Geofísica Internacional (2026) **65-3**: 2269- 2296

An innovative weighted fuzzy soft set-based model for multi-criteria groundwater quality assessment in urban areas: A case study of Agartala, India

Ajoy Kanti Das[1,*,a], Nandini Gupta[2,b], Suman Patra[3,c], Carlos Granados[4,d] and Suman Das[5,e]

Abstract

This study presents an innovative weighted fuzzy soft set-based multi-criteria decision-making model ($\mathfrak{B}$-model) for evaluating groundwater quality and identifying contamination sources in urban settings, with a case study in Agartala, Tripura, North-East India. Rapid urbanization and anthropogenic pressures have significantly impacted groundwater quality, demanding advanced methodologies for effective assessment and management. The proposed model incorporates ten critical groundwater quality parameters (GWQP—pH, electrical conductivity, iron concentration, dissolved oxygen, total hardness, total alkalinity, total dissolved solids, calcium, magnesium, and turbidity), transformed into fuzzy soft sets for comprehensive analysis. Groundwater samples were systematically collected from ten strategic locations across Agartala Municipal Corporation during three distinct seasons: pre-monsoon (March–May), monsoon (June–September), and post-monsoon (October–November) of 2023–2024. The analysis revealed seasonal variations in groundwater quality, with notable degradation during the pre-monsoon season due to reduced aquifer recharge and increased evaporation, while monsoon improvements were limited, primarily influenced by rainfall dilution. Specific areas consistently exhibited high pollution levels, highlighting localized contamination sources and elevated health risks. The model's weighted pollution scores enabled precise identification of pollution sources, facilitating targeted intervention strategies. This research underscores the potential of fuzzy soft set-based approaches for robust groundwater quality assessment and emphasizes their integration into sustainable water management practices and urban planning to mitigate contamination risks in rapidly urbanizing regions like Agartala. Comparative analyses with existing methods validate and underscore the advantages of our approach.

Key words: Fuzzy set, Multi-Criteria Decision-making, Soft Set, Groundwater risk assessment, Water resource management, Environmental Management.

Resumen

Este estudio presenta un innovador modelo de toma de decisiones multicriterio basado en conjuntos difusos suaves ponderados (modelo $\mathfrak{B}$) para evaluar la calidad de las aguas subterráneas e identificar las fuentes de contaminación en entornos urbanos, con un estudio de caso en Agartala, Tripura, noreste de la India. La rápida urbanización y las presiones antropogénicas han afectado significativamente la calidad de las aguas subterráneas, lo que exige metodologías avanzadas para su evaluación y gestión eficaces. El modelo propuesto incorpora diez parámetros críticos de calidad de las aguas subterráneas (GWQP: pH, conductividad eléctrica, concentración de hierro, oxígeno disuelto, dureza total, alcalinidad total, sólidos disueltos totales, calcio, magnesio y turbidez), transformados en conjuntos difusos suaves para un análisis integral. Las muestras de aguas subterráneas se recolectaron sistemáticamente en diez ubicaciones estratégicas de la Corporación Municipal de Agartala durante tres estaciones distintas: premonzón (marzo–mayo), monzón (junio–septiembre) y posmonzón (octubre–noviembre) de 2023–2024. El análisis reveló variaciones estacionales en la calidad de las aguas subterráneas, con un deterioro notable durante la estación premonzónica debido a la reducción de la recarga de los acuíferos y al aumento de la evaporación, mientras que las mejoras observadas durante el monzón fueron limitadas y estuvieron influenciadas principalmente por la dilución causada por las precipitaciones. Determinadas áreas mostraron de manera constante altos niveles de contaminación, destacando fuentes localizadas de contaminación y elevados riesgos para la salud. Las puntuaciones ponderadas de contaminación obtenidas mediante el modelo permitieron una identificación precisa de las fuentes de contaminación, facilitando estrategias de intervención específicas. Esta investigación destaca el potencial de los enfoques basados en conjuntos difusos suaves para una evaluación sólida de la calidad de las aguas subterráneas y enfatiza su integración en prácticas sostenibles de gestión del agua y planificación urbana para mitigar los riesgos de contaminación en regiones de rápida urbanización como Agartala. Los análisis comparativos con métodos existentes validan y destacan las ventajas de nuestro enfoque.

Palabras clave: Conjunto difuso, Toma de decisiones multicriterio, Conjunto suave, Evaluación del riesgo de las aguas subterráneas, Gestión de los recursos hídricos, Gestión ambiental.

Received: February 1, 2025; Accepted: May 1, 2026; Published on-line: July 1, 2026. https://doi.org/10.22201/igeof.2954436xe.2026.65.3.1855

Editorial responsibility: Dra. María Aurora Armienta-Hernández, Universidad Nacional Autónoma de México, Instituto de Geofísica.

* Corresponding author: Ajoy Kanti Das, ajoykantidas@gmail.com
[1] Tripura University, Department of Mathematics, Agartala, Tripura, India.
[2] Bir Bikram Memorial College, Department of Environmental Science, Agartala, Tripura, India.
[3] Tripura University, Department of Mathematics, Agartala, Tripura, India.
[4] Universidad Nacional Abierta y a Distancia, Escuela Ciencias de la Educación, Barranquilla, Colombia.
[5] NIT Agartala, Department of Education (ITEP), Jirania-799046, West Tripura, Tripura, India.
ORCID: [a]0000-0002-9326-1677, [b]0009-0000-5450-9783, [c]0009-0003-9795-7887, [d]0000-0002-7754-1468, [e]0000-0001-5682-9334

CC BY NC SA

1. Introduction

Groundwater is an essential resource, particularly in urban areas where surface water is often limited or contaminated. In rapidly developing cities such as Agartala, Tripura, the pressure from urban expansion, industrial activities, and agriculture has escalated, intensifying groundwater demand and raising significant concerns about its sustainability. To address these challenges, this study applies a weighted fuzzy soft set-based approach—a method rooted in fuzzy set theory by Zadeh (1965)—that effectively handles the uncertainties and complexities inherent in environmental data. This model is particularly well-suited for multi-criteria decision-making (MCDM), where various interdependent factors influence outcomes, enhancing its flexibility for tasks like groundwater quality assessment. We employ this model to evaluate groundwater quality in Agartala, focusing on identifying pollution sources and associated health risks, thus supporting more targeted management efforts. Fuzzy logic-based assessments have shown promise in environmental contexts. For instance, Patel and Chitnis (2022) utilized fuzzy methods to assess the Sabarmati River's water quality in Gujarat, considering climate and industrial impacts. Similarly, Kumar and Krishna (2021) developed a geospatial water quality index for coal mining regions, demonstrating the adaptability of fuzzy approaches in complex pollution assessments. Building on these methods, this study offers a novel framework for groundwater risk assessment, providing valuable insights into sustainable water management strategies in urbanizing regions like Agartala.

As urban centers expand, they face mounting challenges from improper waste management, industrial runoff, and excessive groundwater extraction, all of which degrade water quality and pose significant risks to public health. This is especially true for Agartala, the capital of Tripura in North-East India, where rapid urban development has heightened the urgency for effective groundwater quality monitoring and risk assessment systems. Traditional assessment methods often struggle to capture the inherent uncertainties and complexities of urban groundwater systems, making it difficult to accurately identify pollution sources and assess the overall risk. A fuzzy logic-based approach offers a powerful and flexible solution in response to these challenges. Fuzzy logic provides a more nuanced assessment by converting precise measurements into fuzzy numbers, allowing for a better representation of the uncertainty and variability in groundwater quality data. This approach proves particularly useful in urban environments, where groundwater systems are highly dynamic, and pollution sources are often diffuse and complex. Recent advancements in integrating fuzzy logic with decision-making models and machine learning algorithms have further enhanced the capacity to comprehensively evaluate groundwater quality. These advanced methodologies offer the ability to identify pollution sources more precisely and provide insights into potential health risks. For instance, Mallick *et al.* (2021) applied index-based methods to assess groundwater quality in Saudi Arabia, highlighting the utility of these approaches in arid regions for drinking and irrigation purposes. Milovanovic (2007) examined pollution sources along the Axios/Vardar River in Southeastern Europe, illustrating the significant impact of urbanization and industrial activities on water bodies. In India's Shivalik region, Mittal *et al.* (2021) conducted a hydrochemical analysis and health risk assessment, providing key insights into the health impacts of groundwater contamination. Ram *et al.* (2021) integrated the WQI with a GIS framework to offer a spatial perspective on groundwater contamination, demonstrating the value of combining geospatial techniques with WQI for managing urban groundwater resources. Rango *et al.* (2012) investigated the health effects of groundwater quality, particularly focusing on fluoride contamination in the Main Ethiopian Rift, highlighting the critical need for targeted interventions in regions facing high contamination risks. This study builds on such research by employing a fuzzy logic-based groundwater risk assessment framework to identify pollution sources in urban areas like Agartala, providing a novel and more accurate approach for managing groundwater contamination in rapidly urbanizing environments.

In parallel, soft set theory, introduced by Molodtsov (1999), offers a complementary framework for decision-making under uncertainty, providing flexibility in defining decision parameters. The soft set is particularly suited for addressing real-world decision-making problems involving incomplete or uncertain information, making it a valuable tool in groundwater risk assessments. While other mathematical models, such as interval-valued fuzzy sets, fuzzy sets, and rough sets, are used to handle uncertainty, each has its limitations. In contrast, soft set theory provides a more adaptable framework, allowing for greater flexibility in parameter selection, which is crucial when assessing groundwater pollution in urban environments with complex contamination sources. Molodtsov's pioneering work laid the foundation for soft set theory's broad applicability across disciplines, including environmental management, probability theory, game theory, and operations research. The versatility of soft set theory in handling uncertain data makes it ideal for evaluating groundwater quality in dynamic urban systems. Further contributions by Maji *et al.* (2003) extended the soft set theory to include operations like subsets, complements, unions, and intersections, making it more practical for addressing multi-criteria decision-making MCDM problems in environmental studies. Maji *et al.* (2002) further demonstrated soft set's potential in real-world decision-making

problems, a significant step forward in applying soft set theory to complex environmental challenges. Subsequent research by Ali *et al.* (2009) and Alcantud and Santos-García (2017) expanded the use of soft set theory in various decision-making contexts, particularly where evidence is insufficient or ambiguous. Chen *et al.* (2020) extended the soft set theory by introducing generalized vague N-soft sets, offering sophisticated techniques for decision-making in highly uncertain situations. These advancements highlight soft set theory's increasing relevance for environmental uncertainty scenarios, such as groundwater risk assessments in urban areas. By integrating fuzzy logic with soft set theory, this study leverages both approaches to provide a more comprehensive framework for identifying pollution sources in groundwater, addressing the challenges posed by data uncertainty and enabling more informed decision-making for urban water quality management.

Integrating soft set and fuzzy set theories has led to the development of fuzzy soft sets (FSS), which are particularly valuable for decision-making in uncertain and imprecise conditions, such as those encountered in groundwater quality assessments. Maji *et al.* (2001) first introduced the concept of fuzzy soft sets, and their application has since expanded across various fields, proving effective in handling complex decision-making problems with ambiguous information. In the context of groundwater risk assessment, FSSs offer a powerful framework for dealing with uncertainty, making them ideal for evaluating pollution sources in urban areas like Agartala. FSSs enable a nuanced approach to managing imprecise data, especially when multiple groundwater quality parameters are involved, such as pH, electrical conductivity, iron concentration, and dissolved oxygen. Roy and Maji (2007) laid the foundation for using FSS in decision-making problems by developing a framework based on fuzzy soft sets, which has since inspired further advancements. Feng *et al.* (2010) expanded the approach by introducing weighted fuzzy soft sets, allowing adaptability in handling diverse uncertainties and prioritizing decision-maker preferences. This weighted approach is particularly useful in groundwater assessments where different parameters may carry varying significance levels depending on the local pollution sources. Das and Granados (2021) contributed a group decision-making framework based on FSSs, incorporating weighted score functions to facilitate collective decision-making in complex environmental scenarios. Bhardwaj and Sharma (2021) presented an innovative uncertainty measure utilizing fuzzy soft set theory, further enhancing the ability to manage uncertainty in decision-making problems.

Groundwater contamination in urban areas has become a critical concern, driven by industrial discharge, agricultural runoff, and inadequate waste management practices. Conventional methods for assessing groundwater quality often struggle with environmental data's inherent uncertainties and complexities. To address these challenges, an innovative weighted fuzzy soft set-based approach offers a powerful framework for pollution source identification and risk assessment. This approach, leveraging the strengths of fuzzy soft sets, enables a nuanced analysis by effectively handling diverse and uncertain data, thereby enhancing the accuracy and reliability of groundwater quality assessments. By applying this method, decision-makers comprehensively understand pollution sources and associated health risks, allowing for targeted interventions and improved urban groundwater management. Recent developments in fuzzy soft set theories, such as the fuzzy parameterized intuitionistic fuzzy soft multiset theory, have proven instrumental in handling complex environmental decision-making problems by enhancing data interpretability and accuracy (Das and Granados, 2022). In 2022, Das, Granados, and Bhattacharya (2022) explored novel operations on fuzzy soft sets and their applications in decision-making processes. The authors comprehensively analyze these operations, highlighting their potential to enhance decision-making frameworks across various fields. This work contributes to the growing knowledge in fuzzy set theory and its practical implications for addressing complex problems. Furthermore, IFP-intuitionistic multi-fuzzy N-soft sets have shown potential in dealing with hesitant and uncertain information, further strengthening decision-making models (Das and Granados, 2023). The synergy between fuzzy logic and machine learning allows for improved accuracy in predictions and more dynamic analysis of groundwater risks, as demonstrated in similar applications of fuzzy controllers in complex environmental systems (Darvish Falehi and Torkaman, 2023). Additionally, recent work on weighted neutrosophic soft multisets further underscores the value of advanced fuzzy techniques in refining decision-making processes, particularly in multi-criteria evaluations (Granados and Das, 2023). Das *et al.* (2023) introduced the concept of weighted hesitant bipolar-valued fuzzy soft sets, which provide a more nuanced framework for evaluating alternatives. This research aims to build on these foundational studies by offering an innovative, data-driven solution to one of the most pressing environmental issues in urban settings.

