A comparison of optimal doping behaviors between
$d-$ and s^*- wave superconducting ground states

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In a previous work reported in this journal, the thermodynamical properties of d-wave superconducting ground states close to half filling were obtained by using a generalized Hubbard model. In the present work, extended s-wave (s^*) symmetry superconducting ground states are considered within the same model and a comparison is made between these two symmetries for those properties depending on the carrier density (n) and first- (t) and second-neighbor (t') electron hopping parameters. For s^*-wave superconducting states, the electron-electron interaction Hamiltonian parameters is fixed as in the d-wave case with $\Delta t = 0.5$ eV, $\Delta t_3 = 0.05$ eV, and two cases were studied; $U = 0$ and $U = 2$ eV. Without considering the diluted regime ($n \to 0$), in the interval $n \in [0, 1]$, for both symmetries, the maximum critical temperatures ($T_{c,\max}$) at optimal doping (n_{op}) are lower than those attained at high electron densities $n \in [1, 2]$. The superconducting gap at $T = 0$ K and the corresponding ground state energy (E_g), for all the carrier concentrations, were also obtained. The superconductor state with d-wave symmetry does not depend on U and a supremum for $n_{\mathrm{op}}(n_{\mathrm{op,sup}})$, where both $T_{c,\max}(n_{\mathrm{op},t'}$ and $E_g(n_{\mathrm{op},t'}$ are minimum, can be found close to half filling. However, in general, for a s^*-wave superconductor $T_{c,\max}$ and E_g attain their minimum values at different (n_{op}, t') points and a $n_{\mathrm{op,sup}}$ cannot be properly defined.

Keywords: Theories and models of superconducting state; pairing symmetries (other than s-wave); pseudogap regime.

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1. Introduction

The Hubbard model (HM) was proposed by J. Hubbard in 1963 [1] in order to understand the extensive variety of physical properties of transition metals (TM) with narrow electronic bands, and since then till nowadays it has been used as a basic model to study diverse phenomena, such as antiferromagnetism and charge density waves [2], both phenomena also related to high-T_c superconductors. Moreover, it is widely accepted that the single-band Hubbard model is an appropriate starting point to describe the electronic correlations on the copper-oxygen planes of the cuprate superconductors [3]. In particular, the hole superconductivity proposed by J. Hirsch [4], based on a generalized Hubbard model that includes nearest neighbor hopping (t) and the so-called nearest-neighbor correlated hopping interaction (Δt), could explain superconducting states with an extended s-symmetry energy gap. However, the inclusion of the next-nearest-neighbor hopping could play a crucial role in the competition between antiferromagnetism and superconductivity in cuprate superconductors [5]. Moreover, phase sensitive experiments have shown that the cuprate superconductors have a d-wave sym-

metry superconducting gap [6]. In this context, L. A. Pérez and C. Wang showed that when both next-nearest neighbor hopping (t') and next-nearest-neighbor correlated hopping interaction (Δt_3) are taken into account, a d-wave superconducting ground state can be found in a two-dimensional lattice [7]. This generalized Hubbard model allows to describe many properties of superconducting ground states in a square lattice with different superconducting gap symmetries [8-10].

In a previous work [11], we studied how the critical temperature T_c of d-wave superconducting states on a square lattice depends on the electron density (n) and the second-neighbor hopping (t') for a given set of electron-electron interaction parameters ($U, V, \Delta t, \Delta t_3$). In other words, for each value of t', we calculated $T_c(n)$ and the particular value $n_{\mathrm{op}}(t')$ where T_c is maximum is called the optimal density. Moreover, we also studied the particular value $n_{\mathrm{op,sup}}$ where the minimum value of $T_c(n_{\mathrm{op}})$ is found. This value, which corresponds to the minimum value of the critical temperature among the set formed by the maximum values of T_c, has been dubbed the supreme value of T_c ($T_{c,\max,\mathrm{sup}}$). Moreover, we have found that the minimum superconducting ground state energy is attained at this particular value $n_{\mathrm{op,sup}}$ [11].

