Useful ratios between two-body nonleptonic and semileptonic decays of \( B \) mesons

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We compute important ratios between decay widths of some exclusive two-body nonleptonic and semileptonic \( B \) decays, which could be test of factorization hypothesis. We also present a summary of the expressions of the decay widths and differential decay rates of these decays, at tree level, including \( l = 0 \) (ground state), \( l = 1 \) (orbitally excited) and \( n = 2 \) (radially excited) mesons in the final state. From a general point of view, we consider eight transitions, namely \( H \to P,V,S,A, A', T, P(2S), V(2S) \). Our analysis is carried out assuming factorization hypothesis and using the WSB, ISGW and CLFA quark models.

**Keywords:** \( B \) physics; semileptonic decays; nonleptonic decays.

Calculamos varias relaciones importantes entre los anchos de decaimiento de canales exclusivos no leptónicos y semileptónicos del mesón \( B \), las cuales pueden servir como prueba a la hipótesis de factorización. También, presentamos un resumen sobre las expresiones de los anchos de decaimiento y los anchos de decaimiento diferenciales, para estos procesos, a nivel árbol, incluyendo mesones con \( l = 0 \) (sin excitación orbital), \( l = 1 \) (excitados orbitalmente) y \( n = 2 \) (excitados radialmente) en el estado final. Desde un punto de vista general, consideramos ocho transiciones: \( H \to P,V,S,A, A', T, P(2S), V(2S) \). Nuestro análisis se desarrolla asumiendo hipótesis de factorización y utilizando los modelos de quarks WSB, ISGW y CLFA.

**Descriptores:** Física del \( B \); decaimientos semileptónicos; decaimientos no leptónicos.

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1. Introduction

Exclusive semileptonic and two-body nonleptonic decays of heavy mesons offer a good scenario for studying, at theoretical and experimental levels, CP violation and physics beyond the Standard Model. Some of these channels provide methods for determining the angles of the unitarity triangle, allow to study the role of QCD and test some QCD-motivated models (see for example some recent reviews in Ref 1). These topics are of great interest in particle physics and the knowledge of them will be improved with forthcoming experiments at Large Hadron Collider (LHC) [2].

The purpose of this paper is to compute useful ratios between two-body nonleptonic and semileptonic decays of heavy (\( H \)) mesons, at tree level, that could be tested experimentally. Specifically, we work with exclusive \( B \) channels although we also consider a couple of \( B_s \) processes. We assume naive factorization and use the WSB [3], ISGW [4] and CLFA [5] quark models.

It is expected that naïve factorization approach works reasonably well in decays where penguin and weak annihilation contributions are absent or suppressed, such as \( B \to D K \) [6], \( K^0 \to \pi \pi, D^0 \to K^\pm \pi^\mp, D^0 \to K^+ K^-, \pi^+ \pi^- \) and \( B_s \to J/\psi \phi \) [7], \( D^+ \to \bar{K}_0^0 \pi^+ \) and \( D^+ \to f_0^+ \pi^+ \) [8] channels. Also, factorization assumption works well in two-body hadronic decays of \( B_c \) meson (without considering charmless modes) where the quark-gluon sea is suppressed in the heavy quarkonium [9].

We also present an important summary and a general analysis on the expressions of the decay widths and differential decay rates of two-body nonleptonic and semileptonic decays of heavy mesons, respectively, including \( l = 0,1 \) and \( n = 2 \) mesons in the final state. For \( l = 0 \), we have considered pseudoscalar (\( P \)) and vector (\( V \)) mesons, for \( l = 1 \) we have included orbitally excited (\( p \)-wave) scalar (\( S \)), axial-vector (\( A, A' \)) and tensor (\( T \)) mesons, and for \( n = 2 \), we have studied radially excited \( P(2S) \) and \( V(2S) \) mesons (see Table I). We have classified eight transitions, namely \( H \to P,V,S,A, A', T, P(2S), V(2S) \), in three groups. It allows us to manipulate, in an easy way, all these decays.

The paper is organized as follows: In Sec. 2 we present, in a general way, the parametrization of the hadronic matrix element \( \langle M | J_{\mu} | H \rangle \) for eight cases. Sec. 3 contains expressions for \( \Gamma(H \to M_1 M_2) \) and \( d\Gamma(H \to M l \nu)/dt \) and a brief discussion. In Sec. 4, we analyze vector and axial contributions of the weak interaction to \( H \to (P,V,S,A, A', T) l \nu \) decays assuming a meson dominance model. In Sec. 5, we compute some important ratios between decay widths of exclusive \( B \) (and \( B_s \)) decays, which allow us to get tests to factorization approach. Concluding remarks are presented in Sec. 6. Finally, in the appendix we briefly mention the quark models used in this work.
is possible to transform \((F_1, F_0) \rightarrow (f_+, f_-)\) using the relations showed in the appendix.

### 2. Hadronic Matrix Elements

In this section, we present the parametrizations of the eight \(H \rightarrow M\) transitions, where \(H\) denotes a pseudoscalar heavy meson and \(M\) can be a \(P\), \(V\), \(S\), \(A\), \(A', T\), \(P(2S)\), \(V(2S)\) meson, classified in three groups \(^1\). In the first case, the \(M\) meson has \(J = 0\), in the second, \(J = 1\), and in the third group \(J = 2\).

#### 2.1. \(H \rightarrow M(J = 0)\) transition

In this group, there are three transitions if \(M\) is a meson with \(J = 0\) (see Table I): \(M\) can be the pseudoscalar \(P\) meson, or the scalar \(S\) meson, which is an orbitally excited meson, or the radially excited meson \(P(2S)\). The hadronic matrix element \(\langle M|J_\mu|H\rangle\) for \(M = P, S, P(2S)\) has the same Lorentz structure and it is defined as follows \([4]\):

\[
\langle M(p_M)|J_\mu|H(p_H)\rangle \equiv iG_\mu \epsilon^{\nu\rho\sigma}(p_H+p_M)^\rho(p_H-p_M)^\sigma F_\mu + F_- \epsilon^\mu \epsilon^{\nu\rho\sigma}(p_H+p_M)^\rho(p_H-p_M)^\sigma ,
\]

(1)

where \(J_\mu\) is the \(V_\mu - A_\mu\) weak current, \(p_{H(M)}\) is the 4-momentum of the meson \(H(M)\), \(F_+\) and \(F_-\) are form factors. Following the notation displayed in appendix of the ISGW model \([4]\), these form factors are:

- For \(M = P\): \(\langle P|J_\mu|H\rangle \equiv \langle P|V_\mu|H\rangle, F_+ = f_+\) and \(F_- = f_-\).
- For \(M = S\): \(\langle S|J_\mu|H\rangle \equiv -\langle S|A_\mu|H\rangle, F_+ = u_+\) and \(F_- = u_-\).
- For \(M = P(2S)\): \(\langle P(2S)|J_\mu|H\rangle \equiv \langle P(2S)|V_\mu|H\rangle, F_+ = f'_+\) and \(F_- = f'_-\).

