Experimental Synchronization by Means of Observers

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ABSTRACT
In this paper we deal with the experimental synchronization of the Colpitts oscillator in a real-time implementation. Our approach is based on observer design theory in a master-slave configuration thus, a chaos synchronization problem can be posed as an observer design procedure, where the coupling signal is viewed as a measurable output and a slave system is regarded as an observer. A polynomial observer is used for synchronizing the Colpitts oscillator employing linear matrix inequalities. Finally, a comparison with a reduced order observer and a high gain observer is given to assess the performance of the proposed observer.

Keywords: experimental synchronization, polynomial observer, reduced order observer, high gain observer, algebraic observability condition.

RESUMEN
En este artículo se aborda la sincronización experimental del oscilador de Colpitts en tiempo real. Nuestra aproximación se realiza mediante la teoría de diseño de observadores en una configuración maestro-esclavo, por lo que el problema de sincronización caótica puede plantearse como el diseño de un observador. Se utiliza un observador polinomial para la sincronización del oscilador de Colpitts empleando desigualdades matriciales lineales. Se realiza una comparación con el observador de orden reducido y con el observador de alta ganancia con la finalidad de verificar el desempeño del observador propuesto.

1. Introduction

Synchronization in chaotic systems has been investigated since its introduction paper of Pecora and Carrol [1]. This research area has received a great deal of attention among scientist in many fields due to its potential applications mainly in secure communications [2]-[7].

During the last years (almost two decades), many different approaches related to chaos synchronization have been proposed. See for instance, [8]-[10] in which the authors propose the employment of state observers, where the main applications pertain to the synchronization of nonlinear oscillators; in references [11]-[13] use feedback controllers, which allow to achieve the synchronization between nonlinear oscillators, with different structure and order; in [14], [15] use nonlinear backstepping control; in papers [16], [17] consider synchronization time delayed systems; in works [18], [19] consider directional and bidirectional linear coupling; papers [20], [21] use nonlinear control; in [12] use active control; in [13], [22] use adaptive control; in [23]-[24] employ adaptive observers and so on.

Now, we will mention a brief note about observer theory. The design of observers for nonlinear systems is a challenging problem that has received a considerable amount of attention. Since the observers developed by Kalman [25] and Luenberger [26], several years ago for linear systems, different state observation techniques have been propose to handle the systems nonlinearities. A first category of techniques consists in applying linear algorithms to the system linearized around the estimated trajectory. These are known as the extended Kalman and

Luenberger observers. Alternatively, the nonlinear dynamics are split into a linear part and a nonlinear one. The observer gains then are chosen large enough so that the linear part dominates over the nonlinear one. Such observers are known as, high gain observers [27]. And many other approaches such as [28]-[30].

In this work the synchronization method is based on a master-salve configuration [1]. The main characteristic is that the coupling signal is unidirectional, that is, the signal is transmitted from the master systems (transmitter) to the slave system (receiver), the receiver is requested to recover the unknown (or full) state trajectories of the transmitter. Therefore, the terminology transmitter-receiver is also used. Thus, a chaos synchronization problem can be regarded as observer design procedure, where the coupling signal is viewed as the output and the slave system is the observer [31]-[33].

As we can notice, there are several methods to solve the synchronization problem from the control theory perspective, in this work, we study the synchronization in master-salve configuration [1] by means of state observers based on differential algebraic approach. These proposals are applied in this paper to a Colpitts oscillator [34]. The Colpitts oscillator has been widely considered for the synchronization problem, see for instance [24], [35].

In this paper an exponential observer of polynomial type for the synchronization problem is proposed. We also have designed an asymptotic observer of reduced order. Finally, for comparison purposes, we construct a high-gain observer.

2. Receiver operating principle

Let us consider the following nonlinear system,

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= Cx \quad , \; x_0 = x(t_0) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, is the state vector; $u \in \mathbb{R}^{m'}$, is the input vector, $m' \leq n$; $f(\cdot): \mathbb{R}^n \times \mathbb{R}^{m'} \to \mathbb{R}^n$ is locally Lipschitz on x and uniformly bounded on $u; y \in \mathbb{R}$ is the output of the system. To show the relation between observers for nonlinear systems and synchronization we give the following definition.

Definition 1 (Exponential synchronization) The dynamical system with state vector $\hat{x} \in \mathbb{R}^n$

$$\begin{aligned} \dot{\hat{x}} &= \bar{f}(\hat{x},y,u) \\ \hat{y} &= C\hat{x} \quad , \; \hat{x}_0 = \hat{x}(t_0) \end{aligned} \tag{2}$$

is in state of exponential synchronization with system (1) if there exist positive constants $\bar{\kappa}$ and $\bar{\xi}$ such that

$$\|x - \hat{x}\| \leq \bar{\kappa} \exp(-\bar{\xi} t)$$

In the master-slave synchronization scheme, x is viewed as the state variable of the master system and $\hat{x}$ is considered as the state variable of the slave system. Hence, the master-slave system synchronization problem between systems (1) and (2) can be solved by designing and observer for (1). In order to solve the synchronization problem as an observation problem we introduce the following observability property.

Definition 2 [40] (Algebraic observability condition-AOC) A state variable $x_i \in \mathbb{R}$ is said to be algebraically observable if it is algebraic over $\mathbb{R}\langle u,y\rangle$[1], that is, x satisfies a differential algebraic polynomial in terms of $\{u,y\}$ and some of their time derivatives, i.e.,

$$P_i(x_i, u, \dot{u}, \ldots, y, \dot{y}, \ldots) = 0 \tag{3}$$

with coefficients in $\mathbb{R}\langle u,y\rangle$.

The system (1) can be expressed in the following form,

$$\begin{aligned} \dot{x} &= Ax + \Psi(x,u) \\ y &= Cx \qquad x_0 = x(t_0) \end{aligned} \tag{4}$$

Where $\Psi(x,u)$ is a nonlinear vector that satisfies the Lipschitz condition with constant ϕ that is:

$$\|\Psi(x,u) - \Psi(\hat{x},u)\| \leq \phi \|x - \hat{x}\| \tag{5}$$

The observer for system (4) has the next form

$$\dot{\hat{x}} = A\hat{x} + \Psi(\hat{x},u) + \sum_{i=1}^{m} K_i (y - C\hat{x})^{2i-1} \tag{6}$$

Where $\hat{x} \in \mathbb{R}^n$, and $K_i \in \mathbb{R}^n$ for $1 \leq i \leq m$.

