Is the Coordinated Clusters Representation an analog of the Local Binary Pattern?

¿La Representación de Imágenes por Cúmulos Coordinados es análogo al de Patrones Binarios Locales?

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Abstract. Both the Local Binary Pattern (LBP) and the Coordinated Clusters Representation (CCR) are two methods used successfully in the classification and segmentation of images. They look very similar at first sight. In this work we analyze the principles of the two methods and show that the methods are not reducible to each other. Topologically they are as different as a sphere and a torus. In extracting of image features, the LBP uses a specific technique of binarization of images with the local threshold, defined by the central pixel of a local binary pattern of an image. Then, the central pixel is excluded of each local binary pattern. As a consequence, the mathematical basis of the LBP method is more limited than that of the CCR. In particular, the scanning window of the LBP has always an odd dimensions, while the CCR has no this restriction. The CCR uses a binarization as a preprocessing of images, so that a global or a local threshold can be used for that purpose. We show that a classification based on the CCR of images is potentially more versatile, even though the high performance of both methods was demonstrated in various applications.

Keywords: Texture Image Analysis, Classification, Segmentation, Coordinated Clusters Representation, Local Binary Patterns.

Resumen. La Representación de Imágenes por Cúmulos Coordinados (RICC) y el Local Binary Pattern (LBP) son métodos eficazmente usados para la clasificación y segmentación de imágenes. A primera vista éstos parecen muy similares. Con un análisis de los principios de dos métodos demostramos que no son reducibles uno a otro; en términos de topología matemática son tan diferentes como esfera y dona. En la etapa de extracción de características de una imagen, el LBP usa una técnica específica de binarización de imágenes con umbral local, que se define por el píxel central de un patrón local de la imagen. Después, el píxel central se excluye de cada patrón local. Por tanto, el sustento matemático del método de LBP es más limitado que el de la RICC. En particular, la ventana de barrido en LBP siempre tiene dimensiones impares, la de la RICC no tiene esta restricción. La RICC requiere la binarización como una etapa de preprocesado de imagen y, por tanto, puede usarse un umbral global o local adaptable. La clasificación basada en la RICC es más versátil, aunque las eficiencias terminales de clasificación por los dos métodos pueden ser muy cercanas en muchas aplicaciones.

Palabras clave: Análisis de Imágenes de Textura, Clasificación, Segmentación, Representación de Imágenes por Cúmulos Coordinados, Patrones Binarios Locales.

1 Introduction

In general, classification implies the assignment of an object (image) to one of the predefined classes. Classification consists of learning and recognition phases. In the first, features are extracted from a set of texture images with known class labels, each class being characterized by its prototype feature vector. Then, in the recognition phase, a feature vector of test image is calculated and one of the known classifiers is used to assign the image to the class it matches best. Classification is related closely to the following three concepts. By recognition we mean the identification of an image among a set of test images. Clustering distributes images into groups of similar images. Segmentation is the partitioning of an image into a set of regions with similar visual properties. Any classification requires a set of features that permits the discrimination between the images of different type. So the problem of establishing an adequate set of characteristics is of great practical importance. The

techniques of feature extraction for texture description and analysis can be divided into the four mayor groups: statistical, model based, signal processing and structural methods (Tuceryan and Jain, 1993). Once the features of images are selected, a classification can be done by means of one of several known methods (Chen et al., 1996; Duda et al., 2001; Fukunaga, 1990; Young and Fu, 1986).

A natural texture image has usually a significant statistical component in the intensity and color distribution that complicates its classification. Hence statistical techniques, based mainly on correlation moments and co-occurrence matrices, are widely used in the classification of texture images (Berry and Goutsias, 1989; Chetverikov, 1999; Elfadel and Picard, 1994; Goon and Rolland, 1999; Haralick, 1979; Ohanian and Dubes, 1992; Soh and Tsatsoulis, 1999; Valkealahti and Oja, 1998). Other techniques are based on models of Markov random fields (see, for example (Chellappa and Chatterjee, 1987)), window transforms (Turner, 1986; Azencott and Wang, 1997; Gonzalez-Garcia *et al*., 2007). Among the statistical methods of feature extraction and classification of images, the recently developed LBP/C and CCR were proven to be practical and efficient. At first sight they look similar. The purpose of this work is to analyze the principles of the two methods and show that they are not reducible to each other; then, to show limitations and potential advantages of each one. This analysis provides a deeper understanding of these methods and helps to find out its most efficient applications. The paper is organized as follows. Section 2 describes briefly the CCR method followed by the description of the LBP/C in Section 3. Comparative analysis of the LBP and the CCR is given in Section 4. Conclusions are presented in Section 5.

