OSCILLATING TEMPERATURE PROFILE MODEL FOR A POURED EARTH WALL

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RESUMEN
Actualmente la tierra vertida se puede utilizar como una solución sustentable en la construcción de viviendas, sin embargo, sus propiedades no se han estudiado a profundidad y sus beneficios han derivado solamente de análisis empíricos. Hay varias características de los materiales entre los que la resistencia a la transferencia de calor es notable. En este documento se propone un modelo matemático para describir la variación temporal de la temperatura a lo largo de una pared cuando se expone al calor de la luz solar. Se encontró que el modelo puede estar integrado con una ecuación exponencial que incluye las funciones simples de seno y coseno y con una temperatura exterior igual a $T_{x1} = T_A + A(\sin tw)$.

Palabras clave: Perfil de temperatura, tierra vertida, transferencia de calor, modelo oscilante.

ABSTRACT
Currently the poured earth can be used as a sustainable solution in the construction of homes, yet their properties have not been studied in depth and its benefits have been derived only from empirical analysis. There are various material characteristics among which resistance to heat transfer is notable. In this paper a mathematical model is proposed to describe the temporal variation of temperature along a wall when it is exposed to sunlight heat. It was found that the model can be integrated with an exponential equation which includes the sine and cosine simple functions and with an external temperature equals to $T_{x1} = T_A + A(\sin tw)$.

Keywords: Temperature profile, poured earth, heat transfer, oscillating model.

RESUMO
Atualmente, a terra vertida pode ser usada como uma solução sustentável para a construção de casas, no entanto, as suas propriedades não foram estudadas em profundidade e os seus benefícios foram derivados apenas de análise empírica. Existem várias propriedades do material, entre as quais a resistência à transferência de calor é notável. Este documento propõe um modelo matemático para descrever a variação temporal da temperatura ao longo de uma parede, quando é exposta ao calor da luz solar. Verificou-se que o modelo pode ser integrado com uma equação exponencial que inclui funções seno e cosseno simples e uma temperatura exterior é igual a $T_{x1} = T_A + A(\sin tw)$.

Palavras chave: Perfil de temperatura, terra vertida, transferência de calor, modelo oscilante.

1. INTRODUCTION

Poured earth is a technique consisting of creates monolithic structures from earth, water and stabilizers such as cement (Houben and Guillard 1994). This is rarely used today but is seen as a sustainable solution (Pacheco-Torgal and Jalali, 2012), (Bui *et al.*, 2009). There are various studies that have been made to this material, mainly in the compressive strength and resistance when certain stabilizers are added and deformation by axial forces and effect of air and water on soils (Suárez-Domínguez et al., 2013). Another property is a low coefficient of heat transfer that is directly related to the comfort for users within the structures built with this material, which can be exploited in several countries such as Spain or Egypt (Cid-Falceto *et al.*, 2012; Sameh, 2014), although advantage can be taken alongside the design by

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Artículo recibido el 15 de septiembre del 2014 y aprobado el 27 de mayo del 2015.

placing doors and windows in adequate positions, as well as the earth-based material used (Fernandez *et al.*, 2014; Dong e*t al.*, 2014).

Although, studies on the durability of earth-based material are relevant (Bahar *et al.*, 2014) there are few models that can actually be found to appreciate the effects of the sun on the housing, even though the importance of this knowledge is recognized in design (Dong, 2014), but it is increasingly recognized the possibility of using earth-based materials instead of concrete (Ronsoux, 2012).

On the other hand, there has been the oscillatory effect and delay in heat transfer (Orosa and Oliveira, 2012) is necessary to have a model that does not depend on continuous measurements on the outside and inside of the building elements and allows knowing *a priori* the effects of changes in average temperature environment inside homes.

1. THEORICAL MODEL

Because the study temperature, by sun exposure of the walls, shows an oscillatory behavior throughout the day, the problem to solve is a one-direction heat transport through a solid by the conduction mechanism, in which the system is in a non-steady state and where the outside temperature has periodic oscillations. For a time value less than zero the temperature of the wall is equal to the outside temperature, which is considered constant. For a time greater than zero outside temperature varies periodically.

