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Rossby wave propagation tracks in southern hemisphere mean basic flows associated to generalized frosts over southern South America

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RESUMEN

A partir de los estudios observacionales previos acerca de los eventos extremos fríos en el sudeste de América del Sur, surge la hipótesis la cual plantea que los patrones de gran escala condicionan la frecuencia de ocurrencia de heladas, a través de la propagación de ondas de Rossby excitadas remotamente. Aplicando los conceptos de la teoría lineal de propagación de ondas de Rossby, el objetivo en este artículo es analizar la propagación de tales ondas en dos estados básicos diferentes referidos a los inviernos con una máxima y mínima frecuencia de ocurrencia de heladas generalizadas en la Pampa Húmeda (centro-noreste de Argentina). Basado en los conceptos teóricos acerca de la dispersión de onda y el trazo de trayectorias, se identifican los caminos preferenciales de las ondas de Rossby que alcanzan América del Sur a través de la técnica conocida como trazado de rayos. El análisis del flujo básico desde una perspectiva teórica, basado en los cálculos de las trayectorias, permite comprobar que las ondas de Rossby excitadas en forma remota son el mecanismo que favorece la máxima ocurrencia de heladas generalizadas. Siendo los lugares de excitación de tales ondas condicionados por el estado básico en que se propagan. Ellas son excitadas en lugares determinados de la atmósfera, desplazándose hasta América del Sur a lo largo de los jets que actúan como guías de dichas ondas, propiciando la generación de heladas generalizadas. En suma, este artículo presenta una revisión de la técnica de trazado de rayos y como puede ser usada para investigar la ocurrencia de importantes eventos sinópticos tales como las heladas en una región específica y su relación con la propagación de ondas planetarias de gran escala.

ABSTRACT

Based on previous observational studies on cold extreme events over southern South America, some recent studies suggest a possible relationship between Rossby wave propagation remotely triggered and the occurrence of frost. Using the concept of linear theory of Rossby wave propagation, this paper analyzes the propagation of such waves in two different basic states that correspond to austral winters with maximum and minimum generalized frost frequency of occurrence in the Wet Pampa (central-northwest Argentina). In order to determine the wave trajectories, the ray tracing technique is used in this study. Some theoretical discussion about this technique is also presented. The analysis of the basic state, from a theoretical point of view and based on the calculation of ray tracings, corroborates that remotely excited Rossby waves is the mechanism that favors the maximum occurrence of generalized frosts. The basic state in which the waves propagate is what conditions the places where they are excited. The Rossby waves are excited in determined places of the atmosphere, propagating towards South America along the jet streams that act as wave guides, favoring the generation of generalized frosts. In summary, this paper presents an overview of the ray tracing technique and how it can be used to investigate an important synoptic event, such as frost in a specific region, and its relationship with the propagation of large scale planetary waves.

Keywords: Rossby wave, ray tracing, frosts, wet Pampa.

1. Introduction

One of the possible ways of analyzing wave propagation is through linear wave theory in a barotropic atmosphere. In particular, the southern hemisphere offers a unique environment to test the propagation of these waves in observed atmospheric patterns, due to the equivalent barotropic structure and the prevailing zonality of the time-mean flow (van Loon and Jenne, 1972). Previous studies on Rossby wave propagation through a horizontally varying medium (Hoskins and Karoly, 1981; Branstator, 1983; Hoskins and Ambrizzi, 1993), indicate the local applicability of the results obtained from a longitudinally symmetric basic state, as well as to complex asymmetric states, with the condition that the basic state varies slowly with longitude.