By locating $n_{\text{op-sup}}$ for a fixed $\Delta t_3 = 0.05$ eV, critical temperatures of $T_c \approx 40$ K and $T_c \approx 100$ K, can be found that correspond to $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ and $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ with $\Delta t = 0.1$ eV and $\Delta t = 0.5$ eV, respectively [11].

Motivated by this scenario, in this work the critical temperature of systems with an extended s-wave (s^*) superconducting gap as a function of n and t' (for a given set of electron-electron interaction parameters) has been studied. Considering that the d-wave superconducting state arises from the V and Δt_3 interactions, while the s^*-wave one originates from both Δt and Δt_3, V will be set to zero to keep a minimum set of parameters. It is worth mentioning that for d-wave superconductors, U does not change the superconducting critical temperature [7,10], whereas for s^*-wave superconductors U hinders the superconducting gaps and the critical temperatures [7,10]. In contrast with the d-wave case [11], the results show that, for the s^*-wave case, the minimum of $T_c(n_{\text{op}})$ it is not attained at the same value of n_{op} (and t') where the minimum ground state energy is found, although for $U = 2$ eV, the minimum of $T_c(n_{\text{op}})$ and E_g are attained close to half filling but at slightly different values of n_{op}. The different behaviors found for d-wave and s^*-wave symmetries are related to the fact that, for the latter case the superconducting gap has an isotropic (Δ_s) and anisotropic (Δ_{s^*}) amplitudes.

2. The model

Within the single-band Generalized Hubbard Hamiltonian [10], with on-site electron-electron Coulombic interaction (U), inter-site Coulombic interaction (V), first-(Δt) and second neighbor (Δt_3) correlated-hopping interactions, the interaction ($V_{\mathbf{k},\mathbf{k}',\mathbf{q}}$) between electrons with antiparallel spins in the reciprocal space can be written as [10,11]:

$$V_{\mathbf{k},\mathbf{k}',\mathbf{q}} = U + V\beta(\mathbf{k}-\mathbf{k}') + \Delta t(\beta[\mathbf{k}+\mathbf{q}] + \beta[-\mathbf{k}+\mathbf{q}]$$
$$+ \beta[\mathbf{k}'+\mathbf{q}] + \beta[-\mathbf{k}'+\mathbf{q}] + \Delta t_3(\gamma[\mathbf{k}+\mathbf{q},\mathbf{k}'+\mathbf{q}]$$
$$+ \gamma(-\mathbf{k}+\mathbf{q},-\mathbf{k}'+\mathbf{q}))], \tag{1}$$

where $\mathbf{q}$ is the wave vector of the centre of mass of the two interacting electrons with relative wave vectors $\mathbf{k}$ and $\mathbf{k}'$.

For a square lattice with lattice parameter a, we have [11]:

$$\beta(\mathbf{k}) = 2(\cos[k_x a] + \cos[k_y a]), \tag{2}$$

$$\gamma(\mathbf{k} \cdot \mathbf{k}') = 4\cos[k_x a]\cos[k'_y a] + 4\cos[k'_x a]\cos[k_y a]. \tag{3}$$

Within the mean-field BCS formalism [10], we obtain the following two coupled integral equations which determine the superconducting gap $\Delta_\alpha(\mathbf{k})$ with symmetry $\alpha = s^*, d$, and the corresponding chemical potential μ_α for a given temperature T and electron density (electrons per lattice site) n:

$$\Delta_\alpha = -\frac{a^2}{8\pi^2} \iint\limits_{1BZ} V_{\mathbf{k},\mathbf{k}',\mathbf{0}} \frac{\Delta_\alpha(k')}{E_\alpha(k')} \tan$$
$$\times \left(\frac{E_\alpha(\mathbf{k}')}{2k_B T}\right) dk'_x dk'_y, \tag{4}$$

$$n-1 = -\frac{a^2}{4\pi^2} \iint\limits_{1BZ} \frac{\varepsilon(\mathbf{k})\mu_\alpha}{E_\alpha(\mathbf{k})} \tanh\left(\frac{E_\alpha(\mathbf{k}')}{2k_B T}\right) dk_x dk_y. \tag{5}$$

