

# Construction of an Optimal Solution for a Real-World Routing-Scheduling-Loading Problem

## *Construcción de una Solución Óptima para un Problema de Asignación de Rutas, Horarios y Cargas del Mundo Real*

Juan Javier González Barbosa, José Francisco Delgado Orta, Héctor Joaquín Fraire Huacuja,  
José Antonio Martínez Flores and María Lucila Morales Rodríguez

Instituto Tecnológico de Ciudad Madero, México  
{jjgonzalezbarbosa, francisco.delgado.orta}@gmail.com, hfraire@prodigy.net.mx,  
{jose.mtz, lmorales}@gmail.com

*Article received on July 20, 2009; accepted on November 14, 2009*

### **Abstract**

This work presents an exact method for the Routing-Loading-Scheduling Problem (RoSLoP). The objective of RoSLoP consists of optimizing the delivery process of bottled products in a company study case. RoSLoP, formulated through the well-known Vehicle Routing Problem (VRP), has been solved as a rich VRP variant through approximate methods. The exact method uses a linear transformation function, which allows the reduction of the complexity of the problem to an integer programming problem. The optimal solution to this method establishes metrics of performance for approximate methods, which reach an efficiency of 100% in distance traveled and 75% in vehicles used, objectives of VRP. The transformation function reduces the computation time from 55 to four seconds. These results demonstrate the advantages of the modeling mathematical to reduce the dimensionality of problems NP-hard, which permits to obtain an optimal solution of RoSLoP. This modeling can be applied to get optimal solutions for real-world problems.

**Keywords:** Optimization, Routing–Scheduling–Loading Problem (RoSLoP), Vehicle Routing Problem (VRP), rich VRP.

### **Resumen**

Este trabajo presenta un método exacto para el problema de Asignación de Rutas, Horarios y Cargas (RoSLoP). El objetivo de RoSLoP consiste en optimizar el proceso de entrega de productos embotellados en una compañía caso de estudio. El problema RoSLoP, formulado a través del conocido Problema de Enrutado de Vehículos (VRP), ha sido resuelto como una variable VRP enriquecida a través de métodos aproximados. El método exacto usa una función de transformación lineal, la cual permite la reducción de la complejidad del problema a un problema de programación entera. La solución óptima para este método establece las métricas del desempeño para los métodos aproximados, los cuales alcanzan una eficiencia del 100% en distancia recorrida y 75% en vehículos utilizados, objetivos del VRP. La función de transformación reduce el tiempo del cálculo de 55 a cuatro segundos. Estos resultados demuestran las ventajas del modelado matemático para reducir la dimensionalidad de problemas NP-Duros, lo cual permite la obtención de una solución óptima del problema RoSLoP. Este modelado puede ser aplicado para obtener las soluciones óptimas para problemas del mundo real.

**Palabras Clave:** Optimización, Problema de Asignación de Rutas, Horarios y Cargas (RoSLoP), Problema de Enrutado de Vehículos (VRP), Problema VRP Enriquecido.

## **1 Introduction**

The distribution and delivery processes are inherent to many manufacturing companies; in other cases, it is the main function of several service businesses. Though this could be considered in consequential, however, merchandise delivery in due time with the minimum quantity of resources, reduces operation costs, yielding savings between 5 to 20 % in total costs of products [Toth, P. and D. Vigo, 2001]. In recent years, many researchers have approached transportation problems based on real situations in two ways: formulating rich models of solution and developing

efficient algorithms to solve them. RoSLoP, defined in [Cruz, L. et al, 2007a] and extended in [Cruz, L. et al, 2008], is a high-complexity problem due its dimensionality.

RoSLoP formulation, associated with the transportation of bottled products in a company located in north eastern Mexico, satisfies the needs of the logistics group of the company. The application of a meta-heuristic algorithm based on an ant colony system (presented in [Cruz, L. et al., 2008]) to the RoSLoP problem permits to generate feasible solutions. However, performance metrics that measure the quality of the obtained solutions have not been created for this method. This work presents a new formulation for RoSLoP, based on reported methods in the literature for VRP variants, which uses a mathematical artifice that permits reducing the dimensionality of the problem and its solution as an integer programming problem. Therefore, the solution obtained is used as a measure of performance for heuristic algorithms.

This paper shows the exact method based on the solution of 12 VRP variants: CVRP, VRPM, HVRP, VRPTW, SDVRP, sdVRP, VRPMTW, OVRP, sdVRP, CCVRP, DDVRP and MDVRP. These variants and the state of art of rich VRP variants are described in section 2 and 3. Sections 4 and 5 are devoted to describe RoSLoP and the exact method. Section 6 show the experimentation with real-world instances; and section 7 presents the conclusions for future applications of this work.