Urbanization and industrialization have significantly affected environmental dynamics, especially in rapidly developing cities, posing serious challenges to groundwater quality. The rapid expansion of urban areas often intensifies pressures on groundwater resources due to inadequate waste management, industrial runoff, and over-extraction, which threaten water quality. In such environments, robust groundwater monitoring systems are critical, as groundwater serves as a vital resource for urban and agricultural water supplies. Traditional methods of groundwater quality assessment often struggle to handle the complexities of

uncertain, imprecise, and multi-dimensional data inherent in these dynamic systems. To overcome these limitations, advanced analytical methods, such as fuzzy logic-based approaches, are emerging as effective tools for comprehensive and accurate water quality assessments. Recent studies have highlighted the growing trend of integrating Machine-learning (ML) techniques with fuzzy logic-based decision-making models to enhance water quality assessment and management. For instance, Adimalla (2021) utilized the entropy weighted water quality index (EWQI) and pollution index of groundwater (PIG) to evaluate drinking water quality, emphasizing the need for sophisticated methods in diverse environmental settings. Similarly, Ahmed *et al.* (2021) demonstrated the potential of ML techniques in analyzing water quality indices and assessing pollution levels, providing a more nuanced approach to understanding groundwater contamination. Water Quality Index (WQI) models have been widely applied to evaluate surface and groundwater consistency and safety. The WQI developed by Brown *et al.* (1970) provides a comprehensive metric that combines several water quality parameters into a single index, offering a clear overview of water quality status. However, such methods may fall short of capturing the uncertainties and complexities of groundwater systems in urban areas. By incorporating fuzzy logic into these models, researchers can better address the imprecise nature of environmental data. Fuzzy logic is particularly useful in urban groundwater management, where pollution sources are varied and dynamic. In groundwater risk assessment, the fuzzy logic-based approach goes beyond traditional WQI models, allowing for a more flexible evaluation of pollution levels by converting crisp data into fuzzy values that capture the inherent uncertainties in environmental monitoring. Using the Canadian Council of Ministers of the Environment (CCME) Water Quality Index, Wagh *et al.* (2017) categorized groundwater suitability in the Kadva River basin of Maharashtra, India, across pre-monsoon and post-monsoon seasons. Their study revealed that intensive agricultural activities and anthropogenic influences significantly impacted groundwater quality, aligning with findings from other regions where urban and agricultural pressures degrade water resources. Similarly, Galal Uddin *et al.* (2017) extensively evaluated groundwater quality near the Rooppur Nuclear Power Plant in Bangladesh, identifying critical pollution parameters. These studies underscore the importance of advanced methods for water quality monitoring, as traditional approaches may not capture the full spectrum of contamination sources and risks. In urban areas, factors such as land-use changes, industrial development, and climate variability complicate groundwater quality assessments. Miller and Hutchins (2017) reviewed the impacts of urbanization and climate change on flooding and water quality in the UK, synthesizing knowledge to address emerging challenges in urban water resource management. Using the water quality index, Patel *et al.* (2020) investigated how increasing industries and urban areas affected the Sabarmati River in Gujarat. Their work, along with Shah and Joshi's (2017) assessment of the Sabarmati River's WQI in Gujarat, highlights the significant environmental pressures faced by urban water systems. Hutchins *et al.* (2018) explored the integrated effects of climatic stressors and future land-use changes on ground and surface water in the upper Thames River basin in the UK. Recently, Das *et al.* (2025b) developed a fuzzy MCDM model for examining human influences on urban river water quality is one of their noteworthy contributions to environmental MCDM. This research is extended in 2025 with weighted hesitant fuzzy soft set and fuzzy soft set models for rating water pollution and a case study applying fuzzy soft MCDM model to assess pollution levels in the Gomati and Haora Rivers (Das *et al.*, 2024a; 2025a). This study, and similar research point to the growing importance of combining advanced modeling techniques like fuzzy logic with environmental monitoring to manage the increasing risks to water quality. These innovative methods enable a more holistic assessment of pollution sources, particularly in urban areas where traditional monitoring systems may be insufficient. The application of fuzzy logic-based approaches in groundwater risk assessment offers a promising solution to the challenges posed by urbanization and industrialization. By allowing for a more flexible and accurate water quality evaluation, these methods provide valuable tools for managing water resources in complex urban environments, ensuring sustainable water management practices, and safeguarding public health.

Groundwater quality is shaped by various physical, chemical, and biological factors, collectively known as groundwater quality parameters (GWQPs). When groundwater quality is compromised, it poses risks to human health, ecosystems, and economic stability. This makes continuous monitoring and assessment of groundwater essential for ensuring a safe water supply and promoting sustainable resource management. Traditional assessment methods, which rely on crisp measurements and fixed standards, often fall short when addressing environmental data's inherent uncertainties and fluctuations. To address these limitations, this study employs an innovative weighted fuzzy soft set-based approach, enabling a more nuanced assessment of groundwater quality by transforming crisp data into fuzzy soft sets. This methodology better captures the uncertainty and variability in groundwater quality, supporting more flexible and informed decision-making for water resource management.

In this study, we focus on ten critical GWQPs: pH, electrical conductivity, iron concentration, dissolved oxygen, total hardness, total alkalinity, total dissolved solids, calcium, magnesium, and turbidity. The selection of these ten critical groundwater

quality parameters was based on their comprehensive ability to represent the chemical, physical, and biological characteristics of groundwater across varied land use types. These parameters are widely recognized indicators of water quality that reflect both natural geochemical processes and anthropogenic influences such as industrial activities, agricultural runoff, and domestic waste disposal. For instance, pH and electrical conductivity provide insights into water's acidity and overall ionic content, while heavy metals like iron indicate contamination from industrial or urban sources. Parameters such as total hardness, calcium, and magnesium relate to mineral dissolution from soil and rock formations, important for understanding background water quality. Dissolved oxygen and turbidity help assess biological activity and particulate contamination. By including this diverse set of parameters, the study ensures a holistic evaluation of groundwater quality that captures the effects of different land uses—residential, commercial, agricultural, and industrial—thereby enabling accurate identification of contamination sources and assessment of health risks. Groundwater samples were collected from ten strategic locations within the urban areas of Agartala during pre-monsoon, monsoon, and post-monsoon seasons to capture seasonal variations. By applying weighted fuzzy soft sets to analyze this data, we develop a comprehensive view of temporal and spatial fluctuations in groundwater quality, identifying specific sources of contamination through weighted pollution scores (ϖ-scores). These findings underscore the urgent need for enhanced groundwater management strategies in urban environments, particularly in rapid urbanization and climate variability. This research contributes crucial insights into health risks associated with groundwater contamination, offering valuable information for policymakers, urban planners, and stakeholders involved in water resource management. This study aims to guide future policy initiatives and research toward reducing pollution risks and ensuring long-term water security by highlighting the necessity of sustainable groundwater management. Additionally, the study emphasizes the potential for integrating Machine-learning algorithms with fuzzy soft set-based frameworks to strengthen continuous groundwater quality monitoring in urban areas. This integration enhances the capacity of urban management practices to mitigate contamination risks effectively, fostering sustainable and adaptive strategies for water resource management.

The structure of this chapter is organized as follows: Section 2 details the tools and techniques used for data collection and analysis, including an overview of the study area, groundwater sampling procedures, and the application of the weighted fuzzy soft set-based approach. Section 3 provides a comprehensive assessment of groundwater quality in Agartala City, utilizing the weighted fuzzy soft set methodology to identify contamination sources and assess associated risks. Section 4 presents an in-depth analysis of the weighted pollution scores (ϖ-scores) to evaluate groundwater quality and highlight critical areas of concern. Section 5 represents the ranking of groundwater sampling stations. Section 6 offers a comparative analysis with existing methods, highlighting the advantages of our approach. Finally, Section 7 concludes with key findings and offers recommendations for future research and policy actions aimed at sustainable groundwater management in urban areas.

2. Materials and methods

This section outlines the materials and methodologies employed in assessing groundwater quality in Agartala using an innovative weighted fuzzy soft set-based approach for pollution source identification and risk assessment. It includes details on the study area, data collection procedures, and the implementation of the weighted fuzzy soft set methodology to evaluate groundwater quality and identify contamination sources, offering insights into groundwater suitability and associated risks.

2.1. Study area

The study was conducted at ten carefully chosen locations within the Agartala Municipal Corporation in West Tripura District (Table 1; Figure 1). This selection aimed to capture a representative and detailed picture of groundwater quality in Agartala, with a specific focus on high-risk zones prone to pollution due to rapid urbanization and human activities. Agartala, the capital of Tripura, has a subtropical monsoon climate characterized by hot, humid summers, a significant monsoon season with approximately 2,000 mm of annual rainfall, and mild winters. Seasonal weather variations in the city affect groundwater recharge patterns, with the majority of recharge occurring during the monsoon season through rainwater infiltration.

The ten sampling sites were distributed to cover areas with different land-use profiles and reflect various environmental conditions, including residential, commercial, and industrial zones. These sites included high-density residential neighborhoods, bustling commercial districts, and industrial zones known for higher waste discharge. Additionally, some sites were located near agricultural land, where chemical fertilizers and pesticides could contribute to contamination. The diverse land-use patterns around these locations allowed the study to comprehensively assess the groundwater contamination sources and levels across different urban landscapes within Agartala.

Geographically, the study area is situated between longitudes 91.2433°E and 91.2660°E and latitudes 23.7914°N to

Table 1. Sampling stations for groundwater quality assessment in Agartala, Tripura, including location and coordinates

Sample No.	Sampling stations	Location	Coordinates
$s\alpha_1$	Arundhuti Nagar	Agartala, Tripura	23.8162°N, 91.2493°E
$s\alpha_2$	Golbazar Market	Agartala, Tripura	23.8265°N, 91.2855°E
$s\alpha_3$	Shibnagar	Agartala, Tripura	23.8304°N, 91.3030°E
$s\alpha_4$	Battala Market	Agartala, Tripura	23.8283°N, 91.2671°E
$s\alpha_5$	Banamalipur	Agartala, Tripura	23.8367°N, 91.2869°E
$s\alpha_6$	Lake Chowmuhani Market	Agartala, Tripura	23.8419°N, 91.2797°E
$s\alpha_7$	Gurkhabasti	Agartala, Tripura	23.8554°N, 91.2827°E
$s\alpha_8$	Agartala Government Medical College	Agartala, Tripura	23.8603°N, 91.2927°E
sα9	Ushabazar	Agartala, Tripura	23.88610N, 91.24330E
sα10	Maharaja Bir Bikram Airport	Agartala, Tripura	23.89140N, 91.24380E

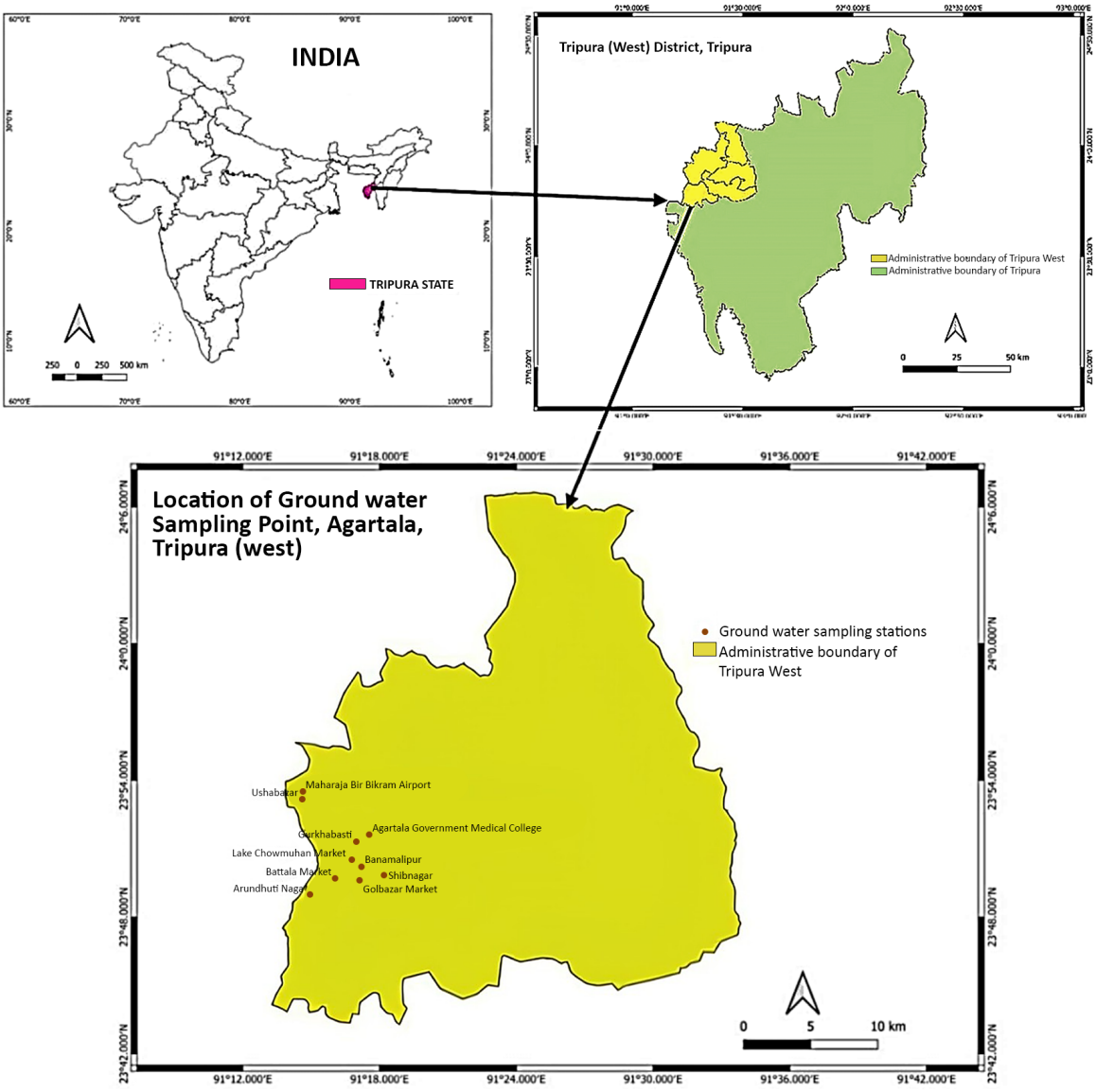

Figure 1. The study area within Agartala Municipal Corporation, Tripura.

23.9075°N. This area is primarily characterized by undulating plains with pockets of hilly terrain, along with valleys that naturally facilitate drainage. The Agartala's unique terrain and drainage system play a significant role in groundwater movement and pollutant dispersal. Rainwater runoff and inadequate waste disposal and unregulated industrial discharges enable pollutants to percolate into the groundwater system, especially in low-lying valley areas.

The population density in Agartala is among the highest in Tripura, with approximately 988,192 residents (as per the 2011 census) residing within a 984 km² area. The city's rapid urban expansion is fueled by increasing residential, industrial, and commercial activities, which collectively contribute to substantial environmental pressures on groundwater resources. Industrial areas, especially those involving small and medium enterprises, often discharge untreated wastewater into nearby drainage systems, exacerbating groundwater contamination. Residential zones contribute to groundwater pollution through domestic sewage, improperly managed waste disposal, and occasional leachate from landfills.

2.2. Hydrogeological Characteristics and Sampling Details

2.2.1. Aquifer Types and Depths

The study area in Agartala primarily features shallow alluvial and weathered fractured aquifers typical of the region's undulating plains and valley systems. The predominant aquifer types at each sampling site are as follows:

- **Residential and Commercial Zones:** Predominantly shallow, unconfined alluvial aquifers with a water table generally located between 5 to 15 meters below ground surface (BGS). The unsaturated zone thickness varies from 3 to 10 meters depending on local topography and soil cover.
- **Industrial Zones:** These areas mostly overlay weathered fractured bedrock aquifers with deeper water levels, ranging from 10 to 25 meters BGS. The unsaturated zone here tends to be thinner (approximately 2 to 6 meters) due to localized excavation and surface disturbances.
- **Agricultural and Peri-Urban Areas:** Characterized by unconfined alluvial aquifers with water tables generally between 3 to 12 meters BGS, and unsaturated zone thicknesses of 4 to 9 meters.

The typical well depths for groundwater extraction across these sites range from 15 to 35 meters, ensuring access to saturated zones for water supply.

2.2.2. Sampling Depth and Procedure

Groundwater samples were collected primarily at the wellhead following standard purging protocols to obtain representative water from the saturated zone at or near the static water level. In some cases, sampling was conducted at depths slightly below the water table within the well bore to ensure minimal surface contamination influence. No surface runoff or shallow soil moisture samples were taken, ensuring that the analyses reflect true groundwater quality.

