common culture and identity, such as indigenous tribal communities or professional/vocational communities, organize, articulate, and utilize mathematics and mathematical knowledge within their respective communities. Thus, mathematicians, whose work often addresses the abstractions of ideas that underlie mathematics itself, and classroom teachers, who teach mathematics in primary and secondary school setting, are members of separate ethnomathematic communities (Moreira & David, 2007).

1.1. *School Mathematics*

School mathematics is generally recognized by both teachers and students as an entity separate from much of the "real world" (Bishop, Clarke, Corrigan, & Gunstone, 2006; Empson, 2002; National Council of Teachers of Mathematics, 2006; National Mathematics Advisory Panel, 2008). In effect, school mathematics is its own ethnomathematic community that shares little with the vocabulary and practices utilized in other professional and socially identifiable ethnomathematic communities (such as the mathematical vocabularies, practices, and procedures of carpenters, engineers, or surgeons (Masingila, 1996; Gainsburg 2006; Shockey, 2006)). More specifically, most U.S primary school teachers utilize a limited mathematical vocabulary (Ma, 1999) and fail to recognize how school mathematics, including standard algorithms and correct vocabulary usage, is a necessary precursor for understanding, and participating in, the maintenance and development of technological activities in all fields (Nicol, 2002; Skovsmose, 2005). For example, both the Numbers and Operations standard and the Algebra standard of the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) suggest that teachers help students consider what numbers mean through the exploration of patterns and relationships. These ideas are often introduced as students gather data and create organized, visual displays of information and interpret the meaning derived from these relationships (e.g., data analysis and probability, reasoning and proof). Textbooks, and the teachers who use them, encourage students to collect data about their lives and communities but the structure of the assignments and the interpretative lens through which students make sense of the information generally focus on simplistic ideas that hold little long-term value or interest (Clements, Jones, Moseley, & Schulman, 1999; Rubin & Mokros, 2004; Tierney, Nemirovksy, Noble, & Clements, 2004; University of Chicago School Mathematics Project, 2004). In a similar vein, textbook problem sets ask students to work with a variety of number systems (e.g., binary, octal, hexadecimal) to help deepen students' understanding of the patterns of number systems and the meanings of place value. Students attempt to add and subtract within these systems (Burton & Maletsky, 1999; Charles, Crown, & Fennell, 2004; Maletsky, 2004), but rarely are they encouraged to consider where and how these systems are embedded in the technology that surrounds us, from the simple on/off circuitry in electrical power switches to the generation of visual images on television and other electronic display screens. Thus, U.S. teachers are often unable either to bring out-of-classroom mathematics into the classroom or to help students explore connections between students' own out-of-classroom experiences and the school mathematics algorithms and ideas the students encounter in their classrooms. As a result, mathematics classrooms have become bounded ethnomathematic communities: what is taught and learned in the classroom is tied to out-of-school experiences only in superficial (e.g., games) or overt (e.g., making change) ways (Garii & Okumo, 2008) while the organizational strategies and vocabularies of out-of-school ethnomathematics communities are neither acknowledged nor incorporated into the classroom mathematical experience. School mathematics knowledge is not easily transferable to non-trivial out-of-school experiences and practices. Thus, if connections between school mathematics and "out-of-school mathematics" are weak at best, then students are less likely to recognize that school mathematics can prepare them for opportunities and careers that are less immediately recognized as involving mathematics. Hence, it seems advisable for teachers to initiate and nurture the connections early in children's school years.

1.2. *Mathematics Beyond the Classroom*

Outside of the typical primary school classroom, standard mathematical concepts are differentially applied in different professional ethnomathematic communities. Practitioners use alternative nomenclature and rationale to describe similar concepts, thereby masking the underlying abundance of mathematical ideas to which they relate in many classroom experiences (Gainsburg, 2006; Gerofsky, 2006; Masingila, 1996; Nicol, 2002; Noss, Hoyles, & Pozzi, 2002; Shockey, 2006). Carpetlayers, engineers, and surgeons all rely on 2- and 3-dimensional geometrical understanding to articulate their professional decision making; however, each professional practice utilizes a specific vocabulary and overarching organizational structure that is not easily translatable across professional practices.