## 2 The Vehicle Routing Problem (VRP)

VRP, defined by Dantzig in [Shaw, P., 1998], is a classic problem of combinatorial optimization. It consists in one or various depots, a fleet of  $m$  available vehicles and a set of  $n$  customers to be visited, joined through a graph  $G(V,E)$ , where:  $V=\{v_0, v_1, v_2, \dots, v_n\}$  is the set of vertex  $v_i$  such that,  $v_0$  is the depot and the rest of the vertex represent the customers; each customer has a demand  $q_i$  of goods to be satisfied by the depot.  $E=\{(v_i, v_j) \mid v_i, v_j \in V, i \neq j\}$  is the set of edges where each edge has an associated value  $c_{ij}$  that represents the transportation cost from  $v_i$  to  $v_j$ .

The VRP consists of obtaining a set  $R$  of routes with a total minimum cost such that: each route starts and ends at the depot, each vertex is visited only once by a route  $v_i \in V - \{v_0\}$  and the length of each route must be less than or equal to  $L$ . So, the main objective is to obtain a configuration with the minimum quantity of vehicles and traveled distance for satisfying all the customer demands.

### 2.1 Variants of VRP

The most known variants of VRP add several constraints to the basic VRP such as capacity of the vehicles (CVRP) [Shaw, P. 1998], independent service schedules at the customers facilities (VRPTW-VRPMTW) [Courdeau, F. et al., 1999], multiple depots to satisfy the demands (MDVRP) [Mingozzi, A., 2003], customers to be satisfied by different vehicles (SDVRP) [Archetti, C., R. Mansini and M.G. Speranza., 2001], a set of available vehicles to satisfy the orders (sdVRP) [Thangiah, S., 2003], customers that can ask and return goods to the depot (VRPPD) [Dorransoro, B., 2005], dynamic facilities (DVRP) [Bianchi, L., 2000], line-haul and back-haul orders (VRPB) [Jacobs, B. and M. Goetshalckx, 1993], stochastic demands and schedules (SVRP) [Tanguiah, S., 2003], multiple use of the vehicles (VRPM) [Fleischmann, B., 1990], a heterogeneous fleet to delivery the orders (HVRP) [Taillard, E., 1996], orders to be satisfied in several days (PVRP) [6], constrained capacities of the customers for docking and loading the vehicles (CCVRP) [Cruz, L. et al, 2007a], transit restrictions on the roads (rdVRP) [3], depots that can ask for goods to another depots (DDVRP) [Cruz, L. et al., 2008] and vehicles that can end its travel in several facilities (OVRP). A rich VRP variant, defined in [Toth, P. and D. Vigo., 2001] as an *Extended Vehicle Routing Problem*, is an application of VRP for real transportation problems. It is based on the Dantzig's formulation; however, it requires the addition of restrictions that represent the combination of many variants in a problem; which increases its complexity, making more difficult the computation of an optimal solution through exact algorithms.

## 3 Related works of rich VRP variants

Recent works have approached the solution of rich VRP problems like the DOMinant Project [Hasle, G., et al., 2007], which solves five variants of VRP in a transportation problem of goods among industrial facilities located in

Norway. Goel [Goel, A. and Gruhn, V., 2005] solves four VRP variants in a problem of sending packages for several companies. Pisinger [Pisinger, D. and S. Ropke., 2005] and Cano [Cano, I., I. Litvinchev, R. Palacios and G. Naranjo., 2005] solve transportation problems with five VRP variants. RoSLoP was formulated initially in [Rangel, N., 2005] with six VRP variants. Due a requirement of the company it was necessary to formulate a VRP with 11 variants in [Herrera, J., 2006]. This new formulation allows the solution of instances of 12 VRP variants. Table 1 details the variants solved by various authors.