It is important to note that the parity operator requires that \(\langle P|A_\mu|H\rangle = 0\) and \(\langle S|V_\mu|H\rangle = 0\).

Reference 3 uses a different parametrization for \(\langle P|J_\mu|H\rangle\) using dimensionless \(F_1\) and \(F_0\) form factors. It is easy to transform \((F_1, F_0) \rightarrow (f_+, f_-)\) using the relations showed in the appendix.
Table II. Differential decay widths of $H \to (P, V, S, A, A', T, P(2S), V(2S))\nu\tau$.

<table>
<thead>
<tr>
<th>$H \to M\nu\tau$</th>
<th>$d\Gamma(H \to M\nu\tau)/dt$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H \to (P, S, P(2S))\nu\tau$</td>
<td>$\zeta [A(t)</td>
</tr>
<tr>
<td>$H \to (V, A, A', V(2S))\nu\tau$</td>
<td>$\zeta t\lambda^{3/2} [H_+^2(t) +</td>
</tr>
<tr>
<td>$H \to T\nu\tau$</td>
<td>$\zeta {\varphi(t)\lambda^{3/2} + \rho(t)\lambda^{3/2} + \theta(t)\lambda^{3/2}}$</td>
</tr>
</tbody>
</table>

where $\epsilon_{\nu\alpha}$ is the polarization tensor of the tensor meson, $p_{H(T)}$ is the momentum of the heavy meson $H(T)$, and $k, k, b_\pm$ are form factors. $k$ is dimensionless and $h, b_\pm$ have dimensions of GeV$^{-2}$.

In the literature [11, 12], there is another parametrization of $(T)J^\mu[H]$, which is constructed in analogy with the parametrization of $(V)J^\mu[H]$ given in Ref. 3, using the tensor polarization $\epsilon_{\nu\mu}$ of the $T$ meson.

3. $d\Gamma(H \to M\nu\tau)/dt$ and $\Gamma(H \to M_1M_2)$

In this section we collect, in a compact form, using the classification of the last section, the expressions, at tree level, of the differential decay rate of $H \to M\nu\tau$ (see Table II) and the decay width of $H \to MM'$ (see Table III), where $H$ is a heavy meson $(D, D_s, B, B_s$ or $B_c)$, and $M$ $(M')$ can be any of the eight mesons $P, V, S, A, A', T, P(2S), V(2S)$.

In the first row of Table II, we display the differential decay rate of the semileptonic $H \to M\nu\tau$ decay, where $M$ is a meson with $J = 0$, i.e., $M = P, S, P(2S)$, using the parametrization given in the WSB model [3]. The second row shows the differential decay rate of $H \to M\nu\tau$, where $M$ is a meson with $J = 1$, i.e., $M = V, A, A', V(2S)$, using parametrizations given in the WSB [3] and ISGW [4] quark models, and in the last row we give the differential decay rate for $H \to T(J = 2)\nu\tau$ using the parametrization of the ISGW model [4].

In Table II, $\lambda = \lambda(m_H^2, m_M^2, t)$, where

$$\lambda = \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

is the triangular function, $t = (p_H - p_M)^2$ is the momentum transfer and $H_{x,0}$ are helicity form factors [3]. The factor $\zeta$ and functions $A(t), B(t), \mathcal{G}(t), \varphi(t), \rho(t), \theta(t), \alpha(t), \beta(t)$ and $\gamma(t)$ are defined by:

$$\zeta = \frac{G_F^2|V_{q'q}|^2}{192\pi^3 m_H^2},$$

$$A(t) = \frac{(t - m_H^2)^2}{t} \left( \frac{2t + m_H^4}{2t} \right),$$

$$B(t) = \frac{3}{2} m_H^2 \left( \frac{t - m_H^2}{t} \right)^2 \frac{(m_H^2 - m_M^2)^2}{t},$$

$$\mathcal{G}(t) = \left[ \frac{2t|V(t)|^2}{(m_H + m_M)^2} + \frac{(m_H + m_M)^2}{4m_H^2} \right] \lambda^{3/2} - \frac{(m_H^2 - m_M^2 - t)^2 A_1(t)A_2(t)}{2m_H^4} \lambda^{3/2} + \frac{|A_2(t)|^2}{4m_H^2 (m_H + m_M)^2} \lambda^{3/2} + 3t(m_H + m_M)^2 |A_1(t)|^2 \lambda^{3/2}, \quad (7)$$

$$\varphi(t) = \frac{s_m^2}{4m_A^2}, \quad (8)$$

$$\rho(t) = \frac{1}{4m_A^2} \left[ r^2 + 8m_A^2 t u^2 + 2(m_H^2 - m_M^2 - t) r s_+ \right], \quad (9)$$

$$\theta(t) = 3t v^2, \quad (10)$$

$$\alpha(t) = \frac{b_\pm^2}{24m_T^2}, \quad (11)$$

$$\beta(t) = \frac{1}{24m_T^2} \left[ k^2 + 6m_T^2 t h^2 + 2(m_H^2 - m_M^2 - t) k b_+ \right], \quad (12)$$

$$\gamma(t) = \frac{5t k^2}{12m_T^2}, \quad (13)$$

where $G_F$ is the Fermi constant, $m_H(P, V, A, T)$ is the mass of the $H(P, V, A, T)$ meson, $m_M$ is the mass of the lepton, $V(t)$ and $A_{1,2}(t)$ are form factors [3], $\varphi(t), \rho(t)$ and $\theta(t)$ are quadratic functions of the form factors $s_+, r$ and $\nu (c_+, l$ and $q)$ for $H \to A(1P_1)\nu \tau (H \to A(3P_1)\nu\tau), \alpha(t), \beta(t)$ and $\gamma(t)$ are quadratic functions [13] of the form factors $k, b_+,$ and $h.$ All these form factors are explicitly given in the appendix B of the Ref. 4.

