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# Neuro-Fuzzy Cognitive Temporal Models for Predicting Multidimensional Time Series with Fuzzy Trends

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**Abstract.** A new type of Neuro-Fuzzy Cognitive Temporal Models (NFCTM) is proposed for predicting multidimensional time series (MTS) taking into account the fuzzy trends of all MTS components. NFCTMS allow predicting the MTS in conditions of non-stochastic uncertainty, non-linearity of mutual influence, partial inconsistency and significant interdependence of the MTS components, as well as in conditions of small samples. This takes into account the direct and indirect mutual influence of all the MTS components with different time lags relative to each other. To carry out temporal changes in specific MTS components the original neuro-fuzzy models of *Rec*ANFIS (Recurrent Adaptive Neuro-Fuzzy Inference System/Model) type are applied, that: firstly, allow to save the predicted values of the MTS components in the range of "liding-window" time series; secondly, identify fuzzy trends of the components of the MTS in the range of "sliding-window" time series; thirdly, adaptively take into account fuzzy trends of the MTS components based on the fuzzy mappings. An original way of a coherent learning NFCTM is described, which lies: firstly, in training *Rec*ANFISs for each concept NFCTM (MTS component) taking into account fuzzy trends; secondly, in coherence of all *Rec*ANFISs between each other to maximize the prediction accuracy of each of the MTS components without compromising the prediction accuracy of at least one of the other MTS components. Experimental studies have been carried out and the results of using the proposed NFCTM for multidimensional forecasting of the urban environment state in Moscow in conditions of a complex epidemiological situation have been obtained.

**Keywords.** Neuro-fuzzy cognitive temporal model, recurrent adaptive neuro-fuzzy inference system/model, multidimensional time series.

## 1 Introduction

Various approaches are used to predict multidimensional time series (MTS), which are usually based on methods for predicting one-dimensional time series [1, 2]. Limitations of these methods are: complexity of accounting for the indirect influence of the interdependent MTS components in the conditions of uncertainty, the non-linear nature of their interaction [3, 4], their lack of consistency, and the complexity of identifying and accounting for the trends of each of the MTS components [5, 6].

Methods for predicting time series using fuzzy and neural network models are being actively developed. However, their limitations are the complexity of accounting for the direct and indirect interaction of the MTS components.

Methods of fuzzy cognitive modeling allow us to take into account the direct and indirect interaction of the components of the MTS [7, 8, 9, 10]. However, the use of these methods to predict the MTS is limited by: the capabilities of the used system dynamics models of the MTS components; the lack of consideration of interference MTS components with their different time lags relative to each other; the lack of approaches to coordinated setting of each of the MTS components; MTS small forecasting samples; identification of the trends of the MTS components.

The study deals with the use of Neuro-Fuzzy Cognitive Temporal Models (NFCTM) for MTS predicting, taking into account the fuzzy trends of each of the MTS components. NFCTM allow predicting the MTS in conditions of non-stochastic uncertainty, non-linearity of interaction, partial inconsistency and significant interdependence of the MTS components in conditions of small samples. This takes into account the direct and indirect mutual influence of all the MTS components with different time lags relative to each other.

Proposed NFCTM include a lot of concepts (relevant to the MTS components), which are connected by subsets of arcs, weighted by fuzzy degrees of influence, arranged in chronological sequence taking into account time lags (delays) of the respective MTS components relative to each other.

To carry out the temporal changes in the individual NFCTM components MTS original neuro-fuzzy models of type *Rec*ANFIS (Recurrent Adaptive Neuro-Fuzzy Inference System/Model) are used, that: firstly, allow to save the predicted values of the MTS components in the range of “sliding-window” time series; secondly, identify fuzzy trends of the MTS components in the range of “sliding-window” time series; thirdly, take into account adaptive fuzzy trends of the MTS components based on the fuzzy mappings.

An original way of a coherent learning NFCTM is described, which lies: firstly, in training *Rec*ANFIS for each concept NFCTM (MTS component) taking into account fuzzy trends; secondly, in coherence of all *Rec*ANFISs between each other to maximize the prediction accuracy of each of the MTS components without compromising the prediction accuracy of at least one of the other MTS components.

Experimental studies have been carried out and the results of using the proposed method were obtained on the example of the problem of multidimensional analysis and forecasting of the urban environment state in Moscow.

## 2 The Problem of Forecasting Multidimensional Time Series

Previously in the work [10] a formalized representation of the MTS, focused on accounting for the mutual influence of the MTS components was proposed:

$$
\begin{gathered}
S=\left(S_1, S_2, \ldots, S_N\right), \\
\forall t \in\{1, \ldots, T, \ldots\}, \; S_t=\left(s_1^{(t)}, s_2^{(t)}, \ldots, s_N^{(t)}\right), \\
s_1^{(t)}=F_1\left(\varphi_{1,1}\left(s_1^{(t-1)}, \ldots, s_1^{\left(t-L_1^1\right)}\right), \ldots, \varphi_{1,N}\left(s_N^{(t-1)}, \ldots, s_N^{\left(t-L_1^N\right)}\right)\right), \\
s_2^{(t)}=F_2\left(\varphi_{2,1}\left(s_1^{(t-1)}, \ldots, s_1^{\left(t-L_2^1\right)}\right), \ldots, \varphi_{2,N}\left(s_N^{(t-1)}, \ldots, s_N^{\left(t-L_2^N\right)}\right)\right), \\
\ldots \\
s_N^{(t)}=F_N\left(\varphi_{N,1}\left(s_1^{(t-1)}, \ldots, s_1^{\left(t-L_N^1\right)}\right), \ldots, \varphi_{N,N}\left(s_N^{(t-1)}, \ldots, s_N^{\left(t-L_N^N\right)}\right)\right),
\end{gathered}
$$

where $S$ – MTS; $S_t=\left(s_1^{(t)}, s_2^{(t)}, \ldots, s_N^{(t)}\right)$ – time “slice” of the MTS at the *t*-th instant of time; $s_j^{(t)}$ – value of the *j*-th MTS component at the *t*-th instant of time; $L_j^i$ – maximum value of the time lag (retrospective) of the *j*-th component relative to the *i*-th; $\varphi_{i,j}$ – operator for accounting for mutual influence of the *j*-th and *i*-th MTS components; $F_i$ – conversion to get $s_i^{(t)}$ taking into account the fuzzy trends of the *i*-th MTS component, $i=1, \ldots, N$, $N$ – the number of MTS components.

The proposed formalized representation of the MTS illustrates the possibility of taking into account the direct and indirect interaction of the MTS components relative to their different time lags.