**Table 1.** Related works about known rich VRP variants

Author	Solved variants												
	CVRP	VRPTW	VRPMTW	OVRP	VRPPD	MDVRP	SDVRP	sdVRP	VRPM	HVRP	CCVRP	DDVRP	rdVRP
Hasle, G., et al 2007	✓	✓	✓						✓	✓			
Goel, A. and Gruhn V., 2005	✓	✓							✓	✓			
Pisinger, D. and S. Ropke, 2000	✓	✓		✓		✓		✓					
Cano I., et al., 2005	✓	✓					✓		✓	✓			
Cruz et al. 2007a	✓	✓				✓	✓		✓	✓			
Cruz et al. 2008	✓	✓	✓			✓	✓	✓	✓	✓	✓	✓	✓
This work	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓

A study of complexity factors of the base case of the problem is presented in Table 2. Rangel’s mathematical formulation [Rangel, N., 2005] requires  $2^6$  integer variables to solve 30 restrictions. Herrera’s approach [Herrera, J., 2006] needs  $2^9$  integer variables to solve 30 restrictions of the formulation. The proposed formulation contains  $2^2$  integer variables and 15 restrictions to solve 12 VRP variants.

**Table 2.** Complexity of the mathematical models created to RoSLoP

Complexity	Number of integer variables	Number of Restrictions	Solved VRP variants	Solver
Element Method				
Rangel, N., 2005	$2^6$	30	6	Heuristic
Herrera, J., 2006	$2^9$	30	11	Heuristic
This work	$2^2$	15	12	Exact

#### 4 Definition of RoSLoP

RoSLoP, immersed in the logistics activity of the company study case, involves a subset of three tasks: routing, scheduling and loading. The mathematical model of RoSLoP was formulated with two classical problems: routing and scheduling through VRP and the loading through the Bin Packing Problem (BPP). Fig. 1 shows RoSLoP and its relation with VRP-BPP.

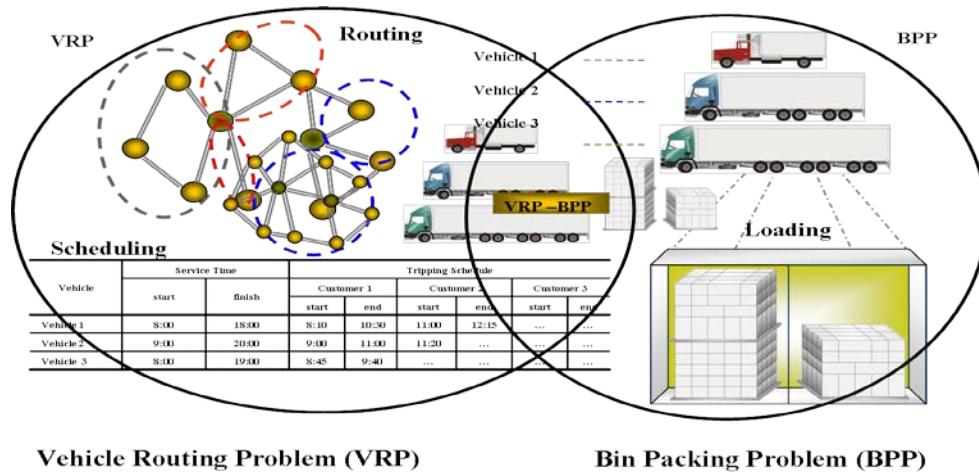


Fig. 1. Definition of the Routing-Scheduling-Loading Problem (RoSLoP)

The case of study contains the next elements:

- A set of *ORDERS* to be satisfied at the facilities of the customers, formed by boxes of products with different attributes such as weight, high, product type, supported weight, packing type and beverage type.
- A set of  $n$  customers with independent service schedules at a facility  $j$  [ $start\_service_j, end\_service_j$ ] and a finite capacity of attention of the vehicles.
- A set of depots with independent schedules, which have the possibility to request goods to other depots.
- A fleet of vehicles with heterogeneous capacity  $Vehicles_d$  to transport goods, with a service time  $service\_time_v$  and a time for attention at the facilities of the customers. The attention time  $tm_{vj}$  depends on the capacity of the vehicle and the available people for docking and loading the vehicles.
- A set of roads represented by the edges of the graph. Each road has an assigned cost  $C_{ij}$ , each one with a threshold of allowed weight  $MAXLoad_{vj}$  for a determined vehicle  $v$  that travels towards a facility  $j$ , and a travel time  $t_{ij}$  from facility  $i$  to  $j$ .

The objective of RoSLoP is to get a configuration that allows the satisfaction of the set of *ORDERS* at the set of the customer facilities, minimizing the number of vehicles used and the distance traveled. This new formulation includes a model with 12 variants of VRP: CVRP, VRPTW, VRPMTW, MDVRP, SDVRP, sdVRP, VRPM, HVRP, CCVRP, DDVRP, rdVRP and OVRP, described in section 2.1.