Here $1BZ$ refers to the first Brillouin zone of a square lattice, defined as

$$\left[\frac{-\pi}{a}, \frac{\pi}{a}\right] \otimes \left[\frac{-\pi}{a}, \frac{\pi}{a}\right]$$

and $E_\alpha(k)$ is the quasi-particle energy given by:

$$E_\alpha(k) = \sqrt{(\varepsilon_{MF}(\mathbf{k}) + \mu_\alpha)^2 + \Delta_\alpha^2(\mathbf{k})}, \tag{6}$$

where $(\varepsilon_{MF}(\mathbf{k}))$ is the mean field (MF) dispersion relation given by

$$\varepsilon_{MF}(\mathbf{k}) = E_{MF} + 2t_{MF}(\cos[k_x a] + \cos[k_y a])$$
$$+ 4t'_{MF}\cos(k_x a)\cos(k_y a), \tag{7}$$

with:

$$E_{MF} = \left(\frac{U}{2} + 4V\right)n, \tag{8}$$

$$t'_{MF} = t' + 2n\Delta t_3, \tag{9}$$

$$t_{MF} = t + n\Delta t, \tag{10}$$

where t and t' are the nearest and next-nearest neighbor hopping parameters. Here we take $t < 0$. Once the electron-electron interaction $V_{\mathbf{k},\mathbf{k}',\mathbf{q}}$ of Eq. (1) is replaced into Eq. (4) and the possible superconducting gap solutions with d- and s^*-wave symmetries are considered (first column of Table I), we obtain the equations shown in the second column of Table I.

TABLE I. Superconducting gap equations for d- and s^*-wave symmetries.

Superconducting gap	Superconducting-gap equation	
d-wave		
$\Delta_d(\mathbf{k}) = \Delta_d(\cos[k_x a] - \cos[k_y a])$	$1 = -\frac{(V-4\Delta t_3)a^2}{4\pi^2} \iint\limits_{1BZ}\left(\frac{[\cos\{k_x a\}-\cos\{k_y a\}]^2}{2E_\alpha(\mathbf{k})}\right)\tanh\left(\frac{E_\alpha(\mathbf{k})}{2k_B T}\right)dk_x dk_y$	(11)
s^*-wave		
$\Delta_{s^*}(\mathbf{k}) = \Delta_s + \Delta_{s^*}(\cos[k_x a] - \cos[k_y a])$	$\Delta_{s^*} = -4\Delta t_3(I_2\Delta_{s^*} + I_1\Delta_s) - 4\Delta t(I_1\Delta_{s^*} + I_0\Delta_s)$	(12)
	$\Delta_s = -U(I_1\Delta_{s^*} + I_0\Delta_s) - 4\Delta t(I_1\Delta_{s^*} + I_1\Delta_s)$	(13)

For the s^*-wave symmetry superconducting gap $\Delta_{s^*}(k) = \Delta_s + \Delta_{s^*}(\cos[k_x a] + \cos[k_y a])$, Δ_s and Δ_{s^*} denote the isotropic and anisotropic gap amplitudes, respectively, and the integrals I_l for $l = 0, 1, 2$, are given by

$$I_l = \frac{a^2}{4\pi^2} \iint_{1BZ} \frac{(\cos[k_x a] + \cos[k_y a])^l}{2E_{S^*}(\mathbf{k})} \tanh\left(\frac{E_{S^*}(\mathbf{k})}{2k_B T} dk_x dk_y\right). \tag{14}$$

The critical temperature (T_c), can be determined from Eq. (11), or from Eqs. (12) and (13) together with Eq. (5) by considering that $\Delta_\alpha(T = T_c) = 0$. In this case, the equations in Table I, can be reduced to those shown in Table II.

TABLE II. Critical temperature equations for d- and s^*-wave symmetries.