2 Coordinated Clusters Representation of Images

The motivating idea of the coordinated cluster transform is that any classification or recognition of an image implies a kind of comparative correlation analysis of image regions (neighborhoods), and those must overlap. An application of the coordinated cluster representation (CCR) to the problems of analysis and classification of binary images was reported in (Kurmyshev and Cervantes, 1996; Kurmyshev and Soto, 1996) for the first time. The origin of the transform dates back to an earlier work (Kurmyshev et al., 1985), though this was done for the characterization and analysis of amorphous solids. Further, the CCR was developed into an efficient method of analysis, recognition and classification of gray level and color texture images (Kurmyshev and Sánchez-Yáñez, 2001; Kurmyshev et al., 2003; Kurmyshev and Sánchez-Yáñez, 2005; Sánchez et al, 2003a, 2003b). Note that the term "coordinated clusters" originates from solid state physics. We present here a resume of the CCR in order to facilitate a conceptual comparison of the CCR with the LBP. The matrix representation of the coordinated cluster transform is given also.

Let $S = [s_{lm}]$ be a matrix of binary image intensities, where $l = 1, 2, .., L$ and $m = 1, 2, .., M$ are the dimensions of the image. Each pixel can take one of the values $(0,1)$. In order to calculate the CCR of a binary image S we first establish a rectangular window of size $N = I$ x J and then scan sequentially, by means of this window, all over the image S with one pixel step. The coordinated cluster transform generates the histogram of occurrence of binary pattern units detected through the scanning window. This histogram is called the coordinated cluster representation of an image. A binary pattern unit (BPU) consists of $N = I$ x J pixels. There are 2^N BPU that describe an image that is $2^9 = 512$ units for the neighborhood of 3x3 pixels. This number defines the length of the histogram. Every BPU is coded by a decimal number. To calculate the code, the BPU matrix is multiplied by the mask of potentials of 2, element by element, and results are summed.

The number 2^N defines the length of the primary CCR histogram that can be reduced by eliminating the BPUs of zero occurrences. When normalized by the number $\tilde{W} = (L - I + 1) \times (M - J + 1)$, the CCR histogram can be considered as a probability distribution function of occurrences: $F_{(I,J)}(b) = \tilde{W}^{-1} H_{(I,J)}(b)$, where $\tilde{W}$ is the total number of occurrences, the subscript *(I,J)* indicates the size of the scanning window and b (= 1, 2, 2^N) is the decimal code of BPU. Figure 1 shows a step of scanning of a binary image and a decimal code of the BPU detected.
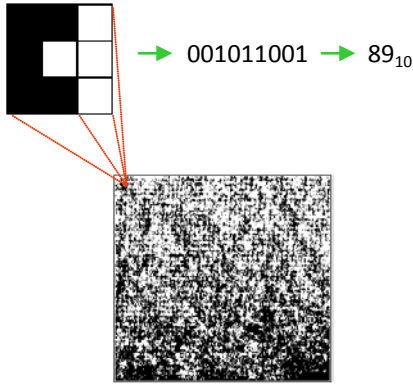
001011001 → 89_{10}

Fig. 1. The CCR calculation of a binary image

The matrix representation of the coordinated cluster transform of an image S is given by the two matrices:

$$A = \begin{bmatrix} 2^0 & 2^J & 2^{2J} & \cdots & 2^{(I-1)J} & 0 & \cdots & 0 \\ 0 & 2^0 & 2^J & \cdots & 2^{(I-2)J} & 2^{(I-1)J} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 2^{(I-1)J} \end{bmatrix} \quad (1)$$

$$B = \begin{bmatrix} 2^0 & 0 & \cdots & 0 \\ 2^1 & 2^0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 2^{J-1} & 2^{J-2} & \cdots & 0 \\ 0 & 2^{J-1} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 2^{J-2} \\ 0 & 0 & \cdots & 2^{J-1} \end{bmatrix} \quad (2)$$

Dimensions of matrices A and B are $(L-I+1)\times L$ and $M \times (M-J+1)$, respectively. The transformed matrix,

$$\tilde{S} = A \cdot S \cdot B \quad (3)$$

has $\tilde{W} = (L-I+1)\times(M-J+1)$ elements and these are decimal codes of binary pattern units. It can be seen that matrices A and B provide a line and a column shift of the scanning window. In addition, they assign a decimal code to every BPU. The CCR histogram, a vector of 2^N components, is obtained by counting the

number of occurrences of each element of the $\tilde{S}$. The b-th component of $H_{(I,J)}(b)$ is the number of occurrences of decimal code b in the matrix $\tilde{S}$.