## 5 Formulation of RoSLoP

### Input sets

- $C$**  Set of customer facilities or vertex of the associated graph
- $ORDERS$**  Set orders to be satisfied
- $D$**  Set of depots to satisfy all the customer demands
- $Vehicles_d$**  Set of available vehicles in a depot  $d$
- $K$**  Set of all existent routes in a graph
- $K_d$**  Set of routes to be covered by a depot  $d \in D$
- $Pallets_v$**  Set of containers of a vehicle  $v$ . Each container has an associated pair  $(h_{palletij}, w_{palletij})$ , which represents the high and weight of a container  $i$  when a customer  $j$  is visited by a vehicle  $v$ .
- $ITEMS_j$**  A set of units of *ORDERS* for a customer  $j$ .

**Parameters**

- Capacity<sub>vj</sub>* Capacity of a vehicle *v* to visit a customer *j*.
- service\_time<sub>v</sub>* Service time of vehicle *v*.
- tm<sub>vj</sub>* Maneuver time of a vehicle *v* at facility *j*, associated to vehicle docking and loading
- MAXLoad<sub>vj</sub>* Upper limit for the load to be assigned to vehicle *v* to visit facility *j*
- MAX<sub>Vehicles<sub>j</sub></sub>* Upper limit for the number of vehicles attended simultaneously at a facility *j*.
- start\_service<sub>j</sub>* The time when a facility *j* starts its operation
- end\_service<sub>j</sub>* The time when a facility *j* ends its operation
- c<sub>ij</sub>* Transportation cost to travel from facility *i* to *j*
- t<sub>ij</sub>* Transportation time to travel from facility *i* to *j*

**Real variables**

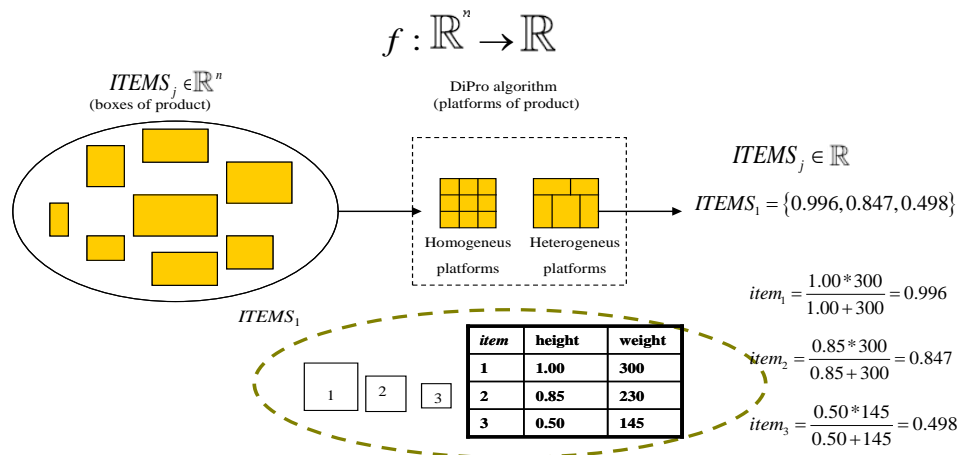
- Load<sub>vj</sub>* Assigned load in a vehicle *v* to visit facility *j*.
- arrive<sub>ik</sub>* Arrival time of a vehicle to facility *i* using route *k*.
- left<sub>ik</sub>* Departure time of a vehicle from a facility *i* using route *k*.
- φ<sub>k</sub>* Associated cost to travel by route *k*.
- t<sub>k</sub>* Travel cost of a vehicle by route *k*.

**Integer variables**

- x<sub>ijk</sub>* 1 if the edge (*i, j*) is visited on route *k*, 0 in otherwise
- y<sub>vk</sub>* 1 if vehicle *v* is assigned to route *k*, 0 in otherwise

**5.1 Preprocessing of the instance of RoSLoP**

The preprocessing of an instance is carried out through a linear transformation function, which normalizes the load objects of the problem. Load objects are defined as *n*-dimensional objects. They are transformed into a set of real numbers where each number represents an *n*-dimensional object. The function of transformation is illustrated in Fig. 2, in which, an order of a customer *j* is transformed. This function represents the relationship between the dimensions of the objects. It consists of two steps: 1) the construction of units of load and 2) the transformation of these units in a representative set of real numbers.