Hoskins and Karoly (1981) used the idea of geometric optics ray tracing and wave propagation in a slow varying medium to study the propagation of stationary planetary waves in a zonally symmetric barotropic atmosphere. Later, Karoly (1983) applied this technique to the propagation of low frequency waves in a zonally symmetric basic state and in the propagation of stationary waves in a basic state which varies zonally. An extension of the barotropic Rossby wave linear propagation analysis in a sphere was done by Hoskins and Ambrizzi (1993), considering zonal and meridional variations in the basic wind field. In this case, results give a qualitative insight of the atmospheric behavior with respect to the preferential stationary Rossby wave propagation regions. The authors identified paths based on the stationary wave number K_s. Results show that the main wave activity takes place in the region of the subtropical and polar jet streams, which act as wave guides, as also shown by Berbery *et al.* (1992) and later by Ambrizzi *et al.* (1995) and Ambrizzi and Hoskins (1997).

In general, most of the analyses found in the literature about Rossby wave propagation consider the stationary case in the calculations. However, Yang and Hoskins (1996) analyzed the propagation of Rossby waves in a non-stationary situation, i.e., with positive and negative frequencies corresponding to easterly and westerly propagation, respectively. The non-zero frequency becomes important when considering local effects of meteorological events, such as blocking and westerly flow fluctuations over mountains. On the other hand, disturbances caused by convective heating in specific geographic regions, present a more stationary oscillation. The authors concluded that the stationary Rossby wave theory is generally enough to explain observed teleconnection patterns in the atmosphere (Hsu and Lin, 1992; Hoskins and Ambrizzi, 1993; Ambrizzi *et al.,* 1995).

The goal of this paper is to apply the linear theory concepts, particularly the ray tracing technique, to analyze the propagation, of Rossby waves in two different basic states based on winters with maximum and minimum generalized frost (GF) frequency of occurrence in the Wet Pampa, Argentina.

2. Theoretical and methodological aspects

Hoskins and Ambrizzi (1993) identified the preferential Rossby wave propagation trajectories for different basic states based on the global distribution of the total wave number (K_s).

$$K = K_s = \left(\frac{\beta^*}{\overline{U}}\right)^{1/2} \tag{1}$$

(see Appendix I)

The signs of $\overline{U}$ and β^* present different possibilities:

$$\overline{U}\begin{cases} >0 & \Rightarrow & K_s \exists \\ <0 & \Rightarrow & K_s \nexists \\ =0 & \Rightarrow & K_s \to \infty \text{(at the equator)} \end{cases} \qquad \beta_*\begin{cases} =0 & \Rightarrow & K_s = 0 \\ <0 & \Rightarrow & K_s \quad \nexists \\ >0 & \Rightarrow & K_s \quad \exists \end{cases}$$

Hence the solution to equation (1) exists when: $\beta^* \geq 0$ and $\overline{U} > 0$.

For a better interpretation, it is useful to make an analogy between a "dynamic refraction" mechanism in the atmosphere and the well known Snell Law in optics, which indicates that a ray of light is refracted when it propagates in an environment with different refraction indices (Branstator, 1983; Hoskins and Ambrizzi, 1993). The outcome of this analogy is that barotropic Rossby waves move apart from regions where the basic flow determines low stationary wave numbers. This can be seen from the analysis below.

The horizontal perturbation scale is considered to be much smaller than the basic field variation scale (Liouville-Green approximation or WKB, Wentzel-Kramers-Brillouin, see Gill, 1982), condition which is approximately satisfied in the meridional structure of the atmosphere, though it is seriously compromised in the zonal direction, mainly in the outflow region of the high level jet streams. When this condition is fulfilled and the wave propagates in an environment which only depends on y, the wave number k in the x direction is constant, but the wave number l in the y direction will vary satisfying the local dispersion relationship. In the case of stationary Rossby waves:

$$k = \text{constant}; \qquad k^2 + l^2 = K_s^2 \tag{2}$$

and the local velocity group is still $c_g = 2\,\overline{U}\cos\alpha$ in the $\hat{K}$ direction. The angle α which $\hat{K}$ forms with the x axis is:

$$\tan\alpha = \frac{l}{k} \tag{3}$$

Given a stationary Rossby wave (e.g. with null phase velocity and group velocity given by (A.6), see Appendix I) which propagates in an environment where K_s varies with y, as this wave propagates in the region where K_s decreases, it can be seen from (2) that l will also decrease. Then, from (3), the tangent of α, and consequently the angle itself, will decrease. In other words, the propagation of the wave becomes more zonal, curving towards the higher stationary wave numbers. Following an analogous reasoning, it can be seen that the propagation in a region where K_s increases will result in the increase of α, leading to a more meridional propagation (curving towards high K_s).