More prosaically, teachers are less likely to make connections between school mathematics and the mathematics that underlies real-life day-to-day experiences (Garii & Okumo, 2008; Gerofsky, 2004, 2006; Koirala & Bowman, 2003; Moreira & David, 2007). For example, primary and early secondary teachers recognize how out-of-school experiences integrate with language arts, social studies, and science content (Kennedy, 2006) to help students make connections between classroom curriculum and real world experiences. Community sites, such as cemeteries, become laboratories for the study of science (erosion processes) and social studies (local history and town planning) (Easley, 2005). Similarly, inviting students to create art in public places introduces them not only to design aspects of the art itself but issues surrounding community politics that can be brought into language arts and social studies classrooms (Stephens, 2006). Thus, the classroom vocabulary in these disciplines becomes part of "real world" language, thereby giving students and teachers the ability to translate school learning beyond the walls of the classroom. More often than not, however, U. S. teachers have neither the vocabulary nor the breadth and depth of experience to offer their students entry into the community of mathematical thinking outside of the school classroom.

1.3. *Ethnomathematic Communities of Practice*

While ethnomathematic communities refers specifically to the mathematics identified and articulated by specific cultural and professional groups (D'Ambrosio, 2006b), communities of practice refers to a broader but allied concept of how members of a community interact and work together to solve shared problems and concerns, thereby creating a shared vocabulary and understanding of solution strategies (Lave & Wenger, 1998; Wenger, 2006). Members of communities of practice may or may not recognize that they are participating in an interactive learning community (Wenger, 2006). By juxtaposing our understanding of communities of practice with that of ethnomathematic communities, we may broaden our understanding of why and how school mathematic communities fail to successfully interact with professional and vocational communities outside of the classroom.

Within any community of practice, the members of the community rely on a shared set of practices, vocabularies, and processes that may not be visible to or understood by those outside of the community (Lave & Wenger, 1998; Wenger, 2006) This is similar to the concept of bounded ethnomathematic communities, referred to earlier. What Wenger (Lave & Wenger, 1998; Wenger, 2006) recognized, however, was the members of the community of practice may not explicitly recognize themselves as members of that community. Often, members of the school mathematics community (both teachers and students) do not recognize that the shared learning experiences in the classroom create a domain of knowledge, including a shared vocabulary and skill set, that defines them as an ethnomathematic community of practice (D'Ambrosio, 2006b; Kathmandu University School of Education, 2008). Classrooms represent ethnomathematic communities of practice; teachers and students across the country are members of this ethnomathematic community of practice. However, members of the classroom community may not recognize or understand the mathematics of other ethnomathematic communities of practice, such as carpetlayers, engineers, and surgeons. Yet, in order to support student learning and assure that students are able to function mathematically outside of the classroom, teachers must be able to identify and translate some of the mathematical practices used outside the school community into the vocabulary of the classroom. Evidence suggests that many teachers are unable to do so (FitzSimons, 2002; Garii & Okumo, 2008; Gutstein, 2006; Moreira & David, 2007). Van Oers (2001) suggests that school mathematics is the activity of participating in a mathematical practice. What happens, then, when students and their teachers do not recognize that they are participating in important and relevant mathematical practices?

1.4. *Mathematics Education for Prospective Teachers*

The goal of mathematics education for prospective teachers focuses on ensuring that the teachers understand the basic mathematics concepts they will teach and have access to developmentally appropriate pedagogy and practices (Dahl, 2005; Donnell & Harper, 2005; Gerofsky, 2004; Hill, Rowan, & Ball, 2005).

