Answer to the Comment on “Continuous groups of transformations and time-dependent invariants”

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In this paper we answer to the Comment on Continuous groups of transformations and time-dependent invariants. Additionally, in order to apply Hegel’s *Aufhebung* concept in physics, some remarks about what we consider the right way to write a “Comment” are given.

**Keywords:** Lie groups; Lorenz group; dynamical systems; Noether’s theorem; infinitesimal transformations.

En el presente trabajo damos respuesta a los comentarios que se hicieron sobre el artículo Continuous groups of transformations and time-dependent invariants. Adicionalmente, para aplicar en física el concepto hegeliano de *Aufhebung*, hacemos algunas observaciones acerca de la que consideramos es la mejor manera de escribir un “Comment”.

**Descriptores:** Grupos de Lie; grupo de Lorentz; sistemas dinámicos; teorema de Noether; transformaciones infinitesimales.

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1. **Errata**

1. For Eq. (6) of [1]

$$\bar{x}^{j} = x^{j} + \xi^{j}_{a}(x) t,$$

read

$$\bar{x}^{j} = x^{j} + \xi^{j}(x) t,$$  \hspace{1cm} (1)

2. For Eq. (8) of [1]

$$\bar{x}^{j}_{1} = \frac{\partial f}{\partial x^{i}} x_{1}^{i},$$

read

$$\bar{x}^{j}_{1} = \frac{\partial f^{j}}{\partial x^{i}} x_{1}^{i}.$$  \hspace{1cm} (2)

3. For

$$\frac{\partial}{\partial x_{1}^{j}} = -x^{x'} \frac{\partial}{\partial x_{1}^{j}} \partial x^{x'}$$

and

$$\frac{\partial}{\partial x_{1}^{j}} = \frac{\partial}{\partial x_{1}^{j}} \partial x^{x'}$$

in the text after Eq. (11) of Ref. 1 read

$$\frac{\partial}{\partial x_{1}^{j}} = -x^{x'} \frac{\partial}{x_{1}^{j} \partial x^{x'}}$$

and

$$\frac{\partial}{\partial x_{1}^{j}} = 1 \frac{\partial}{x_{1}^{j} \partial x^{x'}},$$

respectively.

4. For $-t \bar{e} \cdot \nabla$ in Eq (23) of Ref. 1 read $+t \bar{e} \cdot \nabla$

5. For $\omega = \sqrt{\frac{k}{m}}$ after Eq. (26) of Ref. 1 read $\omega = \sqrt{\frac{k}{m_{0}}}$

2. **Answers**

In the following, we will answer the questions and remarks of Dr. Torres del Castillo concerning the paper “Continuous groups of transformations and time-dependent invariants”.

For the statement: “A very clear and detailed treatment of the subject can be found in Ref. 2.”

We recommend also reading Ref. 2 of this answer.

For the statement: “At the beginning of Sec. 2 a one-parameter group of transformations... but the objects appearing on the right hand side of Eqs. (3) are not defined at all.”

In Refs. 3 to 5 of this answer it is explained exhaustively, why is it possible to do what has been done in Eq. (2) and (3) of [1]. Of special importance is the topic fundamental differential equations of a group. In the first chapter of [6] of this answer this question is also explained.

For the statement: “Similarly, the functions $\xi^{j}_{a}$, appearing in the...”

If the functions $\xi^{j}$ are assumed to be regular in the domain of $x^{j}$, the integrals of Eq. (5) of [1] can be written in the form

$$\bar{x}^{j} = x^{j} + \xi^{j}(x) t + \xi^{j}(x) \frac{\partial \xi^{j}}{\partial x^{j}} \frac{t^{2}}{2} + \ldots$$  \hspace{1cm} (3)
For an infinitesimal value of $t$, neglecting terms of higher order, one obtains (1) or
\[ x^j = x^j + \xi^j (x) \delta t, \]
where $\delta t$ is another representation of the infinitesimal value of $t$ (see also Ref. 6 of this answer).

For the statements: “Equation (11) and the following displayed equations . . . not explained at all.”, and “From Eq. (21) . . . this answer . From Eq. (2) of Ref. 1 it is easy to infer . . .

For the statement: “. . . however, not necessarily each of these terms has to be a constant of motion separately.”

Nowhere in Ref. 1 is it mentioned that each term of (26) is a constant of motion; we have only given an interpretation of each term of the invariant given by Eq. (26) of Ref. 1.

For the statement: “By contrast with the claim in Sec. 4 . . .” To obtain the angular momentum conservation law it is necessary to generalize the method to, at least, two spatial coordinates and Eq. (24) determines the selection of the corresponding $\xi^j$ in Eq. (10). Clearly, Eq. (24) does not represent the angular momentum conservation law but it is the generator of infinitesimal rotations containing components corresponding to spatial rotations in all three coordinate planes and a finite rotation can be obtained by exponentiation of this equation [6]. In addition, Eq. (24) leaves the Hamiltonian unchanged and hence it is associated with a conserved quantity, in this case, the angular momentum.

We shall now give some general comments.

On the one hand, we wish to thank Dr. Torres del Castillo for the observations concerning the typing errors contained in the first section of this answer. On the other hand, since this answer is intended to appear in the “Sección de Enseñanza de la Revista Mexicana de Física”, we finish with a remark about what we consider the right way to write a “Comment”. Under the premise that a researcher must try to interpret the reality and because of the specialization in the scientific work, in our opinion, a “Comment” should contain three main parts:

1. a constructive analysis of the work,
2. the lacks, unfinished proposals, errors, and unsatisfied objectives,
3. an explanation of the achievement of the work in spite of the errors, lacks and other adverse points indicated in the second part.

In addition, the analysis of the work from a different point of view should serve to enrich it. In our opinion the “Comment” of Dr. Torres del Castillo centers mainly on the second point.


10. H. Vázquez Briones (private communication).