Let us consider the following assumptions:

[1] $\mathbb{R}\langle u,y\rangle$ denotes the differential field generated by the field $\mathbb{R}$, the input u, the measurable output y, and the time derivatives of u and y.

Assumption 1. For $\bar{A} := A - K_1 C$, there is a unique symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ which satisfies the following linear matrix inequality (LMI):

$$\begin{bmatrix} -\bar{A}^T P - P\bar{A} - I & \phi P \\ \phi P & I \end{bmatrix} > 0$$

where ϕ is the Lipschitz constant.

Assumption 2. Let us define $M_i := PK_i C$, then:

$$\lambda_{min}(M_i + M_i^T) \geq 0, \ \ for\ 2 \leq i \leq m.$$

Remark 1. By using the Schur complement (see Chapter 11 in [38]) the LMI in Assumption 1 can be represented as an algebraic Riccati equation:

$$\bar{A}^T P + P\bar{A} + \phi^2 PP + I < 0,$$

or for some $\varepsilon > 0$

$$\bar{A}^T P + P\bar{A} + \phi^2 PP + I + \varepsilon I = 0$$

Remark 2. Assumption 2 is used to improve the rate of convergence of the estimation error by injecting additional terms (from 2 to m) which depend upon odd powers of the output error.

In order to prove the observer convergence, we analyze the observer error which is defined as $e = x - \hat{x}$. From Equations 4 and 6, the dynamics of the observer error is given by:

$$\dot{e} = \bar{A}e + F - \sum_{i=2}^{m} K_i (Ce)^{2i-1}$$

where $\bar{A} := A - K_1 C$, and $F := \Psi(x,u) - \Psi(\hat{x},u)$.

Now, we present a lemma which will be useful in the convergence analysis.

Lemma 1 [39]: Given the system (4) and its observer (6) with the error given by $e = x - \hat{x}$. If $P = P^T > 0$ then:

$$2e^T P[\Psi(x,u) - \Psi(\hat{x},u)] \leq \phi^2 e^T PPe + e^T e$$

The following proposition proves the observer convergence.

Proposition 1. Let the system (4) be algebraically observable and Assumption 1 and Assumption 2 hold. The nonlinear system (6) is an exponential polynomial observer of the system (4); that is to say, there are constants $\kappa > 0$ and $\xi > 0$ such that:

$$\|e(t)\| \leq \kappa \|e(0)\| \exp(-\xi t)$$

where $\kappa = \sqrt{\frac{\beta}{\alpha}}$, $\xi = \frac{\varepsilon}{2\beta}$, $\alpha = \lambda_{min}(P)$, and $\beta = \lambda_{max}(P)$.

Proof. We use the following Lyapunov function candidate $V = e^T Pe$

$$\begin{aligned} \dot{V} = \dot{e}^T Pe + e^T P\dot{e} \\ = e^T[\bar{A}^T P + P\bar{A}]e + 2e^T PF \\ - 2e^T P \sum_{i=2}^{m} K_i (Ce)^{2i-1} \end{aligned}$$

using Lemma 1 we obtain:

$$\begin{aligned} \dot{V} \leq e^T[\bar{A}^T P + P\bar{A} + \phi^2 PP + I]e \\ - 2e^T P \sum_{i=2}^{m} K_i (Ce)^{2i-1} \end{aligned}$$

Making some algebraic manipulations on the last term of the above inequality, and taking into account that $Ce \in \mathbb{R}$, we obtain:

$$\begin{aligned} \dot{V} \leq e^T[\bar{A}^T P + P\bar{A} + \phi^2 PP + I]e \\ - 2\sum_{i=2}^{m} (Ce)^{2i-2} e^T PK_i Ce \end{aligned}$$

For simplicity, we define $M_i := PK_i C, \ \ for\ 2 \leq i \leq m$, then we have:

$$\dot{V} \leq e^T[\bar{A}^T P + P\bar{A} + \phi^2 PP + I]e - \{(Ce)^2[e^T M_2 e + (e^T M_2 e)^T] + (Ce)^4[e^T M_3 e + (e^T M_3 e)^T] + \cdots + (Ce)^{2m-2}[e^T M_m e + (e^T M_m e)^T]\}$$

The above expression can be rewritten in a simplified form:

$$\begin{aligned} \dot{V} \leq e^T[\bar{A}^T P + P\bar{A} + \phi^2 PP + I]e \\ - \sum_{i=2}^{m} (Ce)^{2i-2} e^T[M_i + M_i^T]e \end{aligned}$$

From Assumption 2, the second term in the right hand side of above the inequality always will be positive or zero, therefore:

$$\dot{V} \leq e^T[\bar{A}^T P + P\bar{A} + \phi^2 PP + I]e \quad (7)$$

by Assumption 1 (and remark 1), we have:

$$\dot{V} \leq -\varepsilon\|e\|^2 \quad (8)$$

We write the Lyapunov function as $V = \|e\|_P^2$, then by the Rayleigh-Ritz inequality we have that:

$$\alpha\|e\|^2 \leq \|e\|_P^2 \leq \beta\|e\|^2 \quad (9)$$

where $\alpha \coloneqq \lambda_{min}(P)$, and $\beta \coloneqq \lambda_{max}(P) \in \mathbb{R}^+$ (because P is positive definite).

By using (9) we obtain the following upper bound of (8):

$$\dot{V} \leq -\frac{\varepsilon}{\beta}\|e\|_P^2 \quad (10)$$

Taking the time derivative of $V = \|e\|_P^2$ and replacing in inequality (10), we obtain:

$$\frac{d}{dt}\|e\|_P \leq -\frac{\varepsilon}{2\beta}\|e\|_P$$

Finally, the result follows with:

$$\|e(t)\| \leq \kappa\|e(0)\|\exp(-\xi t) \quad (11)$$

where $\kappa = \sqrt{\frac{\beta}{\alpha}}$, and $\xi = \frac{\varepsilon}{2\beta}$.

