The First Rule of Stoic Logic and its Relationship with the Indemonstrables*

Miguel López-Astorga
Instituto de Estudios Humanísticos "Juan Ignacio Molina",
Universidad de Talca, Chile.
milopez@utalca.cl

Abstract

In addition to the indemonstrables, Stoic logic included a number of reduction rules. In this paper, I analyze the first one of such rules in order to prove whether it was formally derived from the indemonstrables or the Stoics could raise it from the use of their natural reasoning abilities. Thus, I try to show that there are reasons to support both possibilities and, based on a semantic approach such as that of the mental models theory, to give arguments in this regard.

Keywords: indemonstrables; formal deduction; logical rules; reasoning; Stoic logic.

Resumen

Además de los indemostrables, la lógica estoica incluye varias reglas de reducción. En este trabajo, analizo la primera de ellas con el fin de comprobar si fue derivada formalmente a partir de los indemostrables o los estoicos pudieron plantearla a partir del uso de sus capacidades naturales de razonamiento. De esta manera, trato de mostrar que tenemos razones para apoyar ambas posibilidades y, basándome en un enfoque semántico como el de la teoría de los modelos mentales, de ofrecer argumentos al respecto.

---

*This paper is a result of the Project N. I003011, "Algoritmos adaptativos e inferencias lógicas con enunciados condicionales," supported by the Directorate for Research of the University of Talca (Dirección de Investigación de la Universidad de Talca), Chile. The author is also the main researcher of that Project".

Recibido: 01 - 06 - 2015. Aceptado: 05 - 08 - 2015.

Palabras clave: indemostrables; deducción formal; reglas lógicas; razonamiento; lógica estoica.

Introduction

As it is well known, Diogenes Laërtius, in *Vitae Philosophorum* 7, 79-81, attributes to Chrysippus of Soli five indemonstrables, which later were named modus ponendo ponens, modus tollendo tollens, modus tollendo ponens, and modus ponendo tollens I and II. There is no doubt that the indemonstrables are the essential elements of the common view of Stoic logic. However, we also know that that logic included other important elements too. Several ancient sources speak about certain reduction rules (θέματα), which allowed reducing the demonstrable syllogisms to the indemonstrables (Boeri and Salles, 2014, 223). The problem is that the sources are not clear in this regard. Galen, in *De Placitis Hippocratis et Platonis* 114, 1-10, states that these rules are four. Nevertheless, the sources explicitly only describe two of them: the first one and the third one. Pseudo-Apuleius mentions the first one in *De Interpretatione* 191, 5-10, and Alexander of Aphrodisias indicates the third one in *Aristotelis Analyticorum Priorum* 278, 11-14 (these three passages are to be found in their original languages in Boeri and Salles, 2014, 230-231, and translated into Spanish in Boeri and Salles, 2014, 217).

I will only address the first rule in this paper. The reason is that it is considered to be the most basic or fundamental rule by Pseudo-Apuleius and, given its structure, enables interesting reflections on it. In particular, what is important about that rule is its possible origin. On the one hand, it can be thought that it was derived from two indemonstrables (from modus tollendo tollens and modus ponendo tollens), which would reveal that Stoic logic was clearly a formal and complex logical system much more akin to the current systems than usually thought. Nevertheless, with the help of a contemporary theory of reasoning, the mental models theory, it can also be thought that the rule was assumed by purely psychological reasons, that common sense leads to it, and that it hence comes from human being's natural inferential abilities.

To clarify which of these two options is the correct one is, undoubtedly, important, since that can provide to us an actual idea of what Stoic logic really was, and of which its true sense was. In this paper, I will try to show that we have arguments supporting both possibilities and to explain which those arguments are. But, before starting, a brief explanation of the general theses of Stoic logic seems to be necessary.

Stoic logic and its reduction rules

We can find several general descriptions of Stoic logic and discussions on some of its most controversial theses in different works (e.g., Barnes, Bobzien, and Mignucci, 1999, 92-176; Bobzien, 1996; Bocheński, 1951, 77-102; Boeri and Salles, 2014, 215-237; Gould, 1970; Kneale and Kneale, 2008, 113-176; Lukasiewicz, 1967; Mates, 1953, Mueller, 1978, O'Toole and Jennings, 2004). Most of such works give a systematic view of Stoic logic, but they do not always offer the same interpretations of the same theses and present very interesting debates and lines of study. However, because the scope of this paper is modest (it is only intended to analyze the true nature of the first reduction rule), it can be enough a brief exposition of the main theses of Stoic logic that are related to the problem that will be reviewed here. In this way, I will only focus on the arguments of the previous works relevant for the aims of this paper.

