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The 1998-1999 eruption of Volcán de Colima, Mexico: an application of Maeda's viscoelastic model

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Resumen

Se presenta una versión modificada del modelo viscoelástico de Maeda y su aplicación a la erupción de 1998-1999 del volcán de Colima. El modelo se ajusta razonablemente bien a los datos observados de volumen emitido suponiendo una cámara magmática con un volumen de 30 km^3 y un radio aproximado de 1.93 km centrado ~ 1.7 km bajo el nivel del mar (~ 5.6 km bajo el cráter). Estas características están de acuerdo con los datos gravimétricos del área y explican en términos generales el proceso de emisión de masa que tuvo lugar en el volcán de Colima durante el periodo estudiado, mismo que consistió en la emisión lenta de lava por 2 meses. En este modelo tal conducta se atribuye a la reología viscoelástica de la roca encajonante, la dimensión del conducto volcánico y la entrada de material a la cámara magmática.

Palabras clave: Volcán de Colima, modelo viscoelástico, modelo de Maeda, tasa de erupción, volcanes de México.

Abstract

A modified version of Maeda's viscoelastic model of mass ejection is applied to the 1998-1999 eruptive period of Volcán de Colima, Mexico. The model fits reasonably well the observed volume and volume rate of the eruption, assuming a magma chamber with a volume 30 km^3 and radius of 1.93 km centered at about 1.7 km below sea level (~ 5.6 km below the summit crater). These figures are roughly in agreement with gravimetric data. The process of mass emission at Colima Volcano during the studied period, consisted of slow emission of lava for 2 months. This behavior is attributed to the viscoelastic rheology of the medium around the volcanic conduit and the input to the magma chamber.

Key words: Volcán de Colima, viscoelastic model, Maeda's model, eruption rate, mexican volcanoes.

Introduction

Volcán de Colima (19°30'45"N, 103°37'W; 3860 masl) is located at the boundary of Jalisco and Colima states. It is the Mexican volcano with the highest historical activity in terms of frequency of its eruptive events and proximity to major cities and towns in Colima and Jalisco. More than 60 eruptions have been reported since 1560, including major events in 1585, 1690, 1818, 1869, 1890, 1903, and 1913 (Medina-Martínez, 1983; De la Cruz-Reyna, 1993, Bretón-González *et al*., 2002). Moderate events occur more frequently and may feature emission of block-lava flows, dome growth, explosions, ashfalls, and generation of block-and-ash flows. The most recent events occurred in 1975-76, 1981-82, 1987, 1991, 1994, 1998-2000 and 2005 (De la Cruz-Reyna, 1993; Navarro-Ochoa *et al*., 2002; Zobin *et al*., 2002a,b). Quantitative information exists for the past decades.

Models of varying complexity have been devised to compute the depth and dimensions of the magma chamber and the rheology of the magma and of the country rock from the mass eruption rate. Maeda (2000) explained the behavior of the 1991-1995 eruption of Mt. Unzen, Japan, from the elastic behavior of the magma chamber and the viscolestic response of the conduit system. The 1998 eruptive episode of Volcán de Colima exhibited a behavior reminiscent of the 1991-1995 Mt. Unzen eruption, but its duration lasted 5 years at Unzen and 2 months at Colima. This difference, however, might be attributed to the amount of magma fed from below.

Estimates of eruption rates for the eruptive period 1998-1999 of Colima Volcano were reported by Navarro-Ochoa *et al*. (2002). An estimate of the location/dimensions of the magma chamber was published by Medina *et al*. (1996) from gravity data.

Eruptive activity of Volcán de Colima

Colima has sustained at least 52 documented eruptions, 29 of them explosive (Luhr and Carmichael, 1980; Medina-Martínez, 1983; De la Cruz-Reyna, 1993; Bretón-González *et al*., 2002). According to Luhr and Carmichael (1980, 1990) Colima volcanic activity has evolved by four eruptive cycles. Each cycle begins with dome formation and ends with a major explosive eruption. The first cycle began in 1576 and ended with the eruption of 1611, the second started in 1749 ending in 1818. The third cycle was characterized by several eruption styles and ended with the 1913 eruption, which produced a Plinian column 23 km high and ashfall as far as 720 km from the volcano (Luhr and Carmichael, 1990; Saucedo *et al*., 2010). The fourth cycle of activity began with the 1961-1962 lava emissions pouring from the crater and continued with lava eruptions in 1975-1976, 1981-1982, 1991 and most recently in 1998-99, 2002-03 and 2004. The andesitic pumice from the cycle-ending eruptions are significantly more basic (57.9-59.2 wt.% SiO_2) than the preceding block-lava flows. The analyses of the andesitic lavas and crater domes extruted since the end of the 1975-76 eruption shows that there is a transition to more basic andesitic compositions (to 58.9 wt.% SiO_2). However this pattern is not simple, due to the presence of compositional sub-cycles, characterized by reversals to more-evolved andesitic magmas in 1975-76 and 1981-82 (Luhr and Carmichael, 1990). Probably several concurrent mechanisms are operating including crystal fractionation, since simple models closely reproduce major element variations in the suites. Intrusion of pulses of relatively basic magma feeding from below the subvolcanic magma system, explain the concentrations of compatible trace elements Cr, Ni, and Zn (Luhr and Carmichael, 1980).

The 1998-2000 period of activity peaked in late 1998 and early 1999. Magma outflow was noticed on 20 November 1998 when the appearance of a dome was observed. The activity had begun much earlier since geochemical and geophysical changes were observed more than a year before. These changes included increases in the S/Cl ratio and δD values at the summit fumaroles by mid-1997; earthquake swarms in the period November–December 1997 and in October–November 1998, inflation of the volcano beginning in November 1997 and continuing until the start of the eruption; increased SO_2 emissions and finally small ash emissions detected by satellite on 22 November 1997 (Zobin *et al*., 2002a).

At 7:30 hrs LT on 21 November, 1998 the dome, with a volume estimated at 3.8 x 10^5 m^3 filled the crater, overtopped its SSW rim at 11:30 hrs and produced block-and-ash flows at intervals of several minutes. The flows ran trough the eastern branch of El Cordobán gully (Navarro-Ochoa *et al*., 2002).

The continuing lava overspill formed a growing coulee that on 22 November reached a volume of 4.6×10^5 m^3. On 27 November its estimated volume was of 4.3x10^6 m^3 and 7.2×10^6 m^3 on 30 November. On 2 December there were three flows from the main Cordoban gully; these had reached lengths of approximately 1170, 1450 and 1090 m. On 18 January 1999, these flows had reached 3400, 3700 and 2300 m. By 8 February the flows, which had almost completely come to a standstill reached lengths about 3500, 3800 and 2800 m respectively. Only the eastern flow continued its advance at a slow rate of 20 m per day (Navarro-Ochoa *et al*., 2002). From early February onwards, volcanic activity shifted from mainly effusive to intermittent and explosive; the 1998 dome as well as parts of older remnant domes were destroyed.

Navarro-Ochoa *et al*. (2002) estimated the daily eruption rate as shown in Fig. 1a. From this plot the cumulative volume of lava erupted can be estimated as appears in Fig. 1b, where the cumulative volume grows almost monotonically in the last part of the eruption. The total erupted volume was estimated by these authors at about 4x$10^7$$m^3$.

Mora *et al*. (2002) based on petrological data found that prior to the eruption the magma was at a temperature of ~900 °C, had an oxygen fugacity of $10^{-11.1}$, and water contents in the rhyolitic melt of ~2 wt%. They suggest that magma probably originated from mixing between two andesitic magmas with different silica content, degree of evolution, and crystal content. One of the magmas was more acidic, had temperatures less than 900 °C, 3 wt% water content in the melt, and stagnated at depth. The other was an andesitic magma at higher temperature, more mafic and intruded into the former magma producing an overpressure in the magmatic system. According to Luhr (2002), Volcán de Colima is located in the four stage of a cycle that began in 1913 and that these eruptive cycles reflect passage of discrete, compositionally zoned magma bodies (upwardly enriched in SiO_2) through the volcanic system. The 1981-82 lava flow was more mafic that the lava erupted in 1976 (SiO_2 ~ 58.8 %). However, andesitic lavas became in a progressive manner richer in SiO_2 during the 1991 and the 1998-99 eruptions (Zobin *et al*., 2002a).