3. Asymptotic reduced order observer

Now, let us consider the nonlinear system described by (1). The unknown states of the system can be included in a new variable $\eta(t)$ and the following new augmented system is considered:

$$\begin{aligned} \dot{x}(t) &= f(x,u,\eta) \\ \dot{\eta}(t) &= \Delta(x,u) \\ y(t) &= h(x) \end{aligned} \quad (12)$$

where $\Delta(x,u)$ is a bonded uncertain function. The problem is to reconstruct the variable $\eta(t)$. This problem is overcome by using a reduced order observer. Before proposing the corresponding observer we introduce some hypotheses:

Assumption 3 $\eta(t)$ satisfies the AOC (Definition 2).

Assumption 4 γ is a C^1 real-valued function.

Assumption 5 Δ is bounded, i.e., $|\Delta| \leq M < \infty$.

Assumption 6. For t_0 sufficiently large, there is $K > 0$, such that, $\limsup_{t\to t_0}\frac{M}{K} = 0$.

The following lemma describes the design of a proportional reduced order observer for system (1).

Lemma 2. If Assumptions 3 to 6 are satisfied, then the system:

$$\dot{\hat{\eta}} = K(\eta - \hat{\eta}) \quad (13)$$

Is an asymptotic reduced order observer of free-model type for system (12), where $\hat{\eta}$ denotes the estimate of η and $K \in \mathbb{R}^+$ determines the desired convergence rate of the observer.

Remark 3. To reconstruct $\eta(t)$ by using an auxiliary state $\hat{\eta}(t)$ sometimes we need to use the output time derivatives, but these may be unavailable. To overcome this fact, an auxiliary function completely artificial γ is defined in such a way that it cancels out all nonmeasurable terms. This action defines a differential equation for γ. This equation is solved, then, γ is substituted in the differential equation of the estimated state and finally the estimate of η is obtained.

We give the following immediate corollary.

Corollary 1. The dynamic system (13) along with

$$\dot{\gamma} = \Psi(x,u,\gamma), \quad \gamma_0 = \gamma(0), \quad \gamma \in C^1$$

constitute a proportional asymptotic reduced order observer for system (12), where γ is a change of variable which depends on the estimated state $\hat{\eta}$, and the state variables.

4. High gain-observer

We present a well-known estimation structure (high gain observer) as a comparison with our proposed schemes. Consider the class of nonlinear systems given by (1). In this case, to estimate the state vector x, we suggested a nonlinear high gain-observer with the following structure:

$$\dot{\hat{x}} = f(\hat{x}, u) + K(y - C\hat{x}) \tag{14}$$

Where $\hat{x} \in \mathbb{R}^n$, $\hat{x}_0 = \hat{x}(t_0)$ and the observer high-gain matrix is given by:

$$K = S_\theta^{-1} C^T, \quad S_\theta = \left(\frac{1}{\theta^{i+j-1}} S_{ij}\right)_{i,j=1,\ldots,n}$$

and the positive parameter θ determines the desired convergence velocity. Moreover, $S_\theta = S_\theta > 0$, should be a positive solution of the algebraic equation:

$$S_\theta \left(E + \frac{\theta}{2} I\right) + \left(E^T + \frac{\theta}{2} I\right) S_\theta = C^T C$$

$$E = \begin{pmatrix} 0 & I_{n-1,n-1} \\ 0 & 0 \end{pmatrix} \tag{15}$$

5. Experimental results

As was previously mentioned, the integrated These proposals are applied to a Colpitts oscillator [34]. The Colpitts oscillator has been widely considered for the synchronization problem, see [24, 35].

In this work we considered the classical configuration of the Colpitts oscillator [36]. The circuit contains a bipolar junction transistor 2N2222A as the gain element (Figure1(b)), and a resonant network consisting of an inductor and two capacitors (Figure 1(a)).

The Colpitts circuit is described by a system of three nonlinear differential equations, as follows:

$$\begin{aligned} L\dot{I}_L &= -Vc_1 - Vc_2 - RI_L + Vcc \\ C_2\dot{V}c_2 &= I_L - I_0 \\ C_1\dot{V}c_1 &= -f(Vc_2) + I_L \end{aligned} \tag{16}$$

where $f(\cdot)$ is the driving-point characteristic of the nonlinear resistor, this can be expressed in the form $I_E = f(Vc_2) = f(-V_{BE})$. In particular, we have $f(Vc_2) = I_s \exp\left(-\frac{Vc_2}{V_T}\right)$.

We introduce the dimensionless state variables (x_1, x_2, x_3), and choose the operating point of (16) to be the origin of the new coordinate system. In particular, we normalize voltages, currents and time with respect to $V_{ref} = V_T$, $I_{ref} = I_0$ and $t_{ref} = 1/\omega_0$, respectively, where $\omega_0 = 1/\sqrt{LC_1C_2/(C_1 + C_2)}$, is the resonant frequency of the unloaded L-C tank circuit. Then, the state equations for Colpitts oscillator can be rewritten in the following form:

$$\begin{aligned} \dot{x}_1 &= -cx_3 - cx_2 - dx_1 \\ \dot{x}_2 &= bx_1 \\ \dot{x}_3 &= -a\, exp(-x_2) + ax_1 + a \end{aligned} \tag{17}$$

where, $a = b\frac{C_2}{C_1}$, $b = \frac{I_0}{\omega_0 C_2 V_T}$, $c = \frac{V_T}{\omega_0 L I_0}$, $d = \frac{R}{L\omega_0}$.