As said, the indemonstrables assigned to Chrysippus of Soli by Diogenes Laërtius are five, and using the denominations that were attributed to them later, they can be expressed as follows:

Modus ponendo ponens:

If (*εἰ*) A, B
But actually (*ἀλλὰ μήν*) A

Therefore (*ἄρα*) B

Modus tollendo tollens:

If (*εἰ*) A, B
But actually not (*οὐκ ἀλλὰ μήν*) B

Therefore not (*οὐκ ἄρα*) A

Modus ponendo tollens I:

Not (*οὐκ*) A and (*καί*) B
But actually (*ἀλλὰ μήν*) A

Therefore not (*οὐκ ἄρα*) B

Modus ponendo tollens II:

Either (*ἤτοι*) A or (*ἤ*) B
But actually (*ἀλλὰ μήν*) A

Therefore not (*οὐκ ἄρα*) B

Modus tollendo ponens:

Either (*ἤτοι*) A or (*ἤ*) B
But actually not (*οὐκ ἀλλὰ μήν*) A

Therefore (*ἄρα*) B

As commented, these arguments were essential in Stoic logic, but they were not the only elements. There were also reduction rules (θέματα). Boeri and Salles (2014, 223) claim that their number, their exact role, and their nature remain unclear. Nonetheless, Galen informs to us, as also said, that they were four, and Pseudo-Apuleius and Alexander of Aphrodisias describe the first one and the third one respectively. Because the first one is the rule that will be mainly analyzed here, it seems opportune to quote the passage written by Pseudo-Apuleius (*De Interpretatione* 191, 5-10) in which it appears:

> *est et altera probatio communis omnium etiam indemonstrabilium, quae dicitur per impossibile appellaturque a Stoicis prima constitutio vel primum expositum. quod sic definiunt: 'si ex duobus tertium quid colligitur, alterum eorum cum contrario illationis colligit contrarium reliquo.'*

> There is also another proof common to all the indemonstrables, which is named by the impossible and the Stoics refer to it as the first statement of the first recorded. It is defined in this way: 'if from two [propositions] a third [proposition] is concluded, from any of them [the propositions] along with the opposite of the conclusion, the opposite of the other one [the other proposition] is concluded.

This passage is important in several senses. On the one hand, Pseudo-Apuleius appears to be the only source that describes this rule. On the other hand, by affirming that it is the first statement (*prima constitutio*) and the first recorded (*primum expositum*), Pseudo-Apuleius seems to mean that it is the most basic and essential rule. What is interesting now is how the Stoics came to it. It could be the result of a deduction more or less similar to those of modern logic. However, it could also be the result of a natural use of the reasoning ability, or, if preferred, of the use of the common sense. As it will be shown, we have reasons for accepting both possibilities. In the next section, I expose the reasons for the first one.

Was the first reduction rule formally derived from the indemonstrables?

As far as this possibility is concerned, I think that it is appropriate to say that works such as that of Bobzien (1996) and that of Barnes et al. (1999, 92-176) can be very interesting for a discussion in this regard, and that all the arguments in this way that I will expose in this paper can be reviewed or even complemented taking works such as those into account. That said, an important point should be firstly noted. We have evidence that Stoic logic admitted conditionalization processes. This means that we can assume the idea that the Stoics thought that an inference with two premises and one conclusion could be considered to be a conditional whose antecedent (ἡγουμένον) consisted of conjunction of the two premises, and whose consequent (λῆγον) was the conclusion (see, e.g., O'Toole and Jennings, 2004, 491-495, for a discussion). A passage of Sextus Empiricus (*Pyrrhoneae Hypotyposes* 2, 137) is very illustrative in this regard:

> τῶν δὲ λόγων οἱ μέν εἰσι συνακτικοὶ οἱ δὲ ἀσυνακτοι, συνακτικοὶ μέν, ὅταν τὸ συνημμένον τὸ ἀρχόμενον μὲν ἀπὸ τοῦ διὰ τῶν τοῦ λόγου λημμάτων συμπεπλεγμένου, λῆγον δέ εἰς τὴν ἐπιφορὰν αὐτοῦ, ὑγιὲς ᾖ, οἷον ὁ προειρημένος λόγος συνακτικός ἔστιν, ἐπεὶ τῇ διὰ τῶν λημμάτων αὐτοῦ συμπλοκῇ ταύτῃ 'ἡμέρα ἔστι, καὶ εἰ ἡμέρα ἔστι, φῶς ἔστιν' ἀκολουθεῖ τὸ 'φῶς ἔστιν' ἐν τούτῳ τῷ συνημμένῳ 'εἰ ἡμέρα ἔστι, καὶ εἰ ἡμέρα ἔστι, φῶς ἔστιν <φῶς ἄρα ἔστιν>' ἀσύνακτοι δὲ οἱ μὴ οὕτως ἔχοντες.
>
> Some arguments are conclusive and other are not conclusive. They are conclusive whenever the conditional starting with the premises of the argument linked and finishing with the conclusion is sound. For example, the argument referred above is conclusive, since, by joining the premises 'it is daytime and if it is daytime, there is light,' 'there is light' can be drawn from the following conditional: 'if [it is daytime and (if it is daytime, there is light)], then there is light.' Those that are different are not conclusive.

From passages such as this one (which is also taken into account by Boeri and Salles, 2014, 231 & 217-218, who present a version of it in Spanish as well), we can say that, given that the indemonstrables all have three propositions (two premises and a conclusion), all of them can be expressed in this way:

A
B

ἄρα (therefore) Γ

Obviously, A refers to the first premise (i.e., 'if A, B' in modus ponendo ponens and in modus tollendo tollens, 'not A and B' in modus ponendo tollens I, and 'either A or B' in modus ponedo tollens II and in modus tollendo ponens), B stands for the second premise (i.e., 'A' in modus ponendo ponens and in modus ponendo tollens I and II, 'not B' in modus tollendo tollens, and 'not A' in modus tollendo ponens), and

Γ represents the conclusion (i.e., 'B' in modus ponendo ponens and in modus tollendo ponens, 'not A' in modus tollendo tollens, and 'not B' in modus ponendo tollens I and II). Thus, the previous passage authored by Sextus Empiricus enables us to state that the five indemonstrables match this structure:

If (A and B), then Γ.

It this is so, because the first reduction rule describes a scenario in which the conclusion is denied and one of the premises is true, we can think about, for example, an inference with these three premises:

[1] If (A and B), then Γ	(premise)
[2] A	(premise)
[3] not Γ	(premise)

But it is evident that these premises allow using one of the indemonstrables, modus tollendo tollens, and deducing:

[4] not (A and B)	(modus tollendo tollens, 1 and 3)

However, step 4 enables to apply another indemonstrable, modus ponendo tollens I, and to come to:

[5] not B	(modus ponendo tollens I, 2 and 4)

So, it can be proved that the first reduction rule is correct by means of the indemonstrables, in particular, by means of two of them. And this means that it is possible to consider Stoic logic to be a formal system more or less akin to the current ones and in which logical form plays a very important role. From this point of view, it could even be said that Stoic logic is the clearest background of the modern deduction systems and the natural deduction calculi such as the one of Gentzen (1935). After all, under this view, Stoic logic could be interpreted as a system with five axioms (the five indemonstrables expressed as conditionals propositions), which allow deducing even important rules (the reduction rules).

Nevertheless, unfortunately, we cannot be absolutely sure about that. Of course, it is possible that the Stoics formally draw their reduction rules from their indemonstrables. Nonetheless, it is also possible that their reasoning abilities led them to the rules without carrying out formal deductions. As said, the mental models theory can help us show that. However, it seems to be opportune to comment the general theses of this theory before explaining how it can describe the mental process that can lead to the rules without making syntactic derivations.

Conjunctions, conditionals, and negations in the mental models theory

We can find an extensive literature on the mental models theory (only some examples of relevant works addressing it are Johnson-Laird, 2006, 2010, 2012; Khemlani, Orenes, and Johnson-Laird, 2012, 2014; Oakhill and Garnham, 1996; Orenes and Johnson-Laird, 2012). That is a psychological reasoning theory trying to describe and predict the actual human inferential activity. It would take a long explanation to expose all its theses. For this reason, I will only mention those that are relevant for the aims of this paper.

Firstly, a very important point of the theory is that human reasoning works by considering combinations of semantic possibilities. According to it, propositions refer to semantic iconic models that stand for all the possible scenarios that can be true given such propositions. In this way, individuals check and compare the models, keeping those that are consistent with each other and rejecting those that lead to a contradiction. Really, the theory considers the models of all the operators of standard logic. Nevertheless, as far as my goals here are concerned, only those of conjunction, the conditional, and the denial are interesting. I begin with conjunction.