On 10 February 1999 a large explosion produced a crater in the 1998–1999 lava dome and marked the beginning of a new explosive stage of activity. Further large explosions occurred on 10 May and 17 July 1999. Sporadic minor explosive activity continued through the year 2000, and a large explosion occurred on 22 February, 2001.

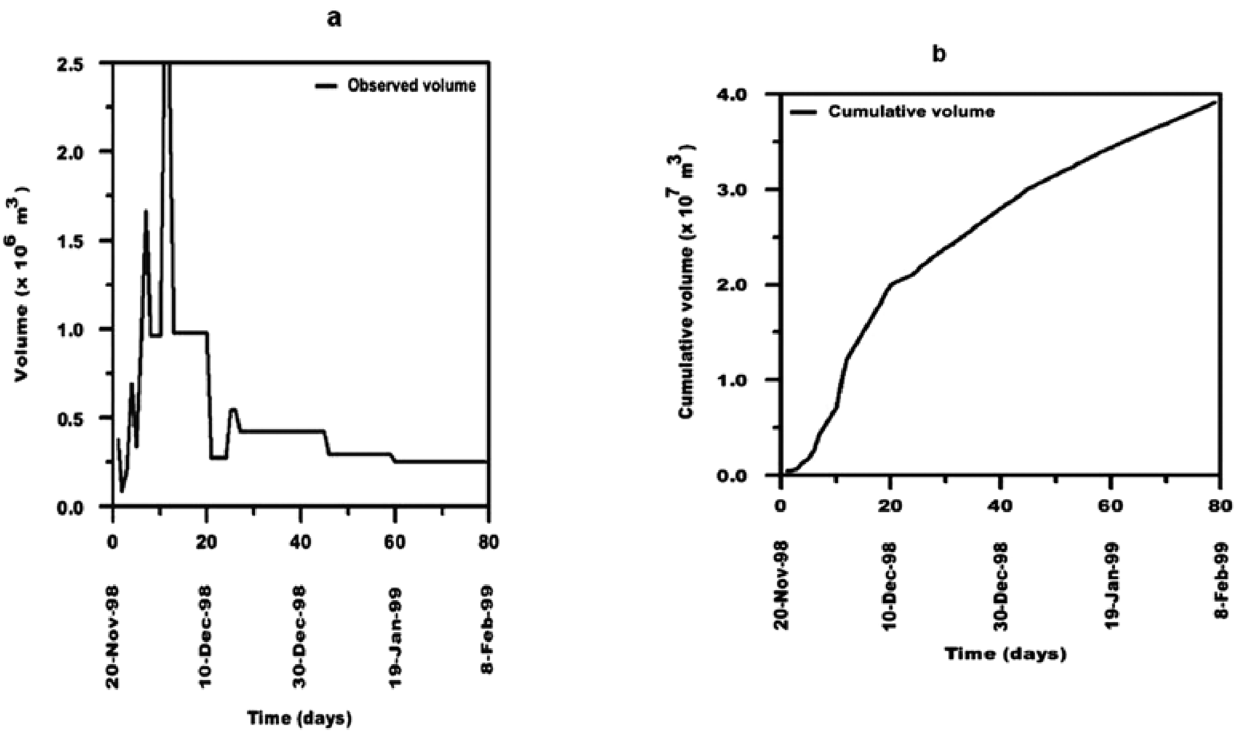

Fig. 1. a) Daily eruption rate and b) cumulative volume for the 1998 – 99 eruption of Volcán de Colima (after Navarro – Ochoa *et al.*, 2002).

Models of mass emission

Volcanic eruptions in polygenetic volcanoes follow an increase of pressure in the magma chamber. The increase is generally due to feeding of the magma chamber from below. The response of the magma chamber to an increase in pressure is deformation followed by fracturing of the country rock until a fracture extending from the magma chamber walls to the surface is produced. The ensuing eruption can be of a variety of types from a slowly proceeding effusive type to a very fast explosive event. The most explosive phase of the 1913 Colima eruption produced about 0.57 km^3 (DRE) of material in 8 hrs (Saucedo *et al.*, 2010). On the other hand during the 1998-1999 period of activity only 0.004 km^3 of material was erupted in two months with a variable eruption rate. The vast difference in eruption rates is related to the amount of volatiles and magma volume in the different cases, but in the effusive case the behavior of the country rock or the conduit system probably plays an important role. Ida (1996) proposed that such behavior is due to oscillations in the conduit radius caused by the viscous deformation of the country rock. He proposed a model of a spherical chamber, fed from below, buried in an elastic medium and connected to the surface through a cylindric conduit in a viscous medium. For constant feeding, this model predicts effusion at regular intervals of the same intensity. Since what is generally observed is a decrease of the discharge with time, Maeda (2000) proposed a similar model except that the medium in the upper conduit is a material of the Maxwell type; this model yields an eruption rate which decreases with time. A viscoelastic rheology of the country rock has been long used to explain several types of phenomena in volcanoes, such as deformation (see Poland *et al.*, 2006 for a review) and premonitory material-failure (De la Cruz-Reyna and Reyes-Dávila, 2001). Scandone and Giacomelli (2001) discussed the behavior of the walls of a magma chamber during volcanic eruptions; they consider that they behave as a rigid body during explosive eruptions because the relaxation time of the country rock is of the order of 10^6 sec. (~12 days), longer than the eruption time in explosive events. The period we are considering lasted 2 months, longer than the relaxation time and therefore time enough for the viscoelastic behavior of the country rock to play a role as assumed by Maeda (2000).

Periodicity in the lava emission could also be an effect of the plumbing system and the magma rheology. Costa *et al.* (2007a,b) proposed a model of a magma chamber coupled to a dyke of elliptic cross section connected to the surface by a cylindrical conduit. The country rock is treated as elastic and the magma rheology depends on composition and crystal fraction content. This model yields a cyclic behavior in mass emission with long and short periods related to the magma chamber and the dyke respectively. The two models may be complementary as they deal with different processes. Volcanic systems evolve with time due to numerous factors; at some point in the system's history, flow changes due to changes in the dyke cross section, since the magma tends to flow faster at the center of the dyke. The dome also affects the course of the eruption but this effect is not accounted by any of the models, which apply after the flow is established.

The pattern of deformation of Volcán de Colima shows stages of inflation and deflation that cannot be fitted by a Mogi point source (Murray and Wooller, 2002, Ramirez *et al*., 2002); using a different type of source is not justified by the available data, which derives from an incomplete deformation network (Murray and Wooller, 2002). Distance changes recorded during different stages of recent activity at Volcán de Colima suggest an inflationary process preceding the 20 November 1998 lava eruption (Ramirez *et al*., 2002). It is reasonable to explore the capability of Maeda's model to explain the eruption rates observed during 1998-1999 at Volcán de Colima.

Maeda's Model

Maeda (2000) proposed a model of a spherical magma chamber in an elastic medium connected to the surface by a cylindrical conduit surrounded by a viscoelastic medium of the Maxwell type (Fig. 2). The chamber is fed from below with a supply rate that can be constant or a function of time. For a simple time dependent rate, a bell shaped function- a soliton- with a constant part was used. The excess mass in the reservoir increases proportionally to the pressure in the chamber, which expands the conduit, through which the magma is discharged with a rate determined by Poiseuille's law.

The equations describing this model are provided by Maeda (2000). We show them here for the sake of completeness.