The dependence of $d\Gamma(H \to M\nu\tau)/dt$ with

$$\lambda(t) = \lambda^{1/2}/2m_H,$$

where $\lambda(t)$ is the three-momentum of the $M$ meson in the $H$ meson rest frame) is given by,

$$d\Gamma/dt \sim \lambda^{1/2},$$

where $l$ is the orbital angular momentum of the wave at which the particles in the final state can be coupled. Assuming conservation of total angular momentum $J$ and a meson dominance model we can find specific values for $l$ in each exclusive $H \to M\nu\tau$ decay. Thus, in $H \to M(J = 0)\nu\tau$ the
particles in the final state are coupled to \( l = 0, 1 \) waves (see the first row in Table II). When \( m_t \approx 0 \) \((l = e, \mu)\), the coefficient \( B(t) \) vanishes, so the contribution of the s-wave is negligible; in \( H \to M(J = 1)\nu \) the particles in the final state can be coupled to \( l = 0, 1, 2 \) waves (see the second row in Table II); and in \( H \to T(J = 2)\nu \) to \( l = 1, 2, 3 \) waves (see the last row in Table II).

It is also possible to write in a compact expression the differential decay rate of the semileptonic \( H \to M l \nu l \) decay, where \( M \) is a p-wave (orbitally excited) meson: scalar, vector-axial or tensor meson, in terms of helicity amplitudes (see Ref. 14). As for two-body nonleptonic decays of heavy mesons, the effective weak Hamiltonian \( \mathcal{H}_{\text{eff}} \) has contributions from current-current (tree), QCD penguin and electroweak penguin operators [15]. In general, \( \mathcal{H}_{\text{eff}} \approx \sum_i C_i(\mu) \mathcal{O}_i \), where \( C_i(\mu) \) are the Wilson coefficients and \( \mathcal{O}_i \) are local operators. The amplitude for the \( H \to M_1 M_2 \) decay is

\[
\mathcal{M}(H \to M_1 M_2) \approx \sum_i C_i(\mu) \langle \mathcal{O}_i \rangle_i.
\]

In the scenario of naive factorization, it is assumed that

\[
\mathcal{M}(H \to M_1 M_2) \approx C_1(\mu) \langle M_2 \rangle \langle J_1 \rangle \langle H \rangle + (M_1 \to M_2),
\]

where \( J_1 \) is the \( V_\mu - A_\mu \) weak current and the hadronic matrix element of a four-quark operator is written as the product of a decay constant and form factors [16].

This factorization presents a difficulty because the Wilson coefficients are \( \mu \) scale and renormalization scheme dependent while \( \langle \mathcal{O}_i \rangle_i \) are \( \mu \) scale and renormalization scheme independent, so clearly the physical amplitude depends on the \( \mu \) scale. The naive factorization disentangles the short-distance effects from the long-distance sector assuming that \( \langle \mathcal{O}_i \rangle_i \), at \( \mu \) scale, contain nonfactorizable contributions in order to cancel the \( \mu \) dependence and the scheme dependence of \( C_i(\mu) \). Thus, the naive factorization is an approximation because it does not consider possible QCD interactions between the meson \( M_2 \) and the \( H \) and \( M_1 \) mesons. In general, it does not work in all two-body heavy meson decays [16].

Assuming naive factorization, we have considered only those decays which are produced by the color-allowed external \( W \)-emission diagram or the color-suppressed internal \( W \)-emission diagram. It is expected that naive factorization works reasonably well in decays where penguin and weak annihilation contributions are absent or negligible, as for example in \( B \to DK [6], K^0 \to \pi^\pm \pi^\mp, D^0 \to K^\pm \pi^\mp, K^+ K^-, \pi^+ \pi^- \) and \( B_s \to J/\psi \phi \) [7], \( D^+ \to K^{0*}\pi^+ \) and \( D_s^+ \to f_0(500)^+ \) [8] channels. Also, factorization assumption works well in two-body hadronic decays of \( B_s \) meson (except in charmless processes, because they are produced only by annihilation contributions) where the quark-gluon sea is suppressed in the heavy quarkonium [9].

We have used the notation \( H \to M_1, M_2 \) [17] to mean that \( M_2 \) is factorized out under factorization approximation, i.e., \( M_2 \) arises from the vacuum. For \( H \to TM \) decays there is not any possibility to produce the \( T \) meson from the vacuum with the \( V-A \) current, because \( \langle T |(V-A)\mu |0 \rangle = 0 \). So, this decay has only the contribution \( H \to T, M \). Recently, it has been reported that it is possible to produce tensor mesons from the vacuum involving covariant derivatives [12, 18].

Using the parametrizations given in Sec. 2 for eight transitions, namely \( H \to M(J = 0, 1, 2) \), we display, in Table III, expressions of decay widths for 40 different types of \( H(q_H q^*_H) \to M_1(q_{1H} q^*_{1H}) M_2(q_{2H} q^*_{2H}) \) decays, which are produced by the \( q_H \to q_{1H} q_{2H} \) transition.

In the first row of Table III, we show the decay width for six different types of channels: \( H \to P, P^*; P, P^* (2S); S, P^*; S, P^* (2S); P(2S), P; P(2S), P^* (2S). \) They are produced by the \( H \to M(J = 0) \) transition. The hadronic matrix elements \( \langle P(S, P(2S)) | J_\mu | H \rangle \), which are necessary in order to calculate the decay width, have the same parametrization. In this case, we have used the parametrization presented in Ref. 3. In these decays the particles in the final state are coupled to a s-wave because \( \Gamma \sim \lambda^{\lambda+1/2} \).