The formalized statement of the problem of multidimensional analysis and forecasting of the state of complex systems and processes for well-coordinated MTS components assumes the possibility of minimizing forecasting errors concurrently for all MTS components and is presented as follows:

$$
\begin{aligned}
&\delta S=\left(\delta S_1, \delta S_2, \ldots, \delta S_N\right), \\
&\delta S \rightarrow \min, \\
&\forall i \in 1, \ldots, N \;\; \delta S_i \rightarrow \min,
\end{aligned}
$$

where $\delta S$ – error of the forecast of the MTS in general; $\delta S_i$ – error in predicting of the *i*-th MTS component; $s_i^{(t)}$ – reference value of the *i*-th MTS component; $s_{i(cur)}^{(t)}$ – the forecast value of the *i*-th MTS component; $N$ – the number of MTS counts. Different metrics can be used to estimate errors in predicting $\delta S_i$ of each of the $s_i^{(t)}$ MTS components, for example, the mean-square deviation:

$$\delta S_i = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(s_i^{(t)} - s_{i\,(cur)}^{(t)}\right)^2},\ i = 1, ..., N.$$

The multi-criteria nature of the MTS prediction makes it necessary to minimize errors concurrently for all the MTS components. However, as a rule, it is impossible to achieve in real conditions of non-stochastic uncertainty, non-linearity of interaction, partial inconsistency and significant interdependence of the MTS components.

Therefore, we apply a compromise approach to the multi-criteria estimation of the forecast error ( $\delta S$ ) of the MTS in general based on the generalized Edgeworth-Pareto principle [11], which, in relation to the problem being solved, is expressed in the fact that it is impossible to maximize the prediction accuracy of any MTS component without deterioration of prediction accuracy of at least one of the other MTS components.

## 3 Description of Neuro-Fuzzy Cognitive Temporal Models

The NFCTM can be presented in the following way:

$$FS = \langle C, W\rangle,$$

$$C = \{C_i \mid i \in 1, ..., N\},\ N = |C|,$$

$$C_i : \tilde{s}_i^{(t)} = F_i\left(\left\{\left(\tilde{s}_j'^{(t-1)}, ..., \tilde{s}_j'^{\left(t-L_i^j\right)}\right) \mid j \in 1, ..., N_i\right\}\right),$$

$$W = \{W_{ij} \mid i,\ j \in 1, ..., N\},$$

$$W_{ij} = \left\{w_{ij}^{\left(t-l_i^j\right)} \mid\ l_i^j = 0, ..., L_i^j\right\},$$

$$\tilde{s}_j'^{\left(t-l_i^j\right)} = \varphi_{ij}\left(w_{ij}^{\left(t-l_i^j\right)}, \tilde{s}_j^{\left(t-l_i^j\right)}\right),\ l_i^j = 0, ..., L_i^j,$$

where $C$ – multiplicity of NFCTM concepts that define the values of the corresponding MTS components; $\tilde{F}_i$ – fuzzy temporal transformation implemented by the corresponding concept $C_i$ taking into account the fuzzy trends of the corresponding MTS component; $N$ – the number of NFCTM concepts; $\tilde{s}_i^{(t)}$ – the predicted fuzzy value of the concept $C_i$ at the *t*-th instant of time; $\left(s_j'^{(t-1)}, ..., s_j'^{\left(t-L_i^j\right)}\right)$ – a subset of the input temporal fuzzy variables of the concept $C_i$, related to the corresponding output temporal fuzzy variables of the concept $C_j$; $N_i$ – the number of NFCTM concepts directly related to the concept $C_i$; $l_i^j$ – time lag (delay) for the corresponding input temporal fuzzy variable $s_j'^{\left(t-l_i^j\right)}$ of the concept $C_i$, $l_i^j = 0, ..., L_i^j$; $W$ – a set of direct influence relations between all pairs of NFCTM concepts; $W_{ij}$ – a subset of fuzzy values that defines a chronologically ordered set of fuzzy influence degrees $w_{ij}^{\left(t-l_i^j\right)}$ of the concept $C_j$ on the concept $C_i$ taking into account the time lag $l_i^j$; $\varphi_{ij}$ – fuzzy operator for accounting for the degree of mutual influence of the output variable of the concept $C_j$ on the input variable of the concept $C_i$, in the case of fuzzy variables, it is advisable to use the T-norm as the operator $\varphi_{ij}$.

Justification of time lags $l_i^j$ and determination of the degree of mutual influence $w_{ij}^{\left(t-l_i^j\right)}$ for the NFCTM concepts is a separate task, which can be solved using various (statistical or expert) methods of analyzing retrospective data, based on the establishment of direct interdependence between the MTS components.

For example, multiple linear regression method can be used to determine the interaction between the time lags of the MTS components [12, 13].

To implement fuzzy temporal transformations $\tilde{F}_i$, the original neuro-fuzzy models of the *Rec*ANFIS (Recurrent Adaptive Neuro-Fuzzy Inference System/Model) are further proposed.

## 4 Method for Designing, Training, and Using Neuro-Fuzzy Cognitive Temporal Models for Predicting Multidimensional Time Series

An example of building and using NFCTM for multidimensional forecasting of the urban environment state in Moscow in conditions of a complex epidemiological situation is under consideration.

The state of the urban environment is characterized by the state of its heterogeneous objects, systems and infrastructure (hereinafter referred to as urban environment objects): of real estate, engineering and transport infrastructure, ecosystem [14, 15]. Therefore, its assessment cannot be reduced to a single comprehensive indicator. The method includes the following stages.

**Stage 1.** Determination of significant MTS components.

Thus, based on the results of previous studies [16], [17] the following most significant factors (MTS components) that characterize the state of the urban environment have been identified: $C_1$ – ecology of the urban environment; $C_2$ – capacity of urban environment infrastructure; $C_3$ – income level of the population; $C_4$ – industrial consumption of fuel and energy resources; $C_5$ – population life quality; $C_6$ – sanitary and epidemiological situation.

**Stage 2.** Determination of the mutual influence of the MTS components for different time lags.

To correctly analyze the mutual influence of the MTS components, the retrospective MTS data is normalized:

$$s_{i\,(norm)}^{\left(t-l_i^j\right)} = \frac{s_i^{\left(t-l_i^j\right)} - s_{i\,(\min)}}{s_{i\,(\max)} - s_{i\,(\min)}},\ \ l_i^j = 0, ..., L_i^j,$$

where $s_i^{\left(t-l_i^j\right)}$ – original value; $s_{i\,(\min)}$, $s_{i\,(\max)}$ – the maximum and minimum values, respectively; $s_{i\,(norm)}^{\left(t-l_i^j\right)}$ – normalized value.

Note. Hereinafter and for normalized values $s_{i\,(norm)}^{\left(t-l_i^j\right)}$ of the MTS components, we will use a notation $s_i^{\left(t-l_i^j\right)}$.