Superconducting gap symmetry	Critical temperature equation	
d-wave	$1 = -\frac{(V-4\Delta t_3)a^3}{4\pi^2} \iint_{1BZ} \left(\frac{[\cos\{k_x a\}-\cos\{k_y a\}]^2}{2|\varepsilon_{MF}(\mathbf{k})-\mu_d|}\right) \tanh\left(\frac{|\varepsilon_{MF}(\mathbf{k})-\mu_d|}{2k_B T_c}\right) dk_x dk_y$	(15)
s^*-wave	$1 = (I_1^2 - I_2 I_0)(4\Delta t_3 - (4\Delta t)^2) + 4\Delta t_3 I_2 + U I_0 + 8\Delta t I_1$	(16)

where

$$I_l = \frac{a^2}{4\pi^2} \iint_{1BZ} \frac{(\cos[k_x a] + \cos[k_y a])^l}{2|\varepsilon_{MF}(\mathbf{k}) - \mu_{s^*}|}$$

$$\times \tanh\left(\frac{|\varepsilon_{MF}(\mathbf{k}) - \mu_{s^*}|}{2k_B T_c}\right) dk_x dk_y. \tag{17}$$

On the other hand, the ground state energy $(E_{g,\alpha})$ per site can be obtained from the following equation [12]:

$$E_{g,\alpha} = \frac{1}{N_s} \langle \psi_{g,\alpha} | \hat{H}_{MF} | \psi_{g,\alpha} \rangle$$

$$= \frac{1}{N_s} \sum_k (\xi_{MF,\alpha}[\mathbf{k}] + E_\alpha[\mathbf{k}])$$

$$+ \frac{1}{N_s} \sum_k \frac{\Delta_\alpha^2(\mathbf{k})}{2E_\alpha(\mathbf{k})} + \mu_\alpha n - \left(\frac{U}{4} + 2V\right) n^2. \tag{18}$$

where $\hat{H}_{MF}$ is the mean field Hamiltonian of the system [10,11], N_s is the number of lattice sites,

$$\xi_{MF,\alpha} = \varepsilon_{MF}(\mathbf{k}) - \mu_\alpha,$$

and

$$\psi_{g,\alpha} = \propto_\mathbf{k} (u_{\mathbf{k},\alpha} + v_{\mathbf{k},\alpha} c_{k,\uparrow}^\dagger c_{-\mathbf{k},\downarrow}^\dagger)|\phi\rangle$$

is the ground state BCS-like wave function for singlet pairs. The probability amplitudes to find the states $\langle \mathbf{k} \uparrow, -\mathbf{k} \downarrow \rangle$ occupied (unoccupied) are $v_{k-\alpha}(u_{k-\alpha})$ are given by:

$$v_{k,\alpha} = \sqrt{\frac{1}{2}\left(1 - \frac{\xi_\alpha(\mathbf{k})}{E_\alpha(\mathbf{k})}\right)},$$

$$u_{k,\alpha} = \sqrt{\frac{1}{2}\left(1 + \frac{\xi_\alpha(\mathbf{k})}{E_\alpha(\mathbf{k})}\right)}. \tag{19}$$

Notice that $v_{k,\alpha}^2 + u_{k,\alpha}^2 = 1$.

3. Results

We investigated the critical temperature as a function of the electron density (n) and the next-nearest neighbor hopping parameter (t') for a system with an extended s-wave (s^*) su-
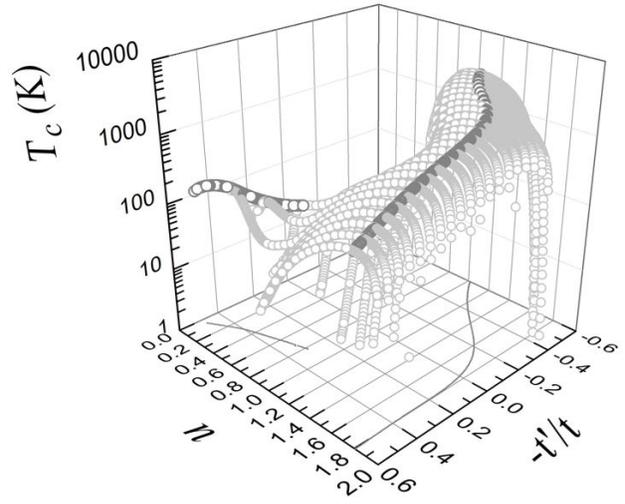
FIGURE 1. The critical temperature (T_c) (open light gray circles) versus the electronic density (n) and the next-nearest neighbor hopping parameter t' for $V = 0$, $\Delta t = 0.5$ eV, $\Delta t_3 = 0.05$ eV and $U = 2$ eV. The gray circles correspond to local maxima $(T_{c,\max})$ of the critical temperature as a function of n for a given t'. The projections of these curves on the plane $n - t'$ are marked with small gray dots and the values of n where $T_{c,\max}$ occur are the optimal electron density or optimal doping n_{op}.

