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Aplicación de las pruebas estadísticas de discordancia y significancia en la comparación del vulcanismo dacítico de la parte central de Cinturón Volcánico Mexicano
Application of discordancy and significance statistical tests for the comparison of dacitic volcanism from the central part of the Mexican Volcanic Belt

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Resumen

Nuestro objetivo es presentar una metodología estadística, junto con dos nuevos programas (DODESSYS y UDASYS). Para esta tarea compilamos una base de datos de 249 muestras de rocas dacitas provenientes de cuatro regiones del cinturón volcánico mexicano (MVB): volcanes monogenéticos de la Sierra de Chichinautzin y el Valle de México, estratovolcán Nevado de Toluca, estratovolcán Iztaccíhuatl y estratovolcán Popocatépetl. Las pruebas estadísticas de discordancia y significancia (ANOVA *–ANalysis Of Variance–*, F de Fisher y t de Student) fueron aplicadas al 99% de nivel de confianza. Se calculó la estadística final para 98 parámetros, incluyendo óxidos mayores, elementos de tierras raras, elementos traza y parámetros adicionales, tales como parámetros de relaciones logarítmicas usados en nuevos diagramas de discriminación tectónica. Estos parámetros fueron tratados como muestras estadísticas univariadas y fueron clasificados en cuatro regiones del MVB. Las pruebas estadísticas de discordancia detectaron datos discordantes en 124 (aproximadamente en el 35%) muestras estadísticas univariadas. La prueba ANOVA mostró diferencias significativas entre todos los grupos en 32 parámetros. Las similitudes y diferencias entre los parámetros de relaciones logarítmicas pueden ser útiles en el futuro para proponer diagramas de discriminación tectónica a partir de una base de datos representativa.

Palabras clave: ANOVA, F de Fisher, t de Student, datos discordantes, datos geoquímicos, manejo estadístico de datos composicionales

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Abstract

Our aim is to show a statistical procedure along with two new computer programs (DODESSYS and UDASYS). For this task we compiled a database of 249 samples of dacite coming from four closely located Mexican Volcanic Belt (MVB) areas: monogenetic volcanoes from the Sierra de Chichinautzin and Valle de México, the Nevado de Toluca stratovolcano, the Iztaccíhuatl stratovolcano and the Popocatépetl stratovolcano. The discordancy and significance (ANOVA – *ANalysis Of Variance*–, Fishers´ F and Student´s t) statistical tests were applied at 99% confidence level. The final statistical was calculated for 98 geochemical parameters, these include major oxides, rare earth elements, trace elements and additional parameters, as well as log-ratio parameters used in new tectonic discrimination diagrams. These geochemical parameters were treated as univariate statistical samples and were classified according with the four MVB regions. Discordancy statistical tests detected discordant outliers in 124 (amount to about 35%) statistical samples. ANOVA tests showed significant differences among all groups in 32 parameters. The similarities and differences between the log-ratio parameters elements may eventually be useful in future to propose tectonic discrimination diagrams from a representative database.

Keywords: ANOVA, Fisher´s F, Student´s t, discordant outliers, geochemical data, statistical handling of compositional data

Introduction

Recently, a new computer programs has been developed, UDASYS (*Univariate Data Analysis SYStem*) [1]. UDASYS is freely available from any of the authors to any scientist interested in correctly processing experimental data. This program, written in Java [2], provides statistical tools pertaining to both robust and outlierbased methods for univariate data. UDASYS also incorporates an updated version of the DODESSYS software [3]. Whereas DODESSYS allowed the application of discordancy tests ([3] for more details on these tests see Table S1 in online Supplementary Material) for statistical sample sizes up to 1000, all discordancy tests can now be applied to statistical sample sizes as large as 30000. Computer programs to enable the application of discordancy tests were practically nonexistent as documented by Barnett and Lewis (1994). Later about 12 years ago, a computer program (SIPVADE) was published by Verma *et al.* (1998), but it is now outdated for several reasons. The most important among them are that SIPVADE uses old, less precise and sometimes even inaccurate critical values then available in the literature (Barnett and Lewis 1994; Verma 2005) and relies on linear interpolation of these values when for a given statistical sample size *n*, the corresponding critical values were not tabulated. Both of these aspects have been shown to cause errors in the final statistical inferences. More importantly, unlike all available software to date (e.g., [4]), UDASYS allows a highly efficient use of significance tests of Fisher's F, Student's t, and ANOVA.

This work illustrates the application of statistical discordancy and significance tests using geochemical data. A geochemical database of major-elements in rocks from the Mexican volcanic belt (MVB) was established long ago by Pal et al. [5]. These authors used their database to objectively characterise for the first time the nature of volcanism in the MVB. This work was later extended by including more analyses of MVB rocks in this database which permitted to highlight the complexity of magmas in the MVB (e.g., [6]). Mean and standard deviation estimates of compositional data were presented by these authors, but this was done without the application of discordancy tests [7]. Similarly in local geochemical studies from this volcanic province (MVB), these two statistical parameters for laboratory analytical data were specifically reported by Verma [8-10] and Verma et al. [11]. Other researchers have used mean and standard deviation estimates for geochemical interpretation [12].

In this work geochemical data are compiled for dacitic rocks from four nearby areas of the MVB. The geochemical parameters are compared through the significance tests such as Fisher´s

F and Student´s t [13] without and with the application of discordancy tests [14-17]. The results highlight the importance of these statistical tests in geosciences.

We searched the published geoscience literature for specific applications of discordancy and significance tests and found that it is not a common practice to apply them in geoscientific studies. Below we list some the reports found that made use of either one of these statistical methodologies.

Rice and Church [18] presented a statistical study on the variability in grain size of sediment from two confluent rivers in northeastern British Columbia (Canada). They stated that it was not appropriate to apply ANOVA test because the statistical samples did not show a normal distribution and their variances were unequal. However, the validity of the first condition can be checked by discordancy tests, whereas the second condition (equal variances) is not a requisite for ANOVA. They applied tests, such as Brown-Forsythe and chi-square, for comparing statistical sample means when sample variances are unequal.