The fundamental properties of the CCR were established in two theorems (Kurmyshev and Cervantes, 1996; Sánchez et al., 2003a) that are given here without proof. The first theorem establishes the structure of the CCR of periodic images. In particular, it helps one to recognize a (pseudo-) periodic texture by means of analysis of the CCR histogram.

Theorem 1. Let S be a binary, translation invariant image with a primitive cell (texton) that has a size τ_1 pixels in one and τ_2 in the other direction. Then any CCR histogram $H_{(I,J)}(b)$ of image S has no more than $T = \tau_1\tau_2$ non-zero values. If the CCR scanning window has the size equal to or larger than the texton size, $I \geq \tau_1$ and $J \geq \tau_2$, then $H_{(I,J)}(b)$ takes $T = \tau_1\tau_2$ non-zero values exactly, each peak of the histogram being the same size.

The second theorem establishes the relation between $H_{(I,J)}(b)$ and the k-th order statistics of a binary image.

Theorem 2. Let $S = [s_{lm}]$ be a binary image and $H_{(I,J)}(b)$ be its CCR histogram. Then, for all l_i and m_i such that $\max_i l_i \leq I$ and $\max_i m_i \leq J$, where $0 \leq i \leq k-1$ and $1 \leq k \leq I \times J$, any autocorrelation function of k-th order,

$$\langle s(l,m)s(l+l_1, m+m_1)...s(l+l_{k-1}, m+m_{k-1})\rangle = \lim_{L,M\to\infty} W^{-1}\sum_{l,m=1}^{L',M'} s(l,m)s(l+l_1, m+m_1)...s(l+l_{k-1}, m+m_{k-1}) \quad (4)$$

can be uniquely reconstructed from the histogram $H_{(I,J)}(b)$, where $W = L\times M$ is an image size, $L' = L - \max l_i$ and $M' = M - \max m_i$.

According to the theorem, the histogram $H_{(I,J)}(b)$ contains all information about the k-point correlation moments of a binary image S if and only if the separation vectors of k pixels fit into the scanning window. This means that a distribution function

$F_{(I,J)}(b)$ provides sufficient information about a k-point joint probability function. Since the correlation moments are important features of an image, the CCR has proven to be highly suitable for recognition and classification of texture images.

The CCR histogram is used as a feature vector in classification and segmentation of textures. In order to use the CCR for the classification of gray level and pseudo-color images, these must be binarized. For this purpose we can use different techniques using global or local adaptable thresholds in accordance with a specific application. It means that a thresholding is a preprocessing phase in the CCR method. A binarized image preserves sufficient amount of information for a primary gray level image to be classified efficiently.

3 Local Binary Pattern

In the paper (Wang and He, 1990) was proposed a technique that characterizes a texture image by its spectrum of texture units. A texture unit (TU) is represented by eight elements of a 3x3 pixel neighborhood, each of which takes one of the three values (0,1,2) depending on whether it is less, equal to or larger than the intensity of the central pixel. It is a three level partition of an image neighborhood, the central pixel being excluded. Hence, there are $3^8 =$ 6561 texture units that describe three level spatial patterns in the 3x3 neighborhoods. The occurrence histogram of TU is called texture spectrum of image. Unfortunately the technique resulted in neither theoretical nor practical significance.