**Fig 2.** Transformation of the orders dataset of the case study

The construction of load units is done through the DiPro is invoked. As a result, two kinds of units are created: homogeneous and heterogeneous platforms. Homogeneous platforms are constituted by products of the same type, while heterogeneous platform are constituted with different types of products with similar characteristics. Both, Homogeneous and Heterogeneous platforms are defined as a set  $ITEMS_j = \{ \forall (w_i, h_i) \}$ . Then, each pair  $(w_i, h_i)$  is transformed into a number  $item_i$ , using the following expression (1). A detailed review of DiPro is presented in [Cruz, L. et al., 2007b].

$$item_i = \frac{h_i w_i}{h_i + w_i} \quad i \in ITEMS_j \quad (1)$$

The capacity  $Capacity_{vj}$  of a vehicle  $v$  to visit node  $j$  is transformed likewise. Each container that belongs to a trailer has two attributes: a high  $hpallet_{ij}$  and weight  $wpallet_{ij}$  of the assigned load to visit customer  $j$ . The width of the load is determined by a categorization of products, asked the company to group the products. This is necessary for adjusting the load to the containers. The transformation of the vehicles dimensions is shown in Fig. 3.

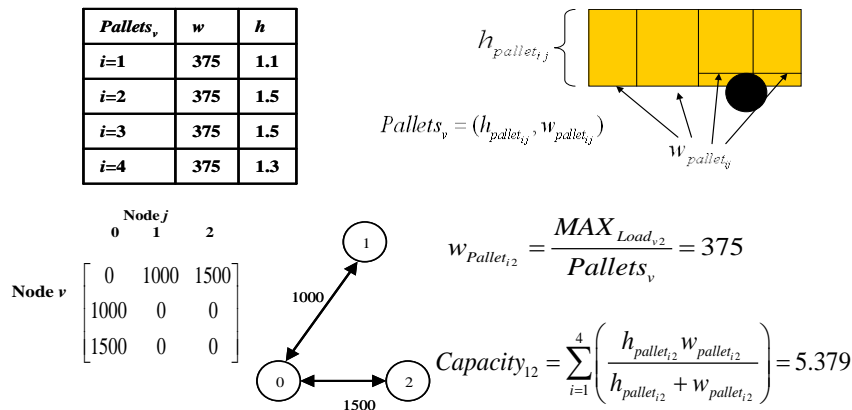


Fig. 3. Transformation of the vehicles dimensions

It is assumed that the weight of the load to be assigned to each container must be uniform. Expression (2) is used to obtain the capacity of the vehicle. This expression ensures that the dimensions of the load objects and the vehicles are equivalent.

$$Capacity_{vj} = \sum_{i=1}^{Pallets_v} \frac{h_{pallet_{ij}} w_{pallet_{ij}}}{h_{pallet_{ij}} + w_{pallet_{ij}}} \quad j \in C, v \in Vehicles_d \quad (2)$$

$$Load_{vj} = \sum_{r \in ITEMS_j} item_r \sum_{i \in C \cup D} x_{ijk} \quad j \in C \cup D, k \in K_d \quad (3)$$

Once defined the input parameters, instance preprocessing is performed. These elements are used to formulate the integer programming model. The combination of these elements generates the solution to the related rich VRP.

### 5.2 Mathematical model to solve the rich VRP

The objective of RoSLoP is to minimize the assigned vehicles and the distance traveled, visiting all the customer facilities and satisfying the demands. Each route  $k$  is constituted by a subset of facilities to be visited and a length  $\phi_k$ .

Expression (4) is used to get the maximum covering set established by the use of variant HVRP. Expressions (5)-(6) permit obtaining the length and the travel time on a route  $k$ .

$$|K_d| = |Vehicles_d| \left\lceil \frac{\max(ITEMS_j)}{\min(Capacity_{vj})} \right\rceil \quad K_d \in K \quad (4)$$

$$\varphi_k = \sum_{j \in C \cup D} \sum_{i \in C \cup D} c_{ij} x_{ijk} \quad k \in K, v \in Vehicles_d \quad (5)$$

$$t_k = \sum_{j \in C \cup D} \sum_{i \in C \cup D} t_{ij} x_{ijk} + \sum_{i \in C \cup D} \sum_{j \in C \cup D} \sum_{v \in Vehicles_d} tm_{vj} x_{ijk} y_{vk} \quad k \in K_d \quad (6)$$

The objective function of the problem, defined by expression (7), minimizes the number of assigned vehicles and the length of all the routes generated. Expressions (8) – (10) are used to generate feasible routes and solve the related TSP problem. Expression (8) restricts each edge  $(i, j)$  on a route  $k$  to be traversed only once. Expression (9) ensures that route  $k$  is continuous. Expression (10) is used to optimize the covering set related with the objective function. These expressions solve variants DDVRP and MDVRP.