Takano et al. [19] made a statistical comparison of inter-laboratory analytical data of fluid samples from crater lake of Maly Semiachik volcano, located in the central part of the Eastern Volcanic Belt of Kamchatka (Japan), obtained from eight different institutions. They used different analytical techniques (atomic absorption spectrometry, atomic emission spectrometry, mass spectrometry, ion chromatography, high performance liquid chromatography, colorimetry, and titrimetry) to compare the measured isotopic data coming from elements such as hydrogen, sulfur, and oxygen. Their comparison consisted of simply calculating the central tendency (mean) and dispersion (coefficient of variation) parameters for each one of these techniques. Experience shows that it would have been worthwhile to apply the discordancy and significance tests for such inter-laboratory evaluations as suggested earlier by several authors [20-21].

Wani and Mondal [22] carried out a geochemical study of shale samples from the Mesoproterozoic-Neoproterozoic Chhattisgarh and Indrāvati basins. They compared chemical compositions of the calcareous and non-calcareous shales of the Chhattisgarh and Indrāvati basins applying only the Student's t at 95% confidence level. They should have applied Fisher's F test prior to the application of the t test since this significance test is sensitive to the presence of discordant outliers. We emphasize once again that discordancy tests should be applied to detect anomalous data in individual statistical samples previous to the comparison and use of significance tests.

The correct statistic application, such as the work we are reporting, tries to promote the evolution of geochemistry towards geochemometrics, where statistics is an essential part of experimental data treatment. In general, in the area of geochemistry is not customary to apply a correct statistics methodology in the processing of databases. For example, Takano et al. [19] assessed the statistic differences in their experimental databases, but failed to apply the methodology based on significance tests and discordancy. However, recently some authors applied successfully this complete methodology in processing geochemical data [17, 23]. Particularly, the discordance tests have been applied in a diversity of scientific and engineering fields, including some branches of earth sciences such as determination of Nernst distribution coefficients [24]; quality control through reference materials [14-16, 23]; geothermal research [25-27]; geochemistry [12, 15, 17]; volcanoes studies [28, 29]; pollution studies [30]; petroleum research [31]; soil research [32]; proteomics research [33]. Also, sensitivity and uncertainty analysis is another important statistical application [34-38].

Method

Database and procedures

Geochemical data for 249 Neogene dacitic rock samples from four closely located areas of the MVB were compiled. The literature sources were as follows: [9, 21, 39-60]. Data are identifiqued as group numbers Gr1 to Gr4 (see locations of these regions on a map presented in Figure 1): Region 1 (Gr1)–diverse locations of the Sierra de Chichinautzin and the southern of the Valle de México (monogenetic volcanoes); Region 2 (Gr2)–the Nevado de Toluca stratovolcano; Region 3 (Gr3)–the Iztaccíhuatl stratovolcano, and Region 4 (Gr4)–the Popocatépetl stratovolcano.

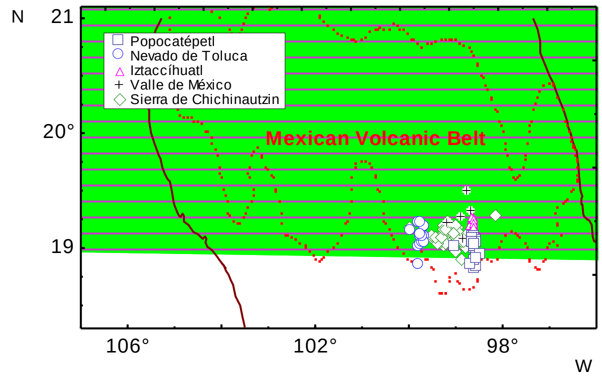

Figure 1. Schematic location of the site under study: Sierra de Chichinautzin, south of Valle de México, Nevado de Toluca, Iztaccíhuatl and Popocatepetl (Mexico).

TAS (*Total Alkalis vs Silica*) diagram was generated by IgRocs sofware [61]; see Figure 2. Geochemical data are concentrated in classification area for dacite rocks.

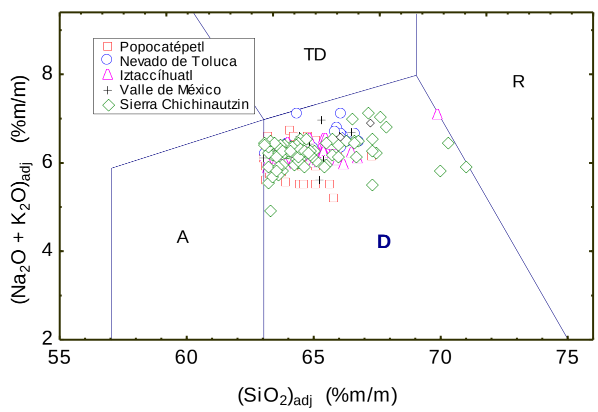

Figure 2. This figure shows a diagram of discrimination TAS. Geochemical data are concentrated in classification area for dacite rocks.

The statistical central tendency (mean) and dispersion (standard deviation) parameters were calculated for several conventional variables, which were 11 major oxides (adjusted values) from $(SiO_2)_{Adj}$ to $(P_2O_5)_{Adj}$, selected normative minerals, Mg-number (or Mg-value), and 6 other indices detailed by [61], followed by 14 rare earth elements from La to Lu, and 22 trace elements from Ba to Zr. In addition to these conventional chemical data, 30 additional parameters were computed and evaluated. These include two ratio parameters defined by Verma [62] called Nb-anomaly with respect to Ba and La and Ta-anomaly with respect to Ba and La, as well as 28 log-ratio parameters of elements used in new multi-dimensional tectonic discrimination diagrams [63-65].

Figure 3 shows the flow diagram of statistical methodology applied in this work. Conventionally, significant test are applied without prior application of discordancy tests. However, because these tests should be applied to normally distributed statistical samples, data for each variable from all individual groups (Gr1-Gr4) were first processed for discordant outliers by single-outlier type discordancy tests (see Table S1 in [1]) at 99% confidence level, and the discordant outlier-free groups were evaluated from the two-sided ANOVA-test and t-test at 99% confidence level (see *Geological implications* in [1] for more details on application of two-sided version of significant tests). The statistical parameters of mean and standard deviation were simply calculated from the discordant outlier-free individual groups.