perconducting gap and $V = 0$, $\Delta t = 0.5$ eV and $\Delta t_3 = 0.05$ eV. Figure 1 shows $T_c(n, t')$ for $U = 2$ eV. Notice that the highest values of T_c occur at high electron densities n. From these data, the maximum of $T_{c,\max}$ as a function of n can be extracted for each value of t' and the particular value of $n(n_{op})$ at which this maximum is attained is recorded. These values of $T_{c,\max}$ at the optimal doping (n_{op}) are marked with open gray circles in Fig. 1. Figure 2 shows the isotropic $\Delta_s(n_{op}(t'))$ (open gray circles) and anisotropic $\Delta_{s^*}(n_{op}(t'))$ (open black circles) superconducting s-wave gap amplitudes at the optimal doping $(n_{op}(t'))$ for each value of t' for the same systems shown in Fig. 1. Figure 3 shows the corresponding ground state energies of s^*-wave superconducting states at the optimal doping for each value of t', i.e., $E_{g,s^*}(n_{op}(t'))$, calculated with
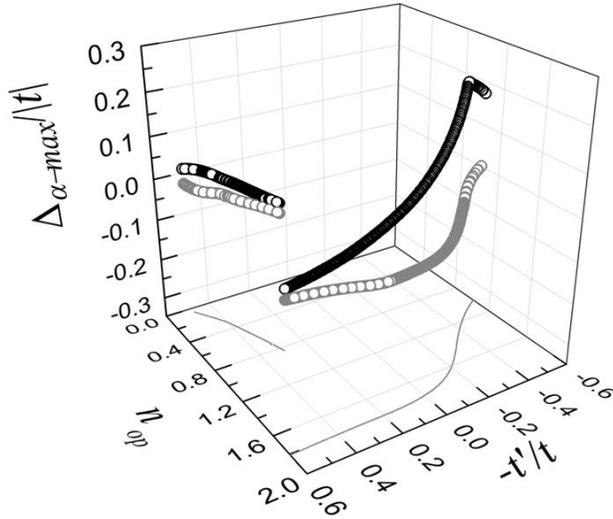
FIGURE 2. Superconducting gap amplitudes $\Delta_{a,\max}$, for $\alpha = s$ (open gray circles) and $\alpha = s^*$ (open black circles) evaluated at the optimal carrier concentration or doping $n_{op}(t')$ considering the same set of parameters as in Fig. 1.
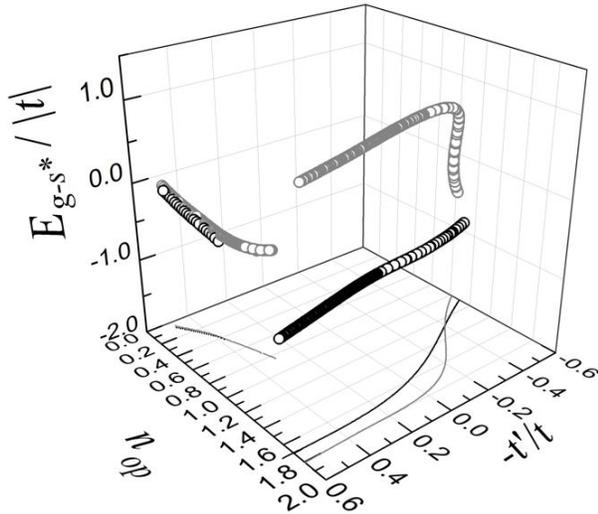
FIGURE 3. Ground state energy (open gray circles) evaluated at the optimal carrier concentration or doping $n_{op}(t')$ considering the same set of parameters as in Fig. 1 together with the case $U = 0$ (open black circles).