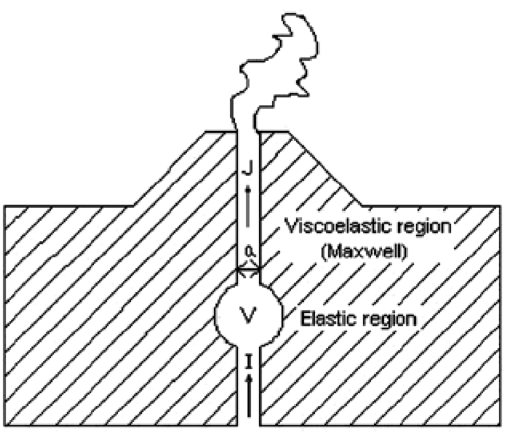

Fig. 2. A sketch of Maeda's model.

Let p be the overpressure in the chamber; it is related to the excess volume v through the equation

$$p = kv \tag{1}$$

where:

$$k = \frac{4\mu K}{(4\mu + 3\ K)V_r}$$

Here V_r is the volume of the magma chamber, and μ and K are the rigidity and bulk constants of the medium. Equation (1) is derived from the Mogi (1958) relation assuming an elastic medium (see Cabrera-Gutiérrez, 2010 for a complete derivation). From consideration of mass conservation the following equation can be derived:

$$\frac{dv}{dt} = I\text{-}J \tag{2}$$

where I and J are the influx and outflux through the conduit respectively. For Poiseulle flow we can derive the following equation:

$$J = \frac{\pi}{8\eta_m}\left(g\ \Delta\rho + \frac{p}{l}\right)a^4 \tag{3}$$

where a is the radius of the conduit, l its length, g the value of gravity, η_m the kinematic viscosity of the magma and $\Delta\rho$ the density contrast between magma and host rock. We shall simplify the problem by assuming constant magma viscosity. Althoung it is a function of temperature, crystal and gas content, we consider that those factors do not change considerably in the course of the eruption, for the period we are analyzing.

The conduit from the chamber to the surface is surrounded by a medium with a Maxwell viscoelastic rheology; therefore the radius of the conduit responds to changes in pressure according to

$$\frac{da}{dt} = \frac{ap}{2\eta_r} + \frac{1}{2\mu}\frac{d(ap)}{dt} \tag{4}$$

where η_r is the viscosity of the country rock.

Maeda (2000) assumed that the magma behaves as an incompressible fluid; however one can consider compressibility though the equation

$$\rho_m = \rho_{m_a}\, e^{\frac{p}{K_m}} \tag{5}$$

where K_m is the bulk modulus of the magma and ρ_{ma} the density of the magma at lithostatic pressure.

Equations 1 to 5 form a complete system of equations suitable for numerical treatment. The solutions obtained are more general and may easily be derived by reducing the number of equations through proper substitution and the introduction of the following dimensionless variables:

$$a = a_0\, a; \qquad p = p_0\, \beta; \qquad t = t_0 \tau\,,$$

where

$$a^4_0 = \frac{16\mu^2\eta_m}{\pi\eta_r kg\Delta\rho}; \quad \gamma = \frac{\eta_m kI}{2\mu^2}; \quad t_0 = \frac{\eta_r}{\mu}; \quad p_0 = 2\mu\,.$$

These changes lead to the following set of dimensionless equations:

$$\frac{d\beta}{d\tau}\left\{1+\frac{2\mu}{K_m}\beta\right\} = \gamma - \alpha^4 - D_r\beta\alpha^4 + Qe^{\frac{2\mu\beta}{K_m}}\alpha^4 \qquad (6)$$

$$\frac{d\alpha}{d\tau} = \frac{\alpha\beta}{1-\beta} + \frac{\alpha}{\{1-\beta\}\left\{1+\frac{2\mu}{K_m}\beta\right\}}\left\{\gamma-\alpha^4-D_r\beta\alpha^4+Qe^{\frac{2\mu\beta}{K_m}}\alpha^4\right\} \qquad (7)$$

where

$$D_r = \frac{2\mu}{lg\,\rho_{r_a}}; \qquad Q = \frac{\rho_{m_a}}{\rho_{r_a}}\,.$$

The above equations were solved with the commercial software ®MATHEMATICA, which uses the Adams-Bashford method to solve the system of differential equations. The input data required for the solution is *I* the input, which is a general function of time of the physical characteristics of magma and country rock, and the dimensions and depth of the magma chamber as well as the initial radius of the conduit. The solution yields the pressure and rate of change of the conduit radius and the discharge rate is computed from equation (3).

The model requires knowing the input rate to the magma chamber. Maeda (2000) considers two cases: a constant supply to obtain the general characteristics of the system, and a time varying input such as a soliton described by the dimensionless function (in dimensionless form):

$$\gamma = \gamma_0 + h\,\mathrm{sech}^2[\omega(t-t_c)]$$

The meanings of the variables in the above equation are shown in Fig. 3. This is a simple representation of the feeding of a magma chamber by patches of magma from below, but the actual feeding must be a more complex function as suggested by the real output rate observed at Colima (Navarro-Ochoa *et al*., 2002).

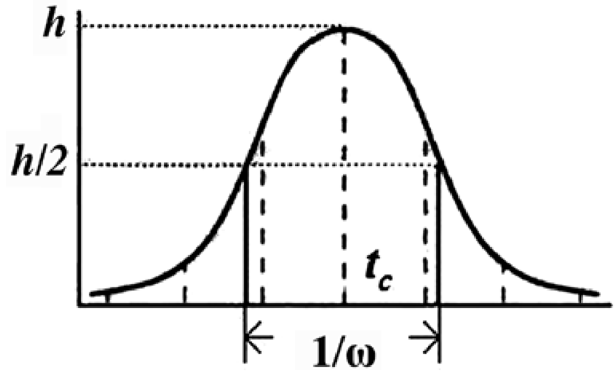

Fig. 3. Dimensions and form of the solitons considered as input. *h* is the soliton height; ω^{-1}, the width of the soliton, is defined as the horizontal distance between points whose height is *h*/2. t_c is the center of the soliton.

Maeda (2000) presented the results of the application of the model to the 1991-1995 eruption of Mt. Unzen; his results are shown in Fig. 4b. This figure shows the comparison between data from Maeda (2000) for Mt. Unzen (Fig 4a) and the results for the algorithm developed in this paper (Fig. 4b) (Cabrera-Gutiérrez, 2010). This last figure was obtained with the values listed in Table 1, whereas Table 2 shows the input to the magma chamber of Mt. Unzen. The results with our code are substantially the same as those shown by Maeda. Before applying the model to the 1998-1999 eruption of Volcán de Colima, the convergence of the model was assessed and the optimal time step was determined. The sensitivity of the model to changes in the variables was also assessed as follows.

Fig. 5 shows the effect of solitons of different shape keeping the total volume and all other variables constant. As can be observed, the output is of the same shape but different amplitude and timing; the cumulative output is less sensitive to the shape of the soliton. Figs. 6 to 12 show output and output rate for different values of the parameters as shown in the inset. The values were chosen to display the differences in the output and output rate. Table 3 shows the percent change from a 100 % change in a given variable. The volume of the magma chamber is by far the most sensitive parameter in the model, next to changes in magma viscosity, rock density contrast, and the rigidity/viscosity ratio of the country rock. The model is less sensitive to changes in the other parameters, such as the rock bulk modulus, the depth, the conduit radius, and the magma bulk modulus. Fig. 13 presents the effect of compressibility of the magma; the difference between the compressible and incompressible cases for a reasonable choice of the bulk modulus of the magma is small. Tables 4 and 5 show the values for the outflow rate and the input , respectively, for the parameter values showed in Fig. 13.