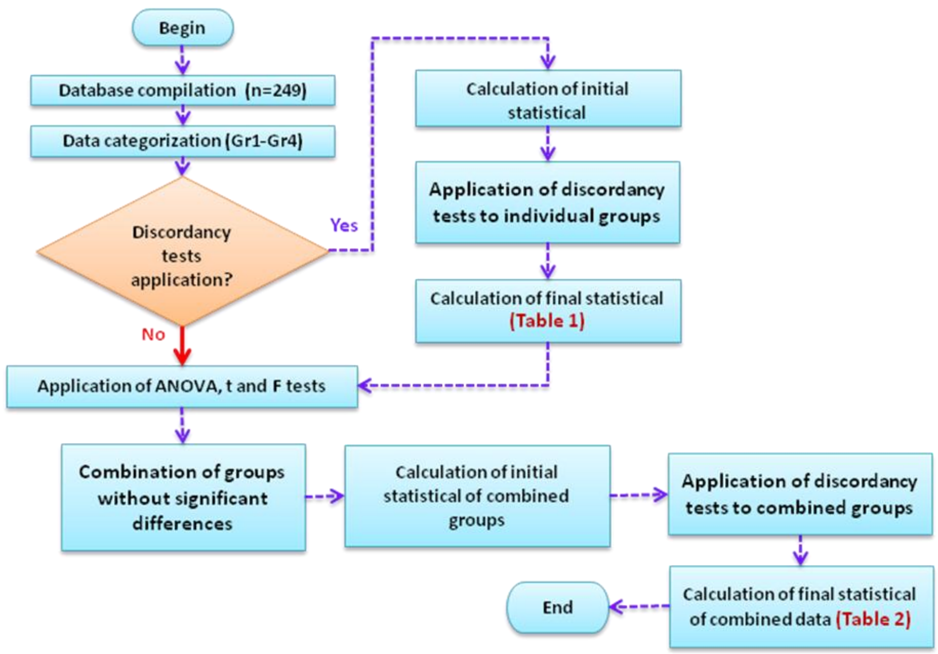

Figure 3. Schematic flow diagram of statistical methodology applied in this work.

We note that, ANOVA test can only be applied to three or more groups or statistical samples [7], therefore this significant test was applied to the data from each group (Gr1-Gr4). The application of ANOVA would result in any of the following: (i) no statistically significant differences among the four regions (Gr1-Gr4); (ii) one –e.g., Gr1– of the four regions showing a statistically significant difference as compared to the other three regions –e.g., Gr2- Gr4–; and (iii) statistically significant differences among the four regions (Gr1- Gr4), which will have to be resolved by Fisher´s F and Student´s t significance tests. When ANOVA detects significant differences among the four regions, the data should be processed thorough the combination of Fisher´s F and Student´s t tests, which are applicable to only two groups at a time [63, 64]. The Fisher´s F test compares the two variances and could result in either the two variances are equal or the two are different. Depending on the result of the F test, the appropriate version of the t test should be applied. The Fisher's F and Student's t tests were applied to each one of the combinations Gr1-Gr2, Gr1-Gr3, Gr1-Gr4, Gr2-Gr3, Gr2-Gr4 and Gr3-Gr4.

It has been suggested that the data from different groups or regions should only be combined after ascertaining that no statistically significant differences exist among them [1]. Thus, for a given chemical parameter or variable, the groups that showed no significant differences were combined and statistical information was obtained for the combined data. Finally, these combined data were once again processed for discordant outliers, and the discordant outlier-free data were used to obtain final statistical (mean and standard deviation).

Resultados

Identification and separation of discordant outliers

Geochemical data for a total of 96 variables o parameters from each group (Gr1-Gr4) were processed in this work. Single-outlier type discordancy tests at a very strict 99% confidence level were then applied to individual groups, outlying observations were separated, and statistical parameters were calculated from discordant outlier-free data. These statistical parameters are reported in Table 1; the first column gives the name of the chemical or ratio parameter, the next columns gives statistical parameters such as statistical sample size (*n*), mean and standard deviation from all individual groups (Gr1-Gr4); i.e. columns 2-4 show stastistical parameters from Sierra de Chichinautzin and Valle de Mexico monogenetic volcanoes. The second column gives the final statistical sample size (*n*) after discordant outlier detection and separation, the third column reports the mean, and the fourth one provided the standard deviation. The number of discordant outliers is represented by a symbol as superscript: α –one–; β –two–; γ –three–; δ –four –; £ –five–; ζ –seven–; η –eight–; λ –ten–. For the total of 350 statiscal samples processed in this work, 124 (35%) samples showed discordant outliers.

Application of ANOVA, t and F tests after elimination of outliers

ANOVA test determined that 32 variables ***did not show statistically significant differences among all groups***, hence they were combined; e.g., $(Na_2O)_{Adj}$, $(K_2O)_{Adj}$, or_{Norm}, ab_{Norm}, an_{Norm} , La, Pr, Nd, Sm, Eu, $\ln((Na^2O)_{Adj}/Si)$, $\ln((K_2O)_{Adj}/Si)$, $\ln((P_2O_5)_{Adj}/Si)$, ln(Nb/Yb), ln(Th/Yb), ln(Y/Yb) and ln(Zr/Yb). ANOVA also ***identified a discordant group*** (Gr2, Gr3 and Gr4 in 3, 17 and 12 variables, respectively) in 32 variables; e.g., the Gr2 group was identified as discordant group in $(P_2O_5)_{Adj}$ variable, therefore, Gr2 group was separated and Gr1, Gr2 and Gr4 groups were combined. Finally, ANOVA ***determined statistically significant differences among the four regions*** in 32 elements, e.g., all groups from $(TiO_2)_{Adj}$ major element were identified as discordant groups and were not combined. Fisher´s F and Student´s t tests were applied to these 32 variables.