A few years later a two level version of the Wang and He method had emerged. The new technique provides a robust description of texture in terms of local binary patterns (LBP) (Ojala et al., 1996a). As in the case of the three level model, the intensity of each of the eight neighboring pixels is compared with the intensity of the central pixel in the 3x3 neighborhood. Neighborhoods overlap, as in the case of the CCR. One of the two values (0,1) is assigned to a pixel in accordance with the rule: $p_i = 0$ if $I_i < I_c$, $p_i = 1$ if $I_i \geq I_c$, where $i = 1,2,..,8$, p_i is the binary value of the i-th pixel of the 3x3 neighborhood, I_i and I_c are intensities of the i-th pixel and the central pixel, respectively. The basic idea of the LBP method is explained in Figure 2, where (a) is the original pattern of a 3x3 neighborhood, (b) is the binary pattern of the

8 pixel neighborhood with the central pixel excluded, (c) is the mask of powers of 2 used for the decimal codification of the binary pattern, (d) is the decimal code of each of the eight binary pixels resulting in the number 93 for the whole neighborhood. In the two level version of texture units, there are only $2^8 = 256$ units, compared to 6561 as in Wang and He method.
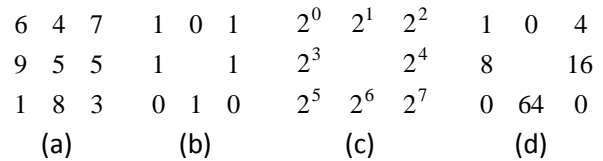
6	4	7		1	0	1		2^0	2^1	2^2		1	0	4
9	5	5		1		1		2^3		2^4		8		16
1	8	3		0	1	0		2^5	2^6	2^7		0	64	0
	(a)				(b)				(c)				(d)	

Fig. 2. Calculation of the LBP = 1+4+8+16+64 = 93 and the contrast C = (6+7+9+5+8)/5 - (4+1+3)/3 = 9.7.

As we can see, the LBP method is, in a certain sense, a method of binarization of neighboring pixels that uses the central pixel intensity of every neighborhood as a local threshold. In this method a texture image is characterized by its histogram of LBP code. The LBP method is invariant to the change of the gray level scale and easily combined with the local contrast measure. The latter is calculated as the difference between the average intensity of pixels that have a binary value 1 and the average value of those that have value 0. In Figure 2, the contrast is calculated as follows: C = (6+7+9+5+8)/5 - (4+1+3)/3 = 9.7. When the contrast is used besides the LBP, the method is called LBP/C. In spite of having only an empirical justification, the LBP/C method of texture analysis has demonstrated a high performance in classification and segmentation of texture images using the LBP/C histogram as a "two dimensional" feature vector (Ojala et al., 1996b; Ojala et al., 2000; Pietikainen et al., 2000; Maenpaa, 2003; Maenpaa and Pietikainen, 2004).

In order to enhance the mathematical validation of the LBP method we prove here an analog of theorem 1 of the CCR. Note that in case of the LBP there is no analog of Theorem 2 proven for the CCR because the "drilled" domains have lost a part of the information about the pixel statistics of different orders.

Theorem 1 (LBP). Let S be a gray scale or color, periodic image with a primitive cell of a size τ_1 pixels in one and τ_2 in the other direction. Then any LBP histogram $H_{(I,J)}(b)$ of image S has no more than

$T = \tau_1\tau_2$ non-zero values, no matter how large is a scanning window.

Proof. Let start one pixel step scanning of a periodic image with a window of a size equal to or larger than a size of primitive cell. Then, after τ_1 steps in one direction we meet the same (simply connected) domain of pixels. In other direction we face with the same pixel configuration after τ_2 steps. None of the domains is repeated in between these steps. Thus, the number of different simply connected domains we meet is equal to $T = \tau_1\tau_2$ exactly. The calculation of the LBPs is based on the comparison of the intensity of neighboring pixels with that of a central one. Because this calculation can transform different gray level neighborhoods into the same LBP, then the number of different LBP neighborhoods is less than or equal to $T = \tau_1\tau_2$. When the size of a sampling window is less than that of a texture primitive cell, even in one of the two directions, then the number of different simply connected domains we face with is equal to $T = \tau_1\tau_2$ at most. The calculation of the LBPs can even reduce this number. That completes the proof of the theorem.

As in the case of the CCR, this theorem helps one to recognize a (pseudo-) periodic texture analyzing its LBP histogram. Some simple examples of 1-D textures are given here in order to illustrate the theorem. Each digit of a sequence represents a pixel intensity.

Example 1. The texture is given by the "infinite" repeating of the five pixel primitive cell (…12223…), $\tau = 5$. Let a 1-D scanning window be 5 pixel size. Then, the histogram of four pixel LBP neighborhoods has five peaks at (0111), (1110), (0000), (1111) and (1011). Those are different LBPs of the five pixel domains (12223), (22231), (22312), (23122) and (31222) respectively. In case of 3 pixel scanning window we have only three different LBPs (01), (11) and (00), because the following three domains (222), (223) and (312) are transformed into the same LBP (11).