2.2.3. Contaminant Mobility and Geochemical Considerations

The ability of contaminants to reach groundwater depends on their chemical nature and interaction with the geological environment:

- **Conservative Contaminants:** Nitrates are highly mobile and persist through the unsaturated zone with minimal attenuation, making them reliable indicators of agricultural runoff or sewage infiltration. Their presence is thus closely monitored, especially near agricultural and densely populated residential areas.
- **Reactive Contaminants:** Phosphates, oils, fats, and many heavy metals exhibit strong adsorption, precipitation, or biodegradation within the soil and unsaturated zone, limiting their mobility. These contaminants tend to accumulate near the surface or in soil layers, and only reach groundwater under specific hydrogeological conditions, such as fractured rock or areas with thin unsaturated zones.

Additionally, the distance from the coastal zone influences salinity intrusion risks and contaminant dispersion, although Agartala's inland location reduces the likelihood of saltwater intrusion.

2.2.4. Health Risk Implications

Understanding contaminant type, concentration, and exposure pathway is critical for assessing health risks:

- **Acute (Short-Term) Exposure:** Typically linked to pathogens, organic contaminants, and high nitrate levels, which can cause immediate health effects such as gastrointestinal illness, methemoglobinemia ("blue baby syndrome"), or poisoning.
- **Chronic (Long-Term) Exposure:** Includes low-level, prolonged exposure to heavy metals, fluoride, and other

toxic substances that may cause cancer, neurological disorders, or developmental issues.

The study's sampling strategy ensures that the groundwater quality data reflect the actual exposure risk to consumers relying on well water, facilitating accurate health risk assessments and mitigation planning.

2.2.5. Rationale for selecting Agartala as the study area

The selection of Agartala as the focal point of this study was based on several key factors:

2.2.5.1. Natural Factors:

- **Geological Composition:** The geological makeup of an area significantly influences groundwater quality. For example, regions rich in limestone typically display elevated hardness levels due to higher concentrations of calcium and magnesium.
- **Weathering and Erosion:** Natural processes such as weathering and erosion can release essential minerals and salts into groundwater systems, potentially affecting their quality.
- **Precipitation and Recharge:** Rainfall plays a critical role in groundwater dynamics by diluting existing contaminants and replenishing aquifers, leading to fluctuations in contaminant concentrations over time.

2.2.5.2. Anthropogenic (Human-Induced) Factors:

- **Agricultural Activities:**
  - **Pesticides and Fertilizers:** The application of agrochemicals in farming can lead to significant groundwater contamination, primarily from nitrates, phosphates, and residual pesticides.
  - **Irrigation Practices:** Excessive irrigation may result in soil salinization, negatively affecting groundwater quality by increasing salinity levels.
- **Industrial and Urban Activities:**
  - **Industrial Waste Disposal:** Improper disposal practices of industrial waste can introduce heavy metals and toxic chemicals into groundwater systems.
  - **Urban Runoff:** Stormwater runoff from urban areas can carry pollutants such as oils, greases, and heavy metals into groundwater, further degrading its quality.
- **Sewage and Wastewater:**
  - **Leaking Septic Systems:** Inadequately managed septic systems can contaminate groundwater with pathogens, nitrates, and other harmful pollutants.
  - **Untreated or Poorly Treated Wastewater:** Insufficient treatment of wastewater can introduce harmful bacteria, viruses, and chemicals into the groundwater supply.
- **Landfills:**
  - Poorly managed landfills produce leachate—a toxic liquid containing harmful chemicals and heavy metals—that can seep into the soil and potentially reach groundwater. The likelihood and extent of groundwater contamination depend largely on the thickness of the unsaturated zone and the depth of the groundwater table; thicker unsaturated zones and deeper water levels generally provide greater natural attenuation, reducing contaminant mobility and risk.
- **Deforestation and Land Use Changes:**
  - **Soil Erosion:** Deforestation increases soil erosion by removing vegetation cover, leading to greater sediment runoff and elevated turbidity in groundwater recharge areas. The impact on groundwater quality depends significantly on the depth of the groundwater table—shallower water tables are more vulnerable to sediment infiltration, while deeper groundwater may be less immediately affected due to natural filtration through the unsaturated zone.
- **Over-Extraction of Groundwater:**
  - **Saltwater Intrusion:** In coastal areas, excessive groundwater extraction can lead to saltwater intrusion, raising salinity levels in freshwater aquifers.
  - **Lowering of the Water Table:** Over-pumping groundwater resources can result in a lowered water table, concentrating contaminants like arsenic and fluoride in remaining water supplies.
- **Climate Change:**
  - **Changes in Precipitation Patterns:** Variability in rainfall can significantly impact groundwater recharge rates and the concentration of contaminants in groundwater.
  - **Temperature Variations:** Increasing temperatures can alter groundwater chemistry by affecting evaporation rates and altering precipitation patterns, impacting recharge and contaminant levels.

This study employs an innovative weighted fuzzy soft set-based approach for groundwater pollution source identification and risk assessment in urban areas. This methodology effectively captures the uncertainty and variability inherent in groundwater

quality data, by integrating various groundwater quality parameters through a weighted fuzzy soft set framework.

Water samples were systematically collected from selected sites across Agartala City, and physicochemical parameters were analyzed. The results were compared against established national and international water quality standards, facilitating the identification of areas in urgent need of improved groundwater management and protection measures. This approach not only enhances the accuracy of groundwater quality assessments but also assists in pinpointing specific pollution sources, thereby providing essential insights for policymakers and urban planners engaged in sustainable water resource management. The study aims to contribute to more effective strategies for mitigating groundwater pollution risks and ensuring long-term water security in urban environments through this innovative framework.

2.3. Method

In this section, we will establish the foundational concepts necessary for understanding the Weighted Fuzzy Soft Set-Based Approach employed for groundwater pollution source identification and risk assessment in urban areas. By introducing key ideas such as weighted fuzzy sets, soft sets, and fuzzy soft sets, we will highlight their critical role in developing our innovative framework. These concepts are essential for effectively managing uncertainty and imprecision in decision-making processes related to groundwater quality analysis.

Let V represent a universal set, or the initial universe, consisting of all possible objects under consideration. The power set of V, denoted as P(V), refers to the set containing all possible subsets of V. Meanwhile, E represents a set of parameters that describe various aspects or attributes of the elements within V.

2.3.1. Fuzzy Sets

A fuzzy set (Zadeh, 1965), as introduced by Zadeh in 1965, extends the classical notion of a set by allowing partial membership, meaning that elements of the set can belong to it with varying degrees. In contrast to traditional binary logic, where an element is either in or out of a set, fuzzy sets permit a gradual assessment of membership using a membership function, which assigns each element a value between 0 and 1.

A fuzzy set Z over a universe V is expressed as $Z=\{(o,\mu_Z(o)):o\in V\}$, where μ_Z: $V\rightarrow[0,1]$ is the membership function, representing the degree of membership for each element $o\in V$. Here, the value $\mu_Z(o)$ indicates how strongly element o belongs to the fuzzy set Z. For example, if $\mu_Z(o)=0$, the element does not belong to the set at all, whereas if $\mu_Z(o)=1$, it fully belongs to the set. Intermediate values between 0 and 1 capture partial membership, providing a powerful way to represent vague or uncertain information.

Fuzzy sets allow for the representation of varying degrees of membership across different groundwater quality parameters, enabling us to assign importance to each parameter based on its relevance to pollution assessment. This is particularly useful in situations where certain factors may have a greater impact on groundwater quality than others, facilitating a more nuanced analysis. The set of all possible fuzzy sets on V is denoted by FS(V).

2.3.2. Soft Sets

A soft set (Molodtsov, 1999) is a mathematical tool for dealing with uncertainty, originally introduced to provide a flexible and simplified framework for addressing vague data, where the information may not be entirely clear or precise. The formal structure is based on the relationship between parameters and subsets of a universe of discourse.

A soft set over a universe V is defined as a couple $(\boldsymbol{F},E)$, where $\boldsymbol{F}$ is a function $\boldsymbol{F}$: $E\rightarrow P(V)$. This function associates each parameter in the set E with a specific subset of V. In simpler terms, for each parameter $p\in E$, the mapping $\boldsymbol{F}(p)$ identifies a subset of elements in V to which the parameter is applicable. This formalizes the idea of associating each parameter with relevant data, while also accommodating uncertainty, as the subsets may not be precisely defined.

Soft sets provide a flexible mathematical framework for dealing with uncertainties associated with groundwater pollution sources. By incorporating various parameters and their associated relationships, soft sets enable us to represent complex decision-making scenarios without the need for fixed membership grades, which is particularly advantageous in the context of environmental assessments.

2.3.3. Fuzzy Soft Sets

Fuzzy soft sets (Maji *et al.*, 2001) combine the flexibility of soft sets with the nuanced membership grading of fuzzy sets. By allowing parameters to be associated with fuzzy sets rather than crisp subsets of V, FSSs offer an enhanced framework for modeling problems with multiple levels of uncertainty and imprecision.

A couple $(\boldsymbol{F},E)$ is said to be an FSS over a universe V, where F: E→FS(V) *a function that assigns a fuzzy* is set to each parameter in **P**. In this setup, for each parameter $p\in E$, the corresponding fuzzy set $F(p)$ provides a degree of membership for each element in V, reflecting the extent to which the element

satisfies or is related to the parameter p. This structure enables the incorporation of both parameter-based uncertainty (through soft sets) and gradual membership (through fuzzy sets), making FSSs well-suited for complex decision-making scenarios where data is not only uncertain but also fuzzy in nature.

To better understand the concepts, consider an example where V is a set of wells, and E represents various parameters related to water quality, such as pH level, pollution, and temperature. Each parameter in E (e.g., "acceptable pH range") can be associated with a fuzzy set over V, where the membership function indicates how well each well satisfies the given condition. The fuzzy soft set then provides a comprehensive and flexible model for assessing water quality, accounting for both uncertainty in the parameters and the fuzziness of the data. Fuzzy soft sets integrate the advantages of both fuzzy sets and soft sets, allowing for a comprehensive representation of uncertainties in groundwater quality data. This approach enhances our ability to model the interactions between different GWQPs, leading to a more robust evaluation of pollution sources and risks.

By integrating these foundational concepts, our proposed weighted fuzzy soft set-based approach establishes a robust framework that effectively tackles uncertainty, complexity, and multi-criteria decision-making, paving the way for future applications in groundwater pollution source identification and risk assessment in urban areas.

Central to this framework is the introduction of the weighted score function ϖ for fuzzy soft sets. This innovative function enables us to quantify the impact of various groundwater quality parameters (GWQPs) on pollution assessment, allowing for a more nuanced evaluation of groundwater contamination levels. The $\mathfrak{B}$-model harnesses this concept to serve as an effective water pollution rating system, systematically assessing the risks associated with different pollution sources.

The application of the ϖ function within the $\mathfrak{B}$-model facilitates the aggregation of data from multiple GWQPs, leading to a comprehensive understanding of groundwater quality in urban environments. This approach not only enhances our capability to identify pollution sources but also assists in prioritizing intervention strategies based on the severity of contamination. By employing this method, we aim to provide actionable insights for policymakers and stakeholders in their efforts to mitigate groundwater pollution and promote sustainable resource management.

2.3.4. Weighted Fuzzy Soft Set

A weighted fuzzy soft set (WFSS) incorporates an additional weight function into the fuzzy soft set structure. Formally, a weighted fuzzy soft set is defined as a triple (F, E, ϖ), where:

- (F, E) is a FSS as described above.
- ϖ: $E \rightarrow [0,1]$ is a weight function that assigns a weight to each parameter $e \in E$, reflecting the relative importance or significance of that parameter in the decision-making or assessment process.
- For each parameter $p_k \in E$, the associated weight is $\varpi_k = \varpi(p_k)$, where ϖ_k lies between 0 and 1. This weight scales the influence of the fuzzy membership values associated with that parameter.

A WFSS allows for the incorporation of both fuzzy set theory, which deals with uncertainty and gradation of membership, and weighted soft set theory, which introduces the concept of relative importance or priority among attributes. This combination enables more nuanced and context-specific modeling and analysis, particularly in decision-making processes where attributes may carry varying degrees of significance. The WFSS framework incorporates the notion of relative importance among parameters, making it particularly useful in cases where attributes have different levels of significance.

2.4. Weighted water pollution score function

Consider (F, E, ϖ) to be a weighted fuzzy soft set over V, where ϖ_k is the weight assigned to each parameter $e_k \in E$. The mapping $\varpi: E \rightarrow [0,1]$ assigns a weight to each parameter e_k, in such a way that $\varpi_k = \varpi(e_k)$.

The weighted water pollution score function W for (F, E, ϖ) is defined as:

$$W(s) = \frac{\sum_{k=1}^{m} \left[\varpi(e_k) \times \mu_{F(e_k)}(s)\right]}{\sum_{k=1}^{m} \varpi(e_k)}, e_k \in E \text{ and } \forall s \text{ in } V \tag{1}$$

- s represents a specific location in the dataset.
- e_k refers to individual GWQPs such as pH, TDS, etc.
- $\mu_{F(e_k)}(s)$ is the fuzzy membership function, which provides a value between 0 and 1, representing the degree to which the sample meets ideal conditions for parameter e_k. A value of 0 indicates the ideal condition, while a value of 1 indicates the worst condition.
- $\varpi(e_k)$ represents the weight assigned to parameter e_k, reflecting its importance in the overall water quality assessment.

2.4.1. Interpretation of W:

- The weighted pollution score W provides a composite score that reflects the overall pollution level for a particular location or sample. A score near 0 (zero) indicates excellent water quality, while a score approaching 1 (one) signifies poor water quality.

- By adjusting the weights $\varpi(e_k)$, the model can emphasize certain parameters more than others. This flexibility is essential because the significance of different GWQPs can vary depending on local environmental conditions or specific water use cases (e.g., drinking, irrigation, industrial use).

This innovative approach is applied within the $\mathfrak{B}$-model to evaluate groundwater contamination resulting from urbanization and anthropogenic activities. By integrating these weighted fuzzy soft sets into our analysis, we enhance the identification of pollution sources and conduct a thorough risk assessment, ultimately guiding effective groundwater management strategies in urban settings.

2.5. Weighted fuzzy soft set based MCDM model ($\mathfrak{B}$-model)

The $\mathfrak{B}$-model is developed to assess groundwater contamination through an innovative approach that leverages weighted fuzzy soft sets. This model is structured to evaluate the sources of groundwater pollution, particularly focusing on impacts from urbanization and anthropogenic activities. The methodology employs the principles of fuzzy set theory, soft sets, and weighted fuzzy soft sets to compute a comprehensive score for water quality.

The following steps outline the $\mathfrak{B}$-model, which integrates these concepts to facilitate the identification of pollution sources and assess the associated risks effectively:

Step 1: *Define the sampling stations, GWQPs, and expert groups.*

Identify key elements needed for the analysis: sampling stations, GWQPs, and expert opinions.

- **Sampling Stations** $S=\{\alpha_1, \alpha_2, \alpha_3,\ldots, \alpha_n\}$: These are the physical locations where groundwater samples are collected for the study. Each station represents a unique site that is being monitored for contamination levels.
- **Groundwater Quality Parameters** $P=\{\beta_1, \beta_2, \beta_3,\ldots,\beta_m\}$**:** These are the specific factors (e.g., pH, dissolved oxygen, total dissolved solids) that are used to evaluate the quality of groundwater. Each parameter has a distinct role in identifying contamination, especially from urbanization and human anthropogenic activities.
- **Expert Group** $E=\{e_1, e_2, e_3,\ldots, e_q\}$**:** Experts in the field (such as environmental scientists, hydrologists, and public health professionals) are involved in providing their opinions on the data, ensuring that the results are validated by experience and domain knowledge.