The system in question is a solid which is in an uniform temperature and equal to T_0 , and which is exposed for a time of zero to a heat source so that for $x = y$ the temperature is equal to T_f for any time value greater than zero. Only in an infinite time the steady-state is reached and all the solid temperature is T_f . To describe the evolution of the temperature profile with respect to time we start with the equation for temperature change (Carslaw and Jeager, 1959):

$$\rho C_p \frac{\partial T_{x,t}}{\partial t} = k\frac{\partial^2 T_{x,t}}{\partial x^2} + U\left(T_{x_1,t} - T_{x,t}\right) \tag{1}$$

where ρ is the density (kg.m^{-3}), C_p is the heat capacity (kcal.kg^{-1}.K^{-1}), k the thermal conductivity coefficient (kcal.m-1.K-1.s-1), U is the heat transfer coefficient on the wall (Kcal.m^{-3}.K^{-1}.s^{-1}) exposed to the heat source and $T_{x,t}$ is temperature at x distance in a time t , given by $T_{x_1,t} = T_A + A\sin(wt)$ and $T_{x,0} = T_A$.

For solution of Equation 1 we define $\alpha = \dfrac{k}{\rho C_p}$ y $\beta = \dfrac{U}{\rho C_p}$ obtaining:

$$\frac{\partial T_{x,t}}{\partial t} = \alpha\frac{\partial^2 T_{x,t}}{\partial x^2} + \beta\left(T_A + A\sin(wt) - T_{x,t}\right) \tag{2}$$

The α parameter is the thermal diffusivity, which represent the rate of solid thermal conductivity and the product of density and heath capacity. It´s value depends on the chemical-physical in the intrinsic characteristics of the material. β parameter involves the heath transfer coefficient and depends on surface characteristics of solid (Bird *et al.,* 2002; Lide, 2011; Revuelta-Acosta e*t al.*, 2010).

To Equation 2 we make the Laplace transform with respect of time to convert our partial differential Equation to an ordinary differential Equation (Borrelli and Coleman, 2005):

$$sT_{x,s} - T_{x,0} = \alpha\frac{\partial^2 T_{x,s}}{\partial x^2} + \beta\left(T_{x_1,s} - T_{x,s}\right) \tag{3}$$

where $T_{x,0} = T_A$ and $T_{x_1,s} = \dfrac{1}{s}T_A + A\dfrac{w}{s^2 + w^2}$.

Equation 3 is solved with respect to x obtaining:

$$T_{x,s} = \frac{1}{s+\beta}\left(T_{x,0} + \beta T_{x_1,s}\right) + C_{17}\exp\left(\frac{x}{\alpha}\sqrt{\alpha(s+\beta)}\right) + C_{18}\exp\left(-\frac{x}{\alpha}\sqrt{\alpha(s+\beta)}\right) \tag{4}$$

In the experimental case for an infinite x the solution is finite, so that $C_{17} = 0$ and

$$T_{x,s} = \frac{1}{s+\beta}\left(T_{x,0} + \beta T_{x_1,s}\right) + C_{18}\exp\left(-\frac{x}{\alpha}\sqrt{\alpha(s+\beta)}\right) \tag{5}$$

By the inverse Laplace transform and simplifying we obtain (Revuelta-Acosta, *et al.*, 2010):

$$L^{-1}\left[\exp\left(-\frac{x}{\sqrt[2]{\alpha}}\sqrt{(s+\beta)}\right)\right] = \exp(-t\beta)\left(\frac{x}{\sqrt{4\alpha\pi}\sqrt{t^3}}\exp\left(-\frac{1}{4t}\frac{x^2}{\alpha}\right)\right) \tag{6}$$

whose solution is:

$$T_{x,t} = T_A + \frac{Aw\beta}{\left(w^2+\beta^2\right)}\left(\exp(-t\beta) - \cos t\sqrt{w^2} + \beta\frac{\sin t\sqrt{w^2}}{\sqrt{w^2}}\right) + C_{18}\exp(-t\beta)\left(\frac{x}{\sqrt{4\alpha\pi}\sqrt{t^3}}\exp\left(-\frac{1}{4t}\frac{x^2}{\alpha}\right)\right) \tag{7}$$

