Student use of vectors in the context of acceleration

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A functional understanding of Newton's second law as a vector equation requires that students be able to reason about forces and acceleration as vectors. In this paper, we present data describing students' conceptual difficulties with vector quantities such as acceleration. These data suggest that after traditional instruction in introductory physics, some students do not recognize the vector nature of this quantity. Other students do not have the requisite procedural knowledge to determine acceleration, and are therefore unable to reason qualitatively about Newton's second law. We describe some specific procedural and reasoning difficulties we have observed in students' use of vectors quantities. In addition, we describe instructional difficulties in mechanics that we observed on the basis of our research into student understanding. Some modifications in the instruction were intended to improve students' understanding of the vector nature of acceleration, and to promote student use of vectors when solving mechanics problems. Finally, we describe initial measurements of the effectiveness of these modifications.

Keywords: Newton's second law; force and acceleration as vectors; traditional and modified instruction.

Un entendimiento funcional de la segunda ley de Newton como una ecuación vectorial requiere que los estudiantes puedan razonar acerca de los vectores fuerza y aceleración. En este artículo se muestran datos para describir las dificultades conceptuales de los estudiantes con cantidades vectoriales como la aceleración. Estos datos sugieren que después de una instrucción tradicional en los primeros semestres de física, algunos estudiantes no reconocen la naturaleza vectorial de la aceleración. Otros estudiantes no tienen el conocimiento procedimental para determinar una aceleración, por lo tanto, no pueden razonar cualitativamente acerca de la segunda ley de Newton. Describimos algunas dificultades de procedimiento y razonamiento que hemos observado con el uso de cantidades vectoriales. Además, describimos dificultades durante la instrucción en mecánica las cuales hemos observado durante nuestra investigación. Algunas modificaciones en la instrucción fueron implementadas con el objeto de mejorar el entendimiento de la naturaleza vectorial de la aceleración por parte del estudiante, y para promover el uso de vectores en la resolución de problemas de mecánica. Finalmente describimos algunas medidas de la efectividad de estas modificaciones.

Descriptores: Segunda ley de Newton, fuerza y aceleración como vectores, instrucción tradicional y modificada.

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1. Introduction

For almost all students enrolled in an introductory physics course, the initial sequence of topics is kinematics, followed by dynamics. This first exposure to physics has Newton's second law –a vector equation– as its central theme. For this reason, students' perception of what physics *is*, and what it means to *do physics*, are strongly influenced by this topic.

In the ideal case, students will learn from this topic that the fundamental principles of physics are powerful general ideas that have broad applicability. Too often, however, students fail to see the connections between the ideas that are presented. Rather than view physics as a subject grounded in a few far-reaching fundamental ideas, they instead form the impression that the subject is a collection of context-specific[1] equations that must be memorized.

Our hope is that research into student understanding of the vector nature of kinematics quantities and forces will promote the development of a curriculum that is more effective in strengthening the students' view of physics as a coherent field of study rather than as a collection of individual facts. In this investigation, results of student understanding of the vector nature of acceleration and of Newton's second law as a vector equation are presented. The research questions we hope to answer are:

1) Can students add and subtract vectors in the contexts of velocity and acceleration?;

2) Can students use these procedures to find the acceleration of an object in different contexts?;

3) Do students recognize that Newton's second law relates acceleration and forces as *vector* quantities?

Most instructors of introductory physics courses recognize that thinking of physical quantities as vectors is difficult for students. Even when instructors consistently model their solutions to problems in Newton's second law by starting with free-body diagrams, many students avoid these diagrammatic tools. There is a tendency, even among fairly capable students, to jump to force components immediately, and to resort to memorizing what these components are in specific cases rather than deriving them from the geometry of the situation.

2. Previous related research

Acceleration is a *kinematical quantity*, defined in terms of a change in velocity: in an introductory algebra-based course, average acceleration is used, and defined as the change in velocity divided by the change in time for a small time interval. The direction of the acceleration is thus determined by the direction of the change-in-velocity vector over the time interval. In a calculus-based course, students work with instantaneous accelerations, defined as the limit of the average acceleration as the time interval approaches zero.