Step 2: *Collect crisp data summaries from experts.*

Gather precise, non-fuzzy (crisp) measurements of GWQPs from each sampling station.

- **Crisp Data Summaries** $DT=\{dt_1, dt_2, dt_3,\ldots, dt_q\}$**:** These are the raw numerical values collected by experts for each GWQP at each station. For example, at a given station, the pH value might be 7.5, and the total dissolved solids might be 300 mg/L.
- These crisp values represent the initial observations and are essential for later steps in converting them into a fuzzy soft set framework, which can incorporate uncertainty in the data.

Step 3: *Enter the standard and ideal values of GWQPs.*

Incorporate regulatory standards or ideal values for each GWQP as a benchmark.

- **Standard and Ideal Values:** Regulatory agencies (such as the World Health Organization or local environmental protection agencies) provide guidelines for acceptable levels of each GWQP. For instance, the ideal pH for drinking water might be between 6.5 and 8.5.
- These values are necessary to define fuzzy membership functions in the next step, which will measure how far the observed data deviates from the ideal or standard values.

Step 4: *Define fuzzy membership functions for GWQPs.*

Convert the crisp data from Step 2 into fuzzy data by defining membership functions for each GWQP.

- **Fuzzy Membership Function** μ_{pt_k}**:**$\mathrm{R}\rightarrow[0,1]$: A fuzzy membership function is created for each water quality parameter $pt_k\in P$. It maps the real-valued data (e.g., pH = 7.5) to a fuzzy value between 0 and 1, where:
  - A value close to 1 indicates that the GWQP is far from ideal (poor water quality).
  - A value close to 0 means the GWQP is near the ideal range (good water quality).
- For example, a pH of 6.5 might map to a membership value of 0.1 (indicating good quality), while a pH of 9 might map to a value of 0.9 (indicating poor quality).

Step 5: *Assign weights to each GWQP.*

Define the relative importance of each GWQP by assigning weights.

- **Weight Function:** This function assigns a weight to each GWQP $\beta_k\in P$, where $\varpi_k=\varpi(\beta_k), \mathrm{k}=1,2,3,\ldots,m$. The weight is a value between 0 and 1, indicating how critical that parameter is in determining overall water quality.

- For example, if total dissolved solids are more important than pH in determining groundwater contamination, it might be assigned a higher weight (e.g., $\varpi_{\text{TDS}} = 0.7$) compared to pH (e.g., $\varpi_{\text{pH}} = 0.3$).

This step allows for prioritization, where more significant parameters contribute more heavily to the final score.

Step 6: *Convert crisp data into WFSSs.*

Combine the crisp data, membership functions, and weights to form weighted fuzzy soft sets (WFSSs).

- The crisp data summaries from Step 2, along with the fuzzy membership functions from Step 4 and weights from Step 5, are now integrated to create WFSSs for each expert.
- These WFSSs (ψ_1, P, ϖ), (ψ_2, P, ϖ), (ψ_3, P, ϖ),…, (ψ_q, P, ϖ) represent the final fuzzy representations of the water quality data, incorporating both fuzziness (due to uncertainty in measurements) and weights (to reflect the importance of different parameters).

Step 7: *Calculate the weighted score for every data collection site.*

Calculate the ϖ-score, which represents the overall groundwater quality at every data collection site.

- The ϖ-score $\Omega_{\varpi}(t)$ is calculated for every data collection site t using the following formula:

$$\Omega_{\varpi}(t) = \frac{\sum_{k=1}^{m} \left[\varpi\left(\beta_k\right) \times \Omega\left(\mu_{\Psi(\beta_k)}(t)\right)\right]}{\Sigma_{k=1}^{m} \varpi\left(\beta_k\right)}, \beta_k \in P \text{ and } \forall t \text{ in } S \tag{2}$$

- This formula computes a weighted average of the fuzzy membership values, taking into account the importance of each parameter (via the weights). It provides a single score for each station, which can be used to compare groundwater quality across all sampling locations.

Step 8: *Interpret the ϖ-score.*

Assess the groundwater quality at each station based on the ϖ-score.

- A lower ϖ-score (closer to 0) indicates that the water quality at a particular sampling station is good, meaning that the contamination levels are low.
- A higher ϖ-score (closer to 1) indicates poor water quality, suggesting significant contamination.
- This final step helps identify which sampling stations have safe water and which ones require intervention for sustainable management strategies.

The $\mathfrak{B}$-model methodically transforms raw water quality data into a fuzzy decision-making framework, accounting for uncertainties and varying levels of importance among parameters. By computing weighted scores for each sampling station, the model provides a robust system for assessing groundwater contamination and identifying areas with degraded water quality. This algorithm enables a nuanced assessment of groundwater contamination, particularly in urban environments affected by urbanization and human anthropogenic activities, and aids in the development of sustainable management strategies.

The flowchart Figure 2 outlines the systematic steps of the $\mathfrak{B}$-model designed to assess groundwater quality, specifically addressing contamination arising from urbanization and anthropogenic activities in Agartala, Tripura, India. Each step emphasizes the integration of an innovative weighted fuzzy soft set-based approach with multi-criteria decision-making principles, facilitating a thorough evaluation of groundwater pollution sources and associated risks. This structured methodology combines expert insights with empirical data, ensuring a nuanced understanding of groundwater contamination levels. By systematically identifying and quantifying pollution sources, the $\mathfrak{B}$-model serves as a valuable tool for guiding effective groundwater management strategies tailored to the specific challenges faced in urban environments. The results derived from this model provide critical information for policymakers and stakeholders aiming to mitigate pollution and protect vital water resources in urban areas.

3. Assessment of groundwater contamination in Agartala City

In this segment, we undertake a comprehensive assessment of groundwater contamination levels in Agartala City, West Tripura, utilizing an innovative weighted fuzzy soft set-based approach specifically designed for identifying pollution sources and conducting risk assessments in urban areas. This methodology is not only crucial for understanding the dynamics of groundwater quality but also plays a vital role in facilitating informed, knowledge-based decision-making. Through this approach, we aim to develop effective strategies that mitigate pollution, ensuring the safety and quality of groundwater resources, and thereby safeguarding public health and environmental integrity.

The $\mathfrak{B}$-model is established as a multi-criteria decision-making framework that seamlessly integrates soft set and fuzzy set theories. Within this framework, the weighted fuzzy soft set

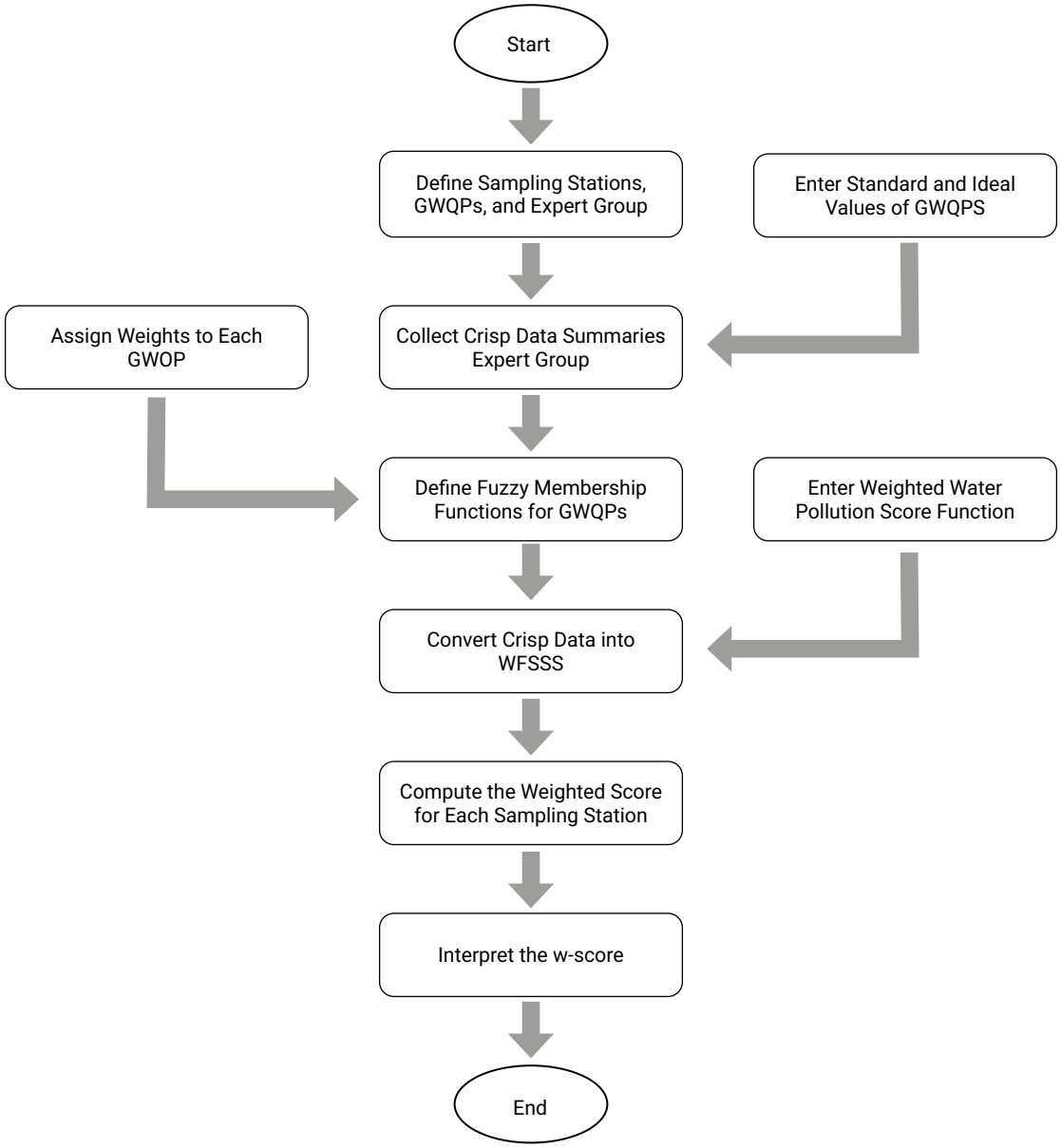

Figure 2. Flowchart for $\mathfrak{B}$-model: Groundwater Quality Assessment.

score (ϖ-score) functions as the primary indicator of groundwater quality. A ϖ-score approaching zero signifies excellent groundwater quality, reflecting minimal contamination, while a score nearing one indicates substantial contamination levels. This model adeptly aggregates multiple groundwater quality parameters, assigns relative weights to each parameter, and produces a comprehensive single parametric value that encapsulates the overall groundwater quality. By allowing for the nuanced interpretation of these scores, the model effectively highlights the critical parameters that influence water safety.

For this study, ten strategically selected groundwater sampling locations across Agartala City were chosen, as illustrated in Figure 1 and detailed in Table 1. These sites were selected based on several criteria, including population density, proximity to urban settlements, and potential sources of contamination. The coordinates of the sampling sites were meticulously recorded using a Garmin eTrex Vista Cx GPS device, ensuring precise location mapping. Samples were collected during the 2023-2024 period, covering different seasonal variations: pre-monsoon, monsoon, and post-monsoon, which are essential for understanding temporal fluctuations in water quality.

Each water sample was collected in triplicate using 1.5-liter polypropylene bottles. To prevent contamination, these bottles were pre-washed with a 10% hydrochloric acid (HCl) solution and thoroughly rinsed with the sample water at the time of collection. The study focused on ten key GWQPs that are critical for evaluating the

impacts of urbanization and anthropogenic activities on groundwater quality. These parameters include pH, Electrical Conductivity, Iron, Dissolved Oxygen (DO), Total Hardness, Total Alkalinity, Total Dissolved Solids, Calcium, Magnesium, and Turbidity. The selection of these parameters was based on their relevance to urban pollution sources and their potential health impacts.

To ensure the reliability and accuracy of the results, all sampling and analysis procedures adhered to the established guidelines outlined by the American Public Health Association (APHA, 2005). The groundwater quality data collected from the ten selected locations were analyzed across the three distinct seasons, with seasonal summaries presented in Tables 2, 3, and 4. This detailed seasonal analysis is vital for capturing the variations in groundwater quality that can occur due to changing climatic conditions and human activities.

The obtained values were meticulously compared to the recommended water quality standards established by the World Health Organization (WHO, 2011), which are summarized in Table 5. This comparison serves as a benchmark for evaluating the degree of contamination and identifying critical areas requiring immediate attention. The results of this thorough analysis provide invaluable insights into the extent of groundwater contamination arising from urbanization and anthropogenic activities.

Furthermore, these findings underscore the pressing need for sustainable management strategies aimed at addressing groundwater pollution effectively. By identifying specific sources of contamination and understanding their impacts on groundwater quality, stakeholders can implement targeted interventions, regulatory measures, and community awareness programs to protect this vital resource. Ultimately, the innovative approach outlined in this study not only enhances our understanding of groundwater dynamics but also contributes to the development of sustainable practices that can mitigate pollution and safeguard public health in urban environments.

Let $S=\{s\alpha_1, s\alpha_2, s\alpha_3, s\alpha_4, s\alpha_5, s\alpha_6, s\alpha_7, s\alpha_8, s\alpha_9, s\alpha_{10}\}$ represent the set of sampling stations within Agartala City, as specified in Table 1. The set P represents the GWQPs being considered, and it is defined as follows:

$$P=\{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10}\}.$$

Table 2. Crisp data summaries of GWQPs for the pre-monsoon season.

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
$s\alpha_1$	6.40	115	1.5	5.4	40	135	180	20	35	3.6
$s\alpha_2$	6.70	125	1.8	5.2	45	130	170	25	40	3.7
$s\alpha_3$	6.95	130	2.0	5.0	50	140	185	30	45	3.5
$s\alpha_4$	6.10	135	1.2	5.3	55	125	160	35	50	3.8
$s\alpha_5$	6.55	120	1.7	5.1	60	145	190	40	55	3.3
$s\alpha_6$	6.75	110	1.3	5.5	65	150	200	45	60	3.4
$s\alpha_7$	6.85	105	1.6	5.6	70	160	210	50	65	3.2
$s\alpha_8$	6.00	130	1.4	5.8	75	170	220	55	70	3.9
$s\alpha_9$	6.50	120	1.9	5.7	80	155	195	60	75	3.6
$s\alpha_{10}$	6.05	115	1.1	5.9	85	140	170	65	80	3.7

Table 3. Crisp data summaries of GWQPs for the monsoon season.

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
$s\alpha_1$	6.60	120	1.4	5.2	32	119	170	15	30	3.4
$s\alpha_2$	6.85	110	1.9	5.3	38	122	180	20	35	3.7
$s\alpha_3$	6.00	125	2.5	5.6	43	145	190	25	40	3.9
$s\alpha_4$	6.90	130	2.2	5.7	48	152	200	30	45	3.5
$s\alpha_5$	6.55	120	1.7	5.5	51	155	210	35	50	3.4
$s\alpha_6$	6.75	115	1.3	5.0	56	160	220	40	55	3.6
$s\alpha_7$	6.85	105	1.6	5.8	62	172	195	45	60	3.1
$s\alpha_8$	6.00	130	1.4	5.7	68	155	180	50	65	3.0
$s\alpha_9$	6.70	125	1.8	5.6	74	141	160	55	70	3.2
$s\alpha_{10}$	6.75	115	2.0	5.5	80	130	140	60	75	3.5

Table 4. Crisp data summaries of GWQPs for the post-monsoon season.