C_{18} constant is determined considering the external temperature value in x_1, in this way:

$$C_{18} = 2\sqrt{\pi}\frac{\sqrt{\alpha}}{x_1 e^{-t\beta}e^{-\frac{1}{4t\alpha}x_1^2}}\left(A\sin tw - Aw\frac{\beta}{\beta^2+w^2}\left(e^{-t\beta} - \cos t\sqrt{w^2} + \beta\frac{\sin t\sqrt{w^2}}{\sqrt{w^2}}\right)\right)\sqrt{t^3} \tag{8}$$

Replacing (Ec. 8) in (Ec. 7) we found the predicted theorical behavior:

$$T_{x,t} = T_A + \frac{Aw\beta}{\left(w^2+\beta^2\right)}\left(\exp(-t\beta) - (\cos tw) + \frac{\beta}{w}(\sin tw)\right)\left(1 - \frac{x}{x_1}\exp\left(-\frac{\left(x^2-x_1^2\right)}{4t\alpha}\right)\right) + A(\sin tw)\frac{x}{x_1}\exp\left(-\frac{\left(x^2-x_1^2\right)}{4t\alpha}\right) \tag{9}$$

Figure 1 shows the predicted temperature behavior with respect time t and the wall thickness x, for general parameters values of model.

Table 1: Parameters assumed for the simulations and construction of figures 1 and 2.

K	A	w	β	α
20	1	1	1	1

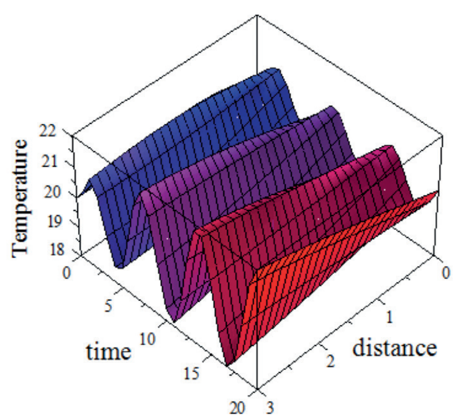


Fig. 1: Spatial and temporal behavior of the wall temperature when the external value oscillates periodically in time for y = 1.

The average of the wall temperature in time $\langle T_x \rangle_t$ can be determined by:

$$\langle T_x \rangle_t = \frac{\int_{x_1}^{X} T_{x,t}\,dx}{\int_{x_1}^{X} dx} \tag{10}$$

So that:

$$\langle T_x \rangle_t = T_A + \frac{Aw\beta}{\left(w^2+\beta^2\right)}\left(\exp(-t\beta) - (\cos tw) + \frac{\beta}{w}(\sin tw)\right)\left(1 - \left(\frac{-2\frac{t}{x_1}\alpha\left(\exp\left(-\frac{1}{4t\alpha}\left(X^2 - x_1^2\right)\right) - 1\right)}{X - x_1}\right)\right)$$
$$+ A(\sin tw)\left(\frac{-2\frac{t}{x_1}\alpha\left(\exp\left(-\frac{1}{4t\alpha}\left(X^2 - x_1^2\right)\right) - 1\right)}{X - x_1}\right) \tag{11}$$

Figure 2 shows the temperature temporal average behavior with respect to the spatial coordinate simulated in figure 1. It can be observed that the model achieves a periodic behavior through time. For this figure the units of temperature and time are arbitrary and we obtain the qualitative behavior of T and t.

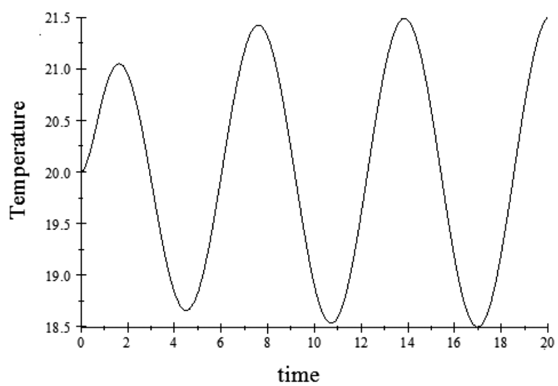


Fig. 2: Temporal behavior of the average temperature for y = 3.