Application of discordancy tests after combining data (significance tests)

Single-outlier type discordancy tests were applied to the combined groups, outlying observations were separated, and statistical parameters were calculated from discordant outlier-free data (see Table 2 in appendix). These discordant outliers were rejected (or separated) and final statistical were calculated and shown in Table 2. Discordant outliers were represented by a symbol as superscript: α –one–; β –two–; γ –three–; δ –four –; £ –five–; ζ –seven–; η –eight–; λ –ten–.

Conclusions

In this work, we have shown a statistical procedure to decipher mean compositions and uncertainty estimates including various regions. For this, geochemical data are compiled for 249 Neogene dacitic rock samples from the four MVB regions.

All single-outlier type discordancy tests and significance (ANOVA *–ANalysis Of Variance–*, Fishers´ F and Student´s t) statistical tests were applied at the very strict 99% confidence level. These statistical tests were applied to each one of the 98 geochemical parameters, which were major oxides, selected normative minerals, rare earth, trace elements, two ratio parameters called Nb-anomaly and Ta-anomaly, as well as 28 log-ratio parameters of elements used in new multi-dimensional tectonic discrimination diagrams.

All geochemical parameters were treated as univariate statistical samples. Final statistical parameters were calculated from discordant outlier-free data. We suggest that the final mean compositions could be used to compare statistically the geochemical data for the same type of igneous rocks, i.e., dacite type, sampled around the world.

Furthermore, significance statistical tests determined significant differences and similarities among various geochemical parameters from the four MVB regions. Particularly, the similarities and differences among the log-ratios parameters could be useful to propose new diagrams to discriminate tectonic settings, with a more representative database.

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Table 1. Final statistical of samples of dacitic rocks from four nearby regions of the Mexican volcanic belt.

Element	Gr1 (Sierra de Chichi-nautzin- Valle de México monogenetic volcanoes)			Gr2 (Nevado de Toluca stratovolcano)			Gr3 (Iztaccíhuatl strato-volcano)			Gr4 (Popocatépetl stra-tovolcano)		
	n	mean	standard deviation	n	mean	standard deviation	n	mean	standard deviation	n	mean	standard deviation
$(SiO_2)_{Adj}$	84^{λ}	64.50	1.05	34	65.44	0.88	54^{α}	64.65	1.09	22	63.93	0.82
$(TiO_2)_{Adj}$	94^{β}	0.661	0.122	34	0.6420	0.0313	53	0.710	0.067	22	0.742	0.050
$(Al_2O_3)_{Adj}$	93^{α}	16.64	0.79	34	16.75	0.47	55	16.34	0.50	21^{α}	16.352	0.321
$(Fe_2O_3)_{Adj}$	94	1.214	0.168	34	1.157	0.081	54^{α}	1.240	0.109	21^{α}	1.3693	0.0414
$(FeO)_{Adj}$	94	3.034	0.420	34	2.893	0.203	54^{α}	3.100	0.274	21^{α}	3.423	0.104
$(MnO)_{Adj}$	90^{δ}	0.0848	0.0146	34	0.0645	0.0100	55	0.0783	0.0095	22	0.0780	0.0136
$(MgO)_{Adj}$	94	2.53	0.80	26^{η}	1.785	0.090	55	2.87	0.66	22	2.94	0.53
$(CaO)_{Adj}$	90^{δ}	4.61	0.51	32^{β}	4.348	0.159	54^{α}	4.551	0.359	22	4.816	0.250
$(Na_2O)_{Adj}$	93^{α}	4.286	0.348	34	4.411	0.131	55	4.246	0.220	22	4.270	0.282
$(K_2O)_{Adj}$	91^{γ}	1.968	0.286	32^{β}	1.991	0.109	55	1.988	0.167	22	1.867	0.173
$(P_2O_5)_{Adj}$	94	0.168	0.052	33^{α}	0.1817	0.0164	55	0.1946	0.0297	22	0.1730	0.0223
q_{Norm}	91^{γ}	17.81	2.59	34	18.75	1.69	55	17.81	2.43	21^{α}	16.19	2.38
or_{Norm}	91^{γ}	11.63	1.69	32^{β}	11.76	0.64	55	11.75	0.99	22	11.03	1.02
ab_{Norm}	93^{α}	36.27	2.94	34	37.32	1.11	55	35.93	1.87	22	36.13	2.39
an_{Norm}	91^{γ}	19.30	2.31	34	19.45	1.01	55	19.15	1.69	21	19.57	1.40
en_{Norm}	93	1.13	1.04	33	0.53	0.59	55	1.15	0.95	22	1.66	1.23
fs_{Norm}	92	0.52	0.47	34	0.311	0.319	55	0.475	0.359	22	0.76	0.51
di_{Norm}	92^{β}	1.62	1.45	33^{α}	0.82	0.87	55	1.63	1.30	22	2.42	1.73
hym_{Norm}	94	5.77	1.78	30^{δ}	4.44	0.47	55	6.62	1.34	22	6.55	1.02
hyf_{Norm}	93^{α}	3.321	0.402	34	3.250	0.277	54^{α}	3.376	0.283	22	3.631	0.254
hy_{Norm}	94	9.10	1.99	32^{β}	7.80	0.84	55	9.97	1.57	22	10.18	1.06
mt_{Norm}	94	1.760	0.244	34	1.678	0.118	54^{α}	1.798	0.159	21^{α}	1.985	0.060
il_{Norm}	94	1.256	0.233	34	1.219	0.059	53^{β}	1.348	0.127	22	1.410	0.094
ap_{Norm}	94	0.388	0.121	33^{α}	0.4210	0.0381	55	0.451	0.069	22	0.401	0.052
Mg#	94	58.4	7.8	26^{η}	52.85	0.98	54^{α}	62.19	3.54	22	60.18	4.40
FeO^t/Mg	86^{η}	1.650	0.394	34	2.007	0.316	54^{α}	1.487	0.229	21^{α}	1.579	0.238
Salic	94	85.35	2.98	31^{γ}	88.12	1.06	54^{α}	84.49	2.34	22	83.49	2.11
Femic	94	13.87	3.19	27^{ζ}	10.94	0.51	55	14.73	2.76	22	15.97	2.20
C.I.	94	25.72	3.84	$29^{\pounds}$	23.37	0.82	55	26.28	3.07	22	27.95	1.93
D.I.	92^{β}	65.97	3.54	34	68.11	2.29	54^{α}	65.24	2.75	20^{β}	63.31	0.85
S.I.	94	19.0	4.8	27^{ζ}	14.67	0.75	55	21.19	3.54	22	21.09	2.76
A.R.	93^{α}	1.849	0.100	33^{α}	1.875	0.054	54^{α}	1.850	0.063	22	1.816	0.076
La	32	18.15	3.58	22	16.31	2.75	---	---	---	32^{δ}	16.06	1.23
Ce	32	40.6	7.9	21^{α}	32.52	3.69	---	---	---	33^{α}	35.1	4.6
Pr	11^{α}	3.93	0.62	16^{α}	4.18	0.52	---	---	---	14	3.66	0.45
Nd	17	18.11	3.61	22	17.40	2.56	---	---	---	13^{α}	16.41	1.51
Sm	14	3.76	0.48	22	3.72	0.52	---	---	---	32^{β}	3.630	0.365
Eu	12^{β}	1.098	0.046	22	1.142	0.125	---	---	---	33^{α}	1.166	0.101
Gd	13	3.418	0.415	17	3.181	0.283	---	---	---	14	3.540	0.348
Tb	14	0.560	0.061	21^{α}	0.4643	0.0394	---	---	---	32^{β}	0.523	0.073
Dy	10^{α}	3.034	0.182	17	2.552	0.164	---	---	---	14	3.112	0.378
Ho	12	0.588	0.078	17	0.4900	0.0260	---	---	---	14	0.599	0.090
Er	12	1.657	0.221	17	1.326	0.097	---	---	---	14	1.750	0.290
Tm	10	0.2180	0.0399	17	0.1971	0.0172	---	---	---	14	0.251	0.053
Yb	14	1.576	0.213	22	1.343	0.223	---	---	---	34	1.560	0.252
Lu	13^{α}	0.2233	0.0400	22	0.2055	0.0332	---	---	---	14	0.2671	0.0278
Ba	56	506	74	22	483	48	45	522	55	43	446	56
Be	7	1.51	0.49	3^{α}	1	0	---	---	---	13	1.31	0.48
Co	27	12.01	2.68	22	11.02	4.48	---	---	---	36^{γ}	13.31	2.03
Cr	45	69.0	36.6	22	57.0	45	42^{γ}	58.5	21.1	40^{α}	87.4	30.6
Cs	9	2.85	1.26	5	1.70	0.71	---	---	---	34	2.88	0.63