$$\min \sum_{k \in K_d} \sum_{v \in Vehicles_d} \varphi_k y_{vk} \quad (7)$$

$$\sum_{i \in C \cup D} x_{ijk} = 1 \quad k \in K_d, j \in C \cup D \quad (8)$$

$$\sum_{i \in C \cup D} x_{ijk} - \sum_{i \in C \cup D} x_{jik} = 0 \quad k \in K_d, j \in C \cup D \quad (9)$$

$$\sum_{i \in C \cup D} \sum_{j \in C \cup D} x_{ijk} \geq 1 \quad k \in K_d \quad (10)$$

Expressions (11)-(14), formulated in [5], calculate the time used by a vehicle assigned to route  $k$ . Expression (14) ensures that the use of a vehicle does not exceed the attention time at facility  $j$ . These expressions permit solving the variants VRPTW, VRPMTW and VRPM. The variants CCVRP and SDVRP are solved using the expression (15), which ensures that two routes  $k$  and  $k'$  do not intersect each other at a facility  $j$ .

$$t_k y_{vk} \leq service\_time_v \quad k \in K_d; v \in Vehicles_d \quad (11)$$

$$left_{jk} \geq arrive_{jk} \sum_{i \in C \cup D} t_{ij} x_{ijk} + \sum_{v \in Vehicles_d} tm_{vj} y_{vk} \quad j \in C \cup D, k \in K_d \quad (12)$$

$$arrive_{jk} \sum_{i \in C \cup D} x_{ijk} = \sum_{i \in C \cup D} t_{ij} x_{ijk} \quad j \in C \cup D, k \in K_d \quad (13)$$

$$start\_service_j \leq arrive_{jk} \leq end\_service_j \quad j \in C, k \in K_d \quad (14)$$

$$arrive_{jk} \leq left_{jk} \leq arrive_{j'k'} \quad k < k', \forall k, \forall k' \in K_d \quad (15)$$

Expressions (16)-(18), formulated in [1] and combined with the linear transformation function, define the restrictions for variants CVRP, sdVRP, rdVRP and HVRP. Expression (16) establishes that a vehicle is assigned to a route  $k$ . Expression (17) ensures that vehicle capacities are not exceeded. Equation (18) establishes that all goods must be delivered and all demands are satisfied. The relaxation of the model that permits the solution of the variant OVRP consists of the reformulation of expression (9) through expression (19).

$$\sum_{v \in \text{Vehicles}} y_{vk} \leq 1 \quad k \in K_d \tag{16}$$

$$\text{Load}_{vj} \leq \text{Capacity}_{vj} y_{vk} \quad k \in K_d; v \in \text{Vehicles}_d \tag{17}$$

$$|\text{ITEMS}_j| - \sum_{j \in C} \text{Load}_{vj} = 0 \quad j \in C \cup D \tag{18}$$

$$\sum_{i \in C \cup D} x_{ijk} - \sum_{i \in C \cup D} x_{jik} \leq 1 \quad k \in K_d, j \in C \cup D \tag{19}$$

The right side of expression (19) is 0 when a route starts and ends at a depot; otherwise, when the right side has the value 1 means that route starts at a depot and finishes at a different facility.

## 6 Experimentation

Real instances were provided by the bottling company. They were solved using the approximate algorithm called Heuristics-Based System for Assignment of Routes, Schedules and Loads (HBS-ARSL) proposed in [3] and an exact method. Both were tested on a set of VRP variants that are present in the test data set: VRPTW, VRPMTW, sdVRP, SDVRP, rdVRP, CCVRP, DDVRP, CVRP y HVRP. The HBS-ARSL algorithm was coded in C# and it was executed during two minutes to observe the time when it reaches the best solution. The implementation of the exact method was coded in C# and uses the LINDO API v4.1. A set of 12 test instances were selected from the database of the company, which contains 312 instances classified by the date of the orders; the database contains also 1257 orders and 356 products in its catalogues. Eight available vehicles were disposed. The results are shown in Table 3.