Eq. (18) with $U = 2$ eV (open gray circles) and $U = 0$ (open black circles). For the case with $U = 2$ eV, $T_{c,\max}$ has a minimum at $n_{op} = 0.55$, whereas E_{g,s^*} has a minimum at $n_{op} = 0.63$. This is in contrast to what happens in the d-wave case where both the minimum of the energy and the

minimum of $T_{c,\max}$ (among the set of maximum $T_{c',s}$) occur at the same value of n_{op} [11]. This could be related to the fact that the s^*-wave superconducting gap has two gap amplitudes (Δ_s, Δ_{s^*}) that at T_c become zero at the same time, whereas at $T = 0$ they could even have different sign. In contrast, the d-wave superconducting gap only has a single superconducting gap amplitude (Δ_d).

Tables III and IV summarize the superconducting gap amplitudes Δ_s and Δ_{s^*}, the Fermi energies (E_F), i e., the chemical potentials at $T = 0$ K, and the ground state energies (E_g) for s^*-wave superconducting states with optimal doping (n_{op}) close to those where the minimum of $T_{c,\max}$ $(n_{op} = 0.55)$ and the minimum of $E_g(n_{op} = 0.63)$ are attained, for systems with $U = 2$ eV, $V = 0$, $\Delta t = 0.5$ eV and $\Delta t_3 = 0.05$ eV. In general, for high electron optimal densities $T_{c,\max}$ is higher than at low electron densities. However, in the case of d-wave superconducting states with large electron densities (n) above have filling $i.e.$ $n \in [1, 2]$, which correspond to low hole densities $(n_h = 2 - n)$, the Fermi energy at the under doped hole regime $(n_h < n_{h,op})$ does not lie within the single-electron band and no Fermi surface is encountered. This electron-density zone could be related to the pseudo gap regime of cuprate superconductors which is characterized by the disappearance of Fermi surface [13]. Figure 4 shows the optimal electron density or doping, n_{op}, for s^*- (open circles) and d-wave (open squares) superconducting states with the same parameters as in Fig. 2 and for different values of t'. Notice that for $t' \in [0.4|t|, 0.5|t|]$, $n_{op}(t')$ coincide for both s^*- and d-wave superconducting states and these occur at low densities, $i.e.$, $n_{op} \in [0, 0.365]$. This is due to the fact that for these values of t', the van Hove singularity of the single-particle electronic density of states (DOS) of a square lattice with first (t) and second neighbor (t') hoppings at the bottom of the band is enhanced, as shown in the inset of Fig. 4, where the DOS for $t' = 0.45t$ and $n = 0.15$ (black line) is depicted. The black arrow in the figure indicates this enhancement in comparison with the DOS for $t' = 0.352|t|$ and $n = 0.55$ (grey line). Due to this increment of the DOS, both superconducting states are enhanced at low carrier concentrations close to the bottom of the band. Also, in Fig. 4, the black solid and the gray solid symbols correspond to the minima of $T_{c,\max}$ and $E_{g,\alpha}$, respectively, for each symmetry α. Observe that for d-wave superconductors these two minima (solid squares) occur at almost the same optimal electron density $(n_{op} = 0.805)$ [11], whereas for s^*-wave superconductors they occur at different optimal doping $n_{op} = 0.55$ and $n_{op} = 0.63$ (solid circles), respectively.

TABLE III. Physical properties for s^*-wave superconducting states close to minimal of $T_{c,\max}$ for $\Delta t = 0.5$ eV, $\Delta t_3 = 0.05$ eV, and $U = 2$ eV.

$-t'/t$	n_{op}	$T_{c,\max}$ (K)	E_g (eV)	Δ_{s^*} (eV)	Δ_s (eV)	E_F (eV)
0.354	0.54	137.15	-0.620463	0.0140545	-0.0192207	-0.759190
0.352	0.55	137.03	-0.623059	0.0138245	-0.01885295	-0.733963
0.35025	0.56	137.04	-0.625388	0.0135944	-0.01849778	-0.708160

TABLE IV. Physical properties for s^*-wave superconducting states close to minimal of E_{g,s^*} for $\Delta t = 0.5$ eV, $\Delta t_3 = 0.05$ eV, and $U = 2$ eV.