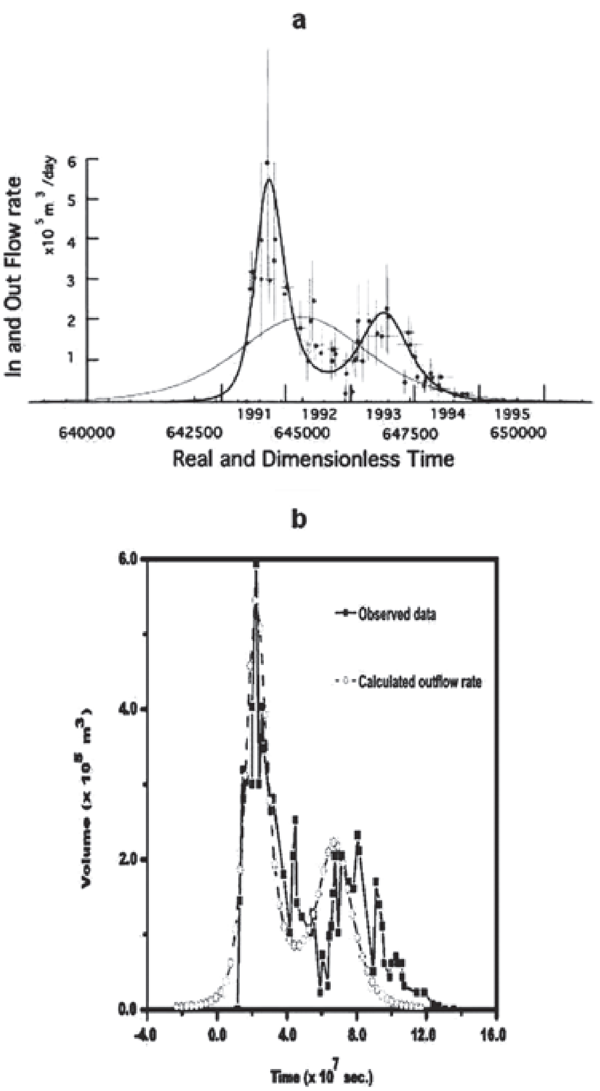

Fig. 4. Comparison between Maeda's resulting data a) and resulting data obtained by an algorithm which was developed for this paper b) for the 1991 – 1995 Mt. Unzen eruption.

Table 1

Values for the outflow rate for Mt. Unzen calculated with Maeda's model.

v, volume of the reservoir	$8.6x10^9$ m^3
l, depth of the reservoir	11000 m
K, bulk modulus of the country rock	$1.0x10^9$ Pa
η_r, country rock viscosity	$2.0x10^{13}$ Pa s
η_m, magma viscosity	$1.4x10^{11}$ Pa s
μ, rigidity of the country rock	$1.0x10^9$ Pa
$\Delta\rho$, density difference	100 kg/m^3

Table 2

Values for the input to the magma chamber of Mt. Unzen. Maeda's model.

t_c, center of the soliton	$64.491x10^4$
h, soliton's height	$2.0x10^{-6}$
ω (ω^{-1}), width of the soliton)	$6.0x10^{-4}$
a_0, scale factor (radius)	4837.21
t_0, scale factor (time)	$2.0x10^4$

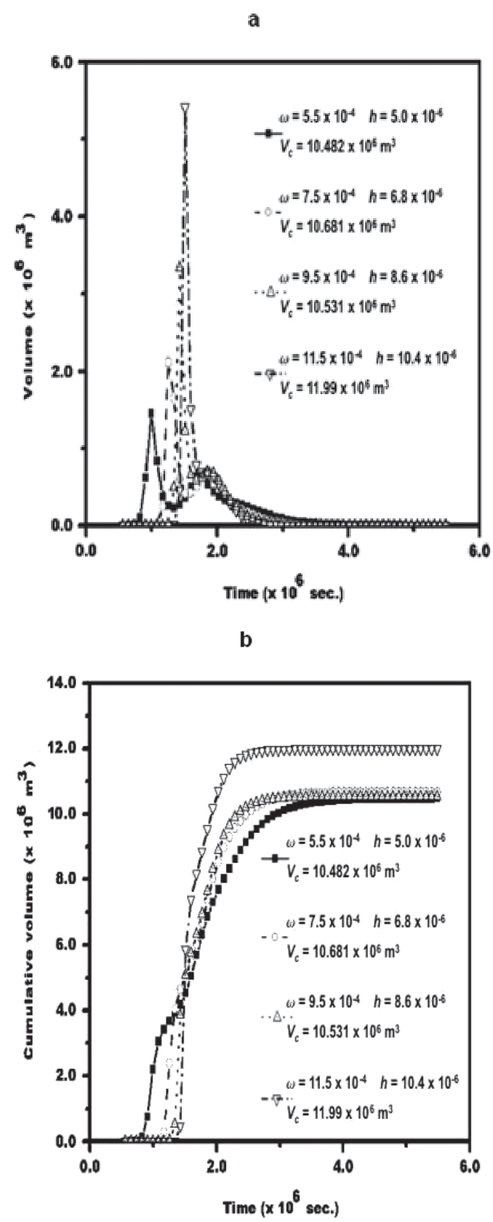

Fig. 5. a) Volume erupted as a function of time b) cumulative volume for different input values. The input values correspond to different values of h (height) and ω (wide) of the soliton. The input volume is roughly the same in all cases. V_C is the final cumulative erupted volume.

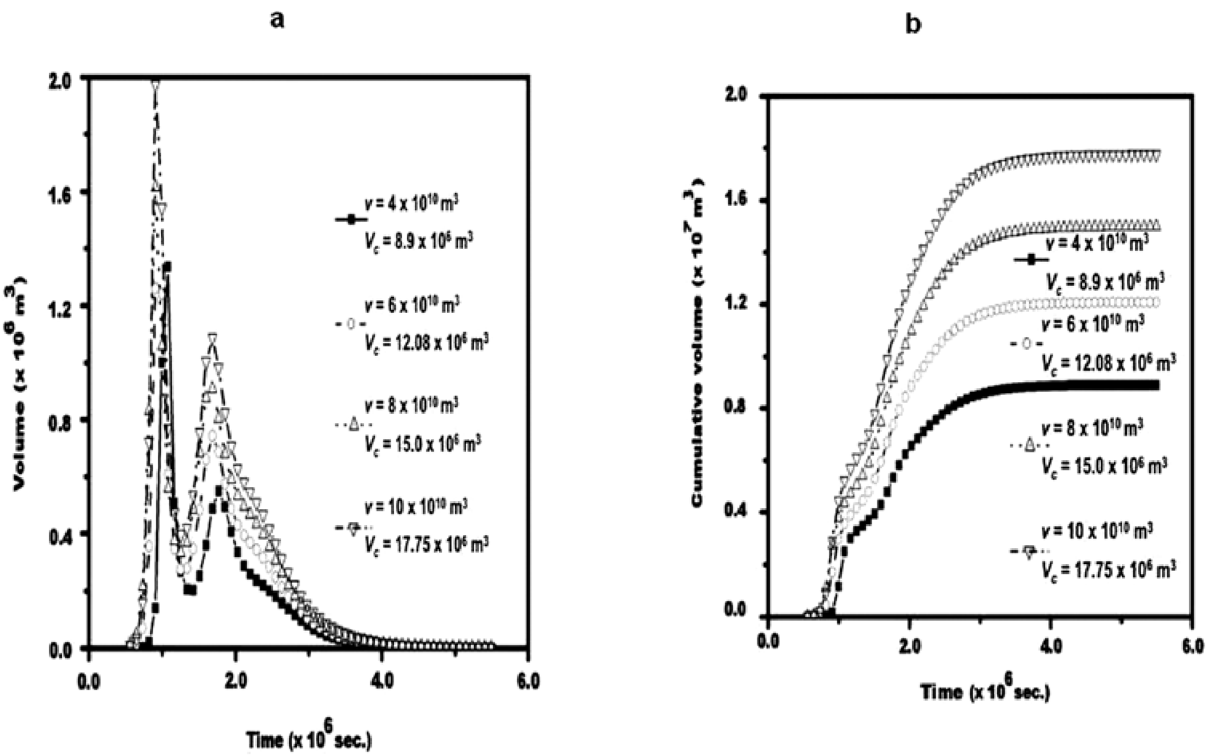

Fig. 6. Outflow rate for the different chamber volumes shown in the inset. (a) Erupted volume as function of time. (b) Cumulative volume.