**Table 3.** Experimentation with real-world instances, provided by the bottling company

Instance	n	K	ORDERS	HBS-ARSL			Exact method		
				Distance Traveled	Vehicles Used	Time (secs)	Distance Traveled	Vehicles Used	Time (secs)
<b>06/12/2005</b>	<b>4</b>	<b>5</b>	158	<b>1444</b>	<b>4</b>	<b>51.33</b>	<b>1444</b>	<b>3</b>	<b>3.09</b>
09/12/2005	5	5	171	1580	5	23.64	1580	5	3.32
12/12/2005	7	9	250	2500	6	38.52	2500	6	5.03
<b>01/01/2006</b>	<b>6</b>	<b>9</b>	286	<b>2560</b>	<b>7</b>	<b>75.42</b>	<b>2560</b>	<b>6</b>	<b>3.42</b>
03/01/2006	4	4	116	1340	4	63.00	1340	4	2.97
<b>07/02/2006</b>	<b>6</b>	<b>9</b>	288	<b>2660</b>	<b>7</b>	<b>83.71</b>	<b>2660</b>	<b>6</b>	<b>4.53</b>
13/02/2006	5	7	208	1980	5	55.57	1980	5	3.85
06/03/2006	6	7	224	1960	5	32.16	1960	5	3.36
09/03/2006	6	9	269	2570	6	76.18	2570	6	3.85
22/04/2006	8	11	381	3358	7	57.35	3358	7	5.24
14/06/2006	7	8	245	2350	6	90.84	2350	6	4.86
04/07/2006	7	9	270	2640	6	72.49	2640	6	4.53
<b>Average</b>	<b>6</b>	<b>8</b>	<b>238.83</b>	<b>2245.16</b>	<b>5.66</b>	<b>55.27</b>	<b>2245.16</b>	<b>5.33</b>	<b>4.00</b>



## 7 Analysis of Results

The exact method obtained the optimal solution for the 12 instances of the test data set; which permits to measure the performance of the algorithm HBS-ARSL when solving the related rich VRP variant. Table 3 shows that HBS-ARSL reaches the optimal solution in 100% of the cases considering the distance traveled and in 75% considering the number of vehicles assigned. These results reveal as consequence, the need of improving the search techniques of HBS-ARLS to reach 100% of efficiency. The best solutions of HBS-ARLS were obtained in 55.27 seconds on average, while the exact method reaches the optimal solutions in 4 seconds, permitting a reduction of 92% in execution time; which reveals the advantages of the transformation function used to reduce the execution time and the computation of the optimal solution.

## 8 Conclusions and Future Works

This work presented a mathematical formulation and a linear transformation function, which make possible to obtain the optimal solution for the rich VRP variant related to RoSLoP. It was demonstrated that, the use of mathematical artifices can be used to reduce the dimensionality generated by the solution of many VRP variants. This allowed the obtaining an optimal solution through the formulated integer problem. It could be advantageous when an exact solution is needed for problems classified as NP-Hard as VRP or BPP. However, this transformation function has the restrictive condition of the dependence of domain for this application. Therefore, it is proposed the construction of a transformation function, which is able to reduce the dimension of some specific problems as BPP to a representative set. This can be used to obtain the optimal solution for other real-world problems.

## References

1. **Archetti, C., Mansini, R. & Speranza, M.G. (2001).** The Vehicle Routing Problem with capacity 2 and 3, General Distances and Multiple Customer Visits. *Operational Research Peripatetic Post-graduate Programme*. 1(1),102-112.
2. **Bianchi, L. (2000).** *Notes on Dynamic Vehicle Routing*. (IDSIA-05-01). Switzerland: Istituto Dalle Molle di Studi sull'Intelligenza Artificiale.
3. **Cano, I., Litvinchev, I., Palacios R., & Naranjo, G. (2005).** Modeling Vehicle Routing in a Star-Case Transportation Network. *Memoria del XIV Congreso Internacional de Computación CIC*, IPN. México DF. 373-377.
4. **Cordeau, J.F., Desaulniers, G., Desrosiers, J., Solomon, M.M. & Soumis, F. (1999).** *The VRP with time windows*. (GERAD G-99-13). Montreal: École des Hautes Études Commerciales de Montréal.
5. **Cruz, L., González, J.J., Romero, D., Fraire, H.J., Rangel, N., Herrera, J.A., Arrañaga, B.A. & Delgado, J.F. (2007a).** A Distributed Metaheuristic for Solving a Real-World Scheduling-Routing-Loading Problem. *Symposium on Parallel and Distributed Processing and Applications ISPA 2007. Lecture Notes in Computer Science*, 4742, 68-77.
6. **Cruz, L., Nieto-Yáñez, D.M., Rangel-Valdez, N., Herrera, J.A., González, J.J., Castilla, G., & Delgado-Orta, J.F. (2007).** DiPro: An Algorithm for the Packing in Product Transportation Problems with Multiple Loading and Routing Variants. *Mexican International Conference on Artificial Intelligence MICAI 2007. Lecture Notes in Computer Science*, 4827, 1078-1088.
7. **Cruz, L., Delgado-Orta, J.F., González, J.J., Torres, J., Fraire, H.J., & Arrañaga, B.A. (2008).** An Ant Colony System to solve Routing Problems applied to the delivery of bottled products. *International Symposium ISMIS 2008. Lecture Notes in Computer Science*, 4994, 329-338.
8. The VRP Web. (2005). Retrieved from <http://neo.lcc.uma.es/radi-aeb/WebVRP>.
9. **Fleischmann, B. (1990).** *The Vehicle routing problem with multiple use of vehicles*. Hamburg: Universitt Hamburg.