Conjunction only refers to one model. Thus, an expression of the kind 'A and B' only enables one possible scenario:

A B

This model represents the situation in which both A and B are true. All the other possibilities are not admitted by conjunction.

On the other hand, the 'Fully Explicit Models' of the conditional, that is, of the expressions of the type 'if A, then B', are, however, three:

A	B
not-A	B
not-A	not-B

Maybe it is necessary to clarify that, as mentioned, these are the Fully Explicit Models of the conditional. This point is important because, according to the theory, people do not always detect these three models. Many times, they only note the first one (i.e., that in which both A and B happen), since noting the three models requires further reflection. But what is relevant in this paper is that the conditional has two models more that conjunction: a model in which the antecedent is false and the consequent is true, and a model in which both of them are false.

Finally, the models of a denied expression are the complement of the entire set of possibilities corresponding to that expression when affirmed. In this way, the models of a denied conjunction are the complement of the models of conjunction. Therefore, if, as said, the only model of conjunction is that related to a scenario in which the two conjuncts are true, the models of a denied conjunction are the following:

A	not-B
not-A	B
not-A	not-B

Perhaps it is also important to note that, while the models described by the theory seem to be akin to the cases in which the operators are true in the truth tables of standard logic, they are not. As commented on, the mental models (I am using the term 'mental' because it is the term used by the proponents of the theory) are iconic and each of them consists of a representation of reality. The idea is that human reasoning is not syntactic, that it mainly considers the content and the meaning of sentences, and that pragmatics can also have an influence on it.

I could indicate here many examples of how the theory works in practice. However, for the aims of this paper, just some of them can be enough. An interesting case in this regard is, undoubtedly, that of certain conditionals. The Fully Explicit Models of the conditional clearly enable, in principle, the use of both modus ponendo ponens and modus tollendo tollens. Given such Fully Explicit Models and, as a second premise, A (modus ponendo ponens), the only possibility is, as the first model informs, B, since in the other two models (the second one and the third one) A is false. On the other hand, if the second premise is not-B (modus tollendo tollens), it is only possible, as established in the last model, not-A, since B is true in the first two models. But, as said, logical form is not the more relevant aspect in human reasoning following the mental models theory. Let us think about, for example, this apparent conditional:

"If she played a musical instrument then she didn't play a flute" (Johnson-Laird, 2010, 204).

This is only an apparent conditional because its Fully Explicit Models do not correspond to those of general conditionals. The only possibilities to which it refers are these ones:

She played a musical instrument.	She did not play a flute.
She did not play a musical instrument.	She did not play a flute.

In this case, the third Fully Explicit Model (she did not play a musical instrument and she played a flute) is impossible because flutes are musical instruments. So, one cannot play a flute without playing a musical instrument at the same time. This is important for two reasons. Firstly, it shows that, although the sentence includes the terms 'if' and 'then', it does not correspond actually to a conditional. Secondly, given that the third Fully Explicit Model is missing now, modus tollendo tollens cannot be applied to this sentence. In this way, the mental models theory can explain why individuals do not often draw a conclusion such as 'she did not play a musical instrument' from premises such as 'if she played a musical instrument then she did not play a flute' and 'she played a flute.' It is true that to derive such a conclusion would be to use modus tollendo tollens correctly, but the models corresponding to the conditional sentence do not allow doing that.

Likewise, the theory solves problems that the psychological theories proposing that the human mind works in accordance with syntactic rules cannot. For example, to draw 'if A, then B' from just the premise 'B' is absolutely correct in standard propositional calculus. Nevertheless, as it can be seen in the literature on cognitive science, individuals often fail to make that deduction. The reason for the mental models theory is obvious. As mentioned, the Fully Explicit Models of the conditional are three, and the last of them (the one in which both A and B are false) is incompatible with the premise, that is, is inconsistent with B (for a discussion on this issue, see, e.g., Orenes and Johnson-Laird, 2012). In this way, according to the proponents of the theory, this is a proof that human reasoning does not work following syntactic rules, and that the human natural way of reasoning is basically semantic.

Based on this general framework, the mental models theory can describe a way to come to the first reduction rule that is not formal and does not require the use of the indemonstrables. This means that, if the mental models theory is correct and exactly describes how human reasoning works in a natural way (without logical background or training), it is not absolutely guaranteed that the Stoics came to their reduction rules from a formal application of the indemonstrables. They could propose those rules because they considered them to be common sense rules that can be obtained by means of a simple and natural use of human reasoning. I account for this idea in details in the next section.