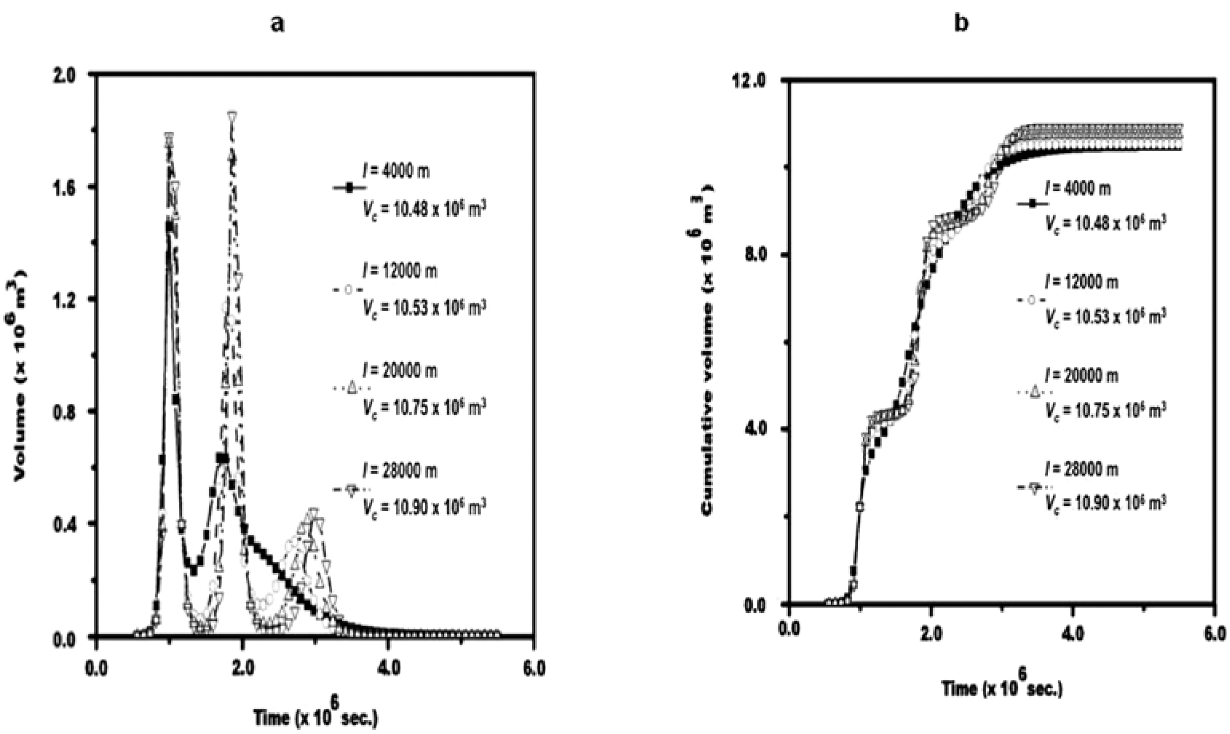

Figure 7. As Fig. 5 for different magma chamber depths.

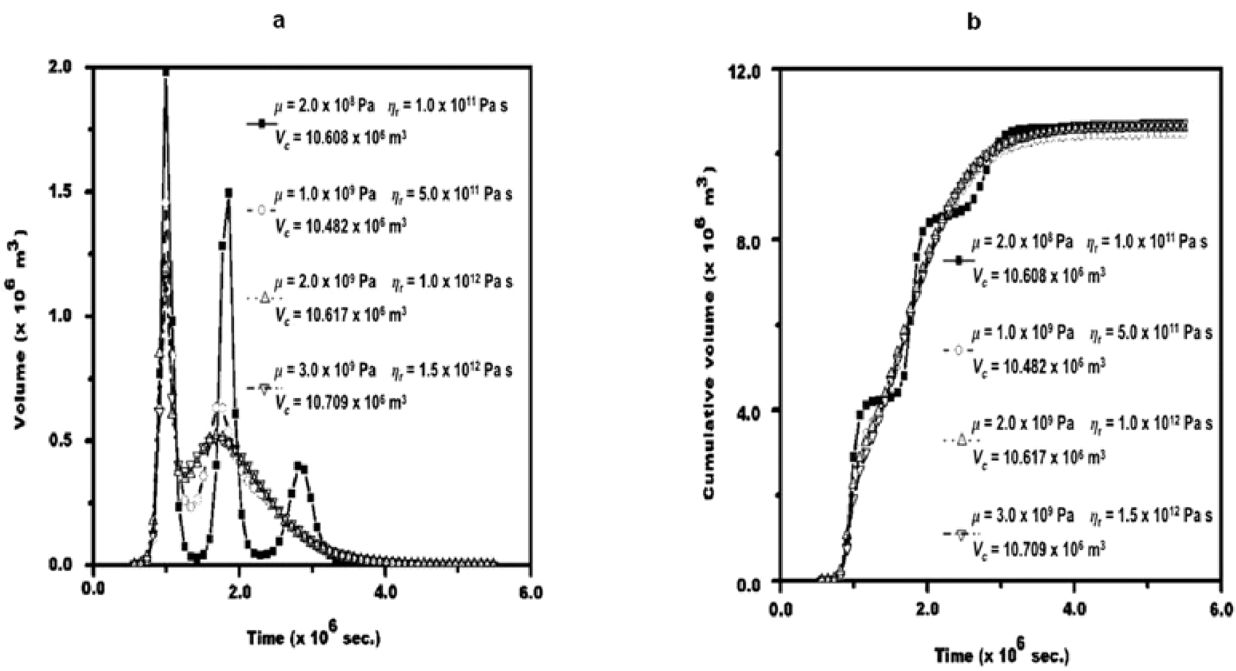

Fig. 8. As Fig. 5 for different values of the rigidity and viscosity of the country rock.