When time tends to infinite:

$$\lim_{t\to\infty}\langle T_x \rangle_t = T_A + \frac{Aw\beta}{\left(w^2+\beta^2\right)}\left(\underbrace{\exp(-t\beta)}_{0} - (\cos tw) + \frac{\beta}{w}(\sin tw)\right)\left(1 - \left(\frac{-2\frac{t}{x_1}\alpha\left(\underbrace{\exp\left(-\frac{1}{4t\alpha}\left(X^2 - x_1^2\right)\right)}_{1} - 1\right)}{X - x_1}\right)\right)$$
$$+ A(\sin tw)\left(\frac{-2\frac{t}{x_1}\alpha\left(\underbrace{\exp\left(-\frac{1}{4t\alpha}\left(X^2 - x_1^2\right)\right)}_{1} - 1\right)}{X - x_1}\right) \tag{12}$$

and

$$\lim_{t\to\infty}\langle T_x \rangle_t = T_A + \frac{Aw\beta}{\left(w^2+\beta^2\right)}\left(\frac{\beta}{w}(\sin tw) - (\cos tw)\right) \tag{13}$$

Finally

$$\langle T_x \rangle_{SS} = T_A + \frac{Aw\beta}{\left(w^2+\beta^2\right)}\left(\frac{\beta}{w}(\sin tw) - (\cos tw)\right) \tag{14}$$

Where the external temperature is given by:

$$T_{x1} = T_A + A(\sin tw) \tag{15}$$

2. METHODOLOGY

In one room house whose walls were entirely built of poured earth the external and internal temperature was monitored in different areas and their temporal behavior was observed. Three times the temperature for seven days during the month of August was recorded, for statistically being the hottest month in the city of Tampico, Tamaulipas. The points in which the superficial temperature, interior and exterior, was measured, of the vertical elements were placed on the central and sunny side of the wall to consider experimentally unilateral heat transfer, for which a Data-logger HobboOn set U-12 Model equipment was used as an input of thermocouples. In the last page of this work it is showed architectural plant of the house walls used in this paper.

Subsequently, a mathematical model from the heat transfer in unsteady conduction of a solid state was made starting with the temperature change Equation and treated as described in the results of this project.

3. RESULTS

It can be seen the oscillatory behavior of the wall temperature that depends on beta parameter, which represents the relationship between the heat transfer coefficient and the heath capacity. In figure 3 behavior of temperature in the wall is observed for different values of beta where the amplitude of these oscillation decreases proportional to this parameter. By the other hand, as we can see in experiments, it is predicted that decrease of the heath transfer coefficient or the growth of the heath capacity of the material are factors that positively affect the comfort of homes.

For the construction of figures 3 and 4, the parameters shown on Table 2 were considered.

Table 2: Parameters assumed for the simulation and construction of figures 3 and 4.

K	A	w	α	Ta
20	5	1	1	25

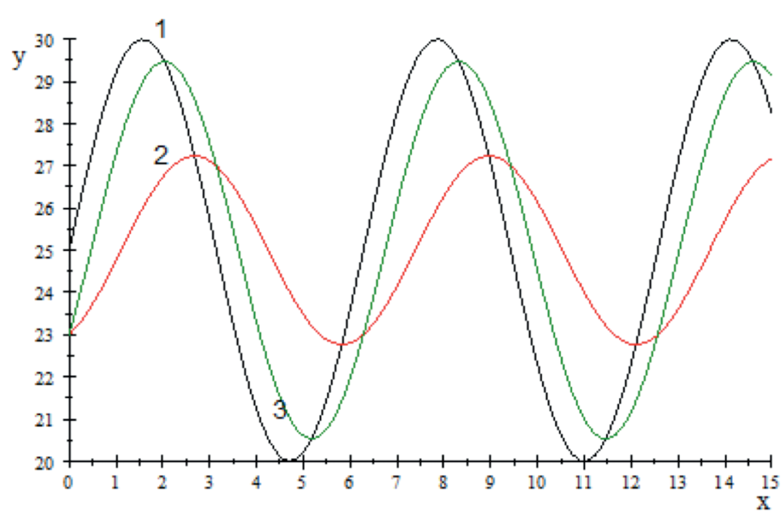


Fig. 3: Behavior of the external temperature for three different values of β.