This result has a very important implication: barotropic Rossby waves abandon regions where the basic flow determines low stationary wave numbers.

Besides, from equations (2) and (3) it can be seen:

1. At latitudes where K_s is equal to the zonal number k, l is equal to zero (zonal propagation), representing turning points in the wave propagation.
2. At latitudes where β^* and therefore K_s, is equal to zero the meridional wave number becomes imaginary, such that the rays must return before hand from these latitudes.
3. Latitudes where $\overline{U}$ is equal to zero, "critical latitudes", K_s tends to infinity, which implies that the same will occur to l, and consequently α is 90°. From (A.6) and (A.7) (Appendix I) the stationary group velocity will tend to zero and the ray will be "absorbed".

Moreover, the curvature in the Rossby wave propagation will be anti-clockwise if K_s increases equatorward and clockwise if it increases towards the poles.

$$r^{-1} = -k \frac{dk_s^{-1}}{dy} \qquad (4)$$

(see Appendix II)

The ray refraction for different situations is presented schematically in Figure 1.

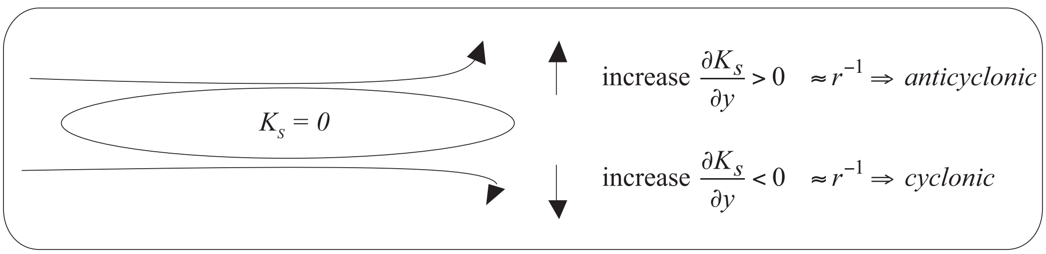

Fig. 1. Trajectories deflection (HS).

3. Ray tracing technique

Ray tracing is defined as a set of points in c_g direction (or c_{estg}, see Appendix I), such that energy propagates along the ray with a velocity equal to the group velocity (Hoskins and Karoly, 1981; Karoly, 1983; Hoskins and Ambrizzi, 1993).

Hoskins and Ambrizzi (1993) derived the expression given by equation (4), which sets the curvature radius of the preferential energy propagation path present in such waves. The authors show the agreement between the curvature radius and the model numerical solutions obtained by them, indicating that the essential characteristics of Rossby wave propagation are represented by the radius. The results of the numerical solutions are known as "ray tracing".

Ray tracing is obtained from the numerical integration of the group velocity:

$$c_g = (u_g, \nu_g) = (d\mathbf{x}/dt, d\mathbf{y}/dt) \qquad (5)$$

Following the derivation from the Appendix I equation (A.4), results:

$$\frac{d\mathbf{x}}{dt} = \frac{2\beta^* k^2}{K^4}, \qquad \frac{d\mathbf{y}}{dt} = \frac{2\beta^* kl}{K^4} \qquad (6)$$

The previous equations are integrated using a second order Runge-Kutta scheme from a given initial position (x_0, y_0), total wave number (K) and frequency (ω). Additionally the relationship between the zonal wave number k and the total wave number K given by $k = K/a$ which is only valid when $l = 0$, where a is the mean Earth radius (see also Yang and Hoskins, 1996).