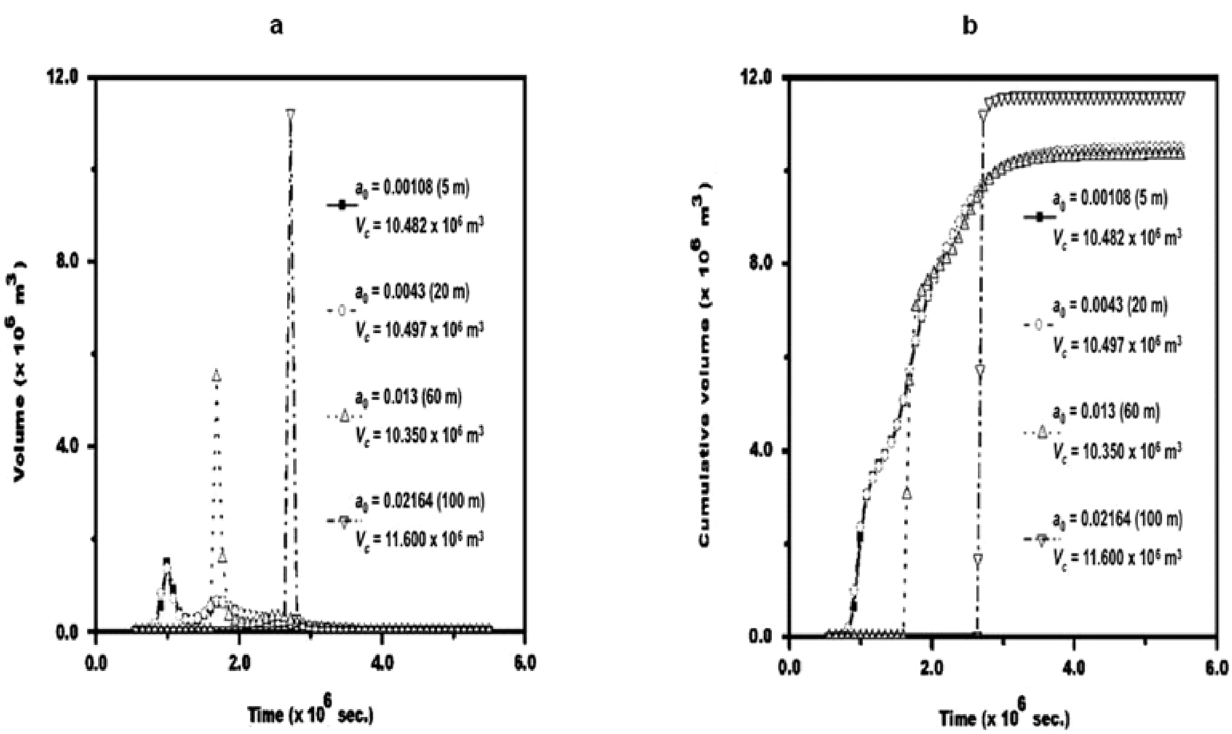

Figure 9. As Fig. 5 for different values of the conduit radius.

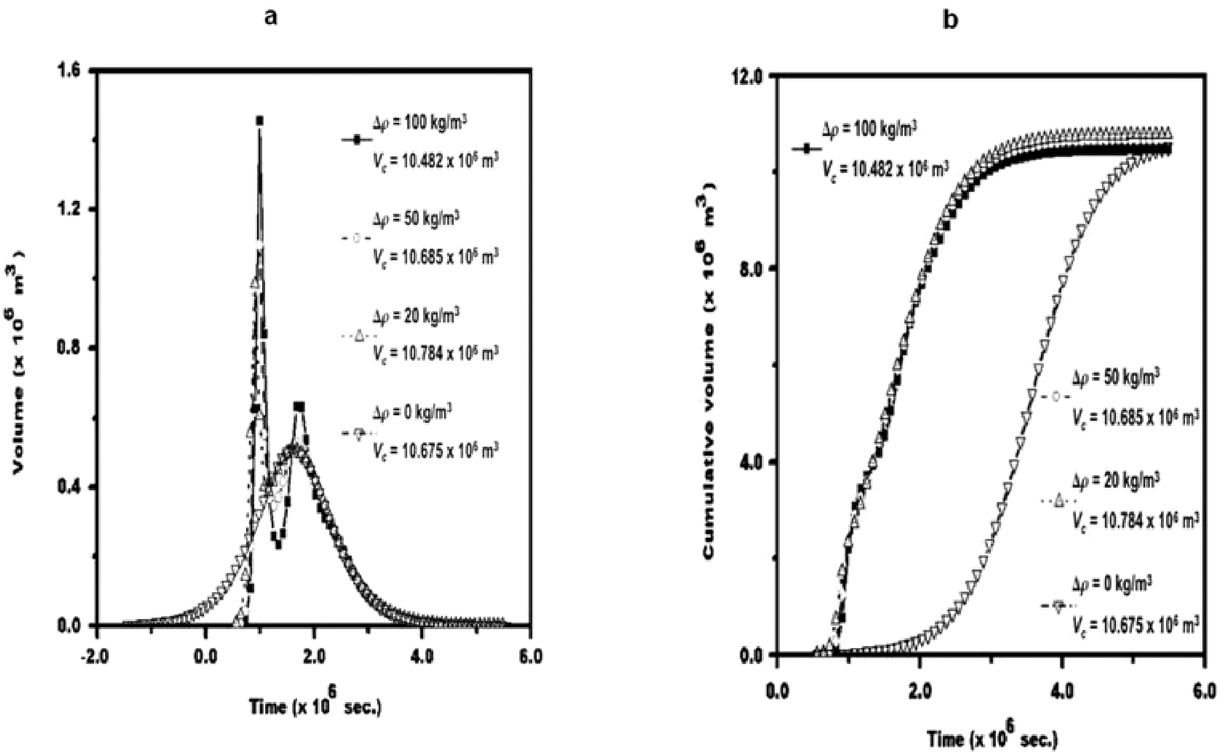

Fig. 10. As Fig. 5 for different density contrasts.

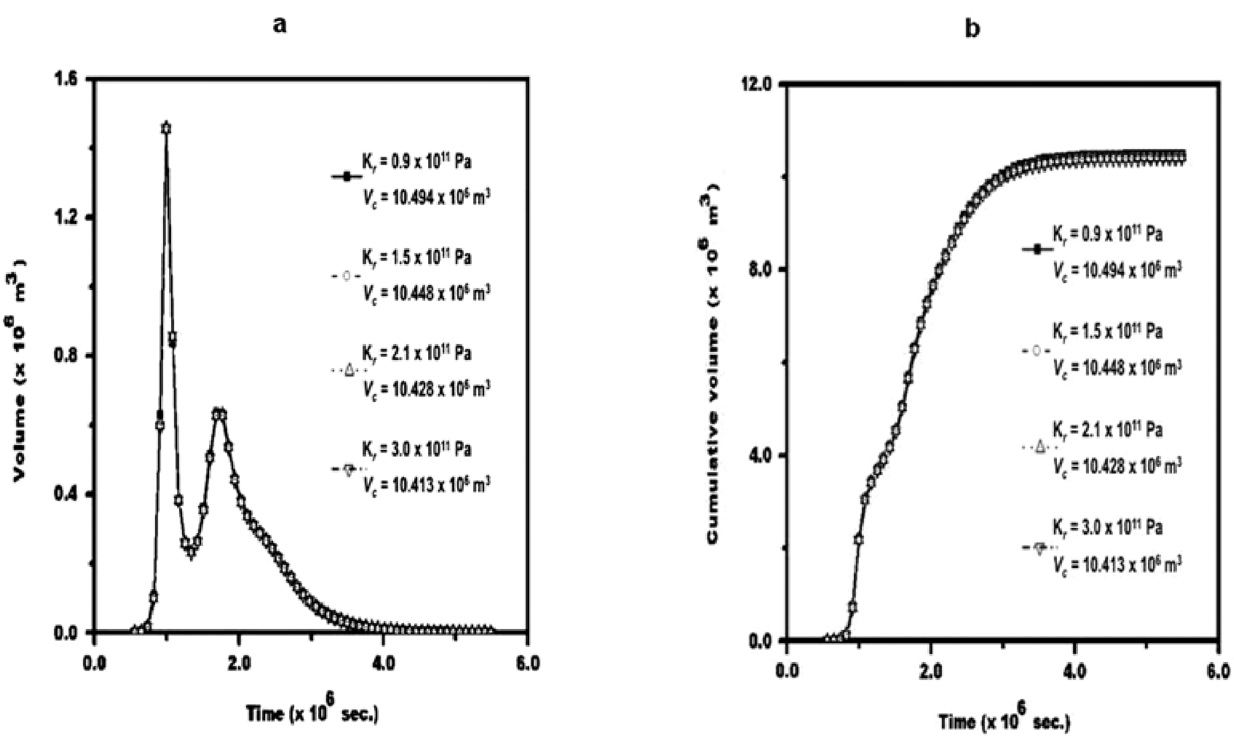

Fig. 11. As Fig. 5 for different values of the rock bulk modulus.