Mental models and the first reduction rule

As indicated, it can be said that the five indemonstrables have this underlying structure:

If (A and B), then Γ.

Thus, we can assume again as premises the latter conditional, A, and not Γ. Given that the first premise is a conditional, in principle, its models would be these:

(A and B)	Γ
not-(A and B)	Γ
not-(A and B)	not-Γ

But the third premise immediately reveals that the only possibility is the third model (Γ is true in the other models and they hence are inconsistent with the third premise). Nonetheless, it can be thought that the third model is not actually one model, but three, because not-(A and B) in turn refers to three possibilities (it is a denied conjunction). In this way, the third model can be displayed as follows:

A	not-B	not-Γ
not-A	B	not-Γ
not-A	not-B	not-Γ

However, given that A is the second premise, only the first one of these three last models can be accepted, which means that, in a scenario in which both not-Γ and A occur, it is only possible that not-B happens (or, if preferred, that B does not happen).

Therefore, it can be said that, if the mental models theory is correct, there are also psychological reasons to assume the first reduction rule of Stoic logic. This is so because the theory shows us that the rule can be concluded in a semantic way without using formal deduction procedures. Of course, one might think that, although the mental models theory is, undoubtedly, a framework with strong experimental supports (the experiments reported in the literature of cognitive science reveal that it can explain and predict participants' majority responses in very different reasoning tasks), it is not absolutely clear that it is the theory that best explains and describes the real way human reasoning works. But, even in that case, there is no doubt that the mental models theory demonstrates that the first reduction rule of Stoic logic can be found by means of processes other than the strict formal deduction. Perhaps the key issue is to review whether the most important aspect in Stoic logic was syntax or semantics.

Conclusions

The reasons why the Stoics assumed their first reduction rule are not clear. It could be the result of an absolutely syntactic and formal derivation, but, as shown by means of the explanation based on the mental models theory, it could also be the result of more semantic processes. Therefore, it can be stated that further research is needed in order to truly know the real scope, sense, and meaning of Stoic logic.

One possible objection to this paper can be that it is only focused on one of the reduction rules, and, as commented, it appears that they were four. In this regard, I can give several responses. Firstly, in the passage quoted above, Pseudo-Apuleius seems to say clearly that the first rule was the main or more important rule. As indicated, that rule was the *prima constitutio vel primum expositum*, and it hence can be thought that it is worth taking into account an analysis of just it without considering the other rules.

Secondly, as also mentioned, we only know for sure one more rule: the third one. Alexander of Aphrodisias describes that rule in *Aristotelis Analyticorum Priorum* 278, 11-14, and it appears that it was akin to the idea of transitivity in modern logic. Indeed, the rule establishes that, if one of the premises can be drawn from external propositions, the conclusion can also be derived from the other premise along with such external propositions. It is very easy to note that this rule can also be formally deduced from the indemonstrables, in particular, using only modus ponendo ponens. Let us suppose that we have these two propositions: 'if (A and B), then Γ' and 'if Δ, then A.' It is obvious that we can infer Γ from B and Δ, since we can obtain A from 'if Δ, then A' and Δ by modus ponendo ponens (and hence Γ from A and B by modus ponendo ponens too).

But, following the approach of the mental models theory, that rule can be easily proposed as well. Because 'if Δ, then A' is a conditional, Δ can only be true in a scenario in which A is also true. Therefore, if Δ happens, A also occurs, and, provided that the premises are again 'if (A and B), then Γ' and 'if Δ, then A,' it is clearly possible to derive Γ from B and Δ under the mental models theory too. The key is that, ultimately, a model including Δ must also include A.

As far as the second and the four rules are concerned, several points can be indicated. On the one hand, although we do not know for sure which these rules were, we do know that, according to the testimony given by Galen in *De Placitis Hippocratis et Platonis* 114, 1-10, the first one and the second one were somehow linked, and there was also a certain relation between the third one and the fourth one. In particular, the passage indicates that some authors or logicians use the first one or the second one, and that other authors or logicians use the third one or the fourth one. In fact, some contemporary authors have tried to rebuild the second and the fourth rules (see, e.g., Bobzien, 1996) and those rebuildings show us that, indeed, there were obvious links between, on the one hand, the first and the second rules, and, on the other hand, the third and the fourth rules. Therefore, in my view, arguments similar to those presented in this paper could be applied to possible reconstructions of the missing rules without great difficulties.