4. Analysis of Rossby wave trajectories

According to the previous discussion, ray tracing technique is highly dependent on the mean zonal flow. In this sense, prior to presenting the results obtained by this method, it is important to understand the departure of the mean zonal wind during the maximum and minimum occurrence of GF in relation to the mean flow. Müller *et al*. (2005) isolated those years of extreme frequency of frost occurrence. A maximum (minimum) frequency of GF in the Wet Pampa is considered when frosts are one standard deviation (σ) above (below) the mean values during the austral winter (June-July-August - JJA) for the period 1961-1990 (GF $+\sigma$ and GF $-\sigma$, or $+\sigma$ and $-\sigma$, respectively). Two sections, one at 170° E and other at 110° W (Figs. 2a, b) depict the seasonal means of the zonal wind at 250 hPa for both extreme winters and the mean zonal climatology of the 1961-1990 period. Figure 2a (170° E) shows that the subtropical jet has its maximum near 28° S, having a slightly higher (lower) intensity for $-\sigma$ ($+\sigma$) with respect to the total mean. The polar jet appears with a maximum near 62° S for $-\sigma$ and at 60° S for $+\sigma$, where the position of the latter coincides with the position of the winter mean maximum.

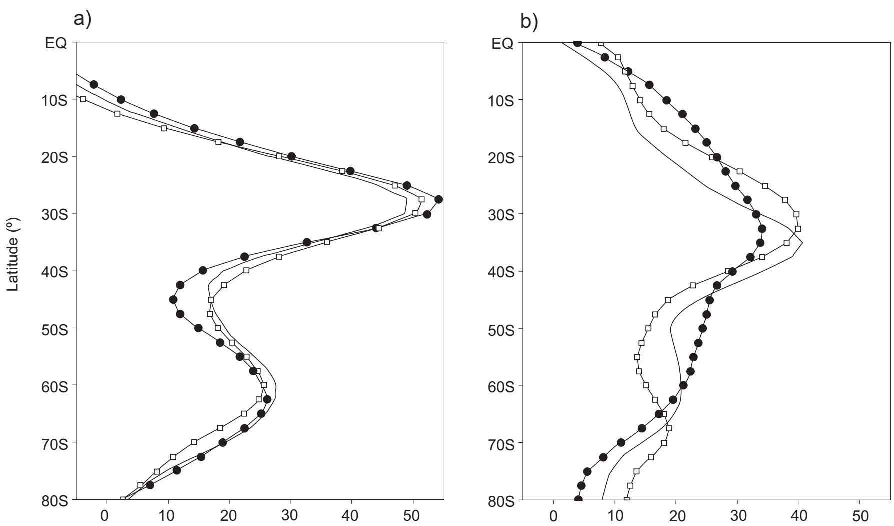

Fig. 2. Meridional profiles corresponding with the zonal mean of the zonal wind component (ms^{-1}) at 170° E a) and 110° W b) at 250 hPa. The full line represents the $+\sigma$ cases, the line with full circles the $-\sigma$ cases and the line with open circles the winter mean.

The section situated at the exit of the subtropical jet (Fig. 2b, 110° W), shows several differences with the analyzed cases. One of them is that the $-\sigma$ basic state shows a less intense subtropical jet stream than the observed in $+\sigma$, both positioned south of the mean flow, approximately at 35° S. The polar jet for the $+\sigma$ is located to the north of the mean flow and it has a larger amplitude while in $-\sigma$ its main position is not clearly defined as shown by Müller and Ambrizzi (2007).

These differences in position and intensity of the jets suggest that they may play an important role in the definition of GF events in the Wet Pampa, bearing in mind that the winters compared differ in the frequency of occurrence of such events. Therefore it is not expected that the winter mean flow profile will be located between the $+\sigma$ and $-\sigma$ composite flows as Fig. 2b clearly indicates. The mentioned jet characteristics, e.g. acting as wave guides and other existent conditions in the basic state represented by the parameter K_s analyzed by Müller and Ambrizzi (2007, see their Figs. 2e, f) are considered in the analysis of the Rossby wave trajectories.