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
$s\alpha_1$	6.50	110	1.3	5.2	34	134	173	21	33	3.6
$s\alpha_2$	6.75	117	1.8	5.5	39	145	182	27	41	3.2
$s\alpha_3$	6.90	113	1.5	5.9	44	162	191	31	47	3.4
$s\alpha_4$	6.85	124	1.2	5.8	49	177	183	35	51	3.3
$s\alpha_5$	6.60	127	1.1	5.2	53	187	210	47	53	3.2
$s\alpha_6$	6.85	124	1.7	5.4	57	179	202	45	63	3.5
$s\alpha_7$	6.90	131	2.5	5.7	62	117	163	50	65	3.8
$s\alpha_8$	6.05	120	3.1	5.5	67	129	128	55	71	3.9
$s\alpha_9$	6.70	125	1.5	5.6	72	121	139	63	75	3.6
$s\alpha_{10}$	6.85	119	1.4	5.1	77	133	132	65	83	3.7

Table 5. Standard and ideal values of the GWQPs according to WHO.

P	Parameter name	Standard value	Ideal value	Recommending agencies
β_1	pH	6.5-8.5	7	WHO (2011)
β_2	Electrical Conductivity (mho/cm)	250	0	WHO (2011)
β_3	Fe (mg/l)	1	0.3	WHO (2011)
β_4	Dissolved Oxygen (mg/l)	5.00	14.6	WHO (2011)
β_5	Total Hardness(mg/l)	500	0	WHO (2011)
β_6	Total Alkalinity (mg/l)	200	0	WHO (2011)
β_7	Total Dissolved Solids (mg/l)	500	0	WHO (2011)
β_8	Calcium (mg/l)	75	0	WHO (2011)
β_9	Magnesium (mg/l)	50	0	WHO (2011)
β_{10}	Turbidity (mg/l)	5	0	WHO (2011)

This set contains the ten key GWQPs under evaluation. The data summaries of these parameters, collected from the ten selected sampling stations across different seasons, are provided in Tables 2, 3, and 4. Additionally, the ideal and standard values for each of the GWQPs are outlined in Table 5.

To calculate the weight α_i for each GWQP $p_i \in P$, we use the following formula:

$$\alpha_i = \frac{1}{V_i}$$

where V_i represents the recommended standard value of the i-th GWQP, $p_i \in P$. For example, the standard value for pH (denoted by β_1) is taken to be 8.5, meaning $V_1 = 8.5$.

Using this formula, the weights α_i allocated to each of the GWQPs in P have been determined as follows:

$\varpi = \{\alpha_1 = \alpha(\beta_1) = 0.118; \alpha_2 = \alpha(\beta_2) = 0.004; \alpha_3 = \alpha(\beta_3) = 1; \alpha^4 = \alpha(\beta_4) = 0.2; \alpha_5 = \alpha(\beta_5) = 0.002; \alpha_6 = \alpha(\beta_6) = 0.005; \alpha_7 = \alpha(\beta_7) = 0.002; \alpha_8 = \alpha(\beta_8) = 0.013; \alpha_9 = \alpha(\beta_9) = 0.02; \alpha_{10} = \alpha(\beta_{10}) = 0.2\}$.

The relative-weights assigned to the GWQPs provide a quantitative measure of the importance of each parameter in evaluating groundwater quality. These weights, combined with the crisp data collected from the sample stations, enable a comprehensive assessment of the groundwater quality in Agartala City.

In this study, crisp numbers are converted into triangular, left-open, and right-open fuzzy numbers. Triangular fuzzy numbers are defined by three parameters: the lower limit a, the peak b, and the upper limit c. The membership function of a triangular fuzzy number $T(a,b,c)$ is defined as follows:

$$\mu_T(x) = \begin{cases} \frac{b-x}{b-a}; & when\ a \le x \le b \\ \frac{x-b}{c-b}; & when\ b \le x \le c \\ 1 & otherwise \end{cases}$$

Where:

- a is the lower bound,
- b is the peak or most likely value,
- c is the upper bound.

Right-open fuzzy numbers have a membership function that is 1 for all values less than a certain point and gradually decreases to 0 for values greater than that point. Conversely, **left-open fuzzy numbers** have a membership function that is 1 for all values greater than a certain point and gradually decreases to 0 for values less than that point.

Drawing from the ideal and standard values of the GWQPs listed in Table 5, we develop fuzzy membership functions to represent the different levels of conformity or deviation of each GWQP relative to the recommended thresholds. These fuzzy membership functions provide a method to quantitatively evaluate how closely the recorded values for each parameter align with the ideal or acceptable benchmarks.

1. **pH:** The pH level of water reflects its acidity or alkalinity, essential for aquatic life. The triangular fuzzy number captures uncertainty around ideal pH values, defined by the membership function:

$$\mu_{\beta_1}(x) = \begin{cases} \frac{7.0-x}{7.0-6.5}; & when\ 6.5 \leq x \leq 7.0 \\ \frac{x-7.0}{8.5-7.0}; & when\ 7.0 \leq x \leq 8.5 \\ 1; & otherwise \end{cases}$$

2. **Electrical conductivity:** Electrical conductivity (EC) is a key parameter reflecting the ability of water to conduct electrical current, which correlates with the concentration of dissolved salts. The membership function for EC is defined as:

$$\mu_{\beta_2}(x) = \begin{cases} \frac{x-0}{250-0}; & when\ 0 \leq x \leq 250 \\ 1; & when\ x \geq 250 \end{cases}$$

This indicates that the ideal EC value is up to 250 µS/cm, after which the membership function remains constant at 1. EC values above this threshold suggest a higher concentration of dissolved ions, which may affect water quality.

3. **Fe:** Iron (Fe) concentration in water is another critical parameter, especially in determining water's potability. The membership function for Fe is given by:

$$\mu_{\beta_3}(x) = \begin{cases} \frac{x-0.3}{1-0.3}; & when\ 0.3 \leq x \leq 1 \\ 1; & otherwise \end{cases}$$

This function describes the acceptable Fe concentration range from 0.3 to 1 mg/L. The membership function equals 1 for Fe concentrations within this range, reflecting the safe levels for water consumption. Higher concentrations may indicate potential contamination, affecting human and aquatic health.

4. **Dissolved oxygen:** Dissolved oxygen is critical for the survival of aquatic organisms. The right-open fuzzy number reflects full membership for lower oxygen levels, tapering off as oxygen increases:

$$\mu_{\beta_4}(x) = \begin{cases} \frac{14.6-x}{14.6-5}; & when\ 5 \leq x \leq 14.6 \\ 1; & when\ x \leq 5 \\ 0; & when\ x \geq 14.6 \end{cases}$$

5. **Total hardness:** Total hardness refers to the concentration of calcium and magnesium ions in water. The left-open fuzzy number signifies that higher hardness levels are more acceptable for certain uses:

$$\mu_{\beta_5}(x) = \begin{cases} \frac{x-0}{300-0}; & when\ 0 \leq x \leq 300 \\ 1; & when\ x \geq 300 \end{cases}$$

6. **Total alkalinity:** Total alkalinity represents the water's capacity to neutralize acids, which is crucial for maintaining stable pH levels. The left-open fuzzy number emphasizes the importance of higher alkalinity values:

$$\mu_{\beta_6}(x) = \begin{cases} \frac{x-0}{200-0}; & when\ 0 \leq x \leq 200 \\ 1; & when\ x \geq 200 \end{cases}$$

7. **Total dissolved solid:** Total dissolved solids indicate the overall concentration of dissolved substances in water. The left-open fuzzy number captures the significance of lower TDS levels for water quality:

$$\mu_{\beta_7}(x) = \begin{cases} \frac{x-0}{500-0}; & when\ 0 \leq x \leq 500 \\ 1; & when\ x \geq 500 \end{cases}$$

8. **Calcium:** Calcium levels affect water hardness and aquatic life. The left-open fuzzy number suggests that higher calcium concentrations are beneficial within certain limits:

$$\mu_{\beta_8}(x) = \begin{cases} \frac{x-0}{75-0}; & when\ 0 \le x \le 75 \\ 1; & when\ x \ge 75 \end{cases}$$

9. **Magnesium:** Magnesium (Mg) is an essential nutrient for aquatic life and plays a significant role in various biological processes. The membership function for magnesium concentration is defined as:

$$\mu_{\beta_9}(x) = \begin{cases} \frac{x-0}{50-0}; & when\ 0 \le x \le 50 \\ 1; & when\ x \ge 50 \end{cases}$$

This function indicates that magnesium concentrations up to 50 mg/L are considered ideal, with the membership function remaining at 1 for any values above this threshold. Higher magnesium concentrations can enhance water quality but may also indicate potential sources of pollution, affecting the overall health of aquatic ecosystems.

10. **Turbidity:** Turbidity is a measure of the cloudiness or haziness of water, primarily caused by suspended particles. It is an important indicator of water quality, influencing light penetration and photosynthesis in aquatic environments. The membership function for turbidity is defined as:

$$\mu_{\beta_{10}}(x) = \begin{cases} \frac{x-0}{5-0}; & when\ 0 \le x \le 5 \\ 1; & when\ x \ge 5 \end{cases}$$

This indicates that turbidity levels up to 5 NTU (Nephelometric Turbidity Units) are acceptable, with the membership function remaining at 1 for turbidity values above this threshold. Increased turbidity can negatively affect aquatic life and reduce water clarity, impacting photosynthesis and habitat quality.

After establishing fuzzy membership functions for each of the GWQPs and considering the associated weight ϖ, the crisp data summaries of the GWQPs are transformed into FSSs (ψ, P, ϖ),(φ, P, ϖ),and (σ, P, ϖ). These fuzzy soft sets capture the relationship between the GWQPs and their fuzzy membership values while accounting for the importance (weight) of each parameter. The corresponding ϖ-score $\Omega_{\varpi}(\alpha_i)$ for each sample station $\alpha_i \in S$ is provided in Tables 6, 7, and 8. These tables showcase how the fuzzy soft sets and the ϖ-scores reflect the degree of water quality at each sampling station, with the weights applied to emphasize the influence of certain parameters on the overall assessment.

4. Detailed result analysis of groundwater quality ϖ-scores

In this section, we discuss the detailed result analysis of groundwater quality ϖ-scores across different seasons. The seasonal variations in physicochemical parameters and their impact on the overall water quality indices are evaluated to understand the temporal changes in groundwater quality for drinking purposes.

4.1. Pre-monsoon analysis

During the pre-monsoon period, groundwater quality is generally poor across all sampling stations. The ϖ-scores range from approximately 0.883325 to 0.956557, indicating high levels of contaminants due to low recharge rates and higher evaporation. This period sees the concentration of pollutants as water

Table 6. WFSS (ψ, P, ϖ) representation of GWQPs for the pre-monsoon season including corresponding ϖ-scores.

S	β_1 0.118	β_2 0.004	β_3 1	β_4 0.2	β_5 0.002	β_6 0.005	β_7 0.002	β_8 0.013	β_9 0.02	β_{10} 0.2	ϖ-score
$s\alpha_1$	1	0.46	1	0.958	0.08	0.675	0.36	0.27	0.7	0.72	0.944504
$s\alpha_2$	0.6	0.50	1	0.979	0.09	0.650	0.34	0.33	0.8	0.74	0.921355
$s\alpha_3$	0.1	0.52	1	1	0.1	0.700	0.37	0.40	0.9	0.7	0.883325
$s\alpha_4$	1	0.54	1	0.969	0.11	0.625	0.32	0.47	1	0.76	0.956557
$s\alpha_5$	0.9	0.48	1	0.990	0.12	0.725	0.38	0.53	1	0.66	0.939664
$s\alpha_6$	0.5	0.44	1	0.948	0.13	0.750	0.40	0.60	1	0.68	0.907270
$s\alpha_7$	0.3	0.42	1	0.938	0.14	0.800	0.42	0.67	1	0.64	0.886515
$s\alpha_8$	1	0.52	1	0.917	0.15	0.850	0.44	0.73	1	0.78	0.955499
$s\alpha_9$	1	0.48	1	0.927	0.16	0.775	0.39	0.80	1	0.72	0.949293
$s\alpha_{10}$	1	0.46	1	0.906	0.17	0.700	0.34	0.87	1	0.74	0.949405

Table 7. WFSS (φ, P, ϖ) representation of GWQPs for the monsoon season including corresponding ϖ-scores.

S	β_1 0.118	β_2 0.004	β_3 1	β_4 0.2	β_5 0.002	β_6 0.005	β_7 0.002	β_8 0.013	β_9 0.02	β_{10} 0.2	ϖ-score
$s\alpha_1$	0.8	0.48	1	0.980	0.064	0.595	0.34	0.20	0.6	0.68	0.925002
$s\alpha_2$	0.3	0.44	1	0.969	0.076	0.610	0.36	0.27	0.7	0.74	0.895391
$s\alpha_3$	1	0.50	1	0.938	0.086	0.725	0.38	0.33	0.8	0.78	0.951692
$s\alpha_4$	0.2	0.52	1	0.927	0.096	0.760	0.40	0.40	0.9	0.70	0.881760
$s\alpha_5$	0.9	0.48	1	0.948	0.102	0.775	0.42	0.47	1	0.68	0.936540
$s\alpha_6$	0.5	0.46	1	1	0.112	0.800	0.44	0.53	1	0.72	0.918692
$s\alpha_7$	0.3	0.42	1	0.917	0.124	0.860	0.39	0.60	1	0.62	0.880824
$s\alpha_8$	1	0.52	1	0.927	0.136	0.775	0.36	0.67	1	0.60	0.932901
$s\alpha_9$	0.6	0.50	1	0.938	0.148	0.705	0.32	0.73	1	0.64	0.909432
$s\alpha_{10}$	0.5	0.46	1	0.948	0.160	0.650	0.28	0.80	1	0.70	0.911106

Table 8. WFSS (σ, P, ϖ) representation of GWQPs for the post-monsoon season including corresponding ϖ-scores.

S	β_1 0.118	β_2 0.004	β_3 1	β_4 0.2	β_5 0.002	β_6 0.005	β_7 0.002	β_8 0.013	β_9 0.02	β_{10} 0.2	ϖ-score
$s\alpha_1$	1	0.440	1	0.979	0.068	0.670	0.346	0.28	0.66	0.72	0.946661
$s\alpha_2$	0.5	0.468	1	0.948	0.078	0.725	0.364	0.36	0.82	0.64	0.897737
$s\alpha_3$	0.2	0.452	1	0.906	0.088	0.810	0.382	0.41	0.94	0.68	0.877064
$s\alpha_4$	0.3	0.496	1	0.917	0.098	0.885	0.366	0.47	1	0.66	0.885068
$s\alpha_5$	0.8	0.508	1	0.979	0.106	0.935	0.420	0.63	1	0.64	0.929763
$s\alpha_6$	0.3	0.496	1	0.958	0.114	0.895	0.404	0.60	1	0.70	0.896608
$s\alpha_7$	0.2	0.524	1	0.927	0.124	0.585	0.326	0.67	1	0.76	0.892347
$s\alpha_8$	1	0.480	1	0.948	0.134	0.645	0.256	0.73	1	0.78	0.958449
$s\alpha_9$	0.6	0.500	1	0.938	0.144	0.605	0.278	0.84	1	0.72	0.920198
$s\alpha_{10}$	0.3	0.476	1	0.990	0.154	0.665	0.264	0.87	1	0.74	0.907145

levels drop, which results in poorer water quality. The lack of dilution and higher rates of extraction during this time contribute significantly to the elevated ϖ-scores, signalling that the water quality is at its lowest.