To analyze the mutual influence of the MTS components a fuzzy extension of the method of multiple linear regression is reasonably chosen [18] due to the peculiarities of the analyzed urban factors:

$$\forall i = 1, ..., N\ \ \tilde{s}_i^{(t)} = \sum_{j=1}^{N} \sum_{l_i^j=1}^{L_i^j} \left( a_j^{\left(t-l_i^j\right)} \tilde{s}_j^{\left(t-l_i^j\right)} + \tilde{b}_j^{\left(t-l_i^j\right)} \right),$$

where $a_j^{\left(t-l_i^j\right)}$ – fuzzy regression coefficients; $\tilde{b}_j^{\left(t-l_i^j\right)}$ – fuzzy free terms (usually equal to 0).

Obtained values of fuzzy coefficients $a_j^{\left(t-l_i^j\right)}$ of regression are then normalized and reduced to a range [0, 1]. After that, time lags whose modal values of fuzzy coefficients are less than a certain threshold are excluded from consideration (for the example under consideration, less than 0.4).

And, thus, a subset of time lags is defined corresponding to these fuzzy values $W_{ij} = \left\{ w_{ij}^{\left(t-l_i^j\right)} \mid l_i^j = 0, ..., L_i^j \right\}$ of the influence indicators of the source-concept $C_j$ on the concept-receiver $C_i$. The formed matrix $W$ of fuzzy relations for the NFCTM concepts of urban environment state is presented in Table 1. The formed matrix W of fuzzy relations of influence between the NFCTM concepts.

**Stage 3.** The formation of the NFCTM structure.

The formation of the NFCTM structure consists in setting subsets of arcs (corresponding to the time lag) between the concepts of NFCTM, weighted with fuzzy values $w_{ij}^{\left(t-l_i^j\right)}$ of their influence on each other. The structure of the NFCTM for multidimensional forecasting of the urban environment state in Moscow is shown in Figure 1.

**Stage 4.** The designing of *Rec*ANFISs $FS_i$ for implementing fuzzy temporal transformations $\tilde{F}_i$.

**Table 1.** The formed matrix W of *fuzzy* relations of influence between the NFCTM concepts

| *W* | $l_i^j$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|---|
| $C_1$ | 1 | 0 | 0.75 | 0 | 0.52 | 0 | 0 |
| | 2 | 0 | 0.84 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0.40 | 0 | 0.40 | 0 | 0 |
| $C_2$ | 1 | 0 | 0 | 0.79 | 1.0 | 0 | 0 |
| | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0.52 | 0.57 |
| $C_3$ | 1 | 0.55 | 0 | 0.68 | 0.50 | 0.40 | 0.43 |
| | 2 | 0 | 1.0 | 0 | 0.46 | 0 | 0 |
| | 3 | 0.61 | 0 | 0 | 0.88 | 0.99 | 0 |
| $C_4$ | 1 | 0 | 0.48 | 0.67 | 0.79 | 0 | 0 |
| | 2 | 0 | 0.41 | 0 | 0.43 | 0 | 0 |
| | 3 | 0.41 | 0.40 | 0 | 0.54 | 0.49 | 0 |
| $C_5$ | 1 | 0 | 0.68 | 0.62 | 0.42 | 0.45 | 1.00 |
| | 2 | 0 | 0.40 | 0 | 0 | 0.48 | 0 |
| | 3 | 1.00 | 1.00 | 1.00 | 0.47 | 1.00 | 0.54 |
| $C_6$ | 1 | 0 | 0 | 0 | 0 | 0.53 | 0.59 |
| | 2 | 0 | 0 | 0 | 0 | 0.51 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0 | 0 |

**Notes**

1. For clarity, Table 1 shows only modal values of fuzzy degrees of influence (without their blurring degrees) between the time lags of the MTS components.
2. Modal values and the degree of blurring of fuzzy influence indicators are further changed in the process of parametric setting of the NFCTM.

As *Rec*ANFISs $FS_i$, that implement fuzzy temporal transformations $\tilde{F}_i$, original neuro-fuzzy *Rec*ANFISs are offered. These models provide the formation, storage and output of the predicted fuzzy values of the MTS components with the time delays required for NFCTM, taking into account fuzzy trends.

Input temporal fuzzy variables of the *Rec*ANFIS $FS_i$ of the concept $C_i$ are related to the output temporal fuzzy variables of those concepts that directly affect the concept $C_i$. In the process, the input temporal fuzzy variables $C_i$ are pre-"weighted" by the corresponding fuzzy degrees of influence $w_{ij}^{\left(t-l_i^j\right)}$:

$$\tilde{s}'^{\left(t-l_i^j\right)}_j = \left( w_{ij}^{\left(t-l_i^j\right)} \mathrm{T} \; \tilde{s}_j^{\left(t-l_i^j\right)} \right), \; l_i^j = 0, ..., L_i^j .$$

The output temporal fuzzy variables of the *Rec*ANFIS $FS_i$ of the $C_i$ are intended for generating, storing, and displaying predicted values of the *i*-th MTS component corresponding to reasonable time lags (see Table 1).

To designing *Rec*ANFISs $FS_i$ a priori information about the MTS components received from experts can be used, as well as data obtained because of evaluation or measurements.

In the first case, it is assumed that the problem of ensuring the completeness and inconsistency of the database of fuzzy rules of the *Rec*ANFIS $FS_i$ has been solved in advance. If only experimental data is known, then the task is to identify this model. Then to solve this problem, well-known methods can be applied to extract fuzzy rules from training samples, for example, fuzzy clustering [19]; fuzzy self-organizing maps T. Kohonen [20]; adaptive fuzzy associative memory B. Kosko [20]; method of gradually increasing splitting of the feature space [22] and others.