The analysis is done for each basic state $+\sigma$ and $-\sigma$ and different wave numbers. Integrations were done over a period of 15 days with a 4 h time step, using the numerical code developed by Yang and Hoskins (1996). Results are plotted every 12h presented in Figure 3 where the upper left (right) panel refers to the basic state $+\sigma$ ($-\sigma$). Only those rays which reach the South American continent are shown for each wave number. It should be pointed out that in the present study the basic state is an average in time and it varies with latitude and longitude. Previous ray tracing studies (e.g. Karoly, 1983) showed that waves tend to propagate in the same direction as the local wind.

We start the analysis at latitudes where the extratropical intraseasonal variability in the southern hemisphere circulation is highest, i.e. between 40 and 60° S (Jones and Simmonds, 1993). It is there where the smallest wave numbers present a relative maximum according to the results obtained for K_s shown by Müller and Ambrizzi (2007). The dominating wave number is 3 and in a lesser degree 4, as seen in Figure 3a. The left panel in Figure 3a shows a propagation for wave number 3 along 50° S in the $+\sigma$ case. Further southwards, but in the right panel of the same figure, a wavelike trajectory is presented for $-\sigma$. The wave trajectory in the $+\sigma$ case follows the winter storm track climatological position (e.g. Beu and Ambrizzi, 2006). The wave path followed in the $-\sigma$ basic state is closer to the Antarctic continent and away from South America.

However, wave number 4 only propagates at high latitudes for $+\sigma$ as can be seen in the trajectory presented in Figure 3b (left panel). For the same wave number but in the $-\sigma$ basic state (Fig. 3b, right panel) the trajectory is northwards entering the continent at middle latitude and continuing in the northeast direction almost reaching the equator where it is finally absorbed.

The similar propagation characteristics of wave number 3 and 4 in both cases analyzed is given by the fact that they are the most frequent low frequency modes in the southern hemisphere. During the austral winter, most of the variability is due to the propagation of wave numbers 3 and 4 (Mechoso and Hartmann, 1982; Randel, 1987; Randel *et al.,* 1987). In particular wave 3 presents a maximum in the stationary variance during winter (Hansen *et al.,* 1989), while wave 4 patterns are modes associated with baroclinic waves traveling eastwards at high latitudes (Randel, 1987).

Applying a similar analysis to the latitude band associated with the subtropical jet, verifies that the wave numbers which propagate are higher (5, 6, 7 and 8) than those found for high latitudes, as can be seen in Müller and Ambrizzi (2007, see their Figs. 2e, f). In each case a large number of initial points were tested, obtaining that in practically all cases the trajectories that propagate towards lower latitudes are absorbed in the critical latitude band (see Section 2). The exception is wave 7 and occasionally wave 6, which presented trajectories either reaching the South American continent or following the subtropical jet. An example for wave number 7 is shown in Figure 3c. The right panel corresponds to $-\sigma$ case, and it is clearly seen how the trajectory follows the subtropical jet at 30° S. In the $+\sigma$ case (left panel), the propagation path reaches the South American continent and later deviates slightly northwards where it is finally absorbed (see Section 2). The other wave numbers, particularly for $+\sigma$ basic flow, do not reach the continent for they are previously absorbed at the denominated critical latitude, near the equator.