4.2. Monsoon analysis

In the monsoon period, the ϖ-scores show some improvement, with values ranging from about 0.880824 to 0.951692. This slight improvement is attributed to the dilution effect caused by heavy rainfall, which increases water volumes and reduces the concentration of contaminants. However, the improvement is not substantial enough to change the overall classification from poor to good quality. The continuous input of fresh rainwater helps to dilute some of the contaminants, but the presence of surface runoff can still introduce pollutants into the groundwater system.

4.3. Post-monsoon analysis

During the post-monsoon period, ϖ-scores again reflect poor water quality, with values ranging from roughly 0.877064 to 0.958449. The scores are quite similar to those in the pre-monsoon period, indicating that the quality of groundwater does not significantly improve post-monsoon. The slight variations in scores can be due to the settling of contaminants and reduced but ongoing agricultural runoff. The period sees stabilized water levels but still carries the accumulated effects of contaminants from the previous months.

The monsoon period shows a slight improvement in groundwater quality compared to the pre-monsoon period. This improvement is primarily due to the dilution effect of increased rainfall, which helps to lower the concentration of pollutants. However, the introduction of surface runoff during heavy rains can still introduce new contaminants. The post-monsoon period

does not show significant improvement over the monsoon period. The slight decline or stabilization in water quality scores indicates that while the recharge from the monsoon helps maintain water levels, the settling of particles and continuing urbanization and anthropogenic activities keep the quality poor. Throughout the year, groundwater quality remains consistently poor, with only minor improvements during the monsoon period. This suggests that factors such as continuous contamination sources, inadequate dilution during non-rainy periods, and ineffective natural filtration are contributing to the persistently poor water quality.

The analysis of ϖ-scores across different periods reveals that groundwater quality is generally poor regardless of the season. The monsoon period provides a brief improvement due to dilution, but this is not sufficient to ensure good water quality. Effective measures, including continuous monitoring, pollution control, and sustainable water management practices, are essential to address the underlying causes of contamination and improve groundwater quality throughout the year.

5. Ranking of sampling stations according to ϖ-scores in each season

Based on the provided ϖ-scores, here are the rankings of the sampling stations for pre-monsoon, monsoon, and post-monsoon periods (Table 9). Stations are ranked from best to worst groundwater quality, with lower ϖ-scores indicating better quality.

5.1. Summary of Rankings

Pre-Monsoon: Station 7 has the best quality, while Station 4 has the worst.

Monsoon: Station 7 has the best quality, while Station 3 has the worst.

Post-Monsoon: Station 3 again has the best quality, while Station 8 has the worst.

5.2. Overall Analysis

Best stations across seasons: Station 7 generally performs well, ranking highest in both pre-monsoon and post-monsoon periods.

Worst stations across seasons: Stations 4 and 8 consistently rank among the lowest across different seasons, indicating persistent issues with water quality.

This ranking provides a clear view of which sampling stations require more attention and intervention to improve groundwater quality.

6. Comparative Analysis of Water Quality Assessment Models

Groundwater quality assessment is critical for sustainable water resource management and public health. Various water quality assessment models have been developed to evaluate contamination levels, each with its own methodology, sensitivity, and classification criteria. In this study, we compare the Canadian Council of Ministers of the Environment (CCME) Water Quality Index (WQI), Brown's WQI (1970), and the proposed ϖ-score model for groundwater quality evaluation. This comparison provides insights into seasonal variations and highlights differences in quality ratings, emphasizing their significance in environmental planning and pollution control.

6.1. Overview of Water Quality Indices

6.1.1. Brown *et al.* (1970) WQI

- One of the earliest models for water quality assessment.
- Uses a weighted sum of multiple water quality parameters.
- Provides a single composite score to represent overall quality.
- Highly stringent in evaluating pollution, often classifying water as very poor when contamination is even slightly above acceptable limits.
- Useful for assessing the necessity of water treatment measures before consumption.

6.1.2. Canadian Council of Ministers of the Environment (CCME) WQI (2001)

- Developed as a standardized tool for assessing aquatic ecosystem health.
- Uses a fuzzy logic-based mathematical formula to determine water quality.
- Classifies water into five categories: Excellent, Good, Fair, Marginal, and Poor based on scope (number of parameters failing), frequency (frequency of failure), and amplitude (extent of failure).
- More flexible than Brown's WQI and often provides a less strict classification of water quality.

6.1.3. Proposed ϖ-Score Model

- A novel model developed to address seasonal variations and site-specific contamination.
- Uses a fuzzy multi-criteria decision-making approach that integrates expert opinions, uncertainty, and the weighted

importance of different water quality parameters.
- Designed to offer a more adaptive assessment, particularly for highly variable groundwater quality conditions.
- Aligns closely with Brown's WQI in being more conservative in quality assessment.

6.2. Seasonal Variations in Water Quality Assessments

The assessment of water quality across different seasons provides critical insights into how climatic and environmental factors influence groundwater contamination. Tables 9, 10 and 11 present the rankings for water quality across pre-monsoon, monsoon, and post-monsoon periods using the three models.

6.2.1. Pre-Monsoon Season

- ϖ-score model indicates poor groundwater quality with high contamination levels due to low recharge rates, increased evaporation, and pollutant concentration.
- Brown WQI classifies the water as very poor, emphasizing the need for treatment before usage.
- CCME WQI gives slightly better results, classifying the water as marginal to fair, indicating frequent impairments but slightly more optimistic compared to Brown's model.

The spatial distribution of water quality across various sampling points during the pre-monsoon season is visually represented in Figures 3(a), 3(b), and 3(c), corresponding respectively to the $\mathfrak{B}$-model, Brown WQI, and CCME WQI.

6.2.2. Monsoon Season

- All three models show a slight improvement in water quality due to the dilution effect of rainfall.
- ϖ-score model still classifies groundwater quality as poor but acknowledges a minor improvement.
- Brown WQI remains highly conservative, rating the water as very poor, indicating that despite dilution, surface runoff introduces new pollutants.
- CCME WQI provides a more moderate assessment, classifying most locations in the fair category, reflecting its less stringent criteria.

Table 9. Ranking of groundwater sampling stations according to ϖ-scores.

Seasons	Best optimal choice	Ranking according to good quality
Pre-monsoon	$s\alpha_7$	$s\alpha_7 \gg s\alpha_3 \gg s\alpha_6 \gg s\alpha_2 \gg s\alpha_5 \gg s\alpha_1 \gg s\alpha_9 \gg s\alpha_{10} \gg s\alpha_8 \gg s\alpha_4$
Monsoon	$s\alpha_7$	$s\alpha_7 \gg s\alpha_4 \gg s\alpha_2 \gg s\alpha_9 \gg s\alpha_{10} \gg s\alpha_6 \gg s\alpha_1 \gg s\alpha_8 \gg s\alpha_5 \gg s\alpha_3$
Post-monsoon	$s\alpha_3$	$s\alpha_3 \gg s\alpha_4 \gg s\alpha_7 \gg s\alpha_6 \gg s\alpha_2 \gg s\alpha_{10} \gg s\alpha_9 \gg s\alpha_5 \gg s\alpha_1 \gg s\alpha_8$

Table 10. Ranking of groundwater sampling stations according to Brown Water Quality index.

Seasons	Best optimal choice	Ranking according to good quality
Pre-monsoon	$s\alpha_{10}$	$s\alpha_{10} \gg s\alpha_4 \gg s\alpha_6 \gg s\alpha_8 \gg s\alpha_1 \gg s\alpha_7 \gg s\alpha_5 \gg s\alpha_2 \gg s\alpha_9 \gg s\alpha_3$
Monsoon	$s\alpha_6$	$s\alpha_6 \gg s\alpha_8 \gg s\alpha_1 \gg s\alpha_7 \gg s\alpha_5 \gg s\alpha_9 \gg s\alpha_2 \gg s\alpha_{10} \gg s\alpha_4 \gg s\alpha_3$
Post-monsoon	$s\alpha_5$	$s\alpha_5 \gg s\alpha_4 \gg s\alpha_1 \gg s\alpha_{10} \gg s\alpha_3 \gg s\alpha_9 \gg s\alpha_6 \gg s\alpha_2 \gg s\alpha_7 \gg s\alpha_8$

Table 11. Ranking of groundwater sampling stations according to CCME Water Quality index.

Seasons	Best optimal choice	Ranking according to good quality
Pre-monsoon	$s\alpha_{10}$	$s\alpha_{10} \gg s\alpha_4 \gg s\alpha_6 \gg s\alpha_8 \gg s\alpha_1 \gg s\alpha_7 \gg s\alpha_5 \gg s\alpha_2 \gg s\alpha_9 \gg s\alpha_3$
Monsoon	$s\alpha_6$	$s\alpha_6 \gg s\alpha_8 \gg s\alpha_1 \gg s\alpha_7 \gg s\alpha_5 \gg s\alpha_2 \gg s\alpha_9 \gg s\alpha_{10} \gg s\alpha_4 \gg s\alpha_3$
Post-monsoon	$s\alpha_5$	$s\alpha_5 \gg s\alpha_4 \gg s\alpha_1 \gg s\alpha_{10} \gg s\alpha_9 \gg s\alpha_3 \gg s\alpha_6 \gg s\alpha_2 \gg s\alpha_7 \gg s\alpha_8$

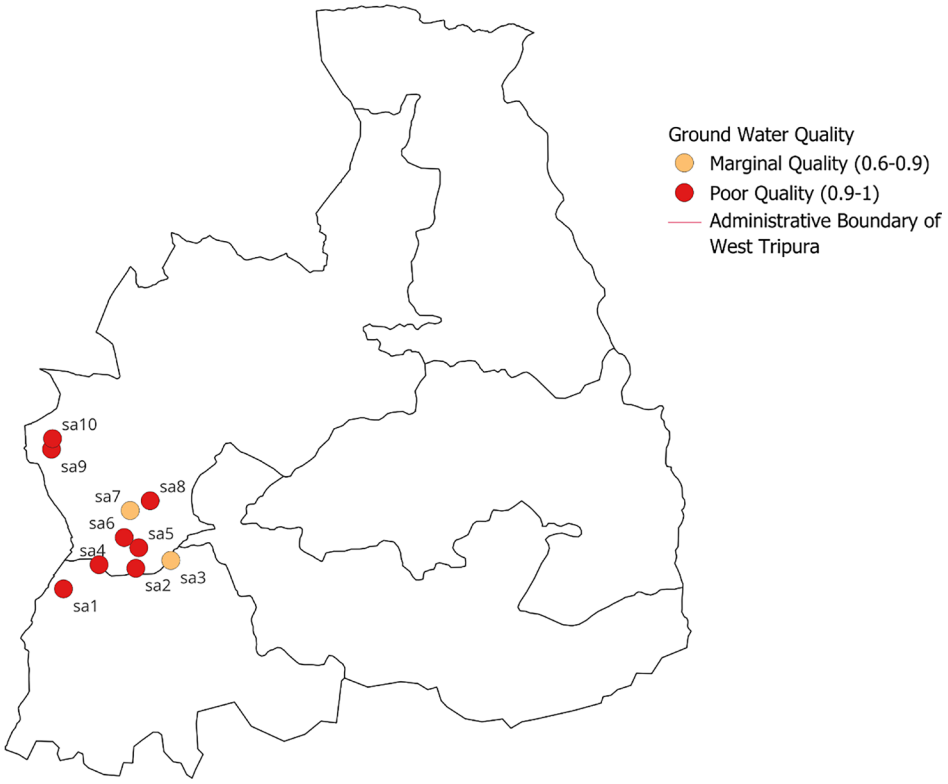

Figure 3(a). Ground water quality in pre-monsoon as per WFSS based 𝔅-model in different water sampling points.

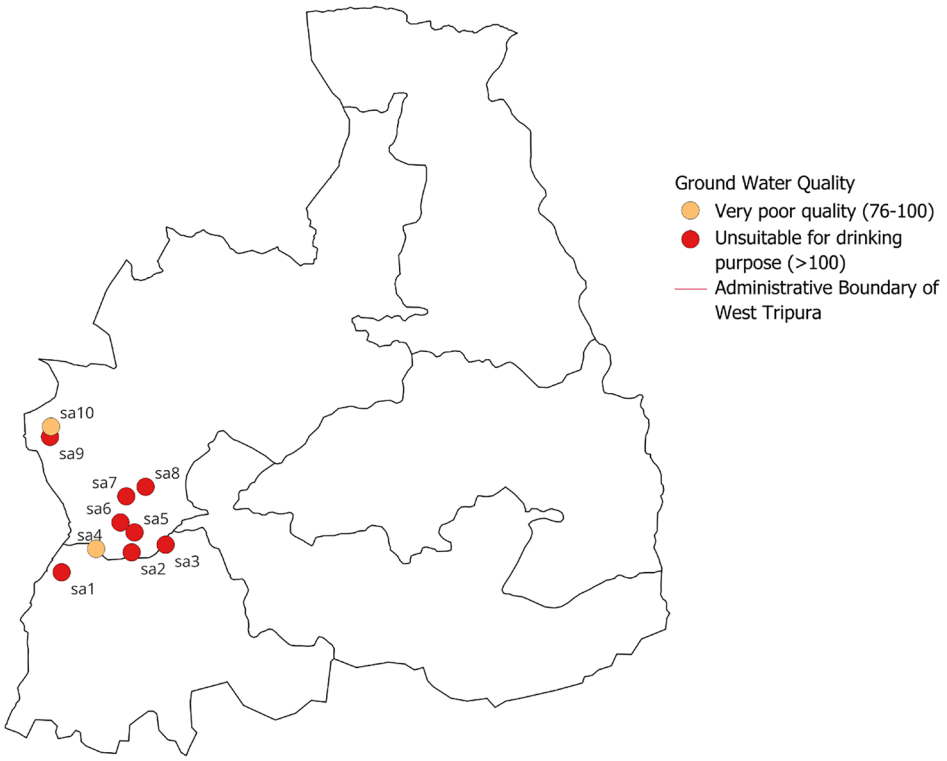

Figure 3(b). Ground water quality in pre-monsoon as per Brown WQI in different water sampling points.