Given information about the velocity of an object along a trajectory as a function of time, it is therefore possible to find the acceleration of the object from the definition. However, it is common for physics and experts alike to invoke *dynamics* – information about the forces acting on an object – to determine acceleration, even when all necessary kinematics information is provided. This often leads to errors.

Reif and Allen[1] examined the interpretation of the concept of acceleration as a kinematics quantity by five expert scientists and by five novice students from the University of California, Berkeley. They asked 13 general questions about acceleration in various situations, and found that most expert scientists correctly answered at least 12 of the questions. However, about half of them used dynamical arguments rather than reasoning about the kinematics alone. The expert whose overall performance was the worst invoked forces frequently. Meanwhile, the expert whose overall performance was the best invoked forces least often and argued primarily on the basis of the acceleration components.

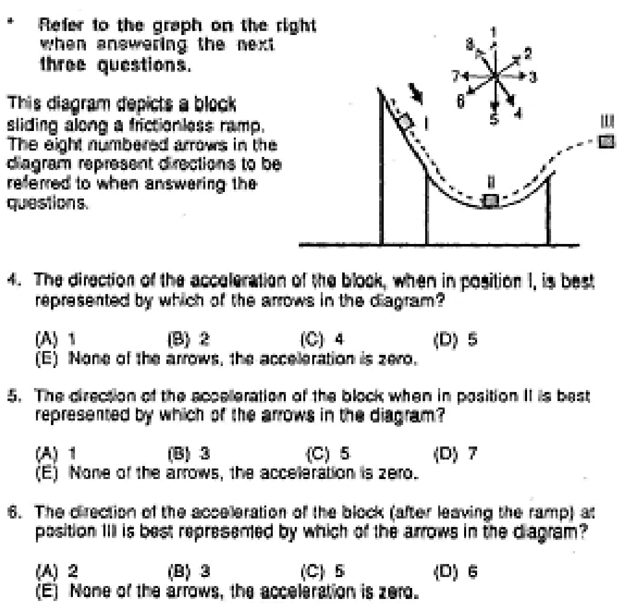


FIGURE 1. Question asked by Hestenes and Wells to probe student conceptual understanding of acceleration.

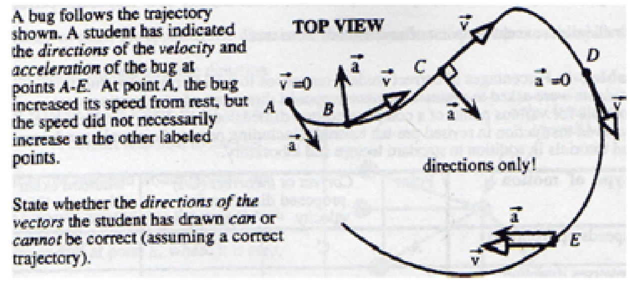


FIGURE 2. Question asked by O'Brien Pride to probe student difficulties with acceleration vectors.

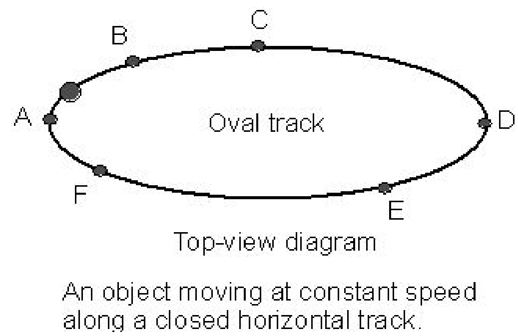


FIGURE 3. Question asked by Shaffer and McDermott to probe student difficulties with velocity and acceleration at several points on a curved path.

Only one of the five novice students solved most problems correctly and three of the five used force arguments. Some students invoked acceleration components, but lacked specific knowledge needed to use them properly.

Hestenes and Wells[2] designed the *Mechanics Baseline Test* to assess students' understanding of basic concepts in mechanics. Figure 1 illustrates a situation used to ask three questions taken from the *Mechanics Baseline Test*. Students were asked which of the arrows from the vector diagram best represents the direction of the acceleration of the block at positions I, II, and III. About two-thirds of 183 students at Harvard University found the correct direction of the acceleration at position I, and about 87% correctly found the direction of the acceleration at position III. However, only 18% correctly identified the direction of the acceleration at position II.