Figure 4 shows the phase diagram for the external temperature vs wall temperature for different β values when is an implicit variable. It can be seen that diagram, exhibits an ellipsoidal shape, where the major axis exhibits an angle decline while β decrease; which indicates a relationship between the exterior temperature and the average interior temperature (or the interior environment).

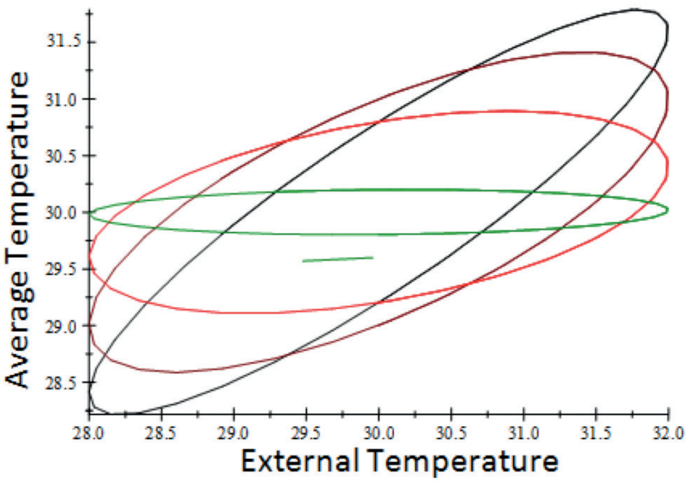
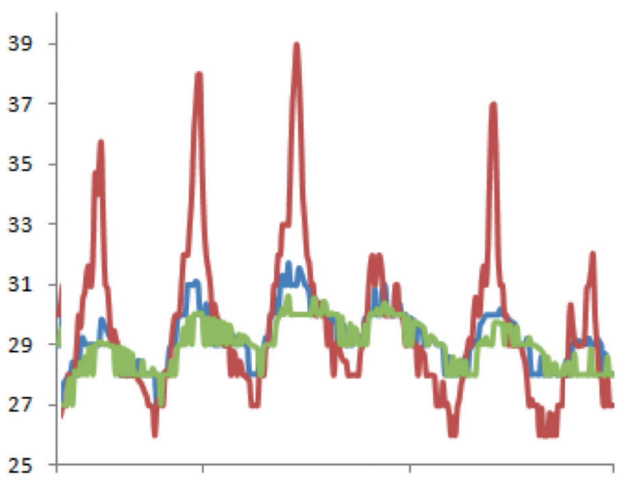


Fig. 4: Phase diagram for different values of β.

These results suggest a qualitative way to evaluate the comfort achieved with certain materials and thickness of the room wall.

Figure 5 shows the experimental results recording along 6 days in august 2013 for two different spaces in a poured earth house. Great differences between exterior temperature (red line) and interior temperature (green line) of over 10 °C can be observed. The blue line represents the interior room temperature. The main difference between room A and room B is the amount of sunlight they receive during the day, being greater in room A due to exposition on two walls, as in the slab, while in room B the sun exposure was limited to one wall and the slab. Nevertheless, on both cases similar interior room temperatures. The maximal average external ambient temperature of those days was 32 °C. It can be seen a deviation of sinusoidal behavior due likely because of a real combination of radiation and convection of heat movement in walls and environmental air.

1.



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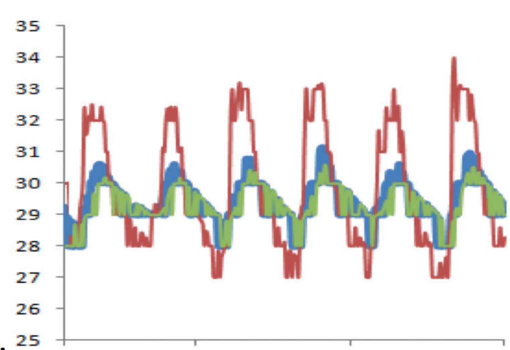


Fig. 5: Experimental results of temperature (°C) wall in a room of poured earth in 6 days for 1. Room A and 2. Room B.

Room and sun temperature on a space, even though it has an intrinsic oscillatory behavior, also exhibits fluctuations, so it is not expected the theoretical behavior predicted with exactitude, but when the phase diagram is built, data points fall within a elliptical region. In this case, the comfort degree can be determined by the inclination of the axis of the ellipse with respect to the horizontal line. In Figure 6 we show the predicted experimental behavior in two areas of a home. In this case it is predicted that room A has a greater comfort to the room B.