In a similar way, in the second row of Table III, we display the decay width of nine different modes: \( H \to P, V; P, A; P, V(2S); S, V; S, A; S, V(2S); P(2S), V; P(2S), A; P(2S), V(2S). \) These nine channels have in common the \( H \to M(J = 0) \) transition. In these decays, the particles in the final state are coupled to a p-wave \( (l = 1). \)

In the third row of Table III, we present the decay width for eight different types of decays: \( H \to V, P; V, P(2S); A, P; A, P(2S); A, P; A, P(2S); V(2S), V; P(2S), P(2S). \) The hadronic matrix elements \( \langle V(A, A'; V(2S)) | J_\mu | H \rangle \), which correspond to the \( H \to M(J = 1) \) transition, have a similar parametrization. The particles in the final state in these decays are coupled to a p-wave \( (l = 1). \) In the fourth row of Table III, we display the decay width for twelve different decays: \( H \to V_1, V_2; V_1, A_2; V_1, V_2(2S); A_1, V_2; A_1, A_2; V_1(2S); A_1, V_2; A_1, A_2; V_1(2S); V_2; V_1(2S), A_2; V_1(2S), V_2(2S). \) They also arise from the \( H \to M(J = 1) \) transition. The two \( J = 1 \) particles in the final state can be coupled to \( l = 0, 1, 2 \) waves.

In the fifth row of Table III, we show the decay width for the \( H \to T, P(P(2S)) \) channels, which are produced by the \( H \to T \) transition. We have used the parametrization for \( \langle T | J_\mu | H \rangle \) given in the Ref. 4. In this case, the particles in the final state can be coupled to a \( l = 2 \) wave. Using the same parametrization, we present in the last row of Table III, the decay width for three different modes: \( H \to T, V \) \( (A, V(2S)) \). In this case, the particles in the final state can be coupled to \( l = 1, 2, 3 \) waves.

In Table III, all form factors and the function \( \lambda \) are evaluated in \( m_{f_2}^2 \) because the momentum transfer \( t = (p_H - p_1)^2 = p_2^2 = m_{f_2}^2 \xi(M_2) \) in the decay constant are given by

\[
\]
Table III. Decay widths of $H \to M_1M_2$, where $M_{1,2} = P, V$, $S$, $A$, $T$, $P(2S)$, $V(2S)$

<table>
<thead>
<tr>
<th>$H \to M_1, M_2$</th>
<th>$\Gamma(H \to M_1, M_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H \to (P_1, S_1, P_2(2S))$, $(P_2, P_2(2S))$</td>
<td>$\xi^{(M_2)}(m_H^2 - m_{M_1}^2) F_{H}^{M_1} (m_{M_2}^2)</td>
</tr>
<tr>
<td>$H \to (P_1, S_1, P_2(2S))$, $(V, A, V(2S))$</td>
<td>$\xi^{(M_2)}</td>
</tr>
<tr>
<td>$H \to (V, A, V(2S))$, $(P, P(2S))$</td>
<td>$\xi^{(M_2)} (t = m_{V_2}^2) \left[</td>
</tr>
<tr>
<td>$H \to T$, $(P, P(2S))$</td>
<td>$\xi^{(M_2)} (1/2M_{2}^2) F^{H \to T} (m_{M_2}^2)</td>
</tr>
</tbody>
</table>
| $H \to T$, $(V, A, V(2S))$ | $\xi^{(M_2)} | \alpha (m_{M_2}^2) | \lambda^{7/2} + | \beta (m_{M_2}^2) | \lambda^{7/2} + | \gamma (m_{M_2}^2) | \lambda^{3/2} |$

Table IV. Vector and axial contributions to semileptonic $H \to (P, V, S, A, T)\nu \overline{\nu}$ decays.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$J^P$ of $W^*$</th>
<th>$H \to Pl\nu$</th>
<th>$H \to Vl\nu$</th>
<th>$H \to Sl\nu$</th>
<th>$H \to Al\nu$</th>
<th>$H \to Tl\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector</td>
<td>$0^+$</td>
<td>$l = 0$</td>
<td>$l = 1$</td>
<td>$l = 1$</td>
<td>$l = 0, l = 2$</td>
<td>$l = 2$</td>
</tr>
<tr>
<td></td>
<td>$1^-$</td>
<td>$l = 1$</td>
<td>$l = 1$</td>
<td>$l = 0$</td>
<td>$l = 2$</td>
<td></td>
</tr>
<tr>
<td>Axial</td>
<td>$0^-$</td>
<td>$l = 1$</td>
<td>$l = 1$</td>
<td>$l = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1^+$</td>
<td>$l = 0, l = 2$</td>
<td>$l = 1$</td>
<td>$l = 1$</td>
<td>$l = 1, l = 3$</td>
<td></td>
</tr>
</tbody>
</table>

Finally, we do not consider decays where a tensor meson, or a scalar meson or an axial-vector meson $1^1P_1$ arises from the vacuum. In the first case, as we mentioned before, $(T) J_\mu (0) \equiv 0$; in the second case, the decay constant of the scalar mesons, defined as $\langle S| J_\mu |0\rangle = f_{SM} \mu$, vanishes or is small (of the order of $m_d - m_u, m_s - m_u, m_d$); and in the last case, the decay constant of the $1^1P_1$ meson vanishes in the SU(3) limit by $G_\pi$-parity [19].

4. Contributions of the vector and axial couplings

In this section, we illustrate how the particles in the final state of $H \to M\nu\overline{\nu}$ and $H \to M_1M_2$ decays can be coupled to specific waves, obtain the quantum numbers of the poles that appear in the monopolar form factors, and explain the correspondence between the form factors and the respective waves in the final state. We show that these numbers depend on the vector and axial couplings of the weak interaction. Let us consider the decay chain $H \to MM^* \to MW^* \to Ml\nu(MM^*)$, where $W^*$ is the off-shell intermediate boson of the weak interaction. We need to combine parity and total angular momentum conservations in the strong $H \to MM^*$ process.