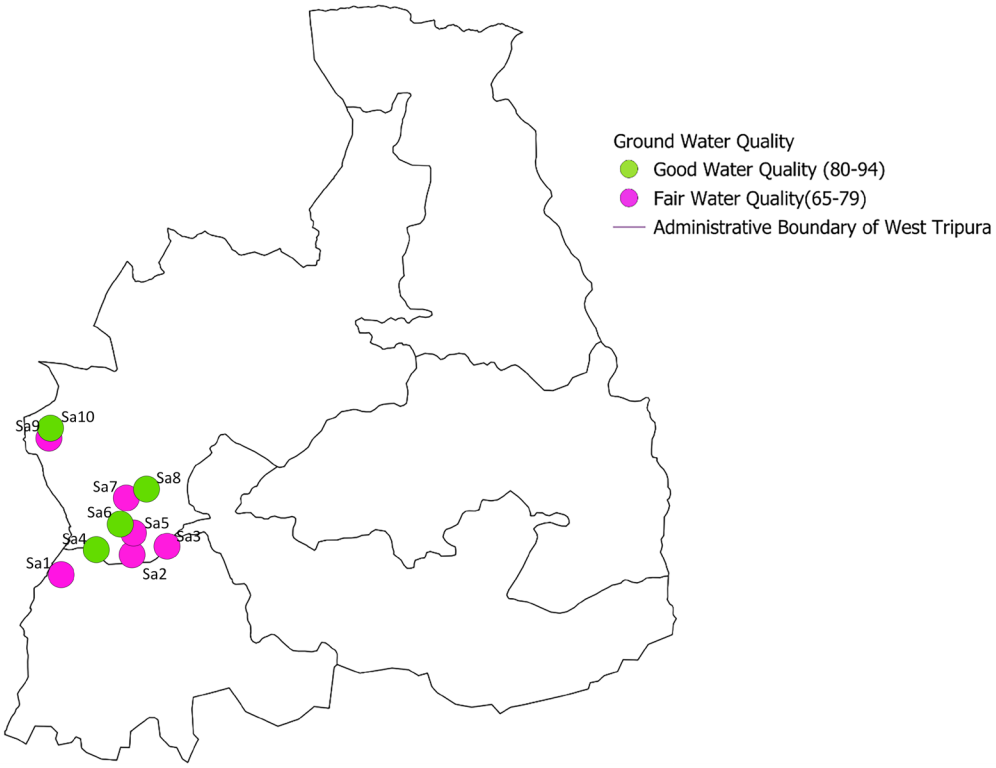

Figure 3(c). Ground water quality in pre-monsoon as per CCME WQI in different water sampling points.

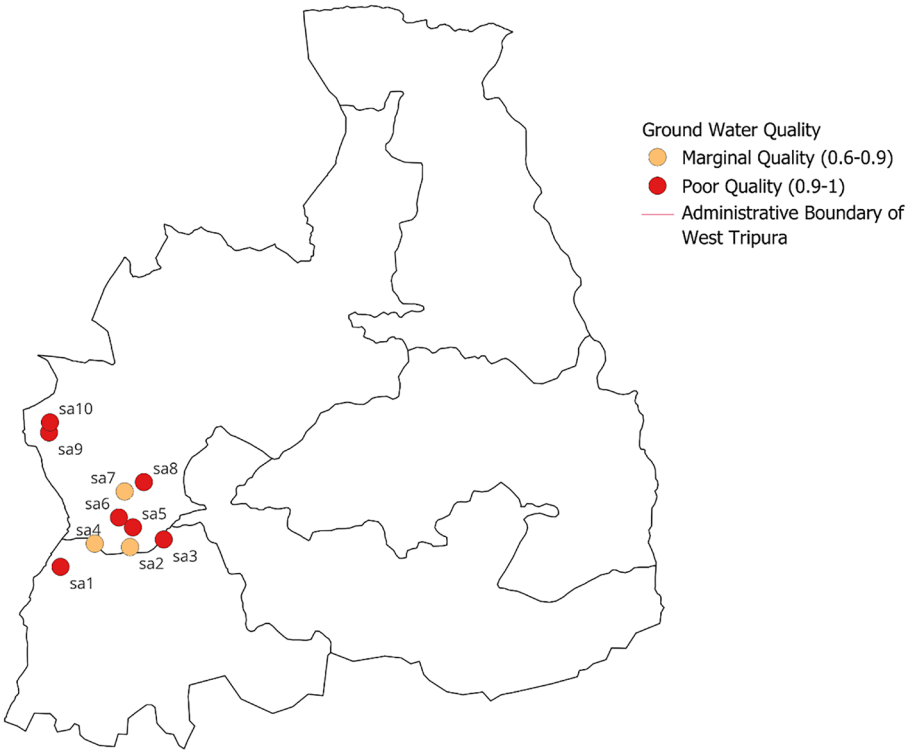

Figure 4(a). Ground water quality in monsoon as per WFSS based 𝔅-model in different water sampling points.

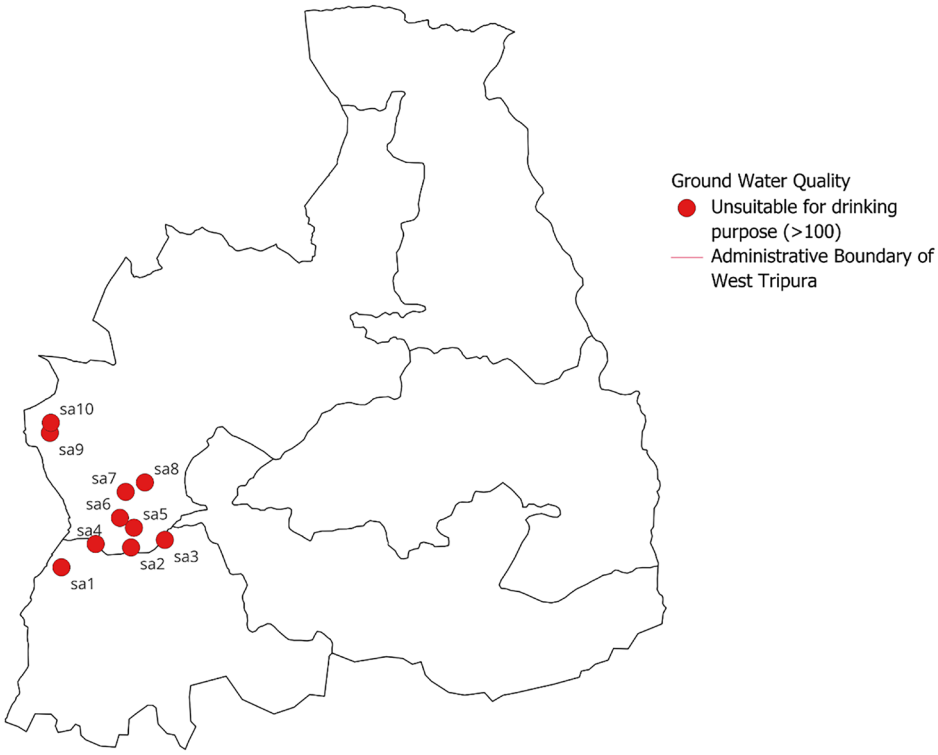

Figure 4(b). Ground Water Quality in Monsoon as per Brown WQI in Different Water Sampling Points.

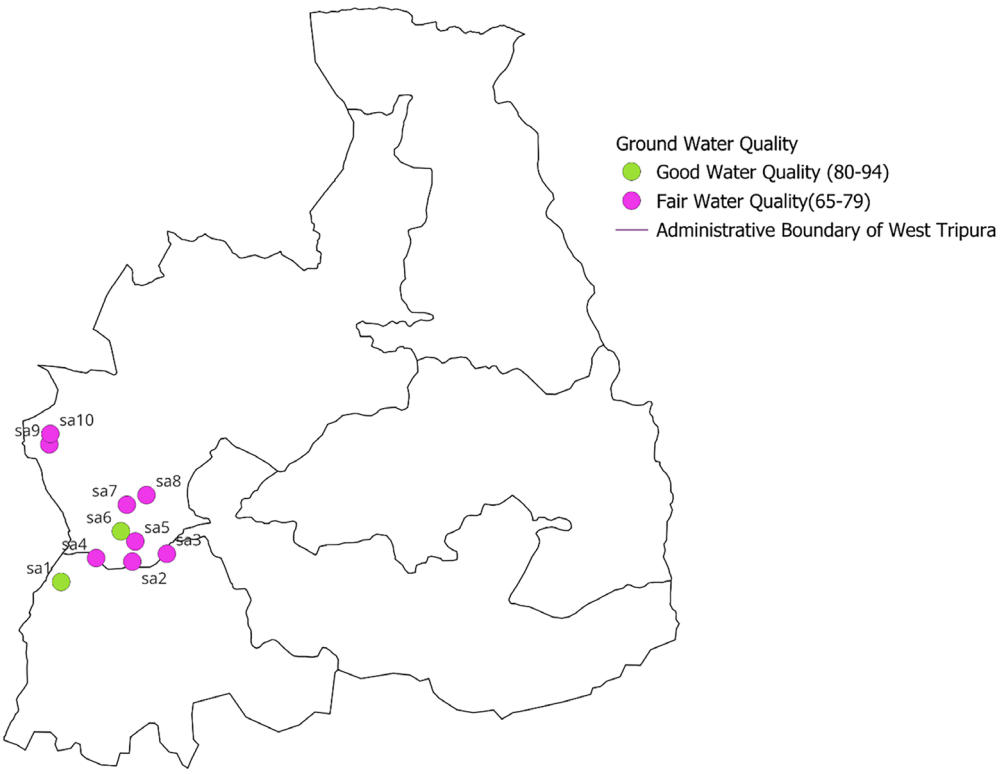

Figure 4(c). Ground Water Quality in Monsoon as per CCME WQI in Different Water Sampling Points.

Figures 4(a), 4(b), and 4(c) show the groundwater quality at different locations during the monsoon season, as measured by the WFSS-based $\mathfrak{B}$-model in 4(a), Brown WQI in 4(b), and CCME WQI in 4(c), clearly comparing how each method rates the water quality.

6.2.3. Post-Monsoon Season

- ϖ-score model and Brown WQI indicate a return to poor quality, similar to pre-monsoon levels.
- CCME WQI gives better scores, rating water as fair to good at some locations.
- The minor improvement noted across models is likely due to settling of suspended solids and reduced urban runoff. However, agricultural runoff and anthropogenic activities continue to degrade quality, leading to persistent contamination.

Figures 5(a), 5(b), and 5(c) show groundwater quality at different locations during the post-monsoon season, as evaluated by the WFSS-based $\mathfrak{B}$-model in 5(a), Brown WQI in 5(b), and CCME WQI in 5(c), highlighting how each method reflects changes in water quality after the monsoon.

6.3. Comparative Analysis of Model Sensitivity

1. Brown's WQI is the most stringent and sensitive model, categorizing water quality as very poor in most cases. This suggests that even minor pollution levels significantly impact the final rating, making it highly useful for identifying critical pollution hotspots and emphasizing preventative water treatment measures.
2. CCME WQI provides a more balanced evaluation by considering scope, frequency, and amplitude of contamination. It tends to classify water less severely than Brown's model, indicating that it tolerates minor exceedances in pollutant levels. This model is useful for regulatory compliance and general water resource management.
3. The ϖ-score model aligns more with Brown's stringent classification, reflecting significant contamination concerns across all seasons. Unlike Brown's WQI, however, it incorporates expert assessments and a fuzzy decision-making approach, making it more adaptive to site-specific conditions. This model is beneficial for targeted groundwater management interventions in urban areas.

6.4. Implications for Groundwater Management

- Brown's WQI suggests that aggressive treatment measures are required before groundwater use, reinforcing the need for immediate pollution control interventions.
- CCME WQI, while more moderate, still indicates that groundwater is frequently impaired, requiring regular monitoring and long-term pollution management strategies.
- The ϖ-score model highlights the importance of seasonal variations, indicating that groundwater contamination is persistent, requiring continuous monitoring rather than seasonal interventions.

The comparative analysis of these models highlights their strengths and limitations in evaluating groundwater quality. Brown's WQI is highly stringent, making it useful for identifying pollution hotspots but sometimes overly restrictive. CCME WQI provides a more flexible assessment, making it useful for broader environmental monitoring. The ϖ-score model aligns more closely with Brown's WQI but integrates fuzzy decision-making techniques, making it a more adaptive and insightful tool for site-specific water management.

The choice of an appropriate water quality index should depend on the specific water management objectives. If the goal is a highly conservative pollution assessment, Brown's WQI is suitable. If a general regulatory framework is needed, CCME WQI is appropriate. However, for detailed groundwater quality assessment in urban environments with complex pollution sources, the ϖ-score model offers the best balance between strict evaluation and adaptive decision-making. The key comparative findings of Brown WQI, CCME WQI, and the ϖ-score model across different seasons are summarized in Table 12.

Table 12. Summary of Comparative Findings.

Model	Pre-Monsoon Rating	Monsoon Rating	Post-Monsoon Rating	Sensitivity to Pollution	Best Use Case
Brown WQI (1970)	Very Poor	Very Poor	Very Poor	Highly sensitive (strictest)	Critical pollution hotspot identification
CCME WQI (2001)	Marginal - Fair	Fair	Fair - Good	Moderately sensitive (more forgiving)	General water management and policy
ϖ-Score Model	Poor	Poor	Poor	Adaptive & conservative	Site-specific groundwater quality monitoring

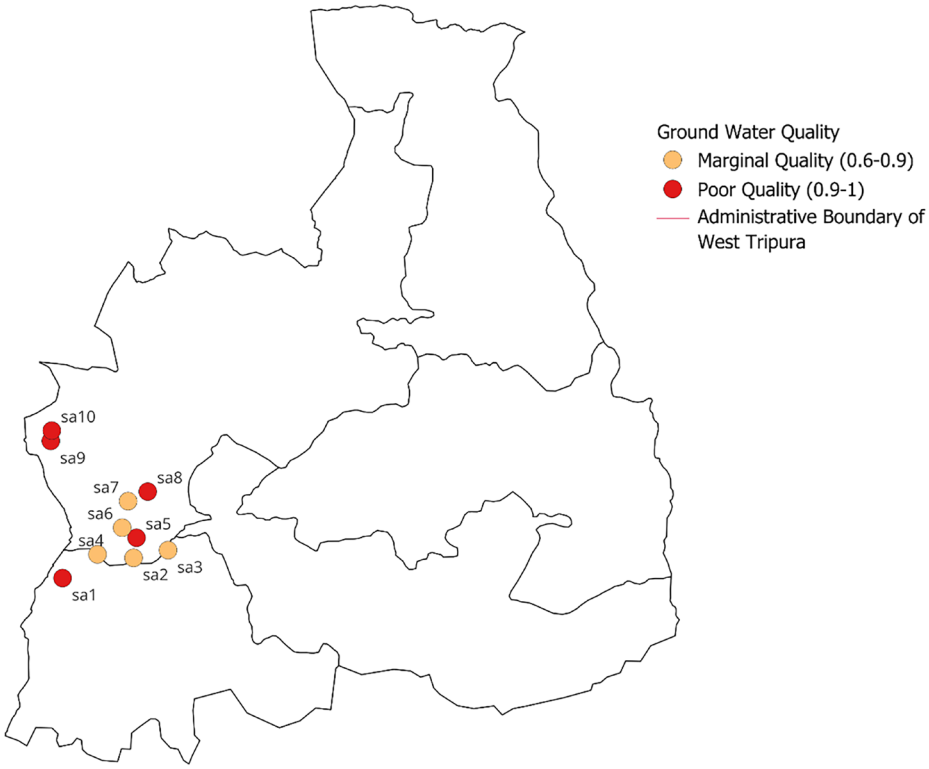

Figure 5(a). Ground water quality in post-monsoon as per WFSS based $\mathfrak{B}$-model in different water sampling points.