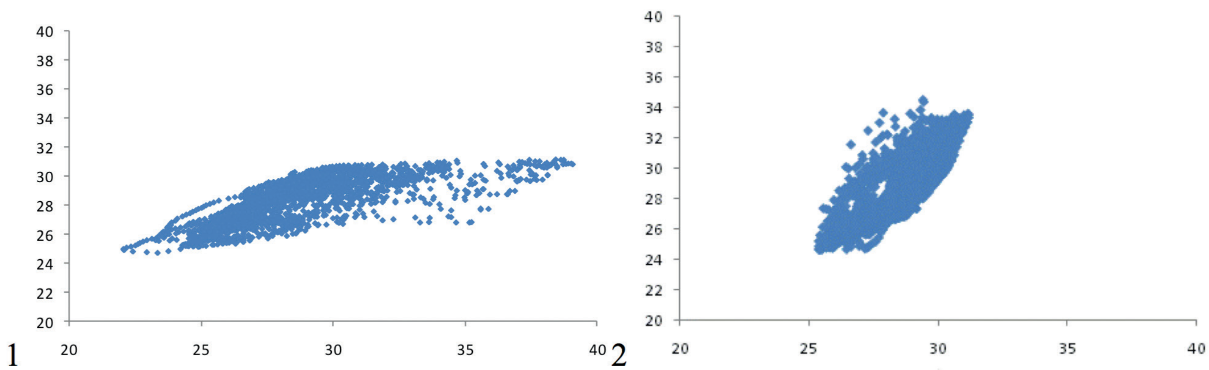


Fig. 6: Phase diagram obtained in: 1. Room A and 2. Room B.

4. CONCLUSIONS

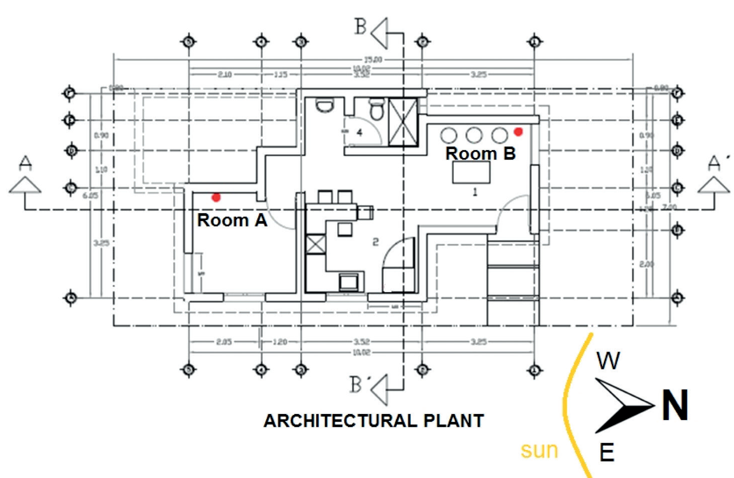
From the phenomenological Equations describing the temperature behavior in heat transport processes, a mathematical model that predicts the behavior of the spatial average temperature wall of a room, considering the intrinsic oscillatory behavior by sun was obtained. From the theoretical results it was found that the phase plane temperature of the surface wall vs room temperature has an elliptical shape where the angle of the axis of the ellipse relative to a coordinate axis is related to the ratio between the coefficient of heat transfer and the ability caloric. This result can be used to characterize the degree of comfort of a plane from the experimentally observed phases.

5. AKNOWLEDGMENT

This research was partially funded by the Mexican company Geo Estratos S.A de C.V. The house used in this research is located in the Facultad de Arquitectura, Diseño y Urbanismo of the Universidad Autónoma de Tamaulipas, its construction was funded by the Consejo Tamaulipeco de Ciencia y Tecnología (COTACYT). Authors thank the support of Dr. José Adan Espuna-Mújica in placing of temperature meters and comments made about our experimental part.

6. ANNEX

Architectural plant of the house used in this work and location of plant A and plant B. Red point indicate the temperature temperature sensor.



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