10. **Goel, A. & Gruhn, V. (2005).** Solving a Dynamic Real-Life Vehicle Routing Problem. *Operations Research Proceedings*, 1(1), 367-372.
11. **Flatberg, T., Hasle, G., Kloster, O., Nilssen, E.J., & Riise, A. (2007).** Dynamic and Stochastic Vehicle Routing in Practice. *Operations Research/Computer Science Interfaces Series*, 38(1), 45-68.
12. **Herrera, J. (2006).** *Development of a methodology based on heuristics for the integral solution of routing, scheduling and loading problems on distribution and delivery processes of products.* MSc Thesis. Instituto Tecnológico de Ciudad Madero, Tamaulipas, México.
13. **Jacobs, B. & Goetshaleckx, M. (1993).** *The Vehicle Routing Problem with Backhauls: Properties and Solution Algorithms.* (MHRC-TR-88-13). Georgia: Georgia Institute of Technology .
14. **Mingozi, A. (2005).** The Multi-depot Periodic Vehicle Routing Problem. *Symposium Abtraction, Reformulation and Approximation SARA 2005. Lecture Notes in Computer Science*, 3607, 347-350.
15. **Pisinger, D. & Ropke, S. (2005).** A General Heuristic for Vehicle Routing Problems. *Computers & Operations Research*, 34(8), 2403-2435.
16. **Rangel, N. (2005).** *Analysis of the routing, scheduling and loading problems in a Products Distributor.* MSc Thesis. Instituto Tecnológico de Ciudad Madero, Madero, Tamaulipas, México.
17. **Shaw, P. (1998).** Using Constraint Programming and Local Search Methods to Solve Vehicle Routing Problems. *Principles and Practice of Constraint Programming – CP98. Lecture Notes in Computer Science*. 1520, 417-431.
18. **Tailard, E. (1996).** *A Heuristic Column Generation Method For the Heterogeneous Fleet VRP.* (CRI-96-03). Switzerland: Istituto Dalle Moli di Studi sull Inteligenza Artificiale.
19. **Thangiah, S. (2003).** *A Site Dependent Vehicle Routing Problem with Complex Road Constraints.* (SRT84-2003) Slippery Rock University, PA.
20. **Toth, P. & Vigo, D. (2001).** *The vehicle routing problem.* *SIAM, Monographs on Discrete Mathematics and Applications.* Philadelphia: Society for Industrial and Applied Mathematics.



**Juan Javier González Barbosa.** He received the Ph.D. in computer science from the Centro Nacional de Investigación y Desarrollo Tecnológico, Cuernavaca, México in 2006. His scientific interests include metaheuristic optimization and machine learning.



**José Francisco Delgado Orta.** He received the MSc degree in computer sciences from the Instituto Tecnológico de Ciudad Madero, Ciudad Madero Tamaulipas, México in 2007. He is a systems consultant with professional activity in Mexico.



**Héctor Joaquín Fraire Huacuja.** *He received the Ph.D. in computer science from the Centro Nacional de Investigación y Desarrollo Tecnológico, Cuernavaca, México in 2005. His scientific interests include metaheuristic optimization and machine learning.*



**José Antonio Martínez Flores.** *He received the Ph.D. in computer science from the Centro Nacional de Investigación y Desarrollo Tecnológico, Cuernavaca, México in 2006. His scientific interests include distributed database design and natural language processing.*



**María Lucila Morales Rodríguez** *is professor at the Instituto Tecnológico de Cd. Madero, Mexico. She received a Ph.D.degree in Informatics from Université Paul Sabatier, France in 2007. She has conducted research in virtual characters and emotional interfaces.*