In a study conducted by O'Brien Pride[3] at the University of Washington, 70 students were asked the bug trajectory question shown in Fig. 2 as part of a midterm examination. Students were asked whether the velocity and acceleration vectors for each point of the trajectory were correct. About 30% of the students gave correct responses for the complete set of points, while another 10% responded correctly for all points except the point of acceleration from rest.

Shaffer and McDermott[4] asked the question shown in Fig. 3 about an object moving at constant speed along a closed, horizontal track to probe student understanding of two-dimensional motion. Sixty-nine hundred introductory calculus-based students from several universities were asked to draw velocity and acceleration vectors at several points. When the speed of the object was constant, about 90% of the students gave correct responses for the velocity, but only 20% did so for the acceleration. Only a small fraction of students drew velocity vectors that were not tangent to the curve. About 20% incorrectly stated that the acceleration is zero for an object moving with constant speed along the oval track.

In the studies described above, many students responded with context-specific rules. For example, when students were asked to find the direction of the acceleration of an object when it is moving in a parabolic path with constant speed, many responded that the direction of the acceleration is toward the center of the trajectory. Some students assumed that there is no acceleration because the speed is constant. These students did not seem to relate the ideas of change in velocity to the limit of the average acceleration in order to construct the concept of acceleration.

3. Research techniques

There are two primary data sources that we used during the spring of 2002 to assess student understanding and to learn about students' ideas about physics topics and about the prevalence of these ideas in a given student population. These are individual student responses to questions in one-on-one interviews and student responses to written questions. We describe each of these in turn.

3.1. Written questions

Our primary source of data for our investigation was student responses to written questions. These questions were asked in their homework (both laboratory and lecture), as laboratory pretests, and in classroom quizzes and examinations. Since we are primarily interested in students' conceptual understanding of physics, the questions we ask are primarily qualitative rather than quantitative. Student responses to these questions are typically analyzed and categorized on the basis of their answer and of the reasoning given for that answer.

In our analysis of these written questions, we are looking for *patterns* of student responses, either correct or incorrect. These patterns may be patterns of incorrect ideas, a common tendency to focus on irrelevant features, patterns of reasoning, or patterns of procedure. Some features of common student responses that seem to lead to correct responses may then form the basis for curriculum exercises that reinforce productive lines of reasoning. Conversely, other patterns of answers may indicate that there is a need for a curriculum that elicits a common misconception or error of procedure and reasoning and then addresses this difficulty.

3.2. Interviews

Interviews were conducted at New Mexico State University. These interviews were audio- or videotaped, and the tapes and students written responses were later analyzed. At NMSU, we interviewed students from the introductory calculus-based mechanics courses intended for engineering majors. All of these students were volunteers. The interviews lasted about 30 minutes. We designed the interviews to probe the students' conceptual reasoning. During the interview students were asked questions about selected topics and were encouraged to explain the reasoning behind their responses.

4. Context for research

The courses used as information sources for this study were:

NMSU: Physics 215 (Introductory calculus-based mechanics).

NMSU: Physics 211 (Introductory algebra-based mechanics).

NMSU: Physics 215 laboratory.

NMSU: Physics 211 laboratory.

Syracuse University: General Physics I (Calculus-based mechanics).

Physics 215 is primarily intended for engineering majors. Instruction in the introductory calculus-based physics courses at New Mexico State University consists of three 50-minute lectures per week. The sequence of topics in the lectures follows the sequence in most textbooks. There is no recitation section.

Physics 211, the algebra-based physics course, covers more topics than the calculus-based course, but at a less rigorous mathematical level. The majors of the students enrolled in Physics 211 are approximately: 30% Engineering Technology, 30% Biology, 10% Agriculture, 5% Education, and 20% Other/Undeclared.

There is an associated 1-credit laboratory, Physics 211L and Physics 215L that is required for some majors. About one-half of the students enrolled in the lecture portion of the course also take the laboratory. The 3-hour laboratory is graded separately from the lecture. All of the laboratory sessions are taught by graduate students. In the laboratory, students work in small groups on materials intended to strengthen connections between observed phenomena and mathematical formalism, to promote scientific reasoning skills, and to foster conceptual understanding. Instead of a laboratory report, students are assigned laboratory homework intended to reinforce and extend the concepts underlying the laboratory. Students are encouraged to *predict*, *compare* or *rank* variables in physical situations. Most of the laboratory sessions for both the calculus-based and the algebra-based course were based on *Tutorials in Introductory Physics*[5].