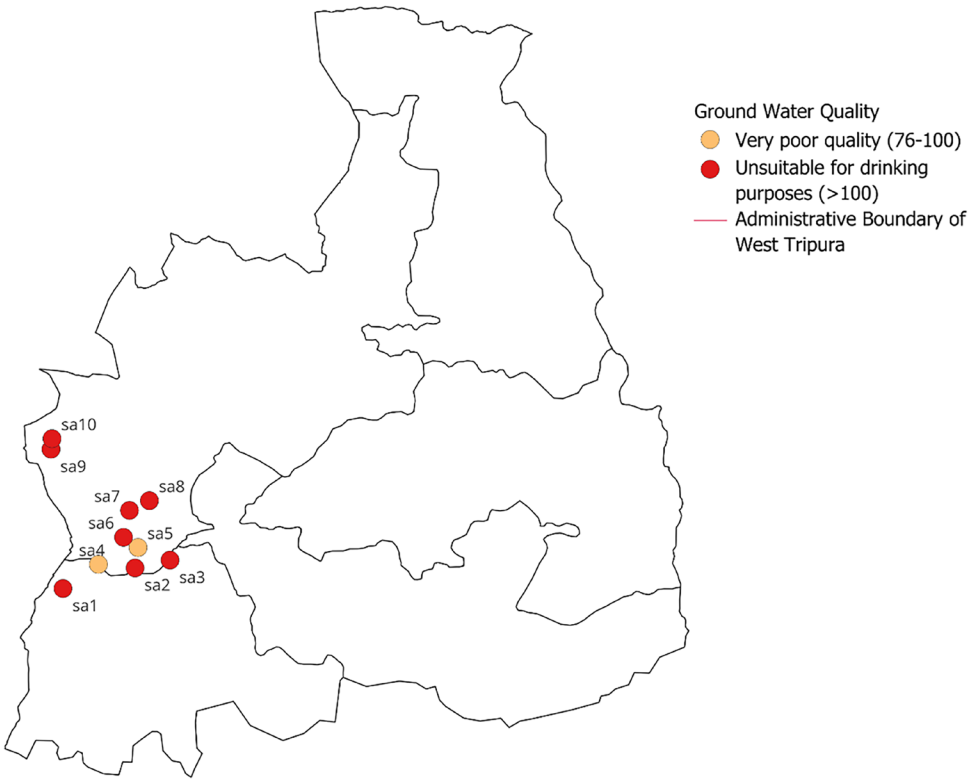

Figure 5(b). Ground Water Quality in Post- Monsoon as per Brown WQI in Different Water Sampling Points.



Figure 5(c). Ground Water Quality in Post- Monsoon as per CCME WQI in Different Water Sampling Points.

7. Conclusions

This study introduces an innovative weighted fuzzy soft set-based approach, known as the $\mathfrak{B}$-model, for groundwater pollution source identification and risk assessment in urban areas. This pioneering methodology provides a refined and adaptable framework for analyzing water contamination, particularly in rapidly urbanizing environments like Agartala. The novelty of this research lies in its integration of weighted fuzzy soft set theory with a multi-criteria decision-making algorithm to calculate pollution scores. This integration facilitates a nuanced evaluation of groundwater quality across different seasons while effectively capturing the inherent uncertainties in environmental data. Such an approach is crucial for understanding the multifaceted impacts of urbanization and industrialization on groundwater resources.

The findings of this study highlight the ongoing challenges regarding groundwater quality in Agartala, with the most severe contamination levels observed during the pre-monsoon season. During this time, reduced water recharge coupled with increased evaporation exacerbates pollutant concentrations. Although the monsoon season provides some relief through dilution from rainfall, the overall improvements in water quality are insufficient to meet acceptable standards. The application of the weighted fuzzy soft set-based MCDM model in groundwater quality assessment, as demonstrated here, reinforces the growing consensus on the necessity of advanced analytical tools for effective environmental management and decision-making.

Future research should aim to expand the capabilities of the $\mathfrak{B}$-model by incorporating real-time data from environmental sensors, thereby enhancing its predictive power and allowing for dynamic assessments of groundwater quality. Additionally, exploring advanced machine learning algorithms, such as deep learning techniques, could further improve the accuracy of pollution predictions generated by the $\mathfrak{B}$-model. It is equally important to investigate the socio-economic implications of groundwater quality degradation, considering its potential impacts on public health and local economies. Interdisciplinary research that integrates hydrology, data science, and urban planning will provide a holistic approach to addressing the environmental challenges associated with rapid urbanization.

The results and methodologies developed in this study are applicable to similar environmental challenges faced by other rapidly urbanizing cities worldwide. The $\mathfrak{B}$-model is versatile and can be adapted to tackle a wide array of environmental issues, including air pollution, soil contamination, and surface water quality assessments. Its capacity to manage uncertainty

and synthesize diverse datasets positions it as a valuable tool for policymakers and urban planners charged with managing natural resources in complex and dynamic environments. By offering a systematic and data-driven approach to environmental monitoring, the 𝔅-model can inform the development of targeted interventions, ensuring that the most vulnerable areas receive the attention and resources necessary for sustainable groundwater management and pollution mitigation.

Funding: No financial support was received.

Data availability: All data supporting the findings of this manuscript are included within the article.

Ethical approval: Any of the authors did not conduct any human or animal experiments for this publication.

Conflict of Interest: The authors declare that they have no conflict of interest regarding the publication of this manuscript.

8. References

Adimalla, N. (2021). Application of the entropy weighted water quality index (EWQI) and the pollution index of groundwater (PIG) to assess groundwater quality for drinking purposes: A case study in a rural area of Telangana State, India. *Archives of Environmental Contamination and Toxicology,* 80, 31–40. doi: **https://doi.org/10.1007/s00244-020-00800-4**

Ahmed, M., Mumtaz, R., & Zaidi, S. M. H. (2021). Analysis of water quality indices and machine learning techniques for rating water pollution: A case study of Rawal Dam, Pakistan. *Water Supply,* 21(6), 3225–3250. doi: **https://doi.org/10.2166/ws.2021.082**

Alcantud, J. C. R., & Santos-García, G. (2017). A new criterion for soft set based decision-making problems under incomplete information. *International Journal of Computational Intelligence Systems,* 10(1), 394–404. doi: **https://doi.org/10.2991/ijcis.2017.10.1.27**

Ali, M. I., Feng, F., Liu, X. Y., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers & Mathematics with Applications,* 57(9), 1547–1553. doi: **https://doi.org/10.1016/j.camwa.2008.11.009**

American Public Health Association. (2005). *Standard methods for the examination of water and wastewater* (21st ed.). American Public Health Association.

Bhardwaj, N., & Sharma, P. (2021). An advanced uncertainty measure using fuzzy soft sets: Application to decision-making problems. *Big Data Mining and Analytics,* 4(2), 94–103. doi: **https://doi.org/10.26599/BDMA.2020.9020020**

Brown, R. M., McClelland, N. I., Deininger, R. A., & Tozer, R. G. (1970). A water quality index-do we dare? *Water & Sewage Works,* 117, 339-343.

Canadian Council of Ministers of the Environment. (2001). *Canadian water quality guidelines for the protection of aquatic life: CCME Water Quality Index 1.0, technical report*. Canadian Council of Ministers of the Environment.

Chen, Y., Liu, J., Chen, Z., & Zhang, Y. (2020). Group decision-making method based on generalized vague N-soft sets. *2020 Chinese Control and Decision Conference (CCDC)*, 4010–4015. IEEE. doi: **https://doi.org/10.1109/CCDC49329.2020.9164602**

Darvish Falehi, A., & Torkaman, H. (2023). Optimal fractional order interval type-2 fuzzy controller for upside-down asymmetric multilevel inverter based dynamic voltage restorer to accurately compensate faulty network voltage. *Journal of Ambient Intelligence and Humanized Computing,* 14, 16683–16701. doi: **https://doi.org/10.1007/s12652-023-04673-y**

Das, A. K., & Granados, C. (2021). An advanced approach to fuzzy soft group decision-making using weighted average ratings. *SN Computer Science,* 2, 471. doi: **https://doi.org/10.1007/s42979-021-00873-5**

Das, A. K., & Granados, C. (2022). A new fuzzy parameterized intuitionistic fuzzy soft multiset theory and group decision-making. *Journal of Current Science and Technology,* 12(3), 547-567. **https://ph04.tci-thaijo.org/index.php/JCST/article/view/295**

Das, A. K., & Granados, C. (2023). IFP-intuitionistic multi fuzzy N-soft set and its induced IFP-hesitant N-soft set in decision-making. *Journal of Ambient Intelligence and Humanized Computing,* 14, 10143–10152. doi: **https://doi.org/10.1007/s12652-021-03677-w**

Das, A. K., Granados, C., & Bhattacharya, J. (2022). Some new operations on fuzzy soft sets and their applications in decision-making. *Songklanakarin Journal of Science and Technology, 44*(2), 440-449. doi: **10.14456/sjst-psu.2022.61**

Das, A. K., Gupta, N., & Granados, C. (2023). Weighted hesitant bipolar-valued fuzzy soft set in decision-making. *Songklanakarin Journal of Science and Technology, 45*(6), 681-690. **https://sjst.psu.ac.th/article/3085**

Das, A. K., Gupta, N., Mahmood, T., Tripathy, B. C., Das, R., & Das, S. (2024a). Assessing anthropogenic influences on the water quality of Gomati River using an innovative weighted fuzzy soft set based water pollution rating system. *Discover Water,* 4, 73. doi: **https://doi.org/10.1007/s43832-024-00136-3**

Das, A. K., Gupta, N., Mahmood, T., Tripathy, B. C., Das, R., & Das, S. (2025a). An efficient water quality evaluation model using weighted hesitant fuzzy soft sets for water pollution rating. In A. Kumar, P. Kumar, S. Rathee, & B. Kumar (Eds.), *Mechatronics: Concepts, tools, applications, and new trends* (1st ed., p. 17). CRC Press. doi: **https://doi.org/10.1201/9781003494478-10**

Das, A. K., Gupta, N., Mahmood, T., Tripathy, B. C., Das, R., & Das, S. (2025b). An innovative fuzzy multi-criteria decision making model for analyzing anthropogenic influences on urban river water quality. *Iran Journal of Computer Science,* 8, 103–124. doi: **https://doi.org/10.1007/s42044-024-00211-x**

Feng, F., Jun, Y. B., Liu, X., & Li, L. (2010). An adjustable approach to fuzzy soft set based decision making. *Journal of Computational and Applied Mathematics,* 234(1), 10-20. doi: **https://doi.org/10.1016/j.**

cam.2009.11.055

Granados, C., Das, A. K., & Osu, B. O. (2023). Weighted neutrosophic soft multiset and its application to decision making. *Yugoslav Journal of Operations Research,* 33(2), 293-308. doi: https://doi.org/10.2298/YJOR220915034G

Hutchins, M. G., Abesser, C., Prudhomme, C., Elliott, J. A., Bloomfield, J. P., Mansour, M. M., & Hitt, O. E. (2018). Combined impacts of future land-use and climate stressors on water resources and quality in groundwater and surface waterbodies of the upper Thames river basin, UK. *Science of the Total Environment,* 631-632, 962–986. doi: https://doi.org/10.1016/j.scitotenv.2018.03.052

Kumar, A., & Krishna, A. P. (2021). Groundwater quality assessment using geospatial technique based water quality index (WQI) approach in a coal mining region of India. *Arabian Journal of Geosciences,* 14, 1126. doi: https://doi.org/10.1007/s12517-021-07474-9

Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets. *Journal of Fuzzy Mathematics, 9*(3), 589–602.

Maji, P. K., Biswas, R., & Roy, A. R. (2002). An application of soft sets in a decision-making problem. *Computers & Mathematics with Applications,* 44(8–9), 1077–1083. doi: https://doi.org/10.1016/S0898-1221(02)00216-X

Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & Mathematics with Applications,* 45(4–5), 555–562. doi: https://doi.org/10.1016/S0898-1221(03)00016-6

Mallick, J., Kumar, A., Almesfer, M. K., Alsubih, M., Singh, C. K., Ahmed, M., & Khan, R. A. (2021). An index-based approach to assess groundwater quality for drinking and irrigation in Asir region of Saudi Arabia. *Arabian Journal of Geosciences,* 14, 157. doi: https://doi.org/10.1007/s12517-021-06506-8

Miller, J. D., & Hutchins, M. (2017). The impacts of urbanization and climate change on urban flooding and urban water quality: A review of the evidence concerning the United Kingdom. *Journal of Hydrology: Regional Studies,* 12, 345–362. doi: https://doi.org/10.1016/j.ejrh.2017.06.006

Milovanovic, M. (2007). Water quality assessment and determination of pollution sources along the Axios/Vardar River, Southeastern Europe. *Desalination,* 213(1-3), 159–173. doi: https://doi.org/10.1016/j.desal.2006.06.022

Mittal, S., Sahoo, P. K., Sahoo, S. K., Kumar, R., & Tiwari, R. P. (2021). Hydrochemical characteristics and human health risk assessment of groundwater in the Shivalik region of Sutlej basin, Punjab, India. *Arabian Journal of Geosciences,* 14, 847. doi: https://doi.org/10.1007/s12517-021-07043-0

Molodtsov, D. (1999). Soft set theory-first results. *Computers & Mathematics with Applications,* 37(4–5), 19–31. doi: https://doi.org/10.1016/S0898-1221(99)00056-5

Patel, A., & Chitnis, K. (2022). Application of fuzzy logic in river water quality modeling for analysis of industrialization and climate change impact on Sabarmati River. *Water Supply,* 22(1), 238–250. doi: https://doi.org/10.2166/ws.2021.275

Patel, P., Bablani, S. & Patel, A. (2020) *Impact assessment of urbanization and industrialization using water quality index on Sabaramati River, Ahmedabad*. Proceedings of the 4th International Conference: Innovative Advancement in Engineering & Technology (IAET). doi: http://dx.doi.org/10.2139/ssrn.3553800

Ram, A., Tiwari, S. K., Pandey, H. K., Chaurasia, A. K., Singh, S., & Singh, Y. V. (2021). Groundwater quality assessment using water quality index (WQI) under GIS framework. *Applied Water Science,* 11, 46. doi: https://doi.org/10.1007/s13201-021-01376-7

Rango, T., Kravchenko, J., Atlaw, B., McCornick, P. G., Jeuland, M., Merola, B., & Vengosh, A. (2012). Groundwater quality and its health impact: An assessment of dental fluorosis in rural inhabitants of the Main Ethiopian Rift. *Environmental International,* 43, 37–47. doi: https://doi.org/10.1016/j.envint.2012.03.002

Roy, A. R., & Maji, P. K. (2007). A fuzzy soft set theoretic approach to decision making problems. *Journal of Computational & Applied Mathematics,* 203(2), 412-418. doi: https://doi.org/10.1016/j.cam.2006.04.008

Shah, K. A., & Joshi, G. S. (2017). Evaluation of water quality index for River Sabarmati, Gujarat, India. *Applied Water Science,* 7, 1349-1358. doi: https://doi.org/10.1007/s13201-015-0318-7

Uddin, M. G., Moniruzzaman, M., & Khan, M. (2017). Evaluating ground water quality using CCME water quality index in the Rooppur Nuclear Power Plant area, Ishwardi, Pabna, Bangladesh. *American Journal of Environmental Protection,* 5(2), 33-43. doi: https://doi.org/10.12691/env-5-2-2

Wagh, V. M., Panaskar, D. B., Muley, A. A., & Mukate, S. V. (2017). Groundwater suitability evaluation by CCME WQI model for Kadava River Basin, Nashik, Maharashtra, India. *Modeling Earth Systems and Environment, 3*, 557–565. doi: https://doi.org/10.1007/s40808-017-0316-x

World Health Organization. (2011). *Guidelines for drinking-water quality* (4th ed.). World Health Organization. https://www.who.int/publications/i/item/9789241549950

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control,* 8(3), 338–353. doi: https://doi.org/10.1016/S0019-9958(65)90241-X