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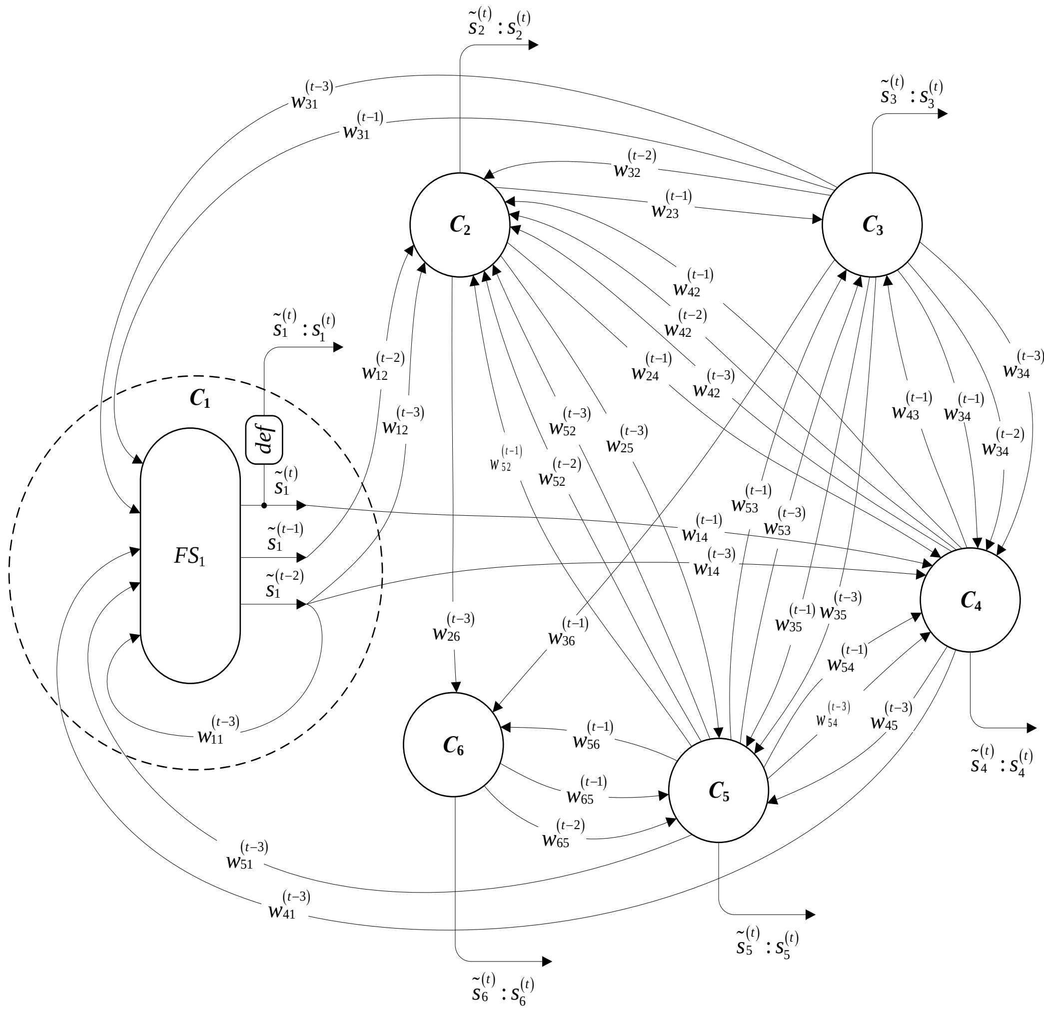


**Fig. 1.** The structure of the NFCTM for multidimensional forecasting of the urban environment state in Moscow

There is often a mixed case when the initial database of model rules is constructed based on heuristic assumptions, and its parametric setting (training) is carried out based on a training sample.

Let's consider this particular case using the example of constructing the structure and then parametric configuration of the *Rec*ANFIS $FS_1$.

The input fuzzy variables of the *Rec*ANFIS $FS_1$ are $S_1' = \left\{ s_3'^{(t-1)}, s_3'^{(t-3)}, s_4'^{(t-3)}, s_5'^{(t-3)}, s_1'^{(t-3)} \right\}$, and its output fuzzy variables are $S_1 = \left\{ \tilde{s}_1^{(t)}, \tilde{s}_1^{(t-1)}, \tilde{s}_1^{(t-2)} \right\}$.

As an example, a separate fuzzy rule is given for the *Rec*ANFIS $FS_1$ for the concept $C_1$ of the NFCTM:

$$\text{IF}\left(\tilde{s}_1'^{(t-1)} \textit{ is } \tilde{L}\right)\text{AND}\left(\tilde{s}_3'^{(t-3)} \textit{ is } \tilde{L}\right)\text{AND}$$
$$\left(\tilde{s}_4'^{(t-3)} \textit{ is } \tilde{M}\right)\text{AND}\left(\tilde{s}_5'^{(t-3)} \textit{ is } \tilde{M}\right)\text{AND}$$
$$\left(\tilde{s}_1'^{(t-3)} \textit{ is } \tilde{H}\right),$$
$$\text{THEN } \left(\tilde{s}_1^{(t)} \textit{ is } \tilde{M}\right)\text{AND}$$
$$\left(\tilde{s}_1^{(t-1)} \textit{ is } \tilde{M}\right)\text{AND}$$
$$\left(\tilde{s}_1^{(t-2)} \textit{ is } \tilde{L}\right),$$

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with $\tilde{L}, \tilde{M}, \tilde{H}$ – fuzzy sets for the rules of the *Rec*ANFIS $FS_1$. The Figure 2 shows an example of the structure of the proposed *Rec*ANFIS $FS_1$.

The structure of *Rec*ANFIS $FS_1$ consists of the following layers of elements.

*Layer 1*. At the output of elements of this layer, the degrees of truth are determined for the current values of input variables relative to the corresponding fuzzy statements assumptions and all model rules:

$$\mu_{\tilde{L}}\left(\tilde{s}_1^{(t-1)}\right) = \tilde{s}_1^{(t-1)} \cap \tilde{L},\ \mu_{\tilde{M}}\left(\tilde{s}_1^{(t-1)}\right) = \tilde{s}_1^{(t-1)} \cap \tilde{M},\ \mu_{\tilde{H}}\left(\tilde{s}_N^{\left(t-L_i^j\right)}\right) = \tilde{s}_N^{\left(t-L_i^j\right)} \cap \tilde{H};$$
$$\dots$$
$$\mu'_{\tilde{L}}\left(\tilde{s}_i^{(t-1)}\right) = \psi_i^{(t-1)}\left(\mu_{\tilde{L}}\left(\tilde{s}_i^{(t-1)}\right), \tilde{T}_i^{(t-1)}\right), \mu'_{\tilde{M}}\left(\tilde{s}_i^{(t-1)}\right) = \psi_i^{(t-1)}\left(\mu_{\tilde{M}}\left(\tilde{s}_i^{(t-1)}\right), \tilde{T}_i^{(t-1)}\right), \mu'_{\tilde{H}}\left(\tilde{s}_i^{(t-1)}\right) = \psi_i^{(t-1)}\left(\mu_{\tilde{H}}\left(\tilde{s}_i^{(t-1)}\right), \tilde{T}_i^{(t-1)}\right);$$
$$\dots$$
$$\mu'_{\tilde{L}}\left(\tilde{s}_i^{(t-\tau)}\right) = \psi_i^{(t-\tau)}\left(\mu_{\tilde{L}}\left(\tilde{s}_i^{(t-\tau)}\right), \tilde{T}_i^{(t-\tau)}\right), \mu'_{\tilde{L}}\left(\tilde{s}_i^{(t-\tau)}\right) = \psi_i^{(t-\tau)}\left(\mu_{\tilde{M}}\left(\tilde{s}_i^{(t-\tau)}\right), \tilde{T}_i^{(t-\tau)}\right), \mu'_{\tilde{H}}\left(\tilde{s}_i^{(t-\tau)}\right) = \psi_i^{(t-\tau)}\left(\mu_{\tilde{H}}\left(\tilde{s}_i^{(t-\tau)}\right), \tilde{T}_i^{(t-\tau)}\right);$$
$$\dots$$
$$\mu_{\tilde{L}}\left(\tilde{s}_N^{(t-1)}\right) = \tilde{s}_N^{(t-1)} \cap \tilde{L},\ \mu_{\tilde{M}}\left(\tilde{s}_N^{(t-1)}\right) = \tilde{s}_N^{(t-1)} \cap \tilde{M},\ \mu_{\tilde{H}}\left(\tilde{s}_N^{\left(t-L_i^j\right)}\right) = \tilde{s}_N^{\left(t-L_i^j\right)} \cap \tilde{H}.$$