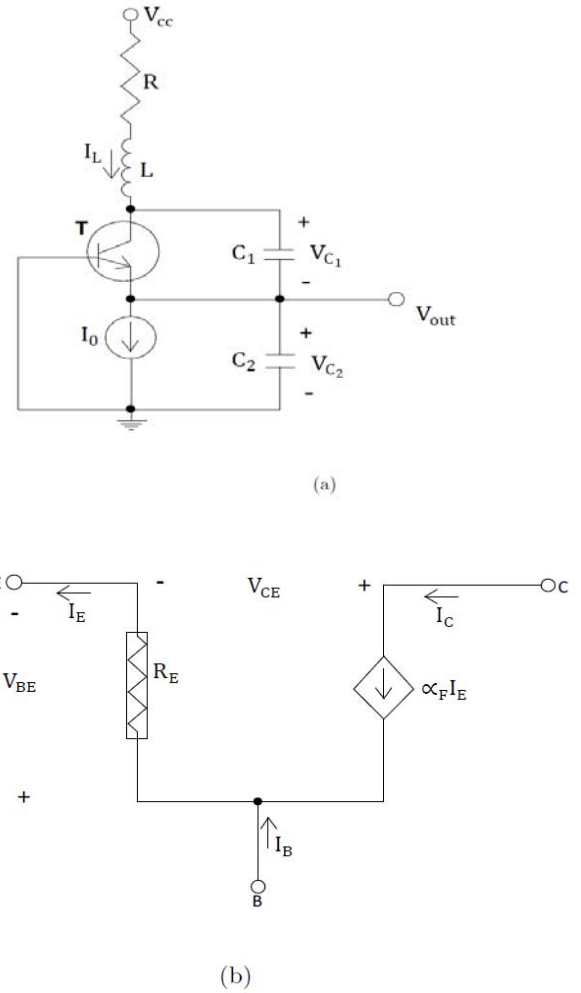


Figure 1. Colpitts oscillator (a) Circuit configuration (b) Model of the Bipolar Junction Transistor (BJT)

According to Definition 2, it is evident that system (17) is algebraically observable with respect to the output $y = x_2$, because the unknown states can be rewritten as:

$$x_1 = \frac{\dot{x}_2}{b} = \frac{\dot{y}}{b} \tag{18}$$

$$x_3 = -\frac{1}{c}\left[\frac{1}{b}\ddot{y} + \frac{d}{b}\dot{y} + cy\right] \tag{19}$$

hence, Colpitts oscillator is algebraically observable with respect to the selected output $y = x_2$.

5.1 Synchronization of the Colpitts oscillator employing the exponential polynomial observer

For the implementation of the observer we first rewrite (17) in the form (4),

$$\dot{x} = \begin{bmatrix} -d & -c & -c \\ b & 0 & 0 \\ a & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -a\exp(-x_2) + a \end{bmatrix} \tag{20}$$

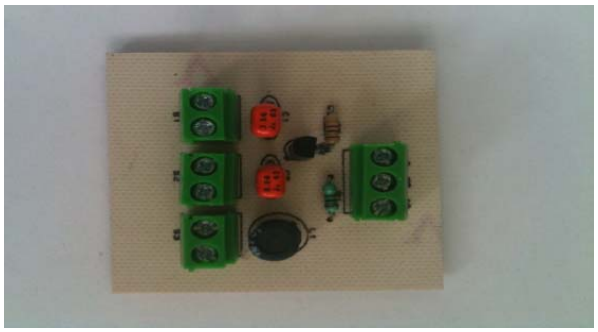


Figure 2. Implementation of the Colpitts circuit (master system)

$$y = [0 \quad 1 \quad 0]\, x$$

Applying Proposition 1, we have

$$\dot{\hat{x}} = \begin{bmatrix} -d & -c & -c \\ b & 0 & 0 \\ a & 0 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 0 \\ -a\exp(-\hat{x}_2) + a \end{bmatrix} + \sum_{i=1}^{m} \begin{bmatrix} k_{1,i} \\ k_{2,i} \\ k_{3,i} \end{bmatrix} ([0 \quad 1 \quad 0]\, e)^{2i-1}$$

hence, the state observer is rewritten as.

$$\dot{\hat{x}}_1 = -c\hat{x}_3 - c\hat{x}_2 - d\hat{x}_1 + k_{3,1}e_{1,2} + k_{3,2}(e_{1,2})^3 + \cdots + k_{3,m}(e_{1,2})^{2m-1}$$

$$\dot{\hat{x}}_2 = b\hat{x}_1 + k_{2,1}e_{1,2} + k_{2,2}(e_{1,2})^3 + \cdots + k_{2,m}(e_{1,2})^{2m-1} \tag{21}$$

$$\dot{\hat{x}}_3 = a\hat{x}_1 - a\exp(-\hat{x}_2) + a + k_{1,1}e_{1,2} + k_{1,2}(e_{1,2})^3 + \cdots + k_{1,m}(e_{1,2})^{2m-1}$$

we verified the real time performance of the exponential observer by using the WINCON platform. To achieve the synchronization in real time, in WINCON the scheme (21) in the master-slave configuration was implemented.

Figure 2 shows the real implementation of the Colpitts circuit. The circuit parameters are:

$L = 100\ \mu H; C_1 = C_2 = 47\ nF, R = 45\ \Omega,\ I_0 = 5\ mA.$ Using the circuit parameter we obtain $a = b = 6.2723$, $c = 0.0797$, and $d = 0.6898$.

The nonlinear term $\Psi(\mathrm{x})$ in (20), satisfies the Lipschitz condition and is considered as follows

$$\Psi(\mathrm{x}) = \begin{bmatrix} 0 \\ 0 \\ -a\exp(-x_2) + a \end{bmatrix}$$

It is necessary to calculate the Lipschitz constant ϕ introduced in (5) over the bounded set

$$\Omega = \{x \in \mathbb{R}^3 |\ |x_1| < M_1, |x_2| < M_2, |x_3| < M_3\} \tag{22}$$

considering the Jacobian of $\Psi(\mathrm{x})$ as

$$\frac{\partial \Psi(\mathrm{x})}{\partial \mathrm{x}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & a\exp(-\mathrm{x}_2) & 0 \end{bmatrix} \tag{23}$$

it can be concluded that[2]

$$\left\| \frac{\partial \Psi(\mathrm{x})}{\partial x} \right\|_{\infty} \leq 3\max\{0, a\exp(-x_2)\}, a \epsilon \mathbb{R}^+ \tag{24}$$

From (22), it is obvious that the following conditions hold for all the points in the bounded set Ω.

$$a\exp(-x_2) < a\exp(M_2) = \max\{a\exp(-x_2)\}, a \in \mathbb{R}^+ \tag{25}$$

hence:

$$\left\| \frac{\partial \Psi(\mathrm{x})}{\partial x} \right\|_{\infty} \leq 3\, a\exp(M_2) \tag{26}$$

[2] Let us consider the matrix $A = [a_{ij}]_{1 \leq i,j \leq n}$, then (see chapter 5 in [38]) $\|A\|_{\infty} := n\max_{1 \leq i,j \leq n} |a_{ij}|$.