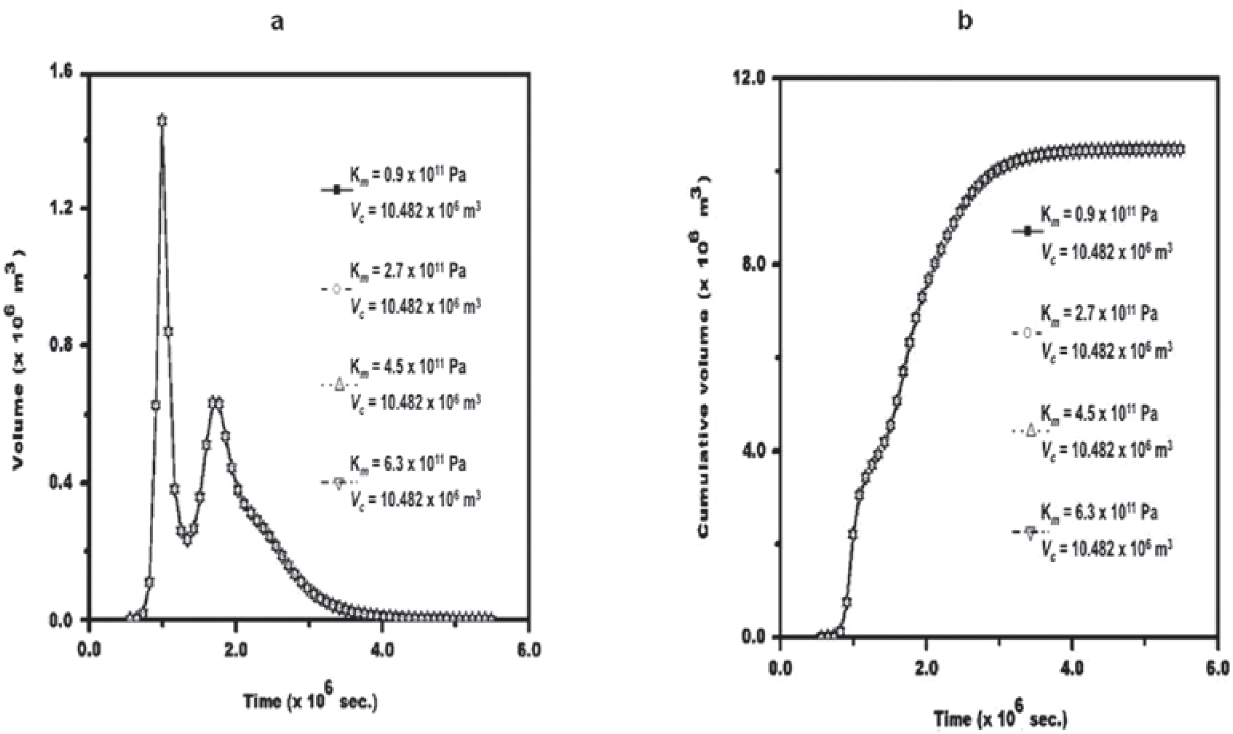

Fig. 12. As Fig. 5 for different values of the magma bulk modulus.

Table 3

Percent change in output volume for a change of 100% in the value of the variable listed.

Parameter	100 %	VC
v, Volume	Increase	68.54 % increase
l, Depth	Increase	0.24 % increase
ηr, Rock Viscosity	Increase	1.29 % increase
η_m, Magma Viscosity	Increase	68.16 % increase
μ, Rigidity	Increase	1.29 % increase
a_0, Radius	Increase	0.05 % increase
$\Delta\rho$, Density difference	Increase	1.90 % diminish
K_m, Magma bulk modulus	Increase	$4x10^{-4}$ % increase
K_r, Rock bulk modulus	Increase	0.53 % diminish

Table 4

Values for the outflow rate calculated with Maeda's compressible model.

v, volume of the reservoir	$5.0x10^{10}$ m^3
l, depth of the reservoir	4000 m
K_r, bulk modulus of the country rock	$1.0x10^{11}$ Pa
K_m, bulk modulus of the magma	$1.0x10^{11}$ Pa
η_r, country rock viscosity	$5.0x10^{11}$ Pa s
η_m, magma viscosity	$3.0x10^{10}$ Pa s
μ, rigidity of the country rock	$1.0x10^9$ Pa
ρ_{ma}, initial magma density	2500 kg/m^3
ρ_{ra}, initial country rock density	2600 kg/m^3

Table 5

Values for the input to the magma chamber. Maeda's compressible case.

t_c, center of the soliton	$64.61x10^4$
h, soliton´s height	$5.0x10^{-6}$
ω (ω^{-1}, width of the soliton)	$5.5x10^{-4}$
a_0, scale factor (radius)	4620.37
t_0, scale factor (time)	500

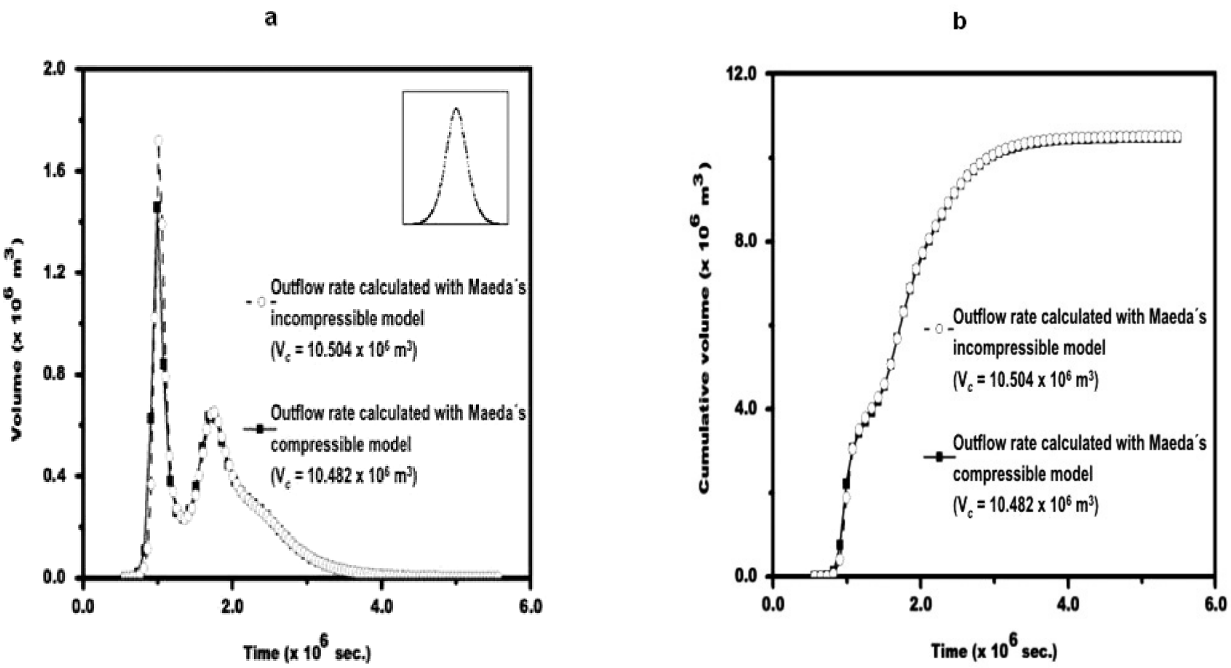

Fig. 13. Comparison between compressible and incompressible Maeda's models. a) Total volume. b) Cumulative volume.

Application of Maeda's model to Colima volcano and discussion

The model is very sensitive to the volume of the Magma chamber, so it is important to have an independent estimate of these parameters. Medina *et al.* (1996) found a negative mass anomaly beneath Colima volcano. They proposed the gravimetric model shown in Fig. 14, a rectangular body 2 km wide, 5 km long and 5 km thick, yielding a volume of 50 km^3. The top of the body is about 1.5 km below sea level (~ 5.4 km below the summit crater). In 1998-1999 the seismic foci occured roughly around this volume, which agrees with the gravity model (Zamora-Camacho *et al.*, 2006). However, the gravity model was not constrained by subsurface density measurements nor other geophysical methods. Since Maeda's model requires a spherical magma chamber we considered a sphere about the center of the rectangular body (Fig. 14). By trial and error the effective viscosity of the magma was set at 10^9 Pa s in agreement with the value obtained by Navarro-Ochoa *et al.*, (2002). The best fit to the observed output is shown in Figs. 15a and 15b with the values listed in Tables 6 and 7, which describe the input to the magma chamber as a time-function formed by several time steps. This form of the time function is required to model the slow emission of lava over several weeks after the main events. Magma apparently continued to feed into the magma chamber at a constant rate after the passage of the peak inflow of the time-varying feeding.