Instruction at Syracuse University (General Physics I) consists of 150 minutes of lecture, a 3-hour required laboratory, and a 50-minute recitation tutorial per week. Students enrolled in this course are primarily engineering majors. In the tutorial, students are encouraged to work in groups on conceptual exercises taken from *Tutorial in Introductory Physics*[5].

In this paper, we describe the instruction in the lecture portion of the course as *traditional* or *modified*. By traditional instruction, we mean instruction that is similar in emphasis and approach to that found in most introductory classrooms. That is, there is no particular emphasis placed on the topics under study in this dissertation, nor is there any modification of the instructional technique used. By modified instruction we mean instruction characterized by an emphasis on conceptual understanding and with particular emphasis placed on the vector topics central to this investigation.

Most of the coursework at NMSU that we describe as modified was taught by Stephen Kanim. He modified the lecture section of the course to increase the emphasis on conceptual understanding. Many homework assignments, exams and exercises are composed of conceptual physical problems. The emphasis of the lectures was modified to focus on vector concepts: the course begins with an introduction of vector addition in the context of force, and velocity and acceleration are first introduced in two dimensions to emphasize their vector nature.

5. Student understanding of vectors in the context of acceleration

A conceptual understanding of Newton's second law requires the ability to reason about vector differences, because acceleration is defined in terms of the limit of a vector difference. Graphical vector a subtraction is taught using two different techniques. The first method consists in adding the opposite of the initial velocity to the final velocity. Then students connect the tail of the final velocity to the head of the opposite of the initial velocity. The second technique consists in placing the vectors tail-to-tail to find the vector that must be added to the initial velocity to find the final velocity. We designed some questions that probe students' conceptual understanding and reasoning ability about vector subtraction in this context.

5.1. Moon question

The *moon question* is shown in Fig. 4. Students were asked to obtain the direction of the change in velocity of the moon when it passes from an initial to a final position in a 7-day time interval. This was a multiple-choice question and the choices include the correct answer (choice e), the vector sum of the two given velocity vectors (choice b), and zero (choice c). A written justification of the answer was also required in some versions.

TABLE I. Results for the answer to the moon question after traditional instruction.

	Traditional instruction NMSU N=132	Modified instruction NMSU N=100	Modified instruction Syracuse N=272
a) ←	3%	1%	1%
b) ↖	52%	8%	37%
c) No change	25%	1%	9%
d) ↑	1%	0%	1%
e) Correct ↙	15%	90%	52%
f) ↓	4%	0%	0%

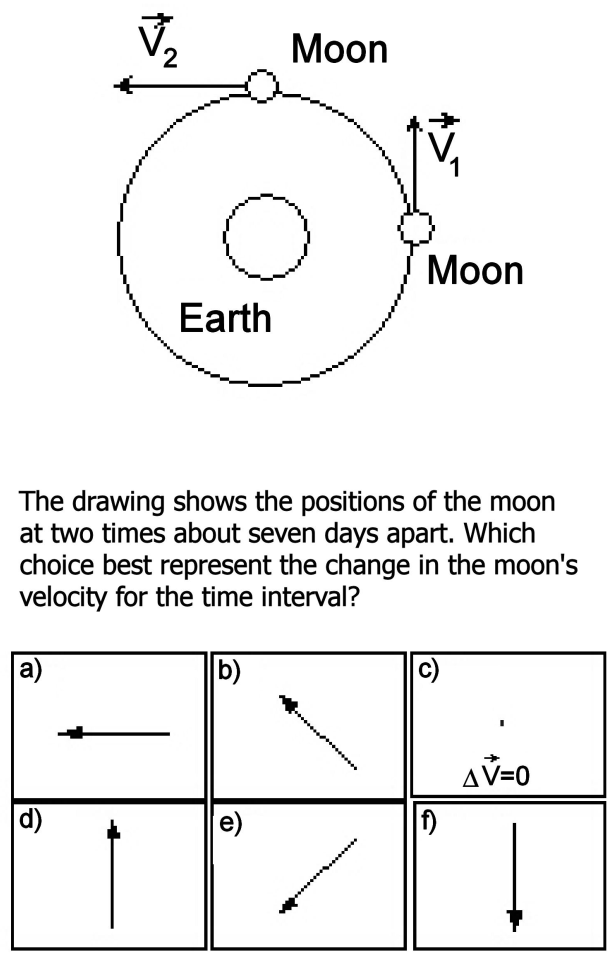


FIGURE 4. The moon question with answer choices.