With (22)-(26) a Lipschitz constant that satisfies the Lipschitz condition (5) is defined as follows

$$\phi = 3\, a \exp(M_2)$$

In this case $M_1 = 3$, $M_2 = 0.1$, $M_3 = 6$, and $a = 6.2723, \Rightarrow \phi = 20.7959$

Following the observer (21), for $m = 2$, and solving the LMI given by assumption 1, the observer gains K_1 and $K_{2,}$ and positive definite matrix P are as follows

$$K_1 = \begin{bmatrix} 10.2130 \\ 16.1211 \\ 10.1500 \end{bmatrix}, K_2 = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix},$$

$$P = \begin{bmatrix} 38.8560 & -36.7794 & 19.4606 \\ -36.7794 & 37.9331 & -20.7898 \\ 19.4606 & -20.7898 & 16.4869 \end{bmatrix} > 0$$

With eigenvalues $\lambda_1(P) = 1.3151$, $\lambda_2(P) = 5.2514$, and $\lambda_3(P) = 86.7095$.

The performance index (quadratic synchronization error) of the corresponding synchronization process is calculated as

$$J(t) = \frac{1}{t + 0.001} \int_0^t |e(t)|^2_{Q_0}, Q_0 = I$$

Figures. 3(a) and (c) show the obtained results by using the exponential polynomial observer (21), it is clear that the synchronization is achieved fairly acceptable even with the noisy measurements. The Colpitts circuit starts in $x(0) = [0 \quad 0 \quad 0]^T$ and the arbitrary initial conditions for the observer are $\hat{x}(0) = [2.1 \quad -0.1 \quad 1{,}506]^T$. Figure 3(d) shows the performance index of the synchronization, which depicts an exponential behavior.

5.2 Synchronization of the Colpitts oscillator by means of the asymptotic reduced order observer

Let us consider the normalized system of the Colpitts oscillator. We assume that the output system is $y = x_2$, therefore, the slave system consists in two estimation structures to achieve synchronization with the master system. Such structures are obtained as follows: Firstly, verify that the master system (Colpitts oscillator) is algebraically observable, and second, by using (13), we construct the observer for the unknown states. Previously, we have verified that master system (Colpitts oscillator) is algebraically observable (see Equations (18) and (19). Then, both unknown states of the master system are algebraically observable, and therefore, we can construct the observers based on Lemma 2 and Corollary 1.

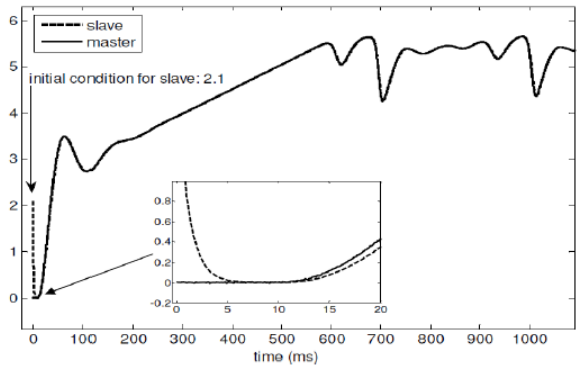
For x_1 the observer is given by:

$$\begin{aligned} \dot{\gamma}_3 &= -\frac{K_3^2}{b} y - K_3 \gamma_3 \\ \hat{x}_1 &= \frac{K_3}{b} y + \gamma_3 \end{aligned} \tag{27}$$

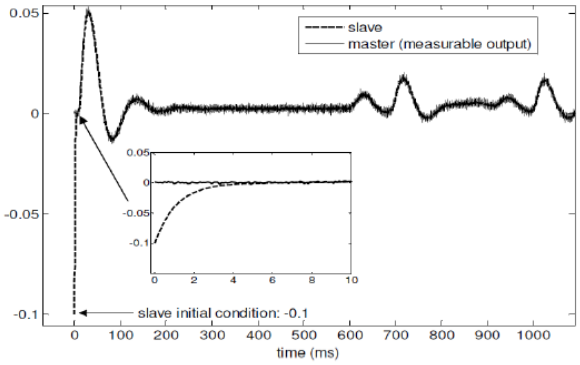
and for x_3, we have

$$\begin{aligned} \dot{\gamma}_4 &= -K_4[\gamma_4 + K_4\, y] \\ \dot{\gamma}_5 &= [K_5 - d]\frac{K_5}{cb}\,[\gamma_4 + K_4\, y] - K_5 y - K_5 \gamma_5 \\ \hat{x}_3 &= -\frac{K_5}{cb}[\gamma_4 + K_4 y] + \gamma_5 \end{aligned} \tag{28}$$

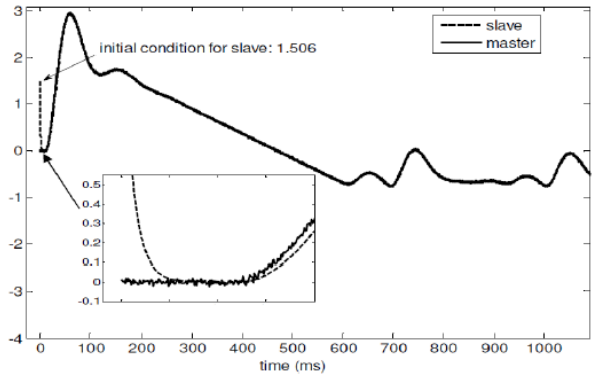
Therefore, (27) and (28) constitute the slave system. Now, we present some experimental results for the synchronization of the Colpitts oscillator by using the asymptotic reduced order observer (27) and (28). Figures 4(a) and (b) show the obtained results for the initial conditions $\hat{x}_1 = -2.498$ and $\hat{x}_3 = 1.506$ in the schemes (27) and (28), respectively. As we can notice, the synchronization results achieved with the reduced order observer are good. Figure 4(c) presents the phase portrait, where the chaotic behavior of the Colpitts oscillator can be clearly observed. Finally, Figure 4(d) illustrates the performance index, which tends to decrease.