Table 1 (continuation). Final statistical of samples of dacitic rocks from four nearby regions of the Mexican volcanic belt.

Element	Gr1 (Sierra de Chichi-nautzin- Valle de México monogenetic volcanoes)			Gr2 (Nevado de Toluca stratovolcano)			Gr3 (Iztaccíhuatl strato-volcano)			Gr4 (Popocatépetl stra-tovolcano)		
	n	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**
Cu	50	12.6	4.9	22	13.3	7.1	---	---	---	20	17.0	6.8
Ga	33	20.30	1.24	---	---	---	---	---	---	14	20.93	1.21
Hf	11	4.214	0.433	22	3.58	0.52	---	---	---	32^{β}	4.307	0.371
Nb	49	6.22	1.57	17	4.447	0.405	45	8.91	2.12	12^{β}	5.24	0.56
Ni	60^{α}	36.5	21.7	18^{α}	21.3	18.0	4^{δ}	25.3	8.7	45	45.9	17.1
Pb	48^{γ}	9.44	1.93	3	6.33	1.53	---	---	---	16^{β}	11.45	2.30
Rb	63	45.1	11.7	22	38.2	5.0	45	58.6	7.2	44	52.5	7.9
Sb	---	---	---	---	---	---	---	---	---	19^{α}	0.167	0.046
Sc	9	11.73	0.82	5	10.42	3.07	---	---	---	35	11.21	1.14
Sr	$59^{£}$	476	62	21^{α}	555	65	37^{η}	420.9	24.1	39^{α}	467	51
Ta	11^{α}	0.405	0.070	19	0.382	0.053	---	---	---	33^{α}	0.472	0.097
Th	35	4.94	1.56	22	3.865	0.442	---	---	---	37	4.84	0.82
U	12	1.74	0.74	20^{β}	1.496	0.109	---	---	---	32^{β}	1.740	0.234
V	25	83.4	13.1	22	70	10.9	44^{α}	91.7	10.7	15	92.5	8.4
Y	51	18.18	2.57	20^{β}	14.61	0.78	43^{β}	21	2.85	14	17.21	1.71
Zn	52^{α}	64.9	8.5	22	71.3	8.8	---	---	---	19^{α}	69.8	7.8
Zr	51	171.8	28.3	22	146.8	11.9	45	161.6	17.8	37	167.0	24.2
Nb/Nb*2	30	0.1778	0.0312	17	0.1339	0.0114	---	---	---	13^{α}	0.1785	0.0109
Ta/Ta*2	8	0.239	0.063	19	0.2013	0.0243	---	---	---	32^{β}	0.2545	0.0315
$\ln((TiO_2)_{Adj}/SiO_2)$	94	-0.4605	0.217	34	-0.4625	0.057	54^{α}	-0.4510	0.116	22	-0.4458	0.069
$\ln((Al_2O_3)_{Adj}/SiO_2)$	93^{α}	-0.1362	0.055	34	-0.13633	0.0322	55	-0.13774	0.0359	21^{α}	-0.13636	0.0220
$\ln((Fe_2O_3)/SiO_2)$	94	-0.3989	0.169	34	-0.4037	0.080	54^{α}	-0.3957	0.104	21^{α}	-0.38427	0.0370
$\ln((FeO)_{Adj}/SiO_2)$	94	-0.3073	0.169	34	-0.3121	0.080	54^{α}	-0.3041	0.104	21^{α}	-0.29264	0.0370
$\ln((MnO)_{Adj}/SiO_2)$	93^{α}	-0.6636	0.205	34	-0.6934	0.157	55	-0.6726	0.142	22	-0.6725	0.209
$\ln((MgO)_{Adj}/SiO_2)$	93^{α}	-0.3291	0.377	27^{ζ}	-0.3600	0.063	53^{β}	-0.3111	0.211	22	-0.3098	0.207
$\ln((CaO)_{Adj}/SiO_2)$	$89^{£}$	-0.2643	0.122	32^{β}	-0.27140	0.0452	54^{α}	-0.2657	0.096	22	-0.2587	0.060
$\ln((Na^2O)_{Adj}/SiO_2)$	$89^{£}$	-0.2708	0.075	34	-0.26973	0.0249	55	-0.2726	0.048	22	-0.2708	0.076
$\ln((K_2O)_{Adj}/SiO_2)$	94	-0.3492	0.149	32^{β}	-0.3494	0.054	55	-0.3487	0.084	22	-0.3537	0.095
$\ln((P_2O_5)_{Adj}/SiO_2)$	86^{η}	-0.5919	0.248	34	-0.5881	0.111	55	-0.5819	0.160	22	-0.5920	0.133
ln(La/Th)	12	1.247	0.252	22	1.433	0.154	---	---	---	34	1.222	0.104
ln(Sm/Th)	12	-0.132	0.311	22	-0.040	0.123	---	---	---	33^{α}	-0.276	0.124
ln(Yb/Th	12	-0.1026	0.342	22	-0.1063	0.166	---	---	---	34	-0.1137	0.221
ln(Nb/Th	21	0.067	0.230	17	0.138	0.091	---	---	---	13^{α}	0.112	0.111
$\ln(Nb/(TiO_2)_{Adj})$	49	-0.7056	0.254	17	-0.7288	0.107	45	-0.6690	0.230	13^{α}	-0.7234	0.121
$\ln(V/(TiO_2)_{Adj})$	24^{α}	-0.4450	0.103	22	-0.4505	0.136	45	-0.4352	0.090	15	-0.4392	0.093
$\ln(Y/(TiO_2)_{Adj})$	48^{α}	-0.5957	0.107	22	-0.6067	0.095	42^{γ}	-0.5808	0.066	14	-0.6071	0.107
$\ln(Zr/(TiO_2)_{Adj})$	50^{α}	-0.3736	0.132	22	-0.3786	0.097	44^{α}	-0.3781	0.108	18	-0.3850	0.144
$\ln(MgO/(TiO_2)_{Adj})$	94	1.298	0.316	26^{η}	1.021	0.046	55	1.380	0.204	22	1.360	0.209
$\ln(P2O5/(TiO_2)_{Adj})$	$89^{£}$	-0.1365	0.230	34	-0.1255	0.103	55	-0.1295	0.156	22	-0.1462	0.127
$\ln(Ni/(TiO_2)_{Adj})$	61	-0.544	0.69	19	-0.590	0.86	45	-0.5579	0.447	22	-0.5141	0.392
ln(La/Yb)	11	2.355	0.246	22	2.496	0.210	---	---	---	34	2.359	0.164
ln(Ce/Yb)	11	3.111	0.217	22	3.211	0.215	---	---	---	34	3.129	0.192
ln(Sm/Yb)	14	0.869	0.174	22	1.023	0.171	---	---	---	34	0.879	0.160
ln(Nb/Yb)	11	1.183	0.193	17	1.224	0.144	---	---	---	14	1.246	0.181
ln(Th/Yb)	12	1.026	0.342	22	1.063	0.166	---	---	---	34	1.137	0.221