Results after traditional instruction at New Mexico State are shown in Table I. Only 15% of the students answered correctly, and about one-half gave choice b as an answer. Most students who used choice b added the velocities. About one-quarter answered that the change in velocity was zero. About one-quarter of students (8 students) who answered that the change in velocity was zero stated that the velocity was constant.

This question was also asked at Syracuse University after modified instruction. An explanation of the answer was required. About half of the 272 students chose the correct answer. Almost 40% selected choice (b). Many of these students stated explicitly that they used vector addition to find the change in velocity vector. Only 9% at Syracuse answered that there was no change in velocity. Figures 5 and 6 show examples of these procedural difficulties.

After modified instruction, 90% of 100 students from New Mexico State answered correctly. Only 8% added the velocities, and only one student answered that the change in velocity was zero (choice c).

5.2. The moon's motion interview

An interview based on the question about the moon's motion was conducted at New Mexico State University. Four students agreed to participate: three whose introductory mechanics course was taught traditionally, and the fourth after modified instruction with emphasis placed on conceptual understanding. Students were asked about the change in the velocity of the moon when passing from an initial to a final position. One interviewed student seemed confused about what is meant by a constant velocity. At three points in the interview, she referred to the circular motion as a constant shape, and concluded that the velocity was therefore constant: "*The velocity is constant because the shape of the path at each point shown creates a circular path*."

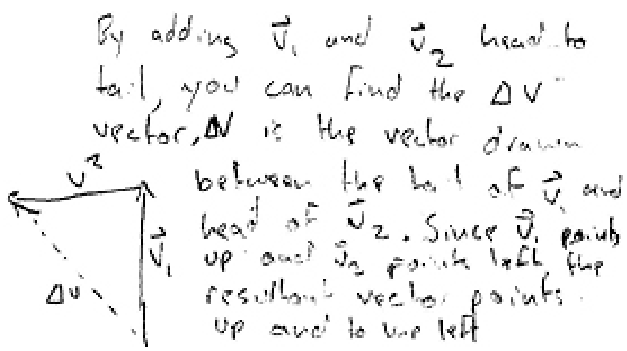


FIGURE 5. Example of a student confusion of vector addition with vector subtraction.

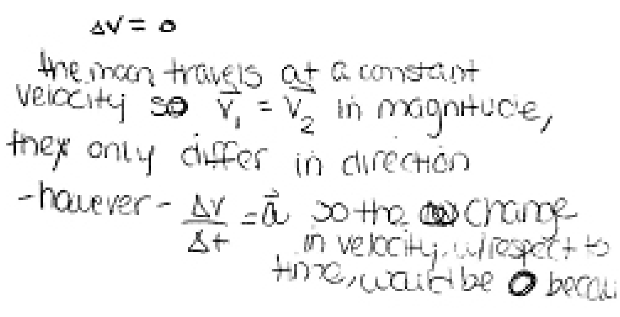


FIGURE 6. Example of reasoning for $\Delta\vec{v} = 0$.

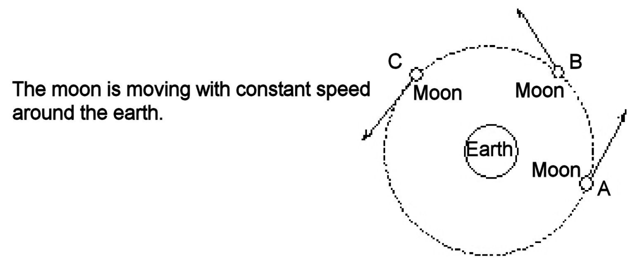


FIGURE 7. Interview question about the moon.