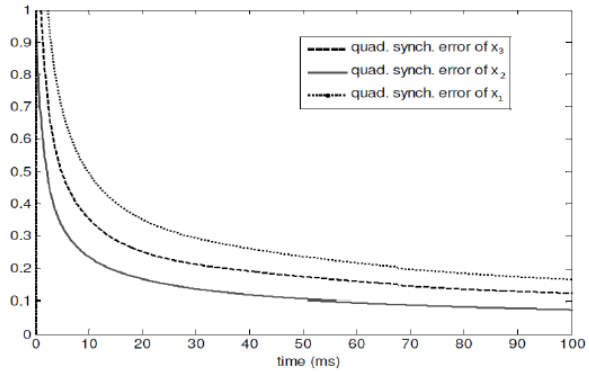
(a)



(b)

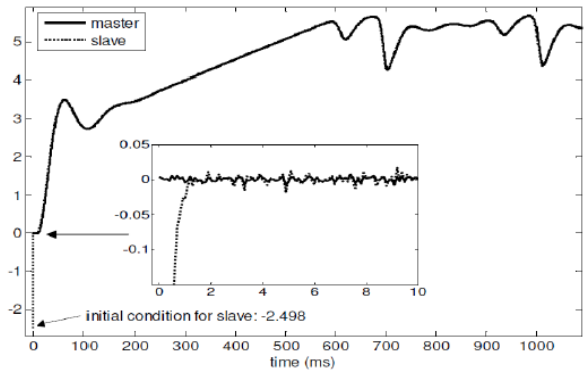


(c)

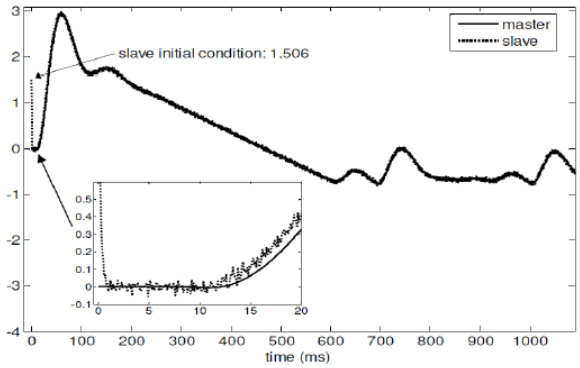


(d)

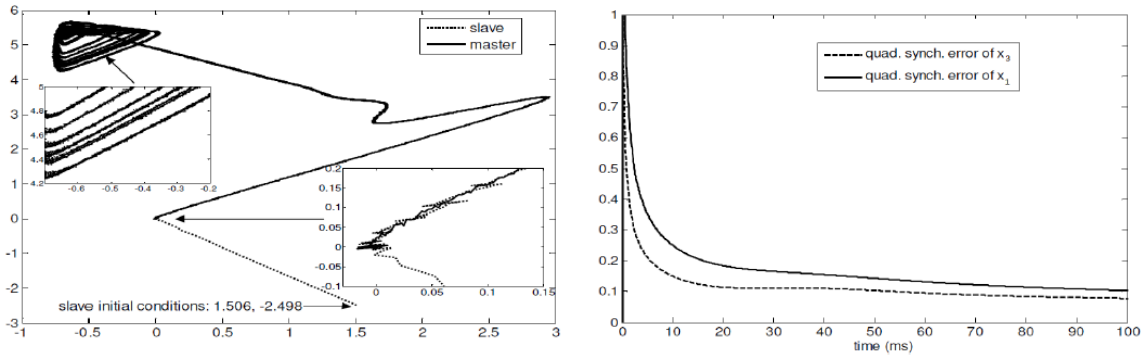
Figure 3. Real-time synchronization of Colpitts oscillator employing observer (21): (a) variables x_1 and $\hat{x}_1$, (b) synchronization of variables x_2 and $\hat{x}_2$, (c) synchronization of variables x_3 and $\hat{x}_3$, and (d) performance index



(a)



(b)



(c) (d)

Figure 4. Real-time synchronization of Colpitts oscillator using reduced-order observer (28) and (27): (a) synchronization of coordinates x_1 and $\hat{x}_1$, (b) synchronization between x_3 and $\hat{x}_3$, (c) Phase portrait of the master system (x_3 (horizontal) versus x_1 (vertical)) and the slave system ($\hat{x}_3$ and $\hat{x}_1$), and (d) performance index

6. Synchronization with the high gain observer

The matrix $S_\theta > 0, S_\theta = S_\theta^T$, that satisfies the algebraic Riccati equation (15) for a third order system $(n = 3)$, is given by:

$$S_\theta = \begin{pmatrix} \frac{1}{\theta} & -\frac{1}{\theta^2} & \frac{1}{\theta^3} \\ -\frac{1}{\theta^2} & \frac{2}{\theta^3} & -\frac{1}{\theta^4} \\ \frac{1}{\theta^3} & -\frac{1}{\theta^4} & \frac{6}{\theta^5} \end{pmatrix} \tag{29}$$

and its corresponding inverse matrix is,

$$S_\theta^{-1} = \begin{pmatrix} 3\theta & 3\theta^2 & \theta^3 \\ 3\theta^2 & 5\theta^3 & 2\theta^4 \\ \theta^3 & 2\theta^4 & \theta^5 \end{pmatrix} \tag{30}$$