Furthermore, there is no doubt that Stoic logic inspired the modern logical systems and that natural deduction calculi such as that of

Gentzen (1935) seem to be, at least in a sense, indebted to its main theses. However, this does not mean that the meaning of the Stoic logical notions was the same as that of standard logic. I think that we do not know the exact sense of those notions yet. So, we cannot say to what extent such notions actually anticipated the rules, principles, and requirements of modern logic.

Maybe what happens is that today we reinterpret the Stoic theses under the framework of standard logic (many authors consider such a perspective to be wrong; an example is Bobzien, 1996, 134, who claims that the latter logic should not be the criterion to evaluate and review the former), and, because of this, we think that Stoic logic is closer to current logic than it is really. In any case, this paper has shown that the mental models theory can be very useful in this regard, since it reveals that certain conclusions that appear to be a result of a formal demonstration can also be obtained by the human mind in a natural way (the semantic mental processes would be the natural mental processes following the mental models theory; in fact, it claims that a naïve individual, i.e., an individual without logical training, only draws conclusions in a semantic way). Of course, as mentioned above, that the mental models theory is not a correct approach to describe human reasoning is a possibility. Nevertheless, at the very least, the theory indicates that there are ways to come to Stoic reduction rules other than a syntactic derivation.

The problem hence is whether the θέματα are a proof that Stoic logic was a formal system or not. Given that, if the mental models theory is assumed, the simple use of the normal human reasoning abilities (or, if preferred, of the common sense) could also have led the Stoics to their reduction rules, this is a point that deserves to be researched in more details. Perhaps only a careful study of the ancient sources can give us the answer.

References

Barnes, J., Bobzien, S., & Mignucci, M. (1999). Logic. *The Cambridge History of Hellenistic Philosophy*. K. Agra, J. Barnes, J. Mansfeld, & M. Schofield (Eds.) (77-176) Cambridge: Cambridge University Press.

Bobzien, S. (1996). Stoic syllogistic. *Oxford Studies in Ancient Philosophy*. C. C. W. Taylor (Ed.) (133-192). Oxford: Clarendon.

Bocheński, I. M. (1951). *Ancient Formal Logic*. Amsterdam: North-Holland.

Boeri, M. D., & Salles, R. (2014). *Los Filósofos Estoicos. Ontología, Lógica, Física y Ética*. Santiago de Chile: Ediciones Universidad Alberto Hurtado.

Gentzen, G. (1935). Untersuchungen über das logische Schließen I. *Mathematische Zeitschrift,* 39, 176-210.

Gould, J. B. (1970). *The Philosophy of Chrysippus*. Albany: State University of New York Press.

Johnson-Laird, P. N. (2006) *How We Reason*. Oxford: Oxford University Press.

____ (2010). Against logical form. *Psychologica Belgica,* 5(3/4), 193-221.

____ (2012). Inference with mental models. *The Oxford Handbook of Thinking and Reasoning*. K. J. Holyoak & R. G. Morrison (Eds.) (134-145). New York: Oxford University Press.

Khemlani, S., Orenes, I., & Johnson-Laird, P. N. (2012). Negation: A theory of its meaning, representation, and use. *Journal of Cognitive Psychology,* 24(5), 541-559.

____ (2014). The negation of conjunctions, conditionals, and disjunctions. *Acta Psychologica,* 151, 1-7.

Kneale, W., & Kneale, M. (2008). *The Development of Logic*. Oxford: Clarendon.

Lukasiewicz, J. (1967). On the history of the logic of propositions. *Polish Logic: 1920-1939*. S. McCall (Ed.) (67-68). New York: Oxford University Press.

Mates, B. (1953). *Stoic Logic*. Berkeley and Los Angeles: University of California Press.

Mueller, I. (1978). An introduction to Stoic Logic. *The Stoics*. J. M. Rist (Ed.) (1-26). Berkeley: University of California Press.

Oakhill, J. & Garnham, A. (Eds.) (1996). *Mental Models in Cognitive Science. Essays in Honour of Phil Johnson-Laird*. Hove: Psychology Press.

Orenes, I. & Johnson-Laird, P. N. (2012). Logic, models, and paradoxical inferences. *Mind & Language*, 27(4), 357-377.

O'Toole, R. R. & Jennings, R. E. (2004). The Megarian and the Stoics. *Handbook of the History of Logic, Volume I. Greek, Indian and Arabic Logic*. M. D. Gabbay and J. Woods (Eds.) (397-522). Amsterdam: Elsevier.