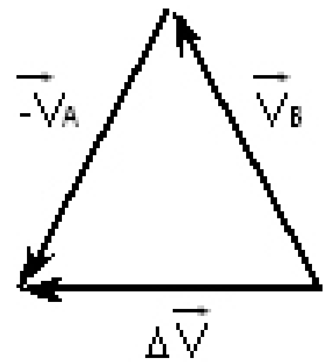


FIGURE 8. Correct answer for the change in velocity question.

As part of this interview, students were asked to find the change in velocity of the moon when it passes from point A to point B as shown in Fig. 7. A correct answer to this question is shown in Fig. 8.

The first student, interviewed after traditional instruction, added the initial and final velocities instead of subtracting them to find the change in velocity of the moon from point A to point B. He did not relate the acceleration to a change in velocity of the moon.

A second student, interviewed after traditional instruction, invoked the correct definition of acceleration. However, he also seemed to be confused by what is meant by the term "constant." For example, while he stated that "*The acceleration is perpendicular to the trajectory,*" he then inferred that "*There is no change in velocity because the acceleration is constant*." During the interview this student could not determine the direction of the acceleration vector for a curved path. In addition, this student seemed to confuse speed and velocity.

A third student was interviewed after modified instruction. She responded that "*The change in velocity is equal to zero because it is moving at constant speed*." In addition, she confused vector subtraction with vector addition in finding the change in velocity. Despite instructional modifications emphasizing vector reasoning, it appears that difficulties with vector concepts persist.

A common feature of these student responses is that students did not recognize that in physics, a vector quantity is constant only if both magnitudes and directions are constant.

TABLE II. Student performance on the car-on-a-hill question.

	Modified instruction NMSU N=75	Modified instruction Syracuse N=248
Correct answer	77%	52%
No acceleration	0%	3%
Acceleration perpendicular to the trajectory	3%	6%
Acceleration toward the center	11%	5%

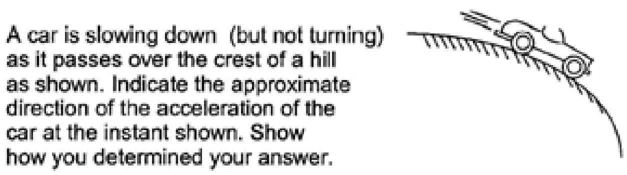


FIGURE 9. The *car-on-a-hill* question.

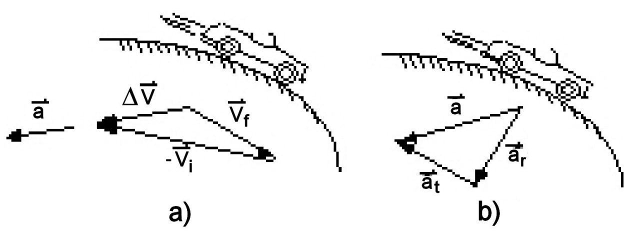


FIGURE 10. A correct answer. a) Velocity subtraction technique. b) Normal and tangential components technique.

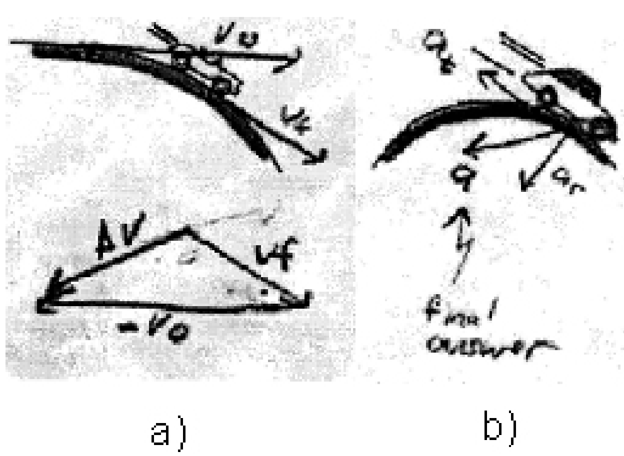


FIGURE 11. Typical student responses to the car-on-a-hill question a) from New Mexico State University, b) from Syracuse University.

5.3. Car slowing down on a hill

As part of a midterm examination, students were asked to find the direction of the acceleration vector of a car when it is moving down a hill and slowing down. The *car-on-a-hill* question is shown in Fig. 9. This question can be answered by subtracting velocity vectors graphically or by reasoning about the tangential and normal components of the acceleration. Figure 10 shows two correct answers to this question. The velocity subtraction technique is shown in part a) and the tangential and radial component of acceleration technique is shown in part b). Responses to this question were collected and analyzed after modified instruction at Syracuse and at New Mexico State University.