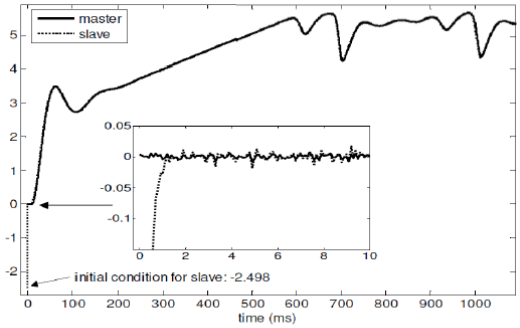
Then the high gain-observer for Colpitts system (17) is as follows:

$$\begin{aligned} \dot{\hat{x}}_1 &= -c\hat{x}_3 - c\hat{x}_2 - d\hat{x}_1 + \frac{5}{b}\theta^3(y - \hat{x}_2) \\ \dot{\hat{x}}_2 &= b\hat{x}_1 + 3\theta^2(y - \hat{x}_2) \\ \dot{\hat{x}}_3 &= a\hat{x}_1 - a\exp(-\hat{x}_2) + a \\ &\quad + \left(-3\theta^2 - \frac{5d}{bc}\theta^3 - \frac{2}{bc}\theta^4\right)(y - \hat{x}_2) \end{aligned} \tag{31}$$

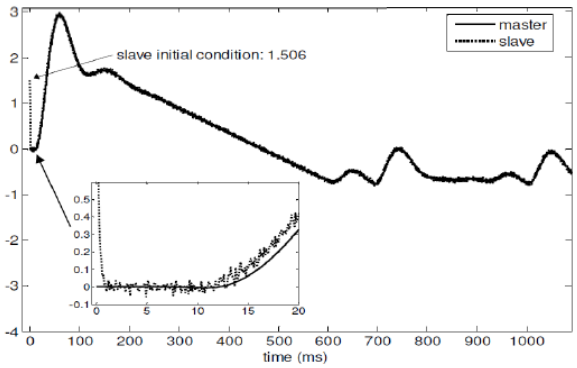
Figure 5 depicts the synchronization between Colpitts oscillator (17) and its high gain observer (31). In order to obtain exponential convergence we have used $\theta = 100$. As we can notice, the performance on the high-gain observer is not good in comparison with the exponential polynomial observer and with the proportional reduced-order observer. An important advantage of the proposed methodologies is that the magnitudes of the observer gains are smaller than the ones used in the high-gain observer.

7. Conclusion

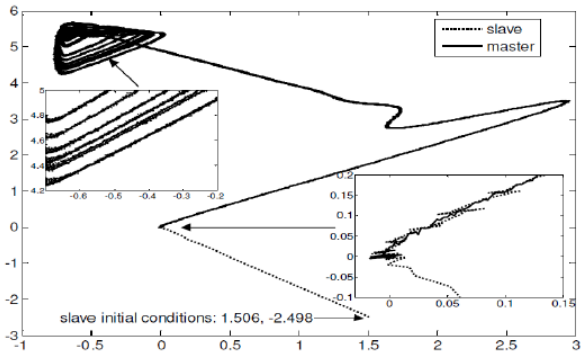
In this paper we tackled the synchronization problem based upon observer's theory. As well as, its main contributions, we show the real-time synchronization in the Colpitts oscillator by using two observer structures: An exponential polynomial observer and an asymptotic reduced order observer. For comparison purposes we implemented a high-gain observer. Finally, some experimental results show the effectiveness of the proposed methodologies.



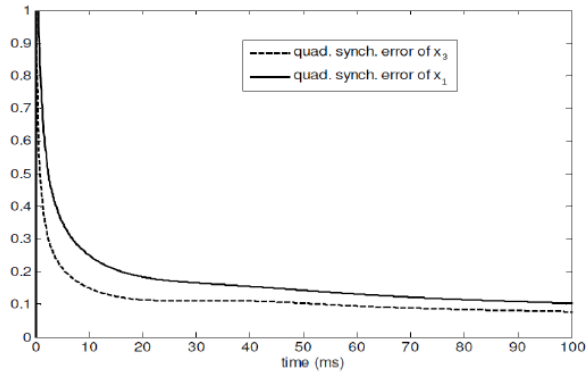
(a)



(b)



(c)



(d)

Figure 5. Synchronization between Colpitts oscillator and its high-gain observer: (a) synchronization of coordinates x_1 and $\hat{x}_1$, (b) synchronization between x_3 and $\hat{x}_3$,(c) Phase portrait of the master system (x_3 (horizontal) versus x_1 (vertical)) and the slave system ($\hat{x}_3$ versus $\hat{x}_1$), and (d) performance index

References

[1] L.M. Pecora, and T.L. Carrol, "Synchronization in chaotic systems", Phys. Rev. A, vol. 64, pp.821-824, 1990.

[2] M. Chen, D. Zhou, and Y. Shang, "A sliding mode observer based secure communication scheme," Chaos, Solitions and Fractals, vol. 25, pp. 573-578, 2005.

[3] M. Feki, Sliding mode control and synchronization of chaotic systems with parametrics uncertainties, Chaos, Solitons Fractals A 41(2009) 1390-1400.

[4] F. Zhu, "Observer-based synchronization of uncertain chaotic systems and its application to secure communications," Chaos, Solitons Fractals, vol.40, no. 5, pp. 2384-2391, Jun. 2009.

[5] H. Serrano-Guerrero, C. Cruz-Hernández, R.M. López-Gutiérrez, L. Cardoza-Avendaño, R.A. Chávez-Pérez, Chaotic Synchronization in Nearest-Neighbor Coupled Networks of 3D CNNs, Journal of Applied Research and Technology, vol. 11, no.1, pp. 26-41, 2013.

[6] R. Núñez Pérez, Measurement of Chua chaos and its applications, Journal of Applied Research and Technology vol 6, no. 1, pp. 45-53, 2009.

[7] R.J. Wai, Y.W. Lin, H.C. Yang, Experimental verification of total sliding-mode control Chua's chaotic circuit, IET Circuits, Devices & Systems vol. 5, 451-461, 2011.

[8] C. Hua and X. Guan, "Synchronization of chaotic systems based on PI observer design," Phys. Lett. A, Vol 334, pp. 382-389, 2005.