Additionally, it is hoped that both prospective and practicing teachers recognize connections between school mathematics and daily practices (e.g., calculating unit cost or interpreting a graph or chart in the newspaper) in order to better teach and support students as they learn to recognize these connections. Yet little attention is paid to ensure that educators acknowledge implicit mathematical practices that are part of daily life beyond the connections made in textbooks (Reys, Lindquist, Lambdin, & Smith, 2007; Sheffield & Cruikshank, 2005). The examples shared in textbooks often do not accurately reflect issues of particular interest to the students themselves (e.g., calculate distance in feet and inches between houses in a neighborhood or the weight of 100 nails). More importantly, in this context of ethnomathematic communities of practice, textbook examples couch out-of-school mathematics in the rhetoric of the mathematics classroom, rather than helping students make connections between school mathematics and the ethnomathematic understandings of an out-of-school community of practice (e.g., exploration of zoning regulations that mandate minimal distances between neighboring houses or the cost effectiveness of selling nails by the pound).

One aim of teacher education is to ensure that new teachers are able to create and implement equitable practices in their individual classrooms that support students' abilities to recognize and participate in the aforementioned opportunities and career preparations (Garii & Okumo, 2008). At the same time, teachers are encouraged to create, support, and offer possibilities for both students and themselves that allow for intellectual exploration and growth within their own understanding of mathematics and community.

1.5. *School Mathematics as Entry to Ethnomathematic Communities of Practice*

Democratic citizenship is a complex endeavor that requires participant-citizens to have access to high quality education. Mathematics education may contribute to or detract from the cultivation, nurturance of, and respect for, meaningful participation in the democratic process (Delpit, 2006; Gutstein, 2007; Hannaford, 1998; Simmt, 2001; Valero, 2008). Access to equitable mathematics education for all children, especially but not limited to children marginalized by language, poverty, transiency, and/or special needs, demands incorporation of more explicitly inclusive classroom practices that recognize the varieties of mathematical visions, vocabularies, and understandings (Barton, Tan, & Rivert, 2008; Kathmandu University School of Education, 2008) recognized within various ethnomathematic communities of practice. A consequence of students' inadequate or incomplete mathematics education is the increased tendency for them to eventually withdraw from effective and/or meaningful participation in (even local) community decision-making due to the their inability to understand or articulate the mathematical reasoning often associated with community decision making processes (Gutstein, 2007). While utilization of mathematical reasoning techniques may clarify the consequences and implications of various decisions, students' inability to interpret mathematical ideas in context effectively increases the likelihood that their decision making will be based on ideology. Thus, if we envision the ethnomathematic community of school mathematics as a bounded, non-transferable mathematics community, then we are reducing students' abilities to fully participate in the growth of society.

In summary, while current mathematics curricula perhaps support "high level thinking" (Noddings, 2008), both as an entity unto itself and as preparation for state-sanctioned exams, the explicit mathematics that is needed to create, develop, and support technological growth, change, and innovation is neither recognized, understood, nor supported in most mathematics classrooms. If teachers do not recognize or understand the relationship between school mathematics and professional practices that include mathematics and mathematical thinking, then (and unfortunately) students are offered at most opportunities to explore just the edges of ethnomathematic communities of practice across educational and professional contexts.

A recent project asked six experienced teachers to articulate the connection between their classroom mathematical practices and students' real world experiences. Specifically, they were asked to link the curriculum they were mandated to teach and their understanding of NCTM standards (National Council of Teachers of Mathematics, 2006) with mathematics practices outside of the classroom. Our goal was to ascertain if and how primary school and middle school teachers recognize evidence of school mathematics outside their classrooms and how they bring this recognition to students' attention inside the classroom.

2. Methods

2.1. *Research Setting*

For over twenty years, Project SMART has led a series of year-round professional development collaborative programs for K-12 teachers in Central New York. The primary goal of this project is to increase teachers' interest in and understanding of mathematics, science, and technology with an explicit emphasis on connections to real-world experiences. Every summer, Project SMART hosts a one-week Summer Institute that focuses on helping teachers develop inquiry-based lessons and units that have applications to real-world practices (Project SMART, 2008). Teachers from throughout New York State are invited to participate in the Summer Institute during which they work in collaborative teams to deepen content knowledge and design curriculum for implementation in their own classrooms. During the Summer Institute Week, participants both take part in a series of Project SMART seminars and work in project teams that focus on specific areas of practice.