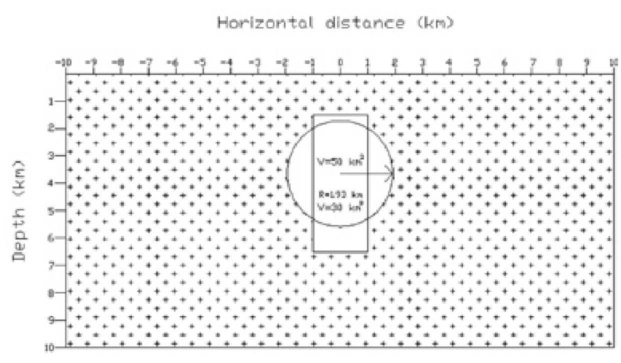

Fig. 14. Magma chamber of Volcán de Colima, after the gravimetric model of Medina *et al.* (1996).

Table 6

Values for the best fit of the calculated outflow rate.

v, volume of the reservoir	$3.0\text{x}10^{10}$ m^3
l, depth of the reservoir	1715 m
K_r, bulk modulus of the country rock	$1.0\text{x}10^9$ Pa
K_m, bulk modulus of the magma	$1.0\text{x}10^9$ Pa
η_r, country rock viscosity	$1.0\text{x}10^{13}$ Pa s
η_m, magma viscosity	$1.8\text{x}10^9$ Pa s
μ, rigidity of the country rock	$2.0\text{x}10^9$ Pa
ρ_{ma}, initial magma density	2500 kg/m^3
ρ_{ra}, initial country rock density	2600 kg/m^3

Table 7

Best fit values for the input to the magma chamber.

i_1, step function for the magma supply rate	$5000\text{H}(\text{t}-t_1)$
t_1, step function abscise	$6.45\text{x}10^5$
t_c, center of the soliton	$64.53\text{x}10^4$
h, soliton´s height	$1.0\text{x}10^{-30}$
ω (ω^{-1}, width of the soliton)	$9.8\text{x}10^{-4}$
a_0, scale factor (radius)	1560.92
t_0, scale factor (time)	5000

From Fig. 15a, the model reproduces roughly the volume erupted as a function of time, consisting of two periods of large emission followed by a continuing small emission of lava. However the theoretical plot is continuous while the observed was constructed with indirect observations made at discrete times and represents average values over periods of time (Navarro-Ochoa *et al*., 2002). The theoretical curves assume a continuous discharge unhampered by surface conditions. Actually, the extrusion of magma is subjected to many processes not considered in the model, but the cumulative volume tends to smooth out these effects as shown Fig. 15b. Notice that the fit is reasonably better than in the previous case. Different values of volume, conduit radius, and depth of the sphere produce results that do not fit the observed data. Fig. 16 shows pressure as a function of time during the eruption: notice that the pressure does not exceed the strength of the country rock, which is believed to be in the range of 30 MPa.

Changes in volume due to input of magma are not accommodated by elastic or viscoelastic deformation as evidenced by the seismicity which suggests that part of the volume change may be accommodated by faulting of the country rock. McGarr (1976) obtained a formula to correlate cumulative seismic moment and volume change, which yielded a figure in the range of 0.14 to 0.18 x 10^5 km^3 (ΣM_0=2.68X10^{22} dynes/cm; Zamora-Camacho, 2003), a small volume compared with the size of the magma chamber required by the model. We conclude that most of the volume change required by the magma input is accomodated by the rheology of the medium, which also influences the mass eruption rate.

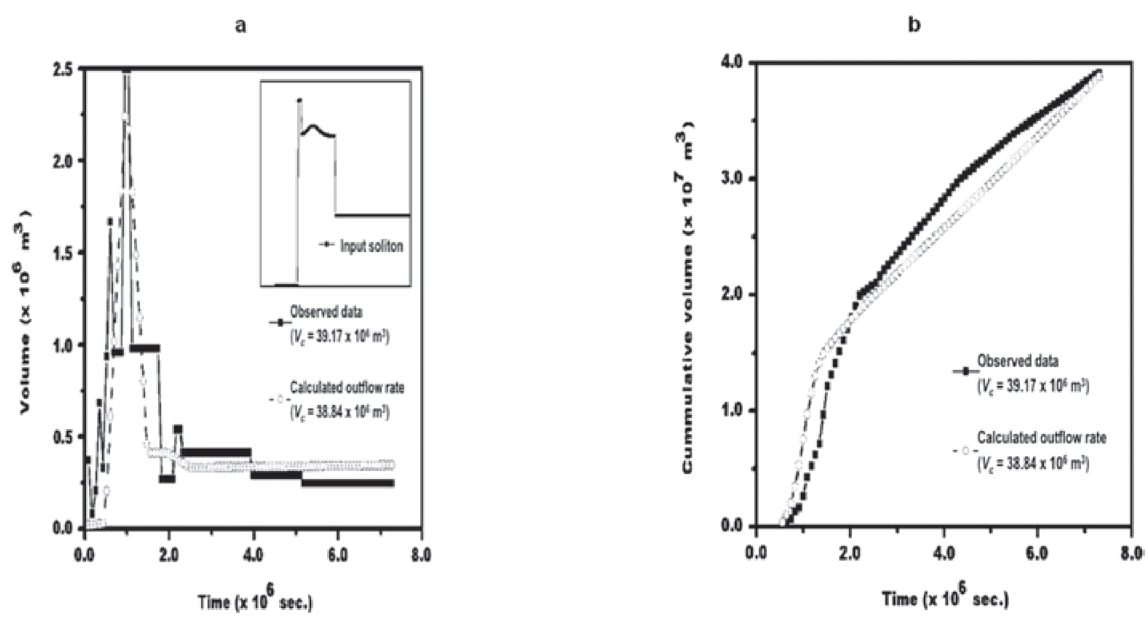

Fig. 15. Best fit to the effusive activity of the 1998 – 1999 Volcán de Colima eruption.

(a) Erupted volume as function of time. (b) Cumulative volume.

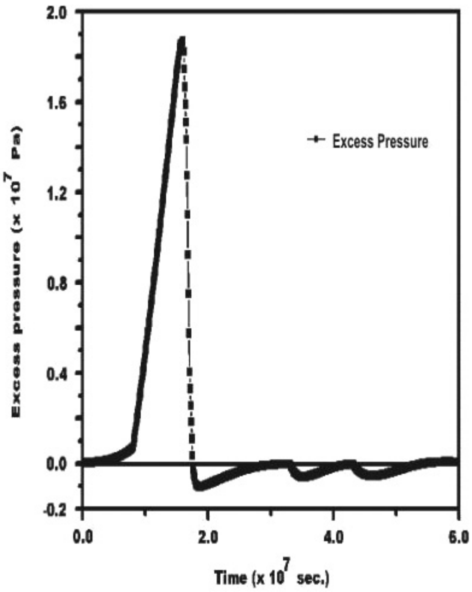

Fig. 16. Excess pressure obtained with Maeda's compressible model for the best fit volume of the effusive activity of Volcán de Colima in 1998 – 99.

Conclusions

The modified model of Maeda (2000) applied to the 1998-1999 eruption of Volcán de Colima fits reasonably well the observed data. The magma chamber is modeled as a sphere at 1.715 km below sea level (5.565 km below the summit crater) with a volume of 30 km^3. The magma chamber evolves through time as it cools during repose and heats up during periods of magma feeding. This model can be applied to future eruptions to investigate its appropriateness to Volcán de Colima; it could serve to determine the mechanism of this type of eruptions and to establish the internal characteristics of the volcanic system at Colima Volcano.

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