where $\tilde{s}_i^{(t-1)}$ – fuzzy temporal variables; $\cap$ – the union operator of fuzzy variables; $\mu_{\tilde{L}}\left(\tilde{s}_i^{(t-1)}\right)$ – linguistic term of a fuzzy input variable; $\tilde{T}_i^{t-\theta}, \theta = 1, .., \tau$ – a fuzzy tendency detected using the element $FT_i^{\theta}$, $\theta \in 1, .., \tau$; $\tau$ – range of "sliding window" of the *i*-th MTS component; $\mu'_{\tilde{L}}\left(\tilde{s}_i^{(t-1)}\right)$ – modified term based on fuzzy tendency; $\psi_i^{\theta}$ – fuzzy mapping operator of a fuzzy variable taking into account the detected fuzzy tendency, $\theta = 1..\tau$; $\tilde{L}, \tilde{M}, \tilde{H}$ – examples of linguistic terms of input variables.

The fuzzy mapping operator $\psi_i^{\theta}$ modifies the fuzzy variable $\tilde{s}_1^{(t-\theta)}$ in accordance with the identified fuzzy tendency in the range $\tau$ of the "sliding window". For example, the following types of fuzzy trends can be represented: growth, weak growth, stability, slight fall, fall. Fuzzy displays with fuzzy trends are illustrated in Figure 3.

*Layer 2.* Layer elements are intended for aggregation based on the T-norm operation (here, min-conjunction) of the truth degrees of rule assumptions. For the *p*-th rule in question ($p = 1, ..., P$).

*Layer 3.* Layer elements activate conclusions of the corresponding rules according to the truth degrees of their assumptions based on the operation.

For the rule in question:

$$\alpha_p = \min\begin{pmatrix} \mu_{\tilde{L}}\left(\tilde{s}'^{(t-1)}_1\right), \mu_{\tilde{L}}\left(\tilde{s}'^{(t-3)}_3\right), \mu_{\tilde{M}}\left(\tilde{s}'^{(t-3)}_4\right), \\ \mu_{\tilde{M}}\left(\tilde{s}'^{(t-3)}_5\right), \mu_{\tilde{H}}\left(\tilde{s}'^{(t-3)}_1\right) \end{pmatrix},$$

$$\mu_{\tilde{M}}\left(\tilde{s}_1^{(t)}\right) = \min\left(\alpha_p, \tilde{M}\right).$$

*Layer 4.* The layer element carries out the max-disjunction operation, accumulating the activated conclusions of all the model rules:

$$\tilde{s}_1^{(t)} = \max\left(\mu_{\tilde{L}}\left(\tilde{s}_1^{(t)}\right), ..., \mu_{\tilde{M}}\left(\tilde{s}_1^{(t)}\right), ..., \mu_{\tilde{H}}\left(\tilde{s}_1^{(t)}\right)\right).$$

*Layer 5*. Elements are designed to detect a fuzzy trend based on values $Z^t\left(\tilde{s}_i^{(t)}\right), ..., Z^{t-\tau}\left(\tilde{s}_i^{(t)}\right)$ with time delay $\theta = 1, ..., \tau$, implemented using elements *Z*:

$$\tilde{T}_i^{t-\theta} = FT_i^{t-\theta}\left(Z^t\left(\tilde{s}_i^{(t)}\right), ..., Z^{t-\theta}\left(\tilde{s}_i^{(t)}\right), ..., Z^{t-\tau}\left(\tilde{s}_i^{(t)}\right)\right),$$
$$\theta \in 1, ..., \tau.$$

**Note:** Hereinafter (as before) for normalized values $s_{i,norm}^{(t)}$ will use the notation $s_i^{(t)}$.

In addition to the above, the value of the output fuzzy variable $\tilde{s}_i^{(t)}$ of the componential temporal model $FS_i$ of each component $C_i$ is defuzzified (reduced to a crisp value $s_i^{(t)}$) using the "center of gravity" method [23].

Thus, a set of values $\left\{s_i^{(t)} \mid i = 1, ..., N\right\}$ at the output of the corresponding models $\left\{FS_i \mid i = 1, ..., N\right\}$ comprehensively characterizes the predicted state of stability of the urban environment at instant of time t.

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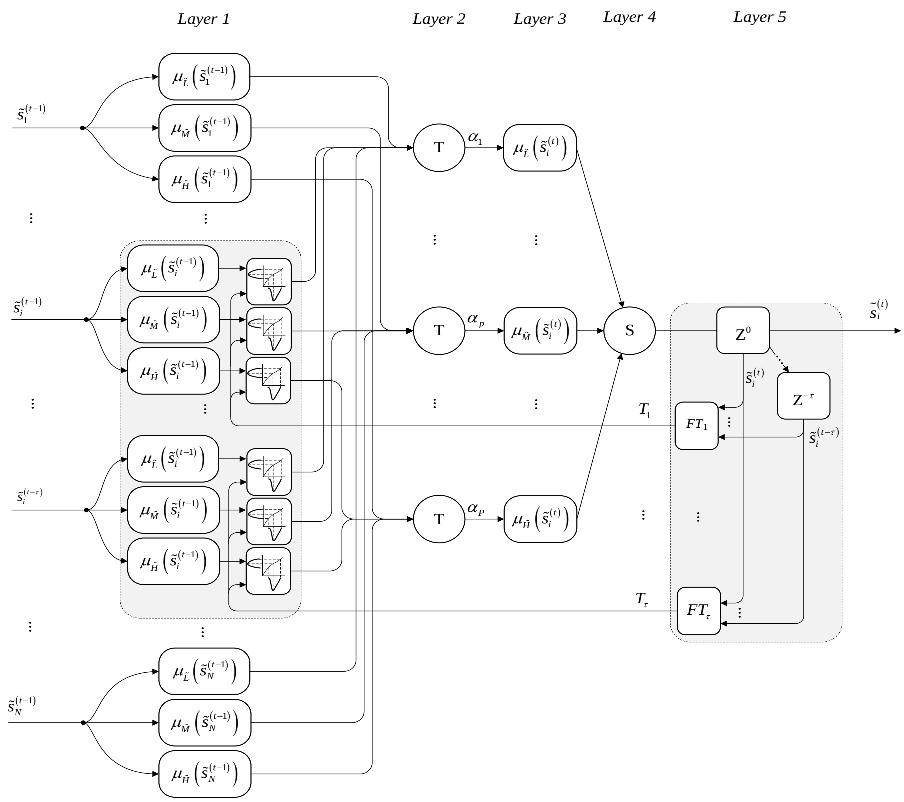