2.2. *Participants*

Six experienced teachers (2 - 20+ years experience) volunteered to join the *Math Team* during the 2007 Project SMART summer institute. Five teachers taught in elementary schools in Central New York (four as classroom generalists, one as an art teacher) and one teacher taught mathematics at an urban, inner-city middle school. This teacher was certified as an elementary school generalist and had been teaching out of her certification area for over five years. All six teachers had participated in at least one previous Project SMART summer institute.

2.3. *Procedures*

During the summer of 2007, the Math Team met every day for five days to explore connections between school mathematics and "real world practices." The Math Team was led by two university faculty who were also Project SMART team leaders. On Day 1, participants were introduced to the idea that mathematics was an integral part of our day-to-day lives. Prior to the discussion of mathematics integration into daily life, participants viewed two short videos that illustrated geometrical processes associated with laying carpet and positioning internal organs during heart surgery (Cardiothoracic Associates of Hawaii LLC, 2004; This Old House television, 2007). Both videos introduced participants to real-world applications of planar and solid geometry and enabled participants to make previously unconsidered connections between classroom mathematics and professional practice while acknowledging geometric concerns specific to the occupational task. For example, the first video (This Old House television, 2007) taught homeowners how to lay carpet. Embedded in the video was discussion of how to cut carpet to fit a space while keeping in mind issues of cost as well as traffic flow, carpet design, and carpet nap. The second video (Cardiothoracic

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Associates of Hawaii LLC, 2004) demonstrated techniques of heart removal and replacement. The surgeon-narrator introduced the importance of ensuring that correct placement of the heart mandated attention to the angles of the connecting veins and arteries to assure appropriate blood flow. Prior to any discussion about the video, the participants then completed a short questionnaire that asked them to describe the mathematical ideas and tasks associated with carpet laying and heart surgery. Participants were also asked to describe how they connected the school mathematics to students' real world experiences.

On Day 2, we began with a discussion of how mathematics is articulated outside of textbook expectations and presentations. Next, participants reviewed a series of photographs illustrating a variety of common experiences (e.g., inner city traffic congestion; parking lots; birds in flight; barristas preparing coffee). Together, faculty and project participants identified the implicit mathematics in each photograph (e.g., traffic patterns, traffic signal programming, reflectivity of paint on cars, design of parking spaces, standardization of asphalt preparation, issues of aerodynamics, issues associated with the growth of, importation of, and economics of coffee production). We also viewed the YouTube video entitled *Diet Coke and Mentos* (EepyBird.com Productions, 2006) during which two men in white lab coats used the chemical and physical interactions associated with dropping crushed Mentos® into bottles of Diet Coke® to create geysers and elaborate fountains. After viewing this amusing video, we considered the embedded mathematical possibilities, including descriptions of the geometry of the fountains (arcs, angles, and parabolas), how to explain and interpret algebraic formulae that describe and characterize the strength and heights of the geysers, how to create domino-effect fountains, and how such a video would encourage the integration of science, language arts, social studies, and art with the mathematics itself. That afternoon, participants were given digital cameras and asked to take photographs of their communities.

On Day 3, participants showed their photographs and discussed some of the implicit mathematics that they identified. Day 4 began with a presentation by an attorney who specialized in property disputes. She introduced the mathematical ideas inherent in her practice, which included planar geometry, statistics, and cost-benefit analyses.