$-t'/t$	n_{op}	$T_{c,\max}$ (K)	E_g (eV)	Δ_{s^*} (eV)	Δ_s (eV)	E_F (eV)
0.33875	0.62	138.17	-0.632258	0.0122655	-0.0162540	-0.555360
0.3370	0.63	138.60	-0.632309	0.0120629	-0.0159037	-0.529490
0.3350	0.64	139.09	-0.631959	0.0118857	-0.0155746	-0.504122
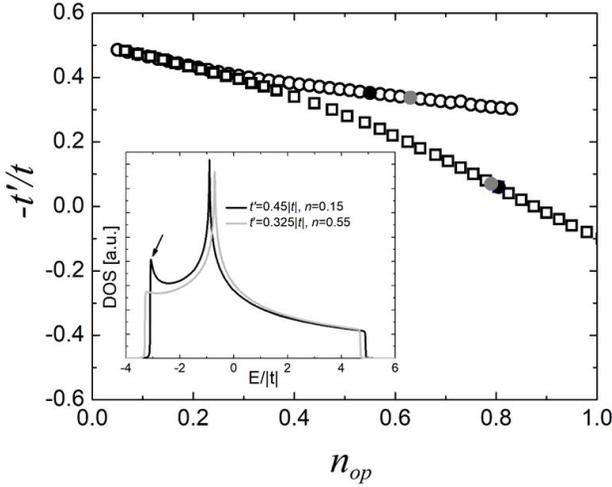
FIGURE 4. Optimal doping as a function of the next-nearest-neighbor hopping ($n_{op}(t')$) for s^*- (open circles) and d-wave (open squares) superconducting states, for the same systems shown in Fig. 2. The solid black and gray circles correspond to the minimum $T_{c,\max}$ and $E_{g,\alpha}$, respectively. Inset: Single-particle electronic densities of states for systems with two different second-neighbor hoppings t' and optimal densities n_{op}.

Figure 5(a) depicts the Fermi surface (FS) for a system with $n = 0.55$, $U = 2.0$ eV, $V = 0$, $\Delta t = 0.5$ eV and $\Delta t_3 = 0.05$ eV which correspond to the minimum of $T_{c,\max}$. Notice that the FS consists of two disconnected parts, one electron-like (circle at the center of the first Brillouin zone) and another one with branches as hole-like FS. Figure 5(b) shows the single-particle excitation energy gap (Δ_0) for this s^*-wave superconducting state as a function of polar angle $\varphi = \tan^{-1}(k_y/k_x)$, which is defined as the minimum value of $E_{s^*}(\mathbf{k})$ (Eq. 6) along the direction in k-space given by ϕ [10,11]. Observe that, although the superconducting gap has an extended s-wave symmetry given by $\Delta_{s^*}(\mathbf{k}) = \Delta_s + \Delta_{s^*}[\cos(k_x a) + \cos(k_y a)]$, $\Delta_0(\phi)$ has a very complex pattern with two types of antinodes with different values, the larger ones at $\phi = 0°$, $90°$, $180°$, $270°$, and the lower ones at $\phi = 45$, $135°$, $225°$, $315°$, and four nodal lines at $\phi = 30$, $60°$, $120°$, $150°$. This complex pattern comes from the interplay between the dispersion relation $\varepsilon_{MF}(k)$ (Eq. 7) and $\Delta_{s^*}(\mathbf{k})$ which appear in $E_{s^*}(\mathbf{k})$.