ln(Y/Yb)	13	2.390	0.113	22	2.422	0.088	---	---	---	14	2.363	0.085
ln(Zr/Yb)	13	4.601	0.136	22	4.703	0.181	---	---	---	34	4.666	0.204

Number of discordant outliers detected: α –one–; β –two–; γ –three–; δ –four –; £ –five–; ζ –seven–; η –eight–; λ –ten–.

Table 2. Final statistical of the combined regions and separated, resulting of application of significance test.

Element	**Combined regions**			**Gr1 (Sierra de Chichi-nautzin- Valle de México monogenetic volcanoes)**			**Gr2 (Nevado de Toluca stratovolcano)**			**Gr3 (Iztaccíhuatl stratovol-cano)**			**Gr4 (Popocatépetl strato-volcano)**		
	N	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**
$(SiO_2)_{adj}$	160	64.47	1.06	---	---	---	34	65.44	0.88	---	---	---	---	---	---
$(TiO2)_{adj}$	125	0.65	0.098	---	---	---	---	---	---	53	0.71	0.067	22	0.742	0.05
	75	0.719	0.064	94	0.661	0.122	34	0.642	0.0313	---	---	---	---	---	---
$(Al_2O_3)_{adj}$	157	16.37	0.49	---	---	---	34	16.75	0.47	---	---	---	---	---	---
$(Fe_2O_3)_{adj}$	180^{α}	1.212	0.136	---	---	---	---	---	---	---	---	---	21	1.3693	0.0414
$(FeO)_{adj}$	180^{α}	3.029	0.339	---	---	---	---	---	---	---	---	---	21	3.423	0.104
$(MnO)_{adj}$	162^{α}	0.0809	0.0113	---	---	---	34	0.0645	0.01	---	---	---	---	---	---
$(MgO)_{adj}$	170^{α}	2.71	0.73	---	---	---	26	1.785	0.09	---	---	---	---	---	---
$(CaO)_{adj}$	175^{α}	4.554	0.414	---	---	---	---	---	---	---	---	---	22	4.816	0.25
$(Na_2O)_{adj}$	$196^{£}$	4.306	0.242	---	---	---	---	---	---	---	---	---	---	---	---
$(K_2O)_{adj}$	198	1.957	0.207	---	---	---	---	---	---	---	---	---	---	---	---
$(P_2O_5)_{adj}$	138^{η}	0.1765	0.0303	---	---	---	---	---	---	55	0.1946	0.0297	---	---	---
q_{Norm}	180	17.99	2.41	---	---	---	---	---	---	---	---	---	21	16.19	2.38
or_{Norm}	198	11.56	1.22	---	---	---	---	---	---	---	---	---	---	---	---
ab_{Norm}	$196^{£}$	36.44	2.05	---	---	---	---	---	---	---	---	---	---	---	---
an_{Norm}	193^{γ}	19.27	1.53	---	---	---	---	---	---	---	---	---	---	---	---
c_{Norm}	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---
dim_{Norm}	168	1.17	0.99	---	---	---	33	0.53	0.59	---	---	---	---	---	---
dif_{Norm}	179	0.451	0.394	---	---	---	---	---	---	---	---	---	22	0.76	0.51
di_{Norm}	165	1.63	1.34	---	---	---	33	0.82	0.87	---	---	---	---	---	---
hym_{Norm}	77	6.6	1.25	94	5.77	1.78	30	4.44	0.47	---	---	---	---	---	---
hyf_{Norm}	180^{α}	3.33	0.342	---	---	---	---	---	---	---	---	---	22	3.631	0.254
hy_{Norm}	76^{α}	10.1	1.33	94	9.1	1.99	32	7.8	0.84	---	---	---	---	---	---
mt_{Norm}	180^{α}	1.756	0.197	---	---	---	---	---	---	---	---	---	21	1.985	0.06
il_{Norm}	125	1.234	0.186	---	---	---	---	---	---	53	1.348	0.127	22	1.41	0.094
	75	1.366	0.121	94	1.256	0.233	34	1.219	0.059	---	---	---	---	---	---
ap_{Norm}	138^{η}	0.409	0.07	---	---	---	---	---	---	55	0.451	0.069	---	---	---
Mg#	113^{β}	59	6.4	---	---	---	26	52.85	0.98	54	62.19	3.54	---	---	---
	75^{α}	61.79	3.57	94	58.4	7.8	26	52.85	0.98	---	---	---	---	---	---
FeO^t/Mg	153	1.542	0.28	---	---	---	34	2.007	0.316	---	---	---	---	---	---
Salic	170	84.84	2.75	---	---	---	31	88.12	1.06	---	---	---	---	---	---
Femic	170	14.38	2.98	---	---	---	27	10.94	0.51	---	---	---	---	---	---
C.I.	167^{γ}	26.35	3.09	---	---	---	29	23.37	0.82	---	---	---	---	---	---
D.I.	141^{α}	65.53	2.91	---	---	---	34	68.11	2.29	---	---	---	20	63.31	0.85
S.I.	170	19.94	4.3	---	---	---	27	14.67	0.75	---	---	---	---	---	---
A.R.	200^{α}	1.85	0.079	---	---	---	---	---	---	---	---	---	---	---	---