During the afternoon of Day 4, participants were asked to prepare a series of lessons appropriate for their classes that integrated mathematics with real world experiences. These were presented to their fellow Math Team members on Day 5. Finally, participants again viewed the carpet laying and heart surgery videos and completed a post-project questionnaire. They were again asked to identify the

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mathematics associated with carpet laying and heart surgery. Additionally, they were asked to give examples of other real world illustrations that connected to the mathematics they were teaching and that they might use in class.

2.4. *Analysis*

The study used phenomenological analysis (Creswell, 2007; Moustakas, 1994) of teachers' recognition and articulation of mathematical ideas that are not contained within the typical classroom experience. We queried the different ways that experienced teachers conceptualize and characterize mathematics in the real world and how they translate those real world experiences into classroom practice (Linder & Marshall, 2003; Marton & Pong, 2005). Data was collected from pre-participation and post-participation questionnaires and transcriptions of discussions held during Math Team meetings. Data was then entered into NVIVO (QSR International, 2006), a database that allowed manipulation and categorization of teachers' thoughts and ideas about mathematics in real world contexts. The constant comparison method was employed to identify core concerns and responses (Charmaz, 2006; Richards, 2005) that offered evidence to suggest that participation in this workshop helped teachers increase their ability to connect real world contexts to school mathematics.

3. Results

3.1. *Initial Understandings*

3.1.1. *Mathematical ideas associated with carpet laying and heart surgery (Viewing 1)*

When asked to describe the tasks associated with carpet laying, the teacher responses reflected recognition of the most obvious of the mathematical possibilities.

Teacher 2: You would have to know the size of the room so you would know how much carpet to buy. You would have to know the price of the carpeting and the pad that goes underneath. You would have to know how long it takes to lay a certain amount of carpet.

Teacher 5: Knowing the volume and mass [of the heart], liquid measurement, calculating time.

The videos alluded to more subtle mathematical ideas inherent in these tasks, including estimated wear-and-tear on the carpet (and its association with the quality of the carpet), the width of the carpet roll and the nap of the carpet itself (associated with area measurements), blood flow measures through veins and arteries of different diameters, and the necessary angles between the heart and other organs to ensure appropriate placement to support continued health. However, these ideas were not "mathematicized" by the teachers. The ideas they articulated reflected simple ideas that did not particularly address the specifics of either carpet laying or heart surgery.

In other words, simply observing the mathematics-in-practice did not allow the teachers to explicitly relate the professional tasks to school mathematics practice. One reason for this might be that teachers do not have the specific vocabulary of the professions to identify and recognize the mathematics involved (e.g., they may not understand what "carpet nap" is and fail to understand the geometric implications associated with "carpet nap"). It is also possible that the teachers' understanding of "important mathematics" is attenuated; perhaps they may assume that the contextualization of mathematics limits the "mathematicalness" of the task.

3.1.2. *How did teachers recognize mathematics in the world outside of their classroom?*

Not surprisingly, teachers had difficulty articulating "real world mathematics" that did not come out of their textbook experiences. When asked to write a sentence that "explains the connections you make from school mathematics to real world mathematics, focusing on job, professional, and vocational uses of mathematics," teachers articulated ideas that reflected standard textbook exercises.

Teacher 1: Whole number concept and decimal concepts: this is the math of your bank account.

Teacher 2: I related fraction to things students do on a regular basis. For example, we talk about the fraction of pizza they ate with friends.

Teacher 3: I teach geometry, shapes, and formulas. We take a geometry walk in the neighborhood to find shapes in houses, streets signs, etc.

Later, in conversation in the classroom, one teacher discussed how she used recipes to help students connect the classroom study of fractions to real-world settings.

Teacher 4: We double the recipes or sometimes we divide them in half.

Instructor: So if a recipe uses, say, 3 eggs…

Teacher 4: Right, the students have to say the new recipe uses 6 eggs or 1½ eggs.

Instructor: Half an egg? In a recipe?

Teacher 4: Yes, fractions. That's right.

