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Respuesta a: Comentarios sobre "Puntos de equilibrio asintóticamente estables en nuevos sistemas caóticos"
Author's reply to: Comments on "Asymptotically stable equilibrium points in new chaotic systems"

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Resumen

Debido a que el Teorema 1 de (Elhadj and Sprott, 2012) es erróneo, algunos de los sistemas encontrados en el artículo (Casas-García *et al*. 2016) pueden tener orbitas homoclínicas o heteroclínicas pudiendo aparecer caos en el sentido de Shilnikov. Sin embargo, la aportación fundamental de nuestro artículo fue encontrar diez sistemas dinámicos simples, en tres dimensiones, con términos no lineales cuadráticos, que presentan un punto de equilibrio asintóticamente estable y son caóticos, lo cual se logró. Estos se obtuvieron usando el método Monte Carlo aplicado específicamente para la búsqueda de estos sistemas.

Palabras clave: sistemas caóticos; equilibrio asintóticamente estable; no existencia del caos de Shilnikov; exponentes de Lyapunov

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Abstract

Since theorem 1 of (Elhadj and Sprott, 2012) is incorrect, some of the systems found in the article (Casas-García *et al*. 2016) may have homoclinic or heteroclinic orbits and may seem chaos in the Shilnikov sense. However, the fundamental contribution of our paper was to find ten simple, three-dimensional dynamic systems with non-linear quadratic terms that have an asymptotically stable equilibrium point and are chaotic, which was achieved. These were obtained using the Monte Carlo method applied specifically for the search of these systems.

***Keywords*:** chaotic systems; asymptotically stable equilibrium; non-existence of Shilnikov chaos; Lyapunov exponents.

We would like to acknowledge the information about Theorem 1 from (Elhadj and Sprott, 2012), used in our article (Casas-García *et al*. 2016) is incorrect, as it was shown in (Algaba *et al*. 2013). This opens up the possibility that the systems that we find have homoclinic or heteroclinic orbits, and may appears chaos in the Shilnikov sense, making its dynamics more interesting, and opening the possibility of deeper analysis for these systems.

On the other hand, we would like to point out that the fundamental contribution of our paper lies in the fact of finding ten simple, three-dimensional dynamic systems with nonlinear quadratic terms, which have an asymptotically stable equilibrium point and present chaos, which was clearly shown in the our work. In addition, the other relevant contribution is the method to find the systems, using the Monte Carlo method, which was first proposed in (Carrillo *et al*. 2013) for this type of search for chaotic systems with special properties, and later used by (Sprott and Xiong, 2015).

In addition, we emphasize that the mention of the dynamic systems of Chen and Lü in the introduction of our article is only to establish some pioneering work in this field of nonlinear dynamics, which is usual in many articles in this area. We do not use them in the rest of the article. Finally, for a broader discussion of comments similar to those made in our paper, we recommend viewing the comments (Algaba *et al*. 2012a), and replies (Zuo-Huan Zheng and Guanrong Chen, 2012), (Guochang *et al*. 2012). For a more recent independent analysis see (Leonov and Kuznetsov, 2015).

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References

Algaba A., Fernández-Sánchez F., Merino M., and Rodríguez-Luis A. J. (2013). Comments on "Non-existence of Shilnikov chaos in continuous-time systems", Applied Mathematics and Mechanics (English Edition), 34(9), 1175-1176.

Algaba A., Fernández-Sánchez F., Merino M., and Rodríguez-Luis A. J. (2012a) "Comment on 'Existence of heteroclinic orbits of the Shilnikov type in a 3D quadratic autonomous chaotic system' [J. Math. Anal. Appl. 315, 106-119 (2006)]". J. Math. Anal. Appl., 392, 99-101.

Algaba A., Fernández-Sánchez F., Merino M., and Rodríguez-Luis A. J. (2012b) “Comment on ‘Analysis and application of a novel three-dimensional energy-saving and emission-reduction dynamical evolution system’ [Energy 40, 291-299 (2012)”. Energy 47, 630-633.

Carrillo, S., Casas-García, K., Flores-Godoy, J. J., Valencia, F. V., and Fernández-Anaya, G. (2015) Study of new chaotic flows on a family of 3-dimensional systems with quadratic nonlinearities. Journal of Physics: Conference Series (Vol. 582, No. 1, p. 12016) IOP Publishing.

Casas-García, K., Quezada-Téllez L. A., Carrillo-Moreno S., Flores-Godoy J. J., and Fernández-Anaya G. (2016). Asymptotically stable equilibrium points in new chaotic systems, Nova Scientia 8(16), 41-58.

Elhadj, Z., and Sprott, J. C. (2012) Non-existence of Shilnikov chaos in continuous-time systems. Applied Mathematics and Mechanics (English Edition), 33(3), 371-374.

Guochang Fang, Lixin Tian, Mei Sun, Min Fu (2012) Reply to: Comments on “Analysis and application of a novel three-dimensional energy-saving and emission-reduction dynamic evolution system” [Energy 40 (2012) 291-299], Energy 47, 634-635.

Leonov, G. A., and Kuznetsov, N. V. (2015) On differences and similarities in the analysis of Lorenz, Chen, and Lu systems, Applied Mathematics and Computation, 256, 334–343.

Sprott, J. C., and Xiong, A. (2015) Classifying and quantifying basins of attraction. Chaos: An Interdiciplinary Journal of nonlinear Science, 25(8), 083101.

Zuo-Huan Zheng and Guanrong Chen (2012) “Author’s reply to: Comment on “Existence of heteroclinic orbits of the Shil’nikov type in a 3D quadratic autonomous chaotic system”. [J. Math. Anal. Appl. 315 (2006) 106–119], J. Math. Anal. Appl., 392, 102.