Example 2. The texture is generated by the five pixel primitive cell (…11223…), $\tau = 5$. Let a 1-D scanning window be 5 pixel size. The four pixel LBPs (0011), (0110), (0000), (1111) and (1111) are the maps of the following five pixel domains (11223), (12231), (22311), (23112) and (31122) respectively. We see that

the LBP histogram has only four peaks because the last two domains have the same LBP (1111).

4 Comparative analyses of the CCR and the LBP

In order to complete a comparison between the two methods, CCR and LBP, we outline the main virtues of both. As shown in Section 1, the intrinsic characteristics of a texture are expressed by virtue of correlation functions. To capture the essence of a texture, the CCR was developed as a transform of binary images that preserves the correlation moments of different orders between image pixels; this is expressed by the two theorems in Section 2. The CCR histogram is used as a feature vector for tasks of recognition, classification and segmentation of images. The extension of the CCR to the grey level and color images is done by means of thresholding in a preprocessing of an image. On the contrary, the LBP is a "synthesized" method in which the binarization (with a local one pixel threshold only) is inseparable from the feature extraction. Features are expressed by means of the spectrum of binary texture units. The CCR method implements the two operations separately. First an image is binarized (or multi thresholded, if necessary) and then features are extracted by means of the coordinated cluster transform. The separation of the two operations provides more flexibility to the CCR method, because the binarization can be done by means of a large variety of techniques with both local and global thresholds.

The difference between the CCR and LBP methods is seen even greater if we apply both transforms to a binary image (an "asymptotic" case of grey level images). The coordinated cluster transform does not change the pattern of a binary neighborhood. In the case of LBP we have: $p_i = 0$ if $I_i < I_c$, $p_i = 1$ if $I_i \geq I_c$ (see Section 3). This rule applied to a binary image gives the following two vary different results. If the value of the central pixel of a neighborhood is $p_c = 1$, then the pattern of the 8 pixel vicinity does not change. Nevertheless, when $p_c = 0$, then every pixel of the LBP vicinity takes the value $p_i = 1$ ($i = 1, 2, ..,$ 8). The latter result is completely different of that given by the CCR (see Figure 3). So, all domains that have the central pixel intensity $p_c = 0$ are

transformed into the unique LBP neighborhood of white pixels only. This effect of a local one pixel threshold of the LBP method can cause some trouble in the analysis and classification of images with a significant number of such domains. In this case we expect an appreciable difference between the rate of classification by means of the CCR and the LBP methods. In practice the LBP transform is applied to gray level images where the variation in the intensity of adjacent pixels is gradual compared to that in a binary image. That diminishes the negative effect.
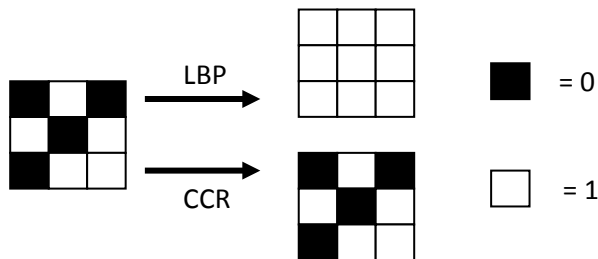

Fig. 3. The CCR and LBP calculation of a binary domain with the central pixel intensity $p_c = 0$

Another aspect distinguishing the CCR from the LBP is that the scanning window of the CCR can have any suitable dimension $I \times J$ and needs not be centrally symmetric. This gives an advantage in dealing with correlation moments of different ranges and orders (short, medium and long range correlations). On the contrary, only odd numbers $I = J$ are used in the LBP, since there has to be a central pixel as a reference for the binarization. In addition, the central pixel is excluded from the vicinity after being used as a threshold; that is the vicinity is a "drilled" cluster, a double connected domain. In terms of mathematical topology, the CCR uses a sphere and the LBP uses a torus to cover a texture pattern. These are not reducible to each other, as are the CCR and the LBP methods.