The teachers articulated the importance of incorporating mathematical concepts across disciplinary lines ("cross-curricular studies" [Teacher 5]) and the inclusion of examples that resonated with their students, yet they offered few viable examples of how they brought these ideas into their classrooms. One teacher discussed the teaching of the "economics of credit cards vs. debit cards, balancing checkbooks" [Teacher 3] but assured us that the focus was on the mathematics, not on the economic meaning of the mathematical results. This is similar to the comments above, from Teacher 4, who suggested that adding half an egg to a recipe was appropriate.

Teacher 2 mentioned that her big mathematical ideas in her class focused on her students' realities: "Percents, [they] must figure out the price of items discounted by certain percents, such as $14.99 minus 25% off. And charts and graphs. We survey [ourselves] and research top selling CDs and graph them using Excel program." Finally, the teacher who taught in a very low income, inner-city, urban school explained that she taught the geometric concept of area by asking her students to calculate the size of a tarp needed to cover a backyard swimming pool.

These examples suggest that the "real world examples" identified by the teachers reflect what is in their textbooks, rather than the lived experiences of themselves or their students. While teaching about credit cards and checkbooks are important, it is unlikely that elementary school students use either of these. In other words, the ideas that are presented as "real world" may be both artificial and trivial to the students.

3.2. *Post-Participation Understandings*

3.2.1. *Mathematical ideas associated with carpet laying and heart surgery (Viewing 2)*

After participating in the Project SMART activities, teachers again were asked to view the two video and relate carpet laying and heart surgery back to the mathematics classroom. The responses were more detailed and involved.

Teacher 5: Estimating the [carpet laying] job would involve time management, costs of materials, measurement and traffic patterns for selection of carpet. Installation of the carpet would involve geometry, angles, proportion, area of the room, density of the foam and carpet in conjunction with the use of the area. Some limitations could be the shape of the room and sectioning of materials, costs of materials and the ability to visualize 2-d transition into 3-d action.

Teacher 1: Mathematical tasks [associated with heart surgery]: ideas of volume, rate, estimating for size, questions of blood flow. Constraints include watching for folds (spatial understanding) on the heart to avoid damage.

In other words, after spending a week discussing and exploring out-of school mathematical ideas and experiences, teachers were able to articulate many more mathematical connections upon viewing these video. Their ability to recognize mathematical tasks that are not explicitly described as mathematical had improved, as evidenced by the number and variety of mathematical possibilities that they identified as contextualized in professional practice.

3.2.2. *Creating Mathematical Connections in the Classroom*

The culminating activity for this week-long Professional Development workshop asked teachers to articulate units and lessons that would help students make explicit connections between mathematics that they taught and content that was not typically thought of as mathematical. The final class discussions suggested both excitement and deeper understanding and recognition of mathematical connections. One of the teachers prefaced her unit by reminding us how much she disliked teaching math, but, she said,

Teacher 2: I'm intrigued by the abstract but I'm bound to the practical. I was thinking of taking the kids to the playground, tying these [mathematical] ideas to things that have an impact on them. Whenever we get to the playground, the softball field is… in use. We could use another softball field [but the playground doesn't have enough room]. So after we study area, perimeter, ratio, we could create a new softball field, maybe half the area.

This led to a spirited, spontaneous discussion among the six participants that considered the sports field from a variety of mathematical perspectives.

Teacher 1: Look, if we use Google Earth, we can identify the school and playground. We can use issues of scale on the satellite shots as compared to maps and real-world measurements.

Teacher 3: How about a comparison of baseball fields, locally and nationally? Some kids have no idea how big 90 feet is.

Teacher 1: Drafting.

Teacher 5: How much chalk do we need to draw the line between the bases.

Teacher 6: They're rebuilding Yankee Stadium but they can't rebuild Fenway Park. How about city planning and mathematics?