In Table IV, we show the specific waves in which particles in the final state of $H \to (P, V, S, A, T)\nu \overline{\nu}$ decays can be coupled and determine if they come from the vector or axial contributions. We must keep in mind that the off-shell $W^*$ boson has spin 0 or 1. Thus, in the vector coupling there are two possibilities: $S_{W^*} = 0$ with $P_{W^*} = +1$, and $S_{W^*} = 1$ with $P_{W^*} = -1$ ($S_{W^*}$ and $P_{W^*}$ denote spin and parity of $W^*$, respectively). In a similar way, in the axial coupling there are two options: $S_{W^*} = 0$ with $P_{W^*} = +1$, and $S_{W^*} = 1$ with $P_{W^*} = +1$. Thus, there are four cases for the $W^*$ boson: $J^P = 0^+, 1^-, 0^-$ and $1^+$. They are displayed in the second column of Table IV. Assuming total angular momentum and parity conservations of the strong $H \to MM^*$ process, we obtain the values of the orbital angular momentum $l$ of the particles in the final state of $H \to Ml\nu$ (see Table IV). These values can be verified with the exponent $l + (1/2)$ of $\lambda$ in the expressions for $d\Gamma/dl$ in Table II. We can see in the third (fourth) and fifth (sixth) columns in Table IV, that the vector and axial contributions interchange their roles in $H \to Pl\nu (H \to Vl\nu)$ and $H \to Sl\nu (H \to Al\nu)$, respectively.

In Table V, we show the respective form factors with the corresponding poles in $H \to P(V)\nu \overline{\nu}$ decays. In the second column, we list the quantum numbers $J^P$ of poles, which are the same $J^P$ options for the off-shell $W^*$ boson (see the second column in Table IV). In this case, we must check the form factors that appear in the parametrization of the hadronic matrix elements $\langle M| V_\nu |H\rangle$ and $\langle M| A_\mu |H\rangle$ for $M = P, V$. Following this idea, we obtain the quantum numbers of the poles for $H \to Ml\nu$ where $M$ is a $p$-wave meson: for $H \to Sl\nu$ the poles are $0^-$ and $1^+$; for $H \to Al\nu$, the poles are $0^+, 1^-$.
and $1^+$, and for $B \to T \ell \nu$ the poles are $1^-, 0^-$ and $1^+$. These values are important if we are interested in constructing a quark model with monopolar form factors for $H \to S, A, T$ transitions.

Let us illustrate, as an example, the situation on $H \to P \ell \nu$ from Tables IV and V. This decay has two contributions: $l = 0$ and $l = 1$ (see exponents of $A$ in Table II) which arise from the vector coupling of the weak current (see Table IV). The respective poles have quantum numbers $0^+$ and $1^-$ and the form factors are $F_0$ and $F_1$ (see Table V).

5. Useful ratios

In this section, we present some ratios between exclusive semileptonic and two-body nonleptonic decays of $B$ and $B_s$ mesons, using the expressions for $d\Gamma(H \to M \ell \nu)/dt$ and $\Gamma(H \to M_1 M_2)$ (see Tables II and III, respectively), that could be a test of factorization hypothesis with forthcoming measurements at LHC. We have worked with decays where it is expected that naive factorization works well. In order to obtain the numerical values presented in this section, we have evaluated the form factors in the WSB [3] and CLFA [5] quark models and taken from the Particle Data Group [20] the values of the CKM factors, branching ratios, masses and mean lifetime of mesons.

5.1 $B \to P, M(c\bar{s})$ decays: Let us consider exclusive two-body nonleptonic $B$ decays with orbitally or radially excited charmonium mesons in the final state, which are produced by the color suppressed $b \to c\bar{s}s$ (d) transition. The following ratio

$$\frac{\Gamma(B^+ \to P^+, M_1(c\bar{s}))}{\Gamma(B^+ \to P^+, M_2(c\bar{s}))} = (\text{kinematical factor}) \left( \frac{f_{M_1}}{f_{M_2}} \right)^2 \left| \frac{F_{0(1)}(m_{M_1}^2)}{F_{0(1)}(m_{M_2}^2)} \right|^2,$$

allows to obtain decay constants of charmonium mesons. The form factor $F_{0(1)}$ corresponds when $M_1$ and $M_2$ are $J = 0(1)$ mesons.

Evaluating the form factors in the CLFA model [5], we obtain

$$\frac{f_{J/\psi}}{f_{\psi(2S)}} = 1.15 \pm 0.07 (1.29 \pm 0.17),$$

$$\frac{f_{J/\psi}}{f_{\chi_{c1}(1P)}} = 1.41 \pm 0.13 (1.51 \pm 0.32),$$

$$\frac{f_{J/\psi}}{f_{J/\psi(2S)}} = 1.65 \pm 1.27,$$

$$\frac{f_{J/\psi}}{f_{\psi(c\bar{s})}} = 2.63 \pm 0.52,$$

and taking ($M_1 = J/\psi$, $M_2 = \psi(2S)$, $P = K(\pi)$, ($M_1 = J/\psi$, $M_2 = \chi_{c1}(1P)$, $P = K(\pi)$, ($M_1 = \eta_c$, $M_2 = \eta_c(2S)$, $P = K$), and ($M_1 = \eta_c$, $M_2 = \chi_{c0}(1P)$, $P = K$), respectively. The most important sources of uncertainties come from experimental values of branching ratios and form factors. However, the error in the last ratios is dominated by the uncertainty in the branching ratios. These quotients between decay constants with orbitally and radially excited charmonium states are a good test of the factorization hypothesis.

On the other hand, taking $f_{J/\psi} = (416.3 \pm 5.3) \text{ MeV}$ [21, 22] and $f_{\eta_c} = (335 \pm 75) \text{ MeV}$ [23], we obtain:

$$f_{\psi(2S)} = 361.7 \pm 22.5 (322.7 \pm 42.7) \text{ MeV},$$

$$f_{\chi_{c1}(1P)} = 295.24 \pm 27.48 (275.7 \pm 58.5) \text{ MeV},$$

$$f_{\eta_c(2S)} = 203.03 \pm 102.12 \text{ MeV},$$

$$f_{\chi_{c0}(1P)} = 127.4 \pm 38.1 \text{ MeV}.$$

From these values we obtain $f_{\eta_c}/f_{\eta_c(2S)} = 1.65 \pm 0.9$ and $f_{J/\psi}/f_{\psi(2S)} = 1.15 \pm 0.07(1.29 \pm 0.17)$ while the Ref. 21 is obtained $f_{\eta_c}/f_{\eta_c(2S)} = f_{J/\psi}/f_{\psi(2S)} = 1.41$.