4. Conclusions

In summary, within the generalized Hubbard model, we have comparatively studied how the critical temperature and the
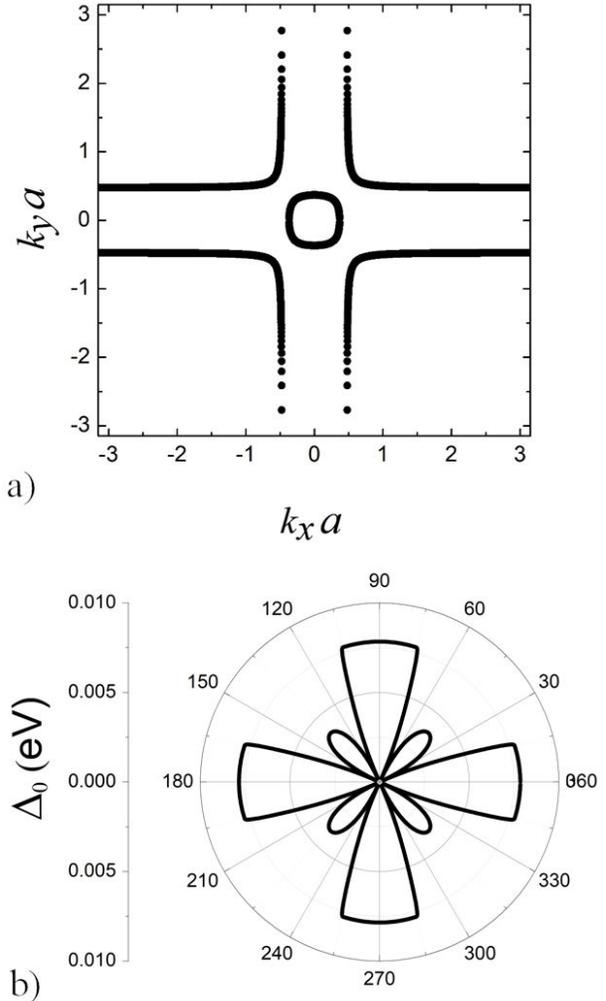
FIGURE 5. a). Fermi surface for a square lattice with $n = 0.55$, $t' = 0.352|t|$, $V = 0$, $\Delta t = 0.5$ eV, $\Delta t_3 = 0.05$ eV and $U = 2$ eV. For this case the Fermi energy is $E_F = -0.734$ eV. b). Single particle excitation energy gap ($\Delta_0(\phi)$) for an s^*-wave superconductor with the same parameters as in Fig. 5. The polar angle (ϕ) is given by $\phi = \tan^{-1}(k_y/k_x)$.

the ground-state energy of d- and s^*-wave superconducting states depend on the electron density n and the first- and second-neighbor hopping parameters t and t'. Within the BCS formalism, the d-wave superconducting properties are calculated by solving the coupled integral Eqs. (5) and (11), whereas the s^*-wave superconducting properties are obtained by means of Eq. (5) together with Eqs. (12) and (13). The

ground state energy (E_g) was obtained for all the optimal electron concentrations (n_{op}) where T_c is maximum for each value of t'. For $t' \in [0.4|t|, 0.5|t|]$, n_{op} is located close to the bottom of the band and it is the same for d- and s^* -wave systems, since for these values of t' the van Hove singularity at the bottom of the band is enhanced. Also, at low electron densities, the behavior of s^*-wave superconducting states differ from that observed in d-wave superconductors where the minimum of the set of maximum critical temperatures (the supremum of $T_{c,\max}$) is attained at the same parameter space point (n, t') where the ground state energy is minimum. This does not occur for s^*-wave superconductors where the supremum of $T_{c,\max}$ is not attained at the same (n, t') where the minimum of the ground-state energy is located and, then, a supremum of n_{op} ($n_{op,sup}$) for s^*-wave can't be defined. For high electron densities close to $n = 2$, similarly to the d-wave case, the superconducting chemical potential lies outside of the single-particle electronic band and the Fermi surface disappears, which could be related to pseudo gap zone experimentally found for high-T_c superconductors [13]. The fact that a $n_{op,sup}$ can be properly defined for d-wave superconductors, and not for s^*-wave ones, deserve a further investigation in relation with other symmetries different from d and s^*-wave, such as p-wave symmetry, to elucidate if this fact is only a peculiarity of d-wave superconducting states or if it could have more profound implications.

In a future work, we will study the possible superconducting symmetries that can be found in a cubic or tetragonal lattice within the generalized Hubbard model.

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