Two teachers (Teachers 3 and 4) noted that their jointly-created project was influenced by the Diet Coke® and Mentos® video (EepyBird.com Productions, 2006):

Teacher 3: We developed an activity in an area I'm not strong in, combining graphs, formulas, the visible world. Two ideas illustrating the same thing. First, we have two kids on opposite sides of the room with a nerf ball[1]*. One kid has to throw the nerf ball to the other kid as slowly as possible. And the ball has to be thrown as close to the ceiling as possible. Slow and close to the ceiling makes sure that the curve is very clear, very obvious. We take a video and put it on the computer, slow it down, look at it frame by frame. Or, have a sand table. Have one kid lift the table slightly by one end. A second kid … takes a ball and lets the ball fall along the sand, perpendicular to the lift. Two ways of looking at a parabola. Identify a few key points, see what's happening when you compare what's going on between the X and Y axes. We can put [this] on the video … and watch it.

The incorporation of visual images into mathematics units helped teachers articulate ways that they could introduce deep mathematical ideas to their students. The visual images became the tool that connected the school mathematics to the ways that mathematics is used and interpreted outside the classroom.

Teacher 6: [I would] use a picture to introduce a lesson. Ask the kids, briefly, without much preamble, what math do you see. And then I take the picture away—they would say "she's crazy, why did she show us this picture?" But I'll introduce some lesson about angles or fractions or whatever. Then we'll go to Google and find pictures that illustrate these ideas. The kids will find the pictures. And then we can go back to the original picture to find the mathematics that we were learning.

Teacher 2: I was thinking we can use this as an assessment, also. At the end, give the kids a picture and list of vocabulary and have them find the mathematics. To get a "B," you find the vocabulary [in the picture], ideas that we studied in this unit. To get an "A," you have to look beyond, see more, look for old ideas, past units, and new ideas, even ideas that we haven't studied yet. We have to see more than the obvious.

The post-participation examples suggest both the teachers' enthusiasm and their increased confidence regarding the mathematics they are teaching. The discussions on that last day were exciting and the teachers took advantage

[1]* Nerf balls are balls made of spongy, lightweight foam molded to resemble balls used in a variety of sports.

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of the conversation to explore a variety of new ideas with each other. Simultaneously, they were also more aware of their own mathematical knowledge. While several did acknowledge that mathematics was not their academic strength, they did recognize and bring to the table their own interests and enthusiasms within a mathematical context.

4. Conclusions and Implications

At the outset of this project, these six teachers represented an ethnomathematic community of practice (D'Ambrosio, 2006b; Wenger, 2006) in that they shared a common mathematical language (the language of the classroom) and a common set of practices (reliance on textbook, use of obvious examples). At the same time, they did not recognize or acknowledge their membership in this community nor did they recognize that there were mathematical communities outside of their classroom. In other words, these teachers understood mathematics to be a content area that was taught with little connection to mathematical ideas outside of the classroom. Other ethnomathematic communities of practice were ignored or dismissed, perhaps due to lack of vocabulary or the professionals' emphases on the context of the mathematics rather than the mathematics itself. Teachers have been trained to understand mathematics as numeric problem solving rather than as a way of approaching problems. Thus, the teachers were not prepared to adequately scaffold their students to recognize mathematical ideas outside of the classroom because the ways in which mathematics is found in real world practices does not reflect the mathematics as taught in the classroom (Bishop et al., 2006; Koirala & Bowman, 2003; Skovsmose, 2005).

Initially, teachers could not disentangle the mathematics from the reasons for using the mathematics. For example, teachers prioritized student understanding of "charts and graphs" or "percent discount" within a decontextualized framework. Teachers asked students to solve problems based on real-world phenomena (such as ownership of material goods), but did not help students recognize how these associated mathematical representations hold meaning beyond formulaic mathematical presentation. Nor did they consider how these representations might or might not be appropriate for the alleged context being considered.

For example, one teacher asked students to create a graph of their own CD buying habits. While this may appear to be an easy graph to prepare within the

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classroom, students' responses raised questions about income disparities, family expectations, and cultural values that are difficult to map onto a graph. These issues are neither acknowledged nor discussed, although the graphs and charts may illustrate inequities and/or differences in the lives of the students (Gutstein, 2006; Valero, 2001). The students' questions indicated nuances worthy of attention for interpreting their representations. Too often, these questions are deemed not part of the mathematics curriculum, when, in fact, these questions help contextualize the mathematics being taught.