5.2 $B^+ \to K^+(\pi^+), J/\psi$ decays: An important test to naive factorization is given by

$$\frac{\Gamma(B^+ \to K^+, J/\psi)}{\Gamma(B^+ \to \pi^+, J/\psi)} = (18.31 \pm 1.51) \left( \frac{F_{B^+\to K}(m_{J/\psi}^2)}{F_{B^+\to \pi}(m_{J/\psi}^2)} \right)^2 = 33.21 \pm 5.14,$$

where the errors come from the numerical values of the CKM and form factors (which are evaluated in the CLFA model [5]). The experimental value of this ratio is $20.7 \pm 1.8$ [20]. This sizable difference means that these exclusive channels have large nonfactorizable contributions [24]. Some authors have explored the possibility of new physics in these decays [25].

5.3 $B_{s} \to D_{s}^+(K^+), K^-(D^-)$ decays: The ratio between the branching ratios of $B_{s} \to D_{s}^+, K^-$ (mediated by the $b \to c\bar{s}s$ transition) and $B_{s} \to K^+, D_-^-$ decays (mediated by the $b \to u\bar{s}s$ transition), which are color favored, is

$$\mathcal{R} = \frac{B(B_{s} \to D_{s}^+, K^-)}{B(B_{s} \to K^+, D_-)} = (3.94) \left( \frac{f_{K^-}}{f_{D^-}} \right)^2 \times \left( \frac{F_{B_{s} \to D_{s}^+}(m_{K^-}^2)}{F_{B_{s} \to K^+}(m_{D_-}^2)} \right)^2.$$

---

**Table V.** Form factors and the vector and axial contributions of the weak interaction to $H \to (P, V)\ell \nu$ decays.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$J^P$ of Pole</th>
<th>$H \to Pl\ell \nu$</th>
<th>$H \to Vl\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector</td>
<td>$0^+$</td>
<td>$F_0(t)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1^-$</td>
<td>$F_1(t)$</td>
<td></td>
</tr>
<tr>
<td>Axial</td>
<td>$0^-$</td>
<td>$A_0(t)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1^+$</td>
<td>$A_1(t), A_2(t), A_3(t)$</td>
<td></td>
</tr>
</tbody>
</table>

This ratio is sensitive to the value of the decay constant $f_{D_s^-}$. Evaluating the form factors in the CLFV model [5], we obtain $R = 9.82 \pm 1.27 (11.34 \pm 1.43)$ with $f_{D_s^-} = 259 \pm 7$ [26] (241 $\pm$ 3 [27]) MeV. The sources of the uncertainty come from the CKM factors, the decay constants and the form factors. The dominant error comes from the value of $V_{ub}$. From the experimental value $B(B_s^0 \rightarrow D_s^+ K^0) = (3.0 \pm 0.7) \times 10^{-3}$ [20] it is obtained $R = 1$ while we compute $R \approx 10$. Thus, with improved measurements, this ratio is a good test to the numerical inputs for $V_{ub}$ and $f_{D_s^-}$.

**5.4 $H \rightarrow P'P$ and $P \rightarrow l\nu_l$ decays:** Let us compare the two-body nonleptonic $H(\tau q_1) \rightarrow P'(\tau q_2), P(\tau q_4)$ and the leptonic $P(\tau q_4) \rightarrow l\nu_l$ decays. It is well known that the decay rate of $P(\tau q_4) \rightarrow l\nu_l$ is

$$\Gamma(P \rightarrow l\nu_l) = \frac{G_F^2 |V_{q_4\tau}|^2 f_P^2 m_P m_l^2}{8\pi} \left(1 - \frac{m_l^2}{m_P^2}\right)^2.$$

The ratio between $\Gamma(H \rightarrow P', P)$ and the last expression is given by

$$\frac{\Gamma(H \rightarrow P', P)}{\Gamma(P \rightarrow l\nu_l)} = \frac{|V_{q_4\tau}|^2 f_{P'}^2 m_{P'} m_l^2}{4} \times \left(\frac{m_H^2 - m_l^2}{m_P^2}\right)^2 \left(1 - \frac{m_l^2}{m_P^2}\right)^2 \times \frac{f_{P'}(m_{P'})^2}{f_P(m_P)^2}.$$  \hspace{1cm} (21)

This quotient is independent of the decay constant $f_P$, and could be used as a test for the form factor $F_{H \rightarrow P'}(m_{P'}^2)$. For some exclusive channels, we obtain

$$|F_{B_s^- \rightarrow D_s^+}(m_{D_s^+}^2)|^2 = 0.301 \pm 0.037 (0.293 \pm 0.053),$$

$$|F_{B_s^- \rightarrow K^+}(m_{K^+}^2)|^2 = 0.765 \pm 0.216 (0.681 \pm 0.197),$$

with $l = \tau^- (\mu^-)$. The error comes basically from the experimental value of the branching ratios. We can see that the value of $|F_{B_s^- \rightarrow D_s^+}(m_{D_s^+}^2)|^2$ is approximately equal when the lepton $l$ is $\tau$ or $\mu$. The situation for $|F_{B_s^- \rightarrow K^+}(m_{K^+}^2)|^2$ is different because the value of $V_{ub}$ also is a source of uncertainty. On the other hand, the value of $F_{B_s^- \rightarrow K^+}$ in $q^2 = 0$ depends strongly on phenomenological models, ranges from 0.23 to 0.31 [28]. Thus, the improvement of these experimental ratios in future experiments, as LHCb, will be a test of the respective form factors.