After modified instruction at Syracuse, about one-half of 248 students answered the question correctly. The most common incorrect response, given by about one quarter of the students, was that the acceleration was tangent to the curve and opposite to the motion. Six percent answered that the acceleration was perpendicular to the trajectory, and about 3% answered that there was no acceleration.

At New Mexico State after modified instruction, about 75% of 75 students answered correctly. Only 4% answered that the acceleration was tangent to the hill and opposite to velocity. These results are shown in Table II.

During instruction at Syracuse students used a tutorial on two-dimensional motion from *Tutorials in Introductory Physics* that provides practice in finding change-in-position and change-in-velocity vectors for objects moving along a curved trajectory[5]. In this tutorial, students analyze the changes in the relative angles of the velocity and change in velocity vectors between two positions, and then draw conclusions about the direction of acceleration with respect to the trajectory. They examine the limiting case when the time interval between the two positions approaches zero. At New Mexico State, modifications to lecture instruction included emphasis on graphical construction of the average acceleration vector. Students were shown graphical vector subtraction in the lectures and were asked to find $\Delta\vec{v}$ vectors in homework assignments.

About 30% of the students at Syracuse who answered correctly reasoned about the angle between velocity and acceleration. About half based their reasoning on the tangential and normal components of acceleration, a technique emphasized in the lecture portion of this course. About 90% of NMSU students who answered correctly subtracted the velocity vectors graphically. Only 5 students from New Mexico State responded on the basis of tangential and normal components. Conversely, only 4 students from Syracuse used vector subtraction arguments in answering the question. Examples of these responses are shown in Fig. 11.

Students at New Mexico State were required to provide an explanation for their response to this question. Most students' reasoning was based on the subtraction of the velocity vectors. Some of the students' procedural difficulties are described below.

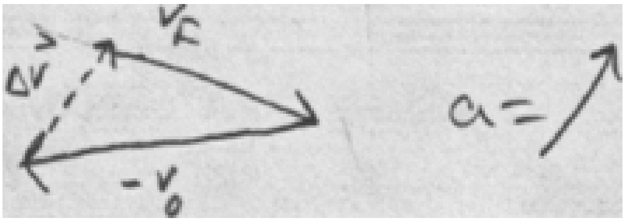


FIGURE 12. Example of a student procedural difficulty using *closing the loop* to subtract the initial and final velocities.

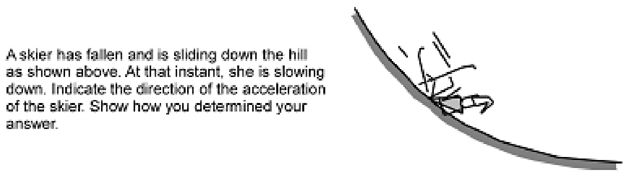


FIGURE 13. The skier slowing down question.

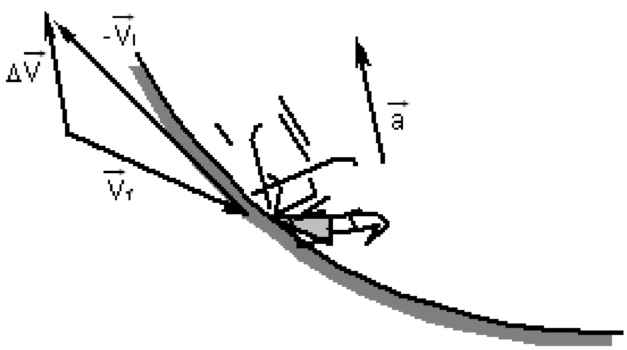
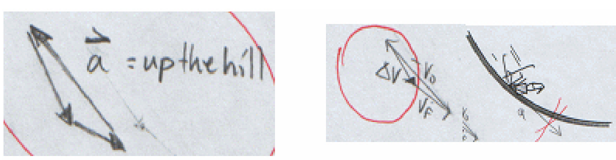


FIGURE 14. A correct solution to the skier slowing down question.