La	84	16.73	2.65	---	---	---	---	---	---	---	---	---	---	---	---
Ce	54	34.11	4.41	32	40.6	7.9	---	---	---	---	---	---	---	---	---

Pr	41	3.93	0.56	---	---	---	---	---	---	---	---	---	---	---	---
Nd	50	17.04	2.23	---	---	---	---	---	---	---	---	---	---	---	---
Sm	67	3.662	0.401	---	---	---	---	---	---	---	---	---	---	---	---
Eu	67	1.146	0.104	---	---	---	---	---	---	---	---	---	---	---	---
Gd	44	3.365	0.372	---	---	---	---	---	---	---	---	---	---	---	---
Tb	46	0.534	0.071	---	---	---	21	0.4643	0.0394	---	---	---	---	---	---
Dy	24	3.08	0.309	---	---	---	17	2.552	0.164	---	---	---	---	---	---
Ho	26	0.594	0.083	---	---	---	17	0.49	0.026	---	---	---	---	---	---
Er	26	1.707	0.26	---	---	---	17	1.326	0.097	---	---	---	---	---	---

Table 2 (continuation)**.** Final statistical of the combined regions and separated, resulting of application of significance test.

Element	Combined regions			Gr1 (Sierra de Chichi-nautzin- Valle de México monogenetic volcanoes)			Gr2 (Nevado de Toluca strato-volcano)			Gr3 (Iztaccíhuatl strato-volcano)			Gr4 (Popocatépetl strato-volcano)		
	n	mean	standard deviation	n	mean	standard deviation	n	mean	standard deviation	n	mean	standard deviation	n	mean	standard deviation
Tm	27	0.2048	0.029	---	---	---	---	---	---	---	---	---	14	0.251	0.053
	24	0.237	0.05	---	---	---	17	0.1971	0.0172	---	---	---	---	---	---
Yb	48	1.565	0.239	---	---	---	22	1.343	0.223	---	---	---	---	---	---
Lu	35	0.2121	0.0364	---	---	---	---	---	---	---	---	---	14	0.2671	0.0278
Ba	120^{α}	506	60	---	---	---	---	---	---	---	---	---	43	446	56
Be	23	1.33	0.47	---	---	---	---	---	---	---	---	---	---	---	---
Co	85	12.31	3.14	---	---	---	---	---	---	---	---	---	---	---	---
Cr	105	59.2	29.1	---	---	---	---	---	---	---	---	---	40	87.4	30.6
Cs	48	2.75	0.85	---	---	---	---	---	---	---	---	---	---	---	---
Cu	90	13.3	5.5	---	---	---	---	---	---	---	---	---	---	---	---
Ga	47	20.49	1.25	---	---	---	---	---	---	---	---	---	---	---	---
Hf	43	4.283	0.385	---	---	---	22	3.58	0.52	---	---	---	---	---	---
Nb	---	---	---	49	6.22	1.57	17	4.447	0.405	45	8.91	2.12	12	5.24	0.56
Ni	105	40.5	20.3	---	---	---	18	21.3	18	41	25.3	8.7	---	---	---
	58	23.3	10.8	60	36.5	21.7	---	---	---	---	---	---	45	45.9	17.1
Pb	---	---	---	48	9.44	1.93	3	6.33	1.53	---	---	---	16	11.45	2.3
Rb	---	---	---	63	45.1	11.7	22	38.2	5	45	58.6	7.2	44	52.5	7.9
Sb	20	0.164	0.047	---	---	---	---	---	---	---	---	---	---	---	---
Sc	49	11.23	1.39	---	---	---	---	---	---	---	---	---	---	---	---
Sr	97	469	54	---	---	---	21	555	65	36	420.8	24.4	---	---	---
Ta	30	0.39	0.06	---	---	---	---	---	---	---	---	---	33	0.472	0.097
	44	0.455	0.095	---	---	---	19	0.382	0.053	---	---	---	---	---	---
Th	72	4.89	1.23	---	---	---	22	3.865	0.442	---	---	---	---	---	---
U	62	1.617	0.257	---	---	---	---	---	---	---	---	---	---	---	---
V	82^{β}	90.4	9.9	---	---	---	22	70	10.9	---	---	---	---	---	---
Y	65	17.97	2.43	---	---	---	20	14.61	0.78	43	21	2.85	---	---	---
Zn	93	67.4	8.8	---	---	---	---	---	---	---	---	---	---	---	---
Zr	130	165.4	22.1	---	---	---	22	146.8	11.9	---	---	---	---	---	---
Nb/Nb*2	43	0.178	0.0266	---	---	---	17	0.1339	0.0114	---	---	---	---	---	---
Ta/Ta*2	26	0.2064	0.0289	---	---	---	---	---	---	---	---	---	32	0.2545	0.0315
	40	0.2514	0.0392	---	---	---	19	0.2013	0.0243	---	---	---	---	---	---
$\ln(Ti/SiO_2)$	125^{α}	-0.4612	0.175	---	---	---	---	---	---	54	-0.451	0.116	22	-0.4458	0.069
	76	-0.4495	0.107	94	-0.4605	0.217	34	-0.4625	0.057	---	---	---	---	---	---