**Fig. 2.** The structure of the *Rec*ANFIS

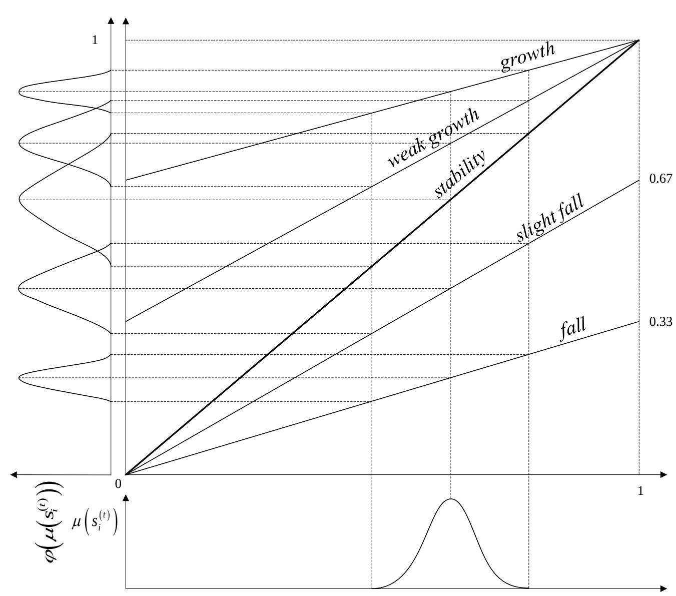


**Fig. 3.** Illustration of fuzzy mappings depending on the fuzzy tendency

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**Table 2.** Fragment of the training sample for the *Rec*ANFIS $FS_1$

| Example number, $k$ | Values of input variables | | | | | | Values of output variable, $\tilde{s}_1'^{(t)}(k)$ |
|---|---|---|---|---|---|---|---|
| | $\tilde{s}_3'^{(t-1)}(k)$ | $\tilde{s}_3'^{(t-3)}(k)$ | $\tilde{s}_4'^{(t-3)}(k)$ | $\tilde{s}_5'^{(t-3)}(k)$ | $s_1'^{(t-3)}(k)$ | $\tilde{T}_1^1$ | |
| 1 | 0.50 | 0.50 | 0.49 | 0.70 | 0.54 | *growth* | 0.49 |
| 2 | 0.50 | 0.50 | 0.50 | 0.69 | 0.54 | *growth* | 0.50 |
| 3 | 0.50 | 0.50 | 0.51 | 0.69 | 0.54 | *fall* | 0.51 |
| 4 | 0.46 | 0.50 | 0.49 | 0.68 | 0.55 | *stability* | 0.49 |
| 5 | 0.46 | 0.50 | 0.49 | 0.69 | 0.54 | *growth* | 0.49 |
| 6 | 0.46 | 0.46 | 0.58 | 0.69 | 0.56 | *growth* | 0.58 |
| 7 | 0.46 | 0.46 | 0.63 | 0.67 | 0.56 | *stability* | 0.63 |
| 8 | 0.44 | 0.46 | 0.62 | 0.68 | 0.56 | *stability* | 0.62 |
| 9 | 0.45 | 0.46 | 0.63 | 0.68 | 0.54 | *fall* | 0.63 |
| 10 | 0.42 | 0.44 | 0.56 | 0.67 | 0.52 | *stability* | 0.56 |
| *K* | 0.58 | 0.58 | 0.55 | 0.89 | 0.49 | *growth* | 0.55 |

**Table 3.** Fragment of the training sample for the *Rec*ANFIS $FS_1$

| № | MTS components | Forecasting error, *MAPE*, % | |
|---|---|---|---|
| | | ANN | NFCTM |
| 1. | Ecology of the urban environment | 7.40 | 6.91 |
| 2. | The infrastructure power of the urban environment | 1.51 | 0.13 |
| 3. | Income level of the population | 8.72 | 9.85 |
| 4. | Industrial consumption of fuel and energy resources | 2.35 | 1.62 |
| 5. | Population life quality | 2.12 | 0.55 |
| 6. | Sanitary and epidemiological situation | 5.35 | 5.31 |

Notice also that in contrast to the Fuzzy Cognitive Maps of B. Kosko [23, 24], the use of the approach suggested above eliminates the need to separately account for positive and negative effects by applying the *Rec*ANFIS for each MTS component.

**Stage 5.** Parametric setting (training) of *Rec*ANFISs.

The training procedure for each *Rec*ANFIS is preceded by a procedure for forming a training sample to configure the corresponding models for detecting fuzzy trends $FT_i^{(t-\theta)}, \theta = 1, ..., \tau$:

$$\left(\tilde{s}_i'^{(t-1)}(k), ..., \tilde{s}_i'^{(t-\tau)}(k), T_i^{t-\theta}(k)\right),$$
$$k = 1..K,\ \theta = 1, ..., \tau.$$

After generating sets of training examples, the construction of models $FS_i^{(t-\theta)}$ for example, fuzzy rule-based models, is carried out:

$$\text{IF}\left(\tilde{s}_i'^{(t)}\, is\ \tilde{L}\right)\text{AND...AND}\left(\tilde{s}_i'^{(t-\tau)}\, is\ \tilde{L}\right)\text{THEN}\left(\tilde{T}\ is\ \text{"}fall\text{"}\right),$$
$$...$$
$$\text{IF}\left(\tilde{s}_i'^{(t)}\, is\ \tilde{M}\right)\text{AND...AND}\left(\tilde{s}_i'^{(t-\tau)}\, is\ \tilde{M}\right)\text{THEN}\left(\tilde{T}\ is\ \text{"}stability\text{"}\right),$$
$$...$$
$$\text{IF}\left(\tilde{s}_i'^{(t)}\, is\ \tilde{H}\right)\text{AND...AND}\left(\tilde{s}_i'^{(t-\tau)}\, is\ \tilde{H}\right)\text{THEN}\left(\tilde{T}\ is\ \text{"}growth\text{"}\right).$$

The identification and use of fuzzy trends eliminates the problem of non-stationarity of the predicted time series, in addition, it is a feedback that gives system stability.