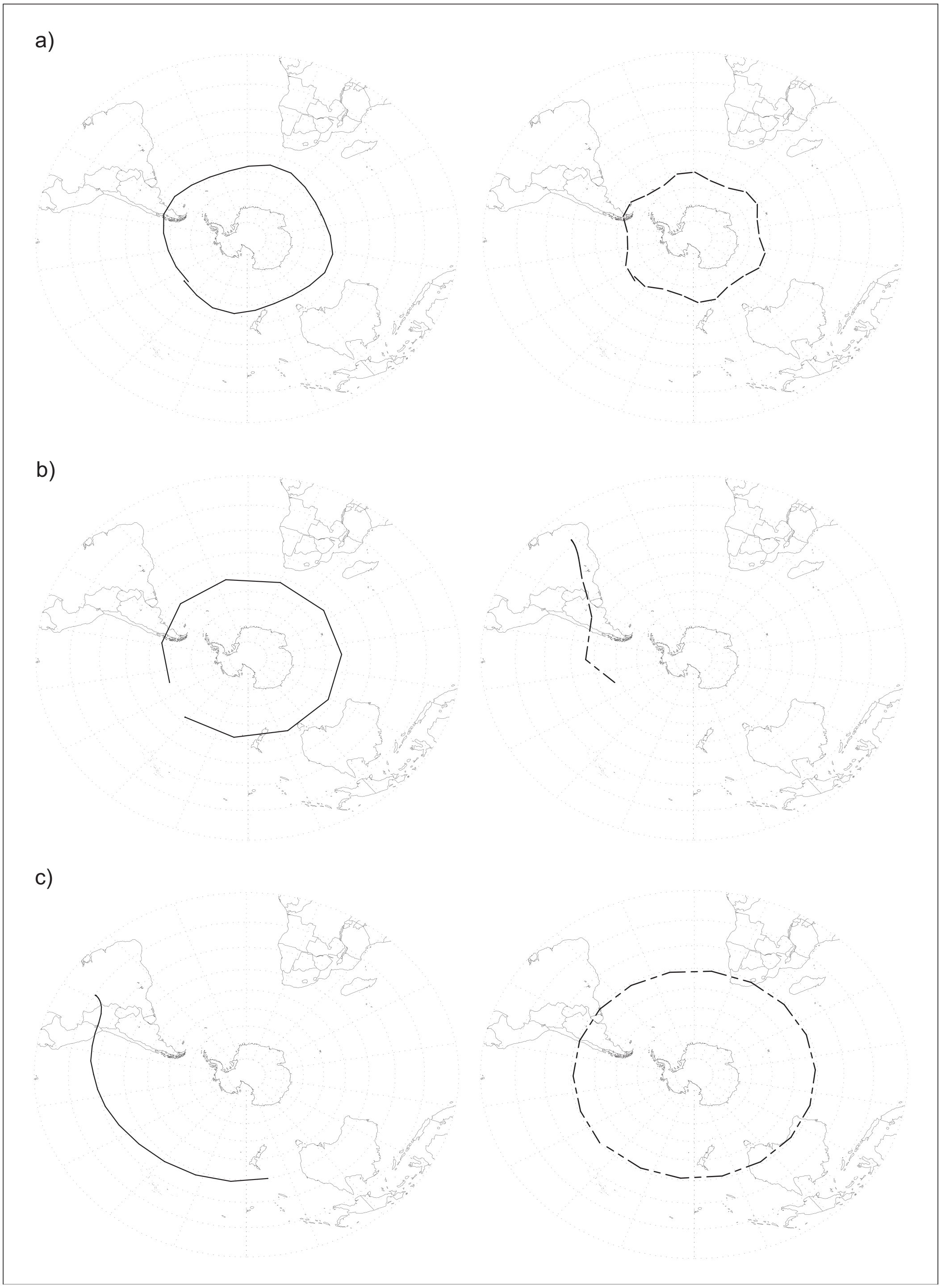

Fig. 3. Preferential trajectories of stationary Rossby wave energy propagation at 250hPa, referred to the composites of $+\sigma$ (left panel) and $-\sigma$ (right panel) winters. Propagation for wave numbers a) 3, b) 4, and c) 7.

5. Conclusions

This study presents further evidence that there is an important relationship between the maximum occurrence of generalized frosts (Müller *et al.*, 2005) and Rossby wave propagation remotely triggered. Using a linear wave tracking scheme which is sensitive to small changes in the basic state, it was possible to confirm that the Southern Hemisphere jets can act as wave guides as previously suggested in the literature (Berbery *et al.*, 1992; Hoskins and Ambrizzi, 1993; and others). Our results are in agreement with those obtained by Müller *et al.* (2008), who run a global baroclinic model using $+\sigma$ and $-\sigma$ basic flows and demonstrated that the GF over Wet Pampa are closely related to the phase of the Rossby waves entering the South American continent. These results clearly indicate that it is the variability of the basic state that determines how the waves will propagate and therefore affect other remote regions once they are triggered.