ln(Al/SiO_2)	200$^{\alpha}$	-0.13671	0.0423	---	---	---	---	---	---	---	---	---	---	---	---
ln(Fe/SiO_2)	180$^{\alpha}$	-0.3987	0.134	---	---	---	---	---	---	---	---	---	21	-0.38427	0.037
ln(FeO/SiO_2)	180$^{\alpha}$	-0.3071	0.134	---	---	---	---	---	---	---	---	---	21	-0.29264	0.037
ln(Mn/SiO_2)	166$^{\beta}$	-0.6677	0.168	---	---	---	34	-0.6934	0.157	---	---	---	---	---	---
ln(Mg/SiO_2)	75	-0.3107	0.209	93	-0.3291	0.377	27	-0.36	0.063	---	---	---	---	---	---
ln(Ca/SiO_2)	165	-0.264	0.109	---	---	---	32	-0.2714	0.0452	---	---	---	---	---	---
ln(Na/SiO_2)	195$^{£}$	-0.2706	0.054	---	---	---	---	---	---	---	---	---	---	---	---
ln(K/SiO_2)	198$^{\beta}$	-0.3498	0.101	---	---	---	---	---	---	---	---	---	---	---	---
ln(P/SiO_2)	194$^{\gamma}$	-0.5873	0.175	---	---	---	---	---	---	---	---	---	---	---	---
ln(La/Th)	34	1.367	0.21	---	---	---	---	---	---	---	---	---	34	1.222	0.104
	45	1.215	0.128	---	---	---	22	1.433	0.154	---	---	---	---	---	---
ln(Sm/Th)	32$^{\beta}$	-0.037	0.155	---	---	---	---	---	---	---	---	---	33	-0.276	0.124
	44	-0.253	0.169	---	---	---	22	-0.04	0.123	---	---	---	---	---	---
ln(Yb/Th)	68	-0.1094	0.233	---	---	---	---	---	---	---	---	---	---	---	---

Table 2 (continuation)**.** Final statistical of the combined regions and separated, resulting of application of significance test.

Element	Combined regions			Gr1 (Sierra de Chichinautzin- Valle de México monogenetic volcanoes)			Gr2 (Nevado de Toluca stratovolcano)			Gr3 (Iztaccíhuatl stratovolcano)			Gr4 (Popocatépetl stratovolcano)		
	n	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**	**n**	**mean**	**standard deviation**
ln(Nb/Th)	51	0.102	0.167	---	---	---	---	---	---	---	---	---	---	---	---
ln(Nb/TiO_2)	30	-0.7265	0.114	49	-0.7056	0.254	---	---	---	45	-0.669	0.23	---	---	---
ln(V/TiO_2)	61	-0.4456	0.12	---	---	---	---	---	---	45	-0.4352	0.09	---	---	---
ln(Y/TiO_2)	36	-0.6069	0.098	48	-0.5957	0.107	---	---	---	42	-0.5808	0.066	---	---	---
ln(Zr/TiO_2)	130$^{\beta}$	-0.3775	0.117	---	---	---	---	---	---	---	---	---	---	---	---
ln(MgO/TiO_2)	170$^{\alpha}$	1.338	0.263	---	---	---	26	1.021	0.046	---	---	---	---	---	---
ln(P_2O_5/TiO_2)	172$^{\gamma}$	-0.132	0.162	---	---	---	---	---	---	---	---	---	22	-0.1462	0.127
ln(Ni/TiO_2)	125$^{\beta}$	-0.542	0.54	---	---	---	19	-0.59	0.86	---	---	---	---	---	---
ln(La/Yb)	67	2.403	0.202	---	---	---	---	---	---	---	---	---	---	---	---
ln(Ce/Yb)	67	3.153	0.205	---	---	---	---	---	---	---	---	---	---	---	---
ln(Sm/Yb)	36	0.963	0.186	---	---	---	---	---	---	---	---	---	34	0.879	0.16
	48	0.876	0.162	---	---	---	22	1.023	0.171	---	---	---	---	---	---
ln(Nb/Yb)	42	1.221	0.168	---	---	---	---	---	---	---	---	---	---	---	---
ln(Th/Yb)	68	1.094	0.233	---	---	---	---	---	---	---	---	---	---	---	---
ln(Y/Yb)	49	2.396	0.095	---	---	---	---	---	---	---	---	---	---	---	---
ln(Zr/Yb)	69	4.665	0.187	---	---	---	---	---	---	---	---	---	---	---	---

Number of discordant outliers detected: α –one–; β –two–; γ –three–; δ –four –; £ –five–; ζ –seven–; η –eight–; λ –ten–.