**5.5 $H \rightarrow P_1, P_2(V')$ decays:** Another important ratio is given by the decay widths of $H \rightarrow P_1, P_2$ and $H \rightarrow P_1, V'$, where $P_2$ and $V'$ have the same quark content with $P_1 = P, S, P(2S), P_2 = P, P(2S)$ and $V' = V, A_1^0 (P_1), V(2S)$. Using the expressions given in Table III and monopolar form factors with the fact that $F_{H \rightarrow P_1}(0) = F_{H \rightarrow P_1}(0)$ [3], we obtain:

$$\Gamma(H \rightarrow P_1, P_2) = \frac{f_P^2}{f_{V'}} \left[\frac{1 - m_{V,l}/m_{P_1}^2}{1 - m_{V,l}/m_{P_2}^2}\right]^2 \times \left[\frac{\lambda(m_H^2, m_{P_2}^2, m_{P_1}^2)}{\lambda(m_H^2, m_{P_1}^2, m_{V,l}^2)}\right]^{3/2}$$

$$\times \left[\frac{\lambda(m_H^2, m_{P_2}^2, m_{P_1}^2)}{\lambda(m_H^2, m_{P_1}^2, m_{V,l}^2)}\right]^{1/2} = \frac{\lambda(m_H^2, m_{P_2}^2, m_{P_1}^2)}{\lambda(m_H^2, m_{P_1}^2, m_{V,l}^2)}.$$

(22)

This ratio provides information on the quotient $f_{P_2}/f_{V'}$. As an example, we obtain $(f_{\tau\pi}/f_{\mu\nu}) = 0.631 \pm 0.045$ using the $B^0 \rightarrow D^+, \pi^+$ and $B^0 \rightarrow D^+, \rho^+$ decays which branching ratios are $(2.68 \pm 0.13) \times 10^{-3}$ and $(7.6\pm1.3) \times 10^{-3}$, respectively [20]. The main uncertainty arises from these experimental values. On the other hand, taking $f_{\tau\pi}/f_{\mu\nu} = (130.7\pm0.4)$ MeV and $f_{\tau\pi}/f_{\mu\nu} = (216 \pm 2)$ MeV [5] it is obtained $(f_{\tau\pi}/f_{\mu\nu}) = 0.605 \pm 0.006$. So, in this case factorization assumption gives a good approximation to the value of this quotient.

**5.6 $H \rightarrow P', V_{i(2)}$ decays:** In order to obtain $f_{V_1}/f_{V_2}$, we can consider the ratio between the decay rates of $H \rightarrow P'$, $V_{1(2)}(q_1\bar{q}_2)$ and $H \rightarrow P', V_1(2), P(2S)$ and $V_{1,2} = V, A_1^0 (P_1), V(2S)$:

$$\frac{\Gamma(H \rightarrow P', V_1)}{\Gamma(H \rightarrow P', V_2)} = \left(\frac{f_{V_1}}{f_{V_2}}\right)^2 \times \frac{F_{H \rightarrow P'}(m_{V_1})^2}{F_{H \rightarrow P'}(m_{V_2})^2} \left[\frac{\lambda(m_H^2, m_{V_2}^2, m_{V_1}^2)}{\lambda(m_H^2, m_{V_1}^2, m_{V_2}^2)}\right]^{3/2}.$$  \hspace{1cm} (23)

Let us choose, as an application, the $B \rightarrow P, V$ and $B \rightarrow P, A$ processes. From the expressions in Table III and using monopolar form factors [3] we obtain:

$$\frac{\Gamma(B \rightarrow P, V)}{\Gamma(B \rightarrow P, A)} = \left(\frac{f_V}{f_A}\right)^2 \times \frac{1 - m_{A_1}^2/m_{V_2}^2}{1 - m_{A_1}^2/m_{V_1}^2} \left[\frac{\lambda(m_H^2, m_{V_2}^2, m_{A_1}^2)}{\lambda(m_H^2, m_{V_1}^2, m_{A_1}^2)}\right]^{3/2}.$$  \hspace{1cm} (24)

Taking the $B^0 \rightarrow D^+, \rho^+$ and $B^0 \rightarrow D^+, a_1^+$ decays we get $(f_{\rho}/f_{a_1}) = 1.06 \pm 0.31$. The dominant error comes from the experimental value $B(B^0 \rightarrow D^- a_1^+)$ = $(6.0 \pm 3.3) \times 10^{-3}$. With $f_{\rho} = (216 \pm 2)$ MeV [5] it is obtained $f_{a_1} = (0.203 \pm 0.059)$ GeV. This value is smaller than the one reported in the literature. For example, in the Ref. 29, $f_{a_1} = 0.238 \pm 0.010$ GeV while the Ref. 8 gives $f_{a_1} = 0.229$ GeV (extracted from the $\tau^- \rightarrow M^- \nu_{\tau}$ decay) and $f_{a_1} = 0.256$ GeV (from the $B^0 \rightarrow D^{*+}, a_1^+$ and $B^0 \rightarrow D^{++}, \rho^-$ decays). On the other hand, in Ref. 30 obtained $f_{a_1} = 0.215 (0.223)$ GeV for $\theta = 32^\circ (58^\circ)$, where $\theta$ is the mixing angle between the $K_{1A}$ and $K_{1B}$ mesons. As the error in $B(B^0 \rightarrow D^- a_1^+)$ is too big, it is important to get a more precise estimation of this branching in future experiments in order to test hypothesis factorization with these exclusive decays.

It is also possible to obtain the quotient $(f_{\rho}/f_{a_1})$ from $B(B_s^0 \rightarrow D_s^+, \rho^-)/B(B_s^0 \rightarrow D_s^+, a_1^+)$ and

we obtain:

\[ \Gamma(B \rightarrow V, A) = \frac{f_{V}}{f_{A}} \frac{G(m_{V}^{2})}{G(m_{A}^{2})}. \]

Taking the \( B^{0} \rightarrow D^{*-}, \rho^{+} \) and \( B^{0} \rightarrow D^{*-}, a_{1}^{+} \) decays and evaluating \( G \) with appropriate monopole form factors [3], we get \( \langle f_{\rho}/f_{a_{1}} \rangle = 0.81 \pm 0.07 \), where the source of uncertainty are the form factors. This value agrees with the one reported in Ref. 29, although is smaller than the value obtained in previous subsection.

\[ f_{\rho}/f_{a_{1}} \text{ can also be obtained from} \]

\[ \Gamma(B \rightarrow V', V) / \Gamma(B \rightarrow V, A) = \left( \frac{f_{V'}}{f_{A}} \right)^{2} \frac{G(m_{V}^{2})}{G(m_{A}^{2})}. \]  