In order to illustrate, rather than prove differences between the two methods we give here an example of multi class classification using the CCR and the LBP. The same conditions are maintained for both methods. A minimum distance classifier is used to assign test images into 8 classes. Eight color images of the Rosa Porriño granite, which were converted to gray level ones, are used as source (master) images of the classes (see Figure 4).
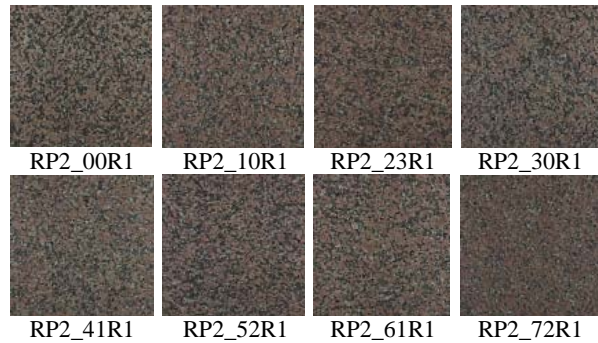

Fig. 4. Eight source images of Rosa Porriño granite

Original 512x512 master images have been shrunk to 204x204 (0.4x0.4 reduction) and 102x102 (0.2x0.2 reduction) pixel size in order to diminish an image scale influence over the classification. 3x3 neighborhoods of the LBP and the CCR are used to create a feature vector (histogram). Note that a similarity of source images is a challenging problem of the classification of subimages randomly extracted of those. Forty subimages extracted randomly from each master image are used to generate a class prototype histogram (the medium of 40 histograms) of both the CCR and the LBP. 300 subimages of the same size randomly extracted from each source image are used as test images, giving in total 3200 test subimages in each experiment.

We made four experiments. In the first, eight source images were of 204x204 pixel size (0.4x0.4 reduction), while prototype and test subimages were 154x154 pixels. In the second, we changed the subimage size for 102x102 pixels remaining the size of source images. The third and fourth experiments were done with the 102x102 source images (0.2x0.2 reduction) and 77x77 and 51x51 subimages, respectively. The classification of a test subimage is considered to be correct when it is assigned to the class of source image, on the contrary it is a misclassification. In the four experiments the average efficiency of classification into the 8 classes is equal to: 1) 97.6% LBP and 100% CCR; 2) 97.4% LBP and 98.4% CCR; 3) 97.5% LBP and 100% CCR; 4) 96.8% LBP and 100% CCR. Thus, in these particular experiments the CCR method has proven to be slightly superior to the LBP.

The algorithm of the coordinated cluster transform is simple and the CCR does not preserve information about the location of scanning window. In this respect

the CCR is similar to the LBP. The matrix representation of the coordinated cluster transform besides the two theorems, given in Section 3, enhances more the mathematical basis of the CCR. On the contrary, the LBP method lacks the mathematical foundation (at least published) that weakens its formal mathematical justification. The main applications of the two methods, the CCR and the LBP, are classification, recognition and segmentation of texture images.

5   Conclusions

The local binary pattern and the coordinated cluster representation are two methods used efficiently for the classification and segmentation of images. Both the CCR and the LBP histograms can be interpreted as a kind of image decomposition. The conceptual analysis of the two methods shows that they are not reducible each to the other. In terms of mathematical topology they are as much different as a sphere and a torus.

The LBP is a technique of statistical feature extraction by means of the binarization of a neighborhood of every image pixel with a local threshold determined by the central pixel, the latter being excluded from the neighborhood. As a consequence there is no analog to Theorem 2 of the CCR method, though an analog to Theorem 1 was proven above. The scanning window of the LBP always has odd dimensions, the CCR does not. Since the CCR method requires the thresholding as a stage of image preprocessing, both global and local threshold methods can be used depending on a potential application. Note that an image binarization with local adaptable threshold can easily fail to detect visual defects because it is used to restore the texture of a defect. So it can not distinguish the real defect, caused by a shade variation, from the effect of non homogeneous illumination.

Matrix representation of the coordinated cluster transform, reported in this work, enhances the mathematical basis of the method and facilitates the use of the powerful computer programs of MATLAB. The CCR and the LBP can be seen loosely as different bases in a feature space of local characteristics of an image. For example, in a linear vector space there exist an infinite set of distinct bases to span any vector of the space. One of the bases can be preferable in order to solve a given mathematical problem. Nevertheless,

every basis will lead to the same solution. This is the case of the CCR and the LBP; they are "bases" of different kind. The efficiency of classification and recognition of images by both methods can be very similar in many cases. Each of the two methods, especially the advanced versions of those, has its own virtues (Ojala et al., 2002). The question is how to get more profit of that. Concerning this question we think that the mathematical foundations of a method provide a deeper understanding of its use.

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