After that, a training sample for *Rec*ANFISs is built based on the prepared training sample together with the identified fuzzy trends.

Using retrospective data, training samples are built for each model $FS_i^{(t-\theta)}$ of the detection of fuzzy trends.

After detecting fuzzy trends, a training sample is built for neuro-fuzzy models of the *Rec*ANFIS. Table 2 shows a fragment of the formed training sample for the fuzzy componential temporal model $FS_1$. Note that, for clarity, Table 2 shows only modal values (without degrees of their blurring) of the input and output temporal variables of the model $FS_1$.

Note that for *Rec*ANFIS $FS_i$, the configurable parameters are the parameters of the membership functions of antecedents and consequents.

**Stage 6.** Reconciliation of all fuzzy *Rec*ANFISs.

Reconciliation of all *Rec*ANFISs $FS_i, i=1,...,N$ of the NFCTM is carried out after their “personalized” parametric setting and consists in such a change in parameters of fuzzy degrees of influence $\left\{w_{ij}^{(t-l_i^j)} \mid l_i^j = 0, ..., L_i^j\right\}$ between NFCTM concepts to ensure the maximization of the prediction accuracy of each of the MTS components without deterioration of prediction accuracy of at least one of the other MTS components. The genetic algorithm can be used for this purpose [25].

**Stage 7.** Prediction of MTS.

Prediction of MTS can be carried out in the following modes:

– multidimensional forecasting for the *t*-th instant of time, i.e. calculating the values of the output variables of models $FS_i, i=1,...,N$ by the corresponding sets of values of the input variables of these;
– self-development and predictive assessment, in which dynamics modeling is carried out from certain situation set by the initial values of all the NFCTM concepts, in the absence of external influences on it.

Predictive assessment in its dynamics are modeled from certain situation set by the initial values of all NFCTM concepts, with external influence on the values of concepts and/or influence relations between NFCTM concepts.

## 5 The Multidimensional Forecasting of the Urban Environment State in Moscow

Experiments have been carried out and the results of using the NFCTM for multi-dimensional forecasting of the urban environment state in Moscow in a complex epidemiological situation have been obtained (Figure 4).

Table 3 presents a comparative assessment of the results of multidimensional forecasting of the urban environment state in Moscow (based on historical data from 2000 to 2020) using an artificial neural network (ANN) and developed by the NFCTM. As a comparison, a multilayer perceptron with a single hidden layer of 16 neurons was used, which showed the best among the various variants of ANN.

The obtained results of the comparative evaluation showed that the use of NFCTM in small samples allows increasing the accuracy of the MTS forecast by compared to the multilayer perceptron with one hidden layer of 16 neurons, which showed the best among various variants of ANN.

The new type of Neuro-Fuzzy Cognitive Temporal Models is proposed. These models are focused on multidimensional prediction of MTS in conditions of non-stochastic uncertainty, non-linearity of interaction, partial inconsistency and significant interdependence of MTS components, as well as in conditions of small samples with fuzzy trends.

To implement fuzzy temporal transformations of concepts, the proposed original *Rec*ANFISs are used, which provide the formation, storage and output of the predicted fuzzy values of the corresponding MTS components with the time delays required for NFCTM, taking into account fuzzy trends.

The method of constructing, training and using NFCTM for MTS predicting taking into account fuzzy trends is considered, which includes:

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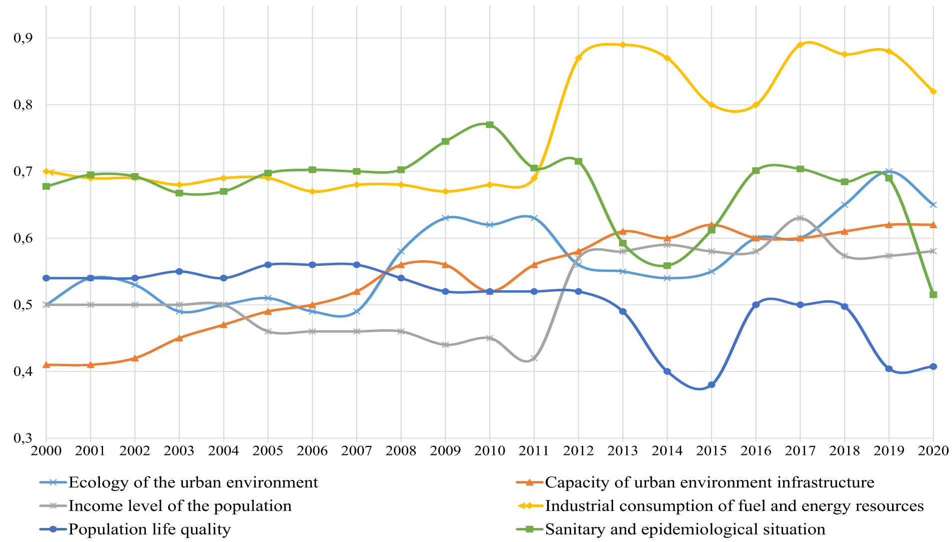


**Fig. 4.** Illustration of the results of multidimensional forecasting of the urban environment state in Moscow

- the formation of the structure of the NFCTM, which consists in setting the structural relations (in the form of displayed time lags) between the concepts of the NFCTM, weighted by fuzzy values of their influence on each other;
- construction and parametric adjustment of RecANFIS for the implementation of fuzzy temporal transformations of MTS components;
- the coordination of all fuzzy componential temporal models of NFCTM, which consists in such a change of parameters of fuzzy degree of influence between concepts NFCTM to ensure the maximization of the prediction accuracy of each of the MTS components without deterioration of prediction accuracy of at least one of other MTS components.

## 6 Conclusion

Prediction of MTS is based on a structurally and parametrically configured NFCTM and can be carried out in the following modes:

- firstly, direct multidimensional forecasting for the t-th instant of time, i.e. calculating the values of the output variables of NFCTM models $FS_i, i = 1, ..., N$ by the corresponding sets of values of the input variables of these models set each time;
- secondly, self-development and predictive assessment, in which dynamics modeling is carried out from certain situation set by the initial values of all the NFCTM concepts, in the absence of external influences on it;
- thirdly, a predictive assessment in which dynamics are modeled from certain situation set by the initial values of all NFCTM concepts, with external influence on the values of concepts and/or influence relations between NFCTM concepts.

Experimental studies have been carried out and the results of using the proposed NFCTM for multidimensional forecasting of the urban environment state in Moscow in conditions of a complex epidemiological situation have been obtained.

Comparative assessment showed that in conditions of small samples the use of NFCTM allows improving the forecast accuracy of MTS compared with the neural network approach, which demonstrated one of the best results in solving this task.