5.7 \( H \rightarrow V_{1}, V_{2(3)} \) decays: Another important ratio in order to compute the quotient \( f_{V}/f_{V_{2}} \) comes from the \( H \rightarrow V_{1}, V_{2}(q_{i}q_{j}) \) and \( H \rightarrow V_{1}, V_{3}(q_{i}q_{j}) \) processes, where \( V_{1} = V, A_{i}^{0}P_{1}, A_{i}^{0}P_{1}, V(2S) \) and \( V_{2,3} = V, A_{i}^{0}P_{1}, V(2S) \). As an example, we consider the \( B \rightarrow V, V' \) and \( B \rightarrow V, A \) decays. From expressions displayed in Table III we obtain:

\[ \Gamma(B \rightarrow V, V') / \Gamma(B \rightarrow V, A) = \left( \frac{f_{V'}}{f_{A}} \right)^{2} \frac{G(m_{V}^{2})}{G(m_{A}^{2})}. \]  

6. Summary

We computed several useful ratios between decay widths of two-body nonleptonic and semileptonic \( B \) and \( B_{s} \) decays, which with improved measurements in forthcoming experiments as LHCb, could be test of factorization approach by means of quotients between form factors or decay constants. The ratios with \( B \) decays considering charmonium states and light mesons in final state (see subsection 5.1) could be the more likely scenario to test the factorization scheme. It is important to mention that divergences from the results obtained assuming the current approximations do not imply a failure of the QCD itself or the factorization approach alone. It would be required a more exhaustive and comprehensive analysis for getting more conclusions on these and possible new physics effects in these decays. We also presented a summary of the expressions for \( \Gamma(H \rightarrow M_{1}, M_{2}) \) and \( d\Gamma(H \rightarrow M_{1} \nu)/dt \), at tree level, including eight types of
mesons in final state: $M_{1.2}$ can be a ground state meson ($l = 0$), or an orbitally excited meson ($l = 1$) or a radially excited meson ($n = 2$), assuming factorization hypothesis and using the parametrizations of $\langle M | J_\mu | B \rangle$ given in the WSB and the ISGW quark models. The form factors were evaluated in the WSB and CLFA quark models. We classified in three groups the $H \rightarrow M_{1.2}$ transitions and explained some aspects related with the dynamics of these processes.

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Appendix

In this appendix, we briefly mention the quark models and their form factors that are used in this work.

1. The ISGW model [4]: it is a hybrid model that combines a nonrelativistic quark potential model with a phenomenological ansatz. It is consistent with heavy quark symmetry at maximum recoil $t_m$. Their form factors are modeled by a gaussian and normalized at $q^2_{max}$. All the form factors in this model are in function of

$$F_n^{H \rightarrow M}(q^2) = \left( \frac{\bar{m}_M}{\bar{m}} \right)^{i/2} \left( \frac{\beta_H \beta_M}{\beta_H \bar{m}_H} \right)^{7/2} e^{-\Lambda(t_m-q^2)^2},$$ (29)

where $\Lambda = m^2_H / (4\kappa^2 \bar{m}_H \bar{m}_M).$ $\bar{m}_{H(M)}$ is the mock mass of the $H(M)$ meson, $\beta$ is a variational parameter and $\kappa = 0.7$ is a relativistic compensation factor of the model. The appendix B of the Ref. [4] has all the required inputs for evaluating the form factors for the $H \rightarrow M(j = 0, 1, 2)$ transition.

2. The WSB model [3]: It gives the form factors in terms of relativistic bound state wave functions taking the solutions from a relativistic harmonic oscillator potential. The form factors are calculated as wave function overlaps in the infinite momentum frame at $q^2 = 0$. The monopolar form factors in this model present a vector meson dominance form

$$F_n^{H \rightarrow M}(q^2) = \frac{F^{H \rightarrow M}(0)}{1 - a(q^2/m_H^2) + b(q^2/m_H^2)^2},$$ (30)

where $m_{J\rho}$ is the mass of the pole. The Ref. 3 provides the values of $F^{H \rightarrow M}(0)$ and $m_{J\rho}$ for the $H \rightarrow M$ transition. We use these form factors in order to compute the numerical values showed in subsections 5.5, 5.6 and 5.7.

We can obtain the form factors of the WSB model [3] in function of the form factors of the ISGW model [4] comparing the parametrizations given in both models for the $H \rightarrow P(V)$ transition. Making $\langle P | J_\mu | H \rangle_{WSB} = \langle P | J_\mu | H \rangle_{ISGW}$ we obtain:

$$F_0(t) = \frac{t}{m_H^2 - m_P^2} f_-(t) + f_+(t),$$ (31)

$$F_1(t) = f_+(t),$$ (32)

and from $\langle V | J_\mu | H \rangle_{WSB} = \langle V | J_\mu | H \rangle_{ISGW}$ it is obtained:

$$A_0(t) = \frac{i}{2m_V} \times [f(t) + ta_-(t) + (m_H^2 - m_V^2) a_+ (t)],$$ (33)

$$A_1(t) = \frac{if(t)}{m_H + m_V},$$ (34)

$$A_2(t) = -i(m_H + m_V) a_+(t),$$ (35)

$$V(t) = -i(m_H + m_V) g(t).$$ (36)

Using these relations it is straightforward to get $d\Gamma(H \rightarrow P(V)\nu\bar{\nu})/dt$ or $\Gamma(H \rightarrow P(V), M)$ with the parametrization of the WSB model from respective expressions in the ISGW model, viceversa.

3. The CLFA model [5]: The relativistic light-front quark model gives a fully treatment of quark spin and the center-of-mass motion of the hadron. In a covariant approach of this model the decay constants and the form factors are calculated by means of Feynman momentum loop integrals which are manifestly covariant [5]. The form factors in the spacelike region are given by the three-parameter form

$$F_n^{H \rightarrow M}(q^2) = \frac{F^{H \rightarrow M}(0)}{1 - a(q^2/m_H^2) + b(q^2/m_H^2)^2}.$$ (37)

We have taken from the Ref. [5] the values of $F^{H \rightarrow M}(0), a$ and $b$ for obtaining the numerical values presented in Subsecs. 5.1, 5.2, 5.3 and 5.9.

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