[9] M. Chen, D. Zhou, and Y. Shang, "A sliding mode observer based secure communication scheme," Chaos, Solitons Fractals, vol. 25, pp.573-578, 2005.

[10] E. Cherrier, M. Boutayeb, and J. Ragot, "Observers-based synchronization and input recovery for a class of nonlinear chaotic models", IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 53, no.9, pp. 1977-1988, Sep. 2006.

[11] A.L. Fradkov, Cybernetical physics: from control of chaos to quantum control, Springer, Berlin; 2007.

[12] N. Noroozi and M. Roopaei, P. Karimaghaee, "Adaptive control and synchronization in a class of partially unknown chaotic systems", Chaos, vol. 19, 023121, 2009.

[13] F. Wang and C. Liu, "A new criterion for chaos and hyperchaos synchronization using linear feedback control", Phys. Lett. A, vol. 360, pp. 274-278, 2006.

[14] A. Harb and W. Ahmad, "Chaotic systems synchronization in secure communication systems," in Proc. World Congress Computer Science Computer Engineering, and Applied Computing, Las Vegas, 2006.

[15] C. Wang and S. Ge, "Adaptive backstepping control of uncertain Lorenz system," Int. J. Bifurc. Chaos, vol. 11, pp. 1115-1119, 2001.

[16] D. Ghosh, A. Chowdhury and P. Saha, "On the various kinds of synchronization in delayed Duffing-Van der pol system," Commun. Nonlinear Sci. Numer. Simulat, vol. 13, pp. 790-803, 2008.

[17] D. Ghosh, S. Banerjee and A. Chowdhury, "Synchronization between variable time-delayed systems and cryptography", Euro. Phys. Lett., vol. 80, 30006: p1-p6, 2007.

[18] S. Garfinkel and G. Spafford, Practical unix and internet security, O' Reilly & Associates Inc., Sebastopol, CA; 1996.

[19] D. Li, J. Lu and X. Wu, "Linearly coupled synchronization of the unified chaotic systems and the Lorenz systems", Chaos, Solitons Fractals, vol. 23, pp.79-85, 2005.

[20] E. Elabasy, H. Agiza and M. El-Dessoky, "Global chaos synchronization for four scroll attractor by nonlinear control," Sci. Res. Essay, vol. 1, pp. 65-71, 2006.

[21] A. Emadzadeh and M. Haeri, "Global Synchronization of two different chaotic systems via nonlinear control," in Proc. ICCAS, Gyeonggi-Do, Korea, 2005.

[22] L. Min and J. Jing, "A new theorem to synchronization of unified chaotic systems via adaptive control," Chaos, Solitons Fractals, vol. 24, pp.1363-1371, 2004.

[23] A. L. Fradkov, B. Andrievsky, and R.J. Evans, "Adaptive observer-based synchronization of chaotic system with first-order coder in the presence of information constraints," IEEE Trans. Circuit Syst. I, Reg. Papers, vol. 55, no. 6, pp. 1685-1694, Jul. 2008.

[24] H. B. Fotsin and J. Daafouz, "Adaptive Synchronization of uncertain chaotic Colpitts oscillator based on parameter identification," Phys. Lett. A, vol. 339, pp. 304-315, 2005.

[25] R. E. Kalman, "A new approach to linear filtering and prediction problems," Trans. ASME – J. Basic Eng., Series D, vol. 82, pp. 35-45, 1960.

[26] D. Luenberger, "An introduction to observers," IEEE Trans. Autom. Contr., vol. 16, pp. 596-602, 1971.

[27] J. Gauthier, H. Hammouri, and S. Othman, "A simple observer for nonlinear systems. Applications to bioreactors," IEEE Trans. Autom. Contr., vol.37, pp.875-880, 1992.

[28] H. Keller, "Non-linear observer design by transformation into a generalized observer canonical form," Int. J. Control, vol 46, pp. 1915-1930, 1987.

[29] A. Levant, "Universal SISO sliding mode controllers with finite time convergence," IEEE Trans. Autom. Contr., vol. 46, pp. 1447-1451, 2001.

[30] J. Young and J. Farrel, "0bserver based backstepping control using online approximation," in Proc. IEEE Amer. Contr. Conf., Chicago, IL, 2000, pp. 3646-3650.

[31] H. Nijmeijer, and I.M.Y. Mareels, "An observer looks at synchronization," IEEE Trans. Circuits Syst. I, Fundam. Theory Appl., vol. 44, pp. 882-890, 1997.

[32] M. Feki, "Observer-based exact synchronization of ideal and mismatched chaotic systems," Phys. Lett. A, vol. 309, pp. 53-60, 2003.

[33] O. Morgüil and M. Feki, "A chaotic masking scheme by using synchronized chaotic systems", Phys. Lett. A, vol. 251, pp. 169-176, 199.

[34] M.P. Kennedy, "Chaos in Colpitts Oscillator", IEEE Trans. Circuits Syst. I, vol.41, pp. 771-774. 1994.

[35] L. Guo-Hui, "Synchronization and anti-synchronization of Colpitts oscillators using active control," Chaos, Solitons Fractals, vol. 26, pp. 87-93, 2005.

[36] G. M. Maggio, O. De Feo, and M.P. Kennedy, "Nonlinear analysis of the Colpitts oscillator and applications to design," IEEE Trans. Circuits Syst. I, vol.46, pp. 1118-1130, 1999.

[37] N. M. Nguyen, "Monolithic Microwave Oscillators and Amplifiers," Ph. D. Thesis, EECS Department, University of California, Berkeley, 1991.

[38] A. Poznyak (2008) Advanced Mathematical Tools for Automatic Control Engineers: Deterministic Techniques, vol. 1. Elservier, pp. 77-212.

[39] S. Raghavan and J. Hedrick, "Observer design for a class of nonlinear systems," Int J. Control, vol. 59, pp. 515-528, 1994.

[40] Rafael Martinez-Guerra and J.L. Mata-Machuca (2014), Fault detection and diagnosis in nonlinear systems: A differential and algebraic viewpoint, understanding complex systems, Springer.