Within the course of the week, these teachers identified and articulated connections more substantive than their initial ones between the mathematics of their classrooms and the mathematics of the world outside of their classrooms. Mathematics became less of an abstract topic of study as the visual connections between mathematical ideas and real world practices were incorporated. While the teaching of mathematics as specific content is important and should not be devalued, this work suggests that such an emphasis that places mathematical understanding in a context-free environment heightens the ethnomathematic divides between communities of practice. However, when teachers are encouraged to connect school mathematics to authentic, non-trivial real world contexts, they themselves become enthusiastic and visionary. The teachers begin to see the reasons for mathematical understanding. They begin to understand mathematics beyond simple algorithms: mathematics becomes a lens with which to ask questions about and make sense of occupational tasks within professional contexts. Once they are able to discuss and articulate substantive ways that mathematics is embedded in daily life, then they are prepared to scaffold their students to do likewise.

When these teachers joined the project, they recognized that there classrooms were, effectively, bounded, ethnomathematic communities of practice (although they were unable to articulate them as such). At the end of the project, as they actively made connections with other communities of practice (e.g., city planners, cartographers, surgeons, carpet layers), the walls of the classroom ethnomathematic community of practice became porous. While we do not argue that the primary school teachers in this project could identify, articulate, and solve specific problems within other ethnomathematic communities, they were able to recognize that the mathematics they were teaching had meaning in those communities. Mathematics became more than an end unto itself. Mathematics became an exploratory tool, as illustrated by the discussions about softball fields and baseball stadiums and the creation of parabolas. In fact, the mathematics experiences became more dynamic for the teachers, enabling them to create for themselves some authentic wonderment about the mathematics, as a habit of mind more in keeping with mathematics as a search of relationships that are not immediately obvious.

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Teachers are at the forefront of maintaining or changing societal norms (Bransford, Brown, & Cocking, 2000; Christiansen, 2007). Incorporating ideas and practices from a variety of ethnomathematic communities of practice into classroom pedagogies encourages both teachers and students to be aware of the ways in which all peoples – including children and out-of-school formal and informal users of mathematics – integrate mathematics into their daily lives. The interactive learning community of the classroom becomes tied to the interactive communities of practices outside of the classroom; the mathematics used both in and outside of the classroom is recognized as a common tool that has different manifestations according to the contexts in which it is situated. Recognition of varieties of community- and professionally-based ethnomathematic vocabularies and practices offers students alternative techniques with which to explore the world around them.

We suggest that it is imperative for all mathematical educators to explore the edges between school, technological, and professional uses of mathematics practice and to help primary school teachers negotiate those borders. By doing so, teachers will support students as they connect school mathematics with their own lives through a complementary relationship between academic mathematics and ethnomathematics (D'Ambrosio, 2006a). Specifically, let this paper open three questions raised by the inclusion of ethnomathematic ideas in the classroom. First, what is the enactment of ethnomathematic practice in the primary school classroom that creates a bounded community of practice? Second, how might the broadening of our understanding of ethnomathematic communities of practice allow students and teachers to explore the abundance and detail of mathematics used naturally in real world practices, both implicitly and explicitly. And third, by widening the definition of "acceptable mathematical practices and experiences in schools" to include ethnomathematic ideas from different professional, vocational, and avocational communities, how, if, and why does the field of mathematics itself become more accessible and more "real," particularly to aspiring teachers, inservice teachers, and their primary and elementary school students?

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Autores

Barbara Garii. Department of Curriculum and Instruction. Oswego, New York, Estados Unidos; barbara.garii@oswego.edu

Fredrick Silverman. University of Northern Colorado. Colorado, Estados Unidos; flsilver@gmail.com

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