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Appendix I

The linear barotropic vorticity equation on a β plane:

$$\frac{\partial}{\partial t}\nabla^2\Psi + \overline{U}\frac{\partial}{\partial x}\nabla^2\Psi + \beta\frac{\partial}{\partial x}\nabla^2\Psi = 0 \qquad \text{(A.1)}$$

where it is assumed that U is a westerly flow and $\nabla^2\Psi$ is the relative vorticity.

The dispersion relationship obtained from equation (A.1) for perturbations in the plane of the form $exp[i\ (kx + ly - \omega t)]$ is:

$$\omega = \overline{U}k - \frac{\beta^* k}{K^2} \qquad \text{(A.2)}$$

where

$$\beta^* = \beta - \frac{\partial^2 \overline{U}}{\partial y^2} \qquad \text{(A.3)}$$

is the meridional gradient of the absolute vorticity, ω is the frequency, $K = (k^2 + l^2)^{1/2}$ is the total wave number and k and l are the zonal and meridional wave numbers respectively. The wave activity in the plane moves with the group velocity:

$$c_{est_g} = \left(u_g, v_g\right) = \left[\frac{\partial\omega}{\partial k}, \frac{\partial\omega}{\partial l}\right] = \left(c, o\right) + \left[\frac{2\beta^*}{K^2}\right]\cos\alpha\hat{K} \qquad \text{(A.4)}$$

where $c = \omega/k$ is the eastward phase velocity, $\hat{K}$ is the unit vector normal to the trough and ridge axis with positive component eastward, and α is the angle the unit vector forms with the eastward direction.

The stationary Rossby waves ($\omega = 0$, $c = 0$), with:

$$K = K_s = \left(\frac{\beta^*}{\overline{U}}\right)^{1/2} \tag{A.5}$$

are possible if the flow is westerly (positive $\overline{U}$) and β^* is positive. From (A.4) and (A.5),

$$c_{est_g} = c_g \hat{K} \tag{A.6}$$

where

$$c_g = 2\overline{U}\cos\alpha \tag{A.7}$$

Hence, for stationary Rossby waves energy propagates normal to the trough and ridge axis with a velocity equal to $2\,\overline{U}\,\cos\alpha$. In particular, the activity of a stationary Rossby wave propagates zonally with a velocity equal to twice the flow speed.

Appendix II

Considering that the Rossby ray paths refract in a similar way to that described by optics Snell Law, when they move with the group velocity c_g, of the equation (2) (see Section 2) the rates of change for k and l are given by:

$$\frac{d_g k}{dt} = 0 \qquad \frac{d_g l}{dt} = \frac{K_s}{l}\frac{d_g K_s}{dt} = \frac{K_s}{l} v_g \frac{dK_s}{dy}$$

but from (A.6) results $v_g/l = c_g/K_s$, then:

$$\frac{d_g l}{dt} = c_g \frac{dK_s}{dy} \tag{A.8}$$

From (3), the ray curvature is given by:

$$\frac{d_g \tan\alpha}{dt} = \frac{1}{k} c_g \frac{dK_s}{dy} \tag{A.9a}$$

or:

$$\frac{d_g\alpha}{dt} = \frac{k}{K_s^2} c_g \frac{dK_s}{dy} \tag{A.9b}$$

Therefore Rossby rays always refract towards latitudes with higher K_s. Defining the curvature radius of the ray path, r, as positive if it curves anticlockwise, the (A.9b) can be written as follows:

$$\frac{d_g\alpha}{dt} = \frac{c_g}{r}$$

Then, the curvature radius of the ray path is given by the simple expression:

$$r = K_s^2 / \left(k \frac{dK_s}{dy} \right) \quad \text{(A.10a)}$$

or:

$$r^{-1} = -k \frac{dK_s^{-1}}{dy} \quad \text{(A.10b)}$$

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