Acceleration opposite to velocity

Acceleration in the same direction of the velocity

FIGURE 15. *Closing the loop* procedure to subtract velocity vectors.

Eight percent of students at NMSU said that when an object moves downward, the acceleration is downward. One of them stated that "*the acceleration is down because it is falling*." There is evidence of similar difficulties on a pretest question answered by more than 20,000 undergraduate students who were asked to draw the direction of the acceleration vector of a ball that is rolling up and down a ramp. On this question, described by Shaffer and McDermott[4], about 20% answered that the ball has an acceleration in the direction of gravity at one or more points.

Seven percent of students answering the car-on-a-hill question at NMSU drew acceleration vectors that were tangent to the hill and opposite to the direction of motion. One example of reasoning given for this response is "*Since it is slowing down, its acceleration is negative, so the vector points backward*."

Five percent of students at NMSU drew initial and final velocities that were the same size and in the same direction. These students subtracted the initial velocity from the final velocity using a graphical method, and found a zero change. They answered that the acceleration at that instant was zero as well.

Seven percent of students "closed the loop" when they subtracted initial velocity from final velocity. These students typically drew the change-in-velocity vector opposite to the correct direction, concluding that acceleration was radially outward. Figure 12 gives one example of this error.

5.4. The skier slowing down question

As part of a midterm examination given to students in two calculus-based courses, the question shown in Fig. 13 was asked to explore students' ability to recognize the direction of the acceleration of an object slowing down along a curvilinear trajectory. We expected students to perform a subtraction of velocity vectors to find the direction of the skier's acceleration. The correct answer is that the direction of the acceleration is toward the inside of the curve and makes an angle greater than 90° with the velocity. A correct answer is shown in Fig. 14.

After traditional instruction at New Mexico State, about 10% of 32 students answered correctly, with 30% answering that the direction of vector acceleration was toward the center of the trajectory. Another 30% answered that the direction of acceleration was opposite to the velocity. After modified instruction, about 35% of 66 students answered correctly. Only about 20% answered that the acceleration was toward the center of the trajectory, and about 40% stated that the acceleration had a direction opposite to the velocity.

Most of the students after traditional instruction did not reason based on the subtraction of velocity vectors. Most of the students after modified instruction used a subtraction of velocity vectors to reason about the direction of the acceleration. Only 3 of these students subtracted the initial velocity from the final velocity using the *closing the loop procedure*. Figure 5 shows two examples of these students' reasoning.

6. Conclusions

After both traditional and modified instruction, many students were unable to determine the direction of the difference between two velocity vectors in order to find the approximate direction of the acceleration. For example, students who were interviewed believed the moon is moving at a constant velocity. None of the students who were interviewed performed a vector subtraction to identify the direction of the moon's acceleration without prompting.

Some students seem to misunderstand what is meant by 'constant'. For example, many students did not recognize that a constant velocity requires both a constant speed and a constant direction. Other students described circular motion as constant because the shape of the path was constant.

Although improvement was made in student performance after modified instruction, responses reveal an inability to add and subtract vectors. Some of these students still have difficulty relating a change-in-velocity to an acceleration. However, a small number of students from New Mexico State had procedural difficulties, compared to Syracuse. Students' strategies for solving certain kinematics questions appear to be influenced by their instruction.

With instructional modifications that provide an emphasis on geometrical vector operations, many students recognized that they needed to subtract vectors. However, most of them still could not find the acceleration for objects moving along curved paths. For example, in the *skier slowing down* question, only one-third answered correctly. Despite most students' using reasoning based on the subtraction between initial and final velocities, many of them made procedural errors that generated incorrect answers.

For the questions in this article presented in a physical context, students still made the same errors that we saw in questions with no context. For example, after modified instruction, some students failed to find the direction of the acceleration for the skier slowing down when they subtracted velocities by using a "closing the loop" procedure. However, the introduction of context created additional opportunities for error and introduced new conceptual errors. In general, as seen in the McDermott and Shaffer[4] study, performance was poorer on vector addition and subtraction questions when they were asked in the contexts of force and acceleration.

Finally, as Flores[6] said, "most of the undergraduate students have a problem understanding the fundamental physics concepts, primarily with vector operations. The developments of the mathematical objects that represent physics concepts determine a cognitive evolution of the student mathematical structures during the learning of physics issues".

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