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## References

1. **Bickel, P.J. & Doksum, K.A. (2015).** Chapman and Hall/CRC, Vol. 1, pp. 1–556.
2. **Box, E., Jenkins, G.M., Reinsel, G.C., & Ljung, G.M. (2015).** *Time Series Analysis: Forecasting and Control.* John Wiley & Sons, pp. 1–712.
3. **Hajek, P. & Prochazka, O. (2018).** Interval-valued intuitionistic fuzzy cognitive maps for supplier selection. **Czarnowski I., Howlett R., Jain L. (eds)** *Intelligent Decision Technologies 2017. IDT 2017. Smart Innovation, Systems and Technologies,* Vol. 72. DOI:10.1007/978-3-319-59421-7_19.
4. **Pedrycz, W., Lu, W., Liu, X., Wang, W., & Wang, L. (2014).** Human-centric analysis and interpretation of time series: a perspective of granular computing. *Soft Computing,* Vol. 18, No. 12, pp. 2397–2411. DOI:10.1007/s00500-013-1213-5.
5. **Batyrshin, I., Solovyev, V., & Ivanov, V. (2016).** Time series shape association measures and local trend association patterns. *Neurocomputing,* Vol. 175-B, pp. 924–934. DOI:10.1016/j.neucom.2015.05.127.
6. **Batyrshin, I., Sheremetov, L., & Herrera-Avelar, R. (2007).** Perception based patterns in time series data mining. *Proceeding Studies in Computational Intelligence (SCI, Perception-based Data Mining and Decision Making in Economics and Finance,* Springer, Vol. 36, pp. 85–118. DOI:10.1007/978-3-540-36247-0_3.
7. **Murat, M. & Asan, U. (2021).** A compromis a compromise-based new approach to learning fuzzy cognitive maps. *Advances in Intelligent Systems and Computing, International Conference on Intelligent and Fuzzy Systems,* Vol. 1197, pp. 1172–1180. DOI: 10.1007/978-3-030-51156-2_137.
8. **Papageorgiou, E.I. (2014).** *Fuzzy Cognitive Maps for Applied Sciences and Engineering.* Intelligent Systems Reference Library, Vol. 54, pp. 395.
9. **Averkin, A.N. & Yarushev, S.A. (2017).** Hybrid approach for time series forecasting based on ANFIS and Fuzzy Cognitive Maps. *Proceeding 20th IEEE International Conference on Soft Computing and Measurements (SCM),* pp. 379–381. DOI:10.1109/SCM.2017.7970591.
10. **Borisov, V.V. & Luferov, V.S. (2020).** The method of multidimensional analysis and forecasting states of complex systems and processes based on fuzzy cognitive temporal models. *Systems of Control, Communication and Security,* Vol. 2, pp. 1–23.
11. **Abdalla, A. & Buckley, J.J. (2007).** Monte carlo methods in fuzzy linear Regression. *Proceeding Soft Computing,* Vol. 12, No. 5, pp. 991–996. DOI: 10.1007/s00500-006-0148-5.
12. **Kasa, S.R., Bhattacharya, S., & Rajan, V. (2019).** Gaussian mixture copulas for high-dimensional clustering and dependency-based subtyping. *Bioinformatics,* Vol. 36, No. 2, pp. 621–628. DOI: 10.1093/bioinformatics/btz599.
13. **Zolekar, R.B. & Bhagat, V.S. (2015).** Multi-criteria land suitability analysis for agriculture proceeding hilly zone. Remote sensing and gis approach. *Computers and Electronics in Agriculture,* Vol. 118, pp. 300–321. DOI:10.1016/j.compag.2015.09.016.
14. **Renn, O., Lucas, K., Haas, A., & Jaeger, C. (2019).** Things are different today. The challenge of global systemic risks. *Journal of Risk Research*, Vol. 22, No. 4, pp. 401–415. DOI:10.1080/13669877.2017.1409252.
15. **Pesce, M., Critto, A., Torresan, S., Giubilato, E., Santini, M., Zirino, A., Ouyang, W., & Marcomini, A. (2018).** Modelling climate change impacts on nutrients and primary production in coastal waters. *Science of the Total Environment,* Vol. 628–629, pp. 919–937. DOI:10.1016/j.scitotenv.2018.02.131
16. **Klimenko, V.V., Klimenko, A.V., Tereshin, A.G., & Mitrova, T.A. (2019).** Impact of climate changes on the regional energy balances and energy exports from russia. *Thermal Engineering,* Vol. 66, No. 1, pp. 3–15. DOI: 10.1134/S00406015 1901004X.
17. **Borisov, V.V., Stefantsov, A.G., Bobryakov, A.V., & Luferov, V.S. (2019).** A system of fuzzy cognitive analysis and modeling of system dynamics. *Proceeding of the 21th Int. Conf. Problems of Control and Modeling in Complex Systems,* Vol. 2475, pp. 312–316.
18. **Fahrmeir, L., Kneib, T., & Lang, S. (2009).** *Regression modelle. Methoden und Anwendungen.* DOI:10.1007/978-3-540-33933-5_2.
19. **Bezdek, J.C., Keller, J., Krisnapuram, R., & Pal, N.R. (2005).** *Fuzzy Models and Algorithms for Pattern Recognition and Image Processing*. Springer Science and Business Media.
20. **Bezdek, J.C., Tsao, E.C.K., & Pal, N.R. (1992).** Fuzzy Kohonen clustering networks. *Proceeding IEEE International Conference on Fuzzy Systems,* pp. 1035–1043. DOI:10.1109/FUZZY.1992.258797.

ISSN 2007-9737



21. **Kosko, B. (1992).** Neural networks and fuzzy systems. *The Journal of the Acoustical Society of America 1*, Vol. 103, No. 6, pp. 3131. DOI:10.1121/1.423096.
22. **Nelles, O., Fink, A., Babuška, R., & Setnes, M. (2000)** Comparison of two construction algorithms for Takagi-Sugeno fuzzy models. *International Journal of Applied Mathematics and Computer Science,* Vol. 10, No. 4, pp. 835–855.
23. **Borisov, V.V., Kruglov, V.V., & Fedulov, A.S. (2018).** *Fuzzy models and networks*. Stereotype, Moscow, HotLine-Telecom.
24. **Kosko, B. (1986).** Fuzzy cognitive maps. *International Journal of Man-Machine Studies*, Vol. 24, pp. 65–75.
25. **Stach, W., Kurgan, L., Pedrycz, W., & Reformat, M. (2005).** Genetic learning of fuzzy cognitive maps. *Fuzzy Sets and Systems,* Vol. 153, No. 3, pp. 371–401. DOI:10.1016